Statistical Analysis of Extreme Precipitation in Wyoming

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ABSTRACT

This study deals with the regionalization of extreme precipitation in Wyoming. The basic methodological framework employed is an index-flood type approach in conjunction with the L-moments. Based on the elevation of rain gauges and sample L-moments of the annual maximum precipitation for different durations, statistical cluster analysis was applied to obtain an initial region delineation over the entire State of Wyoming. Adjustment to regions was then made according to the relationship between the mean annual precipitation and the L-moments of annual maximum precipitation to obtain a region delineation with better defined boundaries. This report describes the procedures and presents some results of region delineation.

Another issue addressed in this study is the selection of the underlying regional distribution for extreme storms in a regional analysis. Here the choice of a probability model for extreme data is discussed. Several selection techniques were proposed and their performance examined.

TABLE OF CONTENTS

STATISTICAL ANALYSIS OF EXTREME PRECIPITATION IN WYOMING

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION AND METHODS

1.1 Problem Statement

The estimate of the largest amount of precipitation that can occur, known as the probable maximum precipitation (PMP), is of great concern in the design and maintenance of major hydrologic structures such as spillways. The PMP is used by engineers to estimate the probable maximum flood (PMF), that might be produced from a watershed. The PMF is often used to determine the necessary capacity in spillway design. Major hydrologic structures such as dams are typically designed to withstand a PMP or PMF event or some percentage of such an event . Designs based on PMP/PMF, however, do not provide any measure of design safety or risk of failure.

It may be more useful to assess the maximum amount of precipitation expected to occur over a given time. This given time is called the *return period* for that event. A probability distribution is used to model the characteristics of the largest expected event for a given return period. This method has the advantage that a risk-based design which incorporates the risk of failure and cost of a design can be considered. It may not be economically feasible to build all structures for the largest flood that could ever occur. A reasonable approach is to design a structure so that there is a tolerably small risk/cost of failure. Furthermore, any estimate of the PMP or PMF is subject to error, so it is appropriate to consider risk-based design. This chapter will review some of the methods which are commonly used in this type of analysis.

1.2 Hydrologic Frequency Analysis

Prediction of extreme hydrologic events presents a difficulty due to a lack of data for very rare events. Statistical estimates require a certain amount of data in order to obtain a desired accuracy. For frequently occurring events the data is easy to obtain and therefore predictions can be quite accurate. In the case of extreme events, data is by definition very rare, and accuracy of predictions will suffer. In addition, extreme events may be generated by unusual circumstances and hence follow a different distribution from that of common events. An example of this would be the precipitation from a large thunderstorm as compared to that of a slow moving storm system.

The data used in this analysis consists of the largest precipitation depth that occurred in a year (called annual maximum precipitation depth) for storm durations of 2, 6, 24 hours, or one 'day' as recorded by an observer. Events that are much larger than the average event are common in this type of data. These extremes cannot be treated as outliers, because they are the events that are of the greatest interest and importance in this analysis. Estimation of distribution parameters from this type of data will have several problems: product moment and maximum likelihood estimators are greatly influenced by the presence of extreme values, and log-transforms of the data give too much weight to the smaller values.

1.3 L-moment Estimation

An alternative approach to estimation by product moments is to use L-moments. L-moments are defined as linear combinations of order statistics (Hosking, 1986). L-moments are analogous to conventional product moments and are estimated by linear combinations of the observed order statistics. They can (in theory) characterize a wider range of distributions and are more robust to the presence of outliers in the data. L-moment estimators tend to be less biased, approximate their asymptotic normal distribution more closely in finite samples, and often give more accurate estimates of the parameters of a fitted distribution. The parameter estimates from L-moments are sometimes more accurate in small samples than are the maximum likelihood estimates (Hosking, 1986). In this analysis L-moments and the L-moment ratios (L-cv, L-skew, and L-kurtosis) are used. L-moment ratios are analogous in interpretation to their product moment ratio equivalents and have favorable small sample qualities.

Hosking (1986, 1989) presents a unified approach to the use of probability weighted moments and L-moments in statistical estimation. He also demonstrates that L-moments are competitive with the conventional product moments and maximum likelihood techniques. A brief description is given herein to provide the basic definitions. Readers are referred to Hosking (1986) for more details.

L-moments are a subset of probability weighted moments which are defined to include order statistics as

$$M_{p,r,s} = E\left[X^{p}\{F(X)\}^{r}\{1 - F(X)\}^{s}\right] \tag{1.1}$$

where $M_{p,r,s}$ is the p^{th} order probability weighted moment of the order statistic with r values less than p and s values greater than p. Here $F(X)$ represents the cumulative distribution function. Let

$$\beta_r = M_{1,r,0} = E[X\{F(X)\}^r] \qquad r=0,1,... \tag{1.2}$$

where β_r is also a probability weighted moment having an unbiased estimator

$$\hat{\beta}_r = b_r = \frac{1}{n}\sum_{j=1}^{n}\frac{\binom{j-1}{r}}{\binom{n-1}{r}}X_{(j)} \qquad r = 0,1,\dots,n-1 \tag{1.3}$$

where $X_{(j)}$ is the j^{th} order statistic. In the case of k less than r, let k *choose* r be equal to zero. The r^{th} L-moment is defined as

$$\lambda_r = \frac{1}{r}\sum_{k=1}^{n}\left[(-1)^k\binom{r-1}{k}EX_{r-k:r}\right] \qquad r=1,2,... \tag{1.4}$$

where $E[X_{r-k:r}]$ is the expectation of the $(r\text{-}k)^{th}$ order statistic out of a sample of r observations. In terms of β_r and b_r, the first four L-moments and their corresponding estimators are, repectively,

$$\begin{aligned}\lambda_1 &= \beta_0\\ \lambda_2 &= 2\beta_1-\beta_0\\ \lambda_3 &= 6\beta_2-6\beta_1+\beta_0\\ \lambda_4 &= 20\beta_3-30\beta_2+12\beta_1-\beta_0\end{aligned} \tag{1.5}$$

and

$$
\begin{aligned}
\hat{\lambda}_1 &= b_0 \\
\hat{\lambda}_2 &= b_1 - b_0 \\
\hat{\lambda}_3 &= 6b_2 - 6b_1 + b_0 \\
\hat{\lambda}_4 &= 20b_3 - 30b_2 + 12b_1 - b_0
\end{aligned}
\tag{1.6}
$$

The iterative form of these estimators are more intuitive. In this form the first four unbiased L-moments estimators can be expressed as

$$
\begin{aligned}
\hat{\lambda}_1 &= \ell_1 = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} X_{(i)} \\
\hat{\lambda}_2 &= \ell_2 = \frac{1}{n\binom{n}{2}}\sum_{i=2}^{n}\sum_{j=1}^{i-1}(X_{(i)} - X_{(j)}) \\
\hat{\lambda}_3 &= \ell_3 = \frac{1}{n\binom{n}{3}}\sum_{i=3}^{n}\sum_{j=2}^{i-1}\sum_{k=1}^{j-1}(X_{(i)} - 2X_{(j)} + X_{(k)}) \\
\hat{\lambda}_4 &= \ell_4 = \frac{1}{n\binom{n}{4}}\sum_{i=4}^{n}\sum_{j=3}^{i-1}\sum_{k=2}^{j-1}\sum_{m=1}^{k-1}(X_{(i)} - 3X_{(j)} + 3X_{(k)} - X_{(m)})
\end{aligned}
\tag{1.7}
$$

where ℓ_r is an unbiased estimator of λ_r.

The L-moments ratios for L-coefficient of variation (τ_2), L-skew coefficient (τ_3), and L-kurtosis coefficient (τ_4) are estimated respectively by

$$\hat{\tau}_2 = t_2 = \frac{\ell_2}{\ell_1}$$
$$\hat{\tau}_3 = t_3 = \frac{\ell_3}{\ell_2} \qquad (1.8)$$
$$\hat{\tau}_4 = t_4 = \frac{\ell_4}{\ell_2}$$

The asymptotic covariance of the L-moment estimators for the generalized extreme value distribution are given by Hosking (1986).

1.4 Hydrologic Regionalization

The limited amount of extreme precipitation data in Wyoming presents a difficulty for statistical estimation. A small number of gauge sites make it necessary to combine machine gauged data and manual observations from weather bureau volunteers. Furthermore, for a given rain-gauge location the available record length is typically short. In Wyoming the average at-site precipitation data available is 30 years. It may be required to predict maximum precipitation depths for return periods of 1000 years or more. This corresponds to the extreme right-hand tail of a distribution. Predictions based on short record lengths will clearly be highly unreliable, and for these short record lengths maximum likelihood estimates do not always perform well.

A solution to the problem of short record lengths is to use a regionalization procedure. The first step is to identify an homogeneous region for a subset of the data, and then determine a statistical distribution which will adequately model the combined data for the region. For compatibility, data from each station is scaled by dividing by the average at-site annual maximum. This is a commonly utilized technique in hydrologic analysis and is know as the index event procedure (Stedinger et al., 1993). Using index event methodology a frequency analysis on the combined data is performed, resulting in a much longer effective record length. Regional analysis, therefore, will yield better parameter estimates and the results are more robust to the presence (or absence) of extreme values than the at-site analysis, even when correlations are present in the data (Hosking and Wallis, 1986).

Some assumptions are necessary for region delineation. The region should be climatically homogenous; All storms in the region being analyzed must have originated from the same type of storm (i.e., storm front, thunderstorm, hurricane). For a more extensive discussion of regional analysis, readers are referred to Schaefer (1982).

1.5 Scope and Objectives of Research

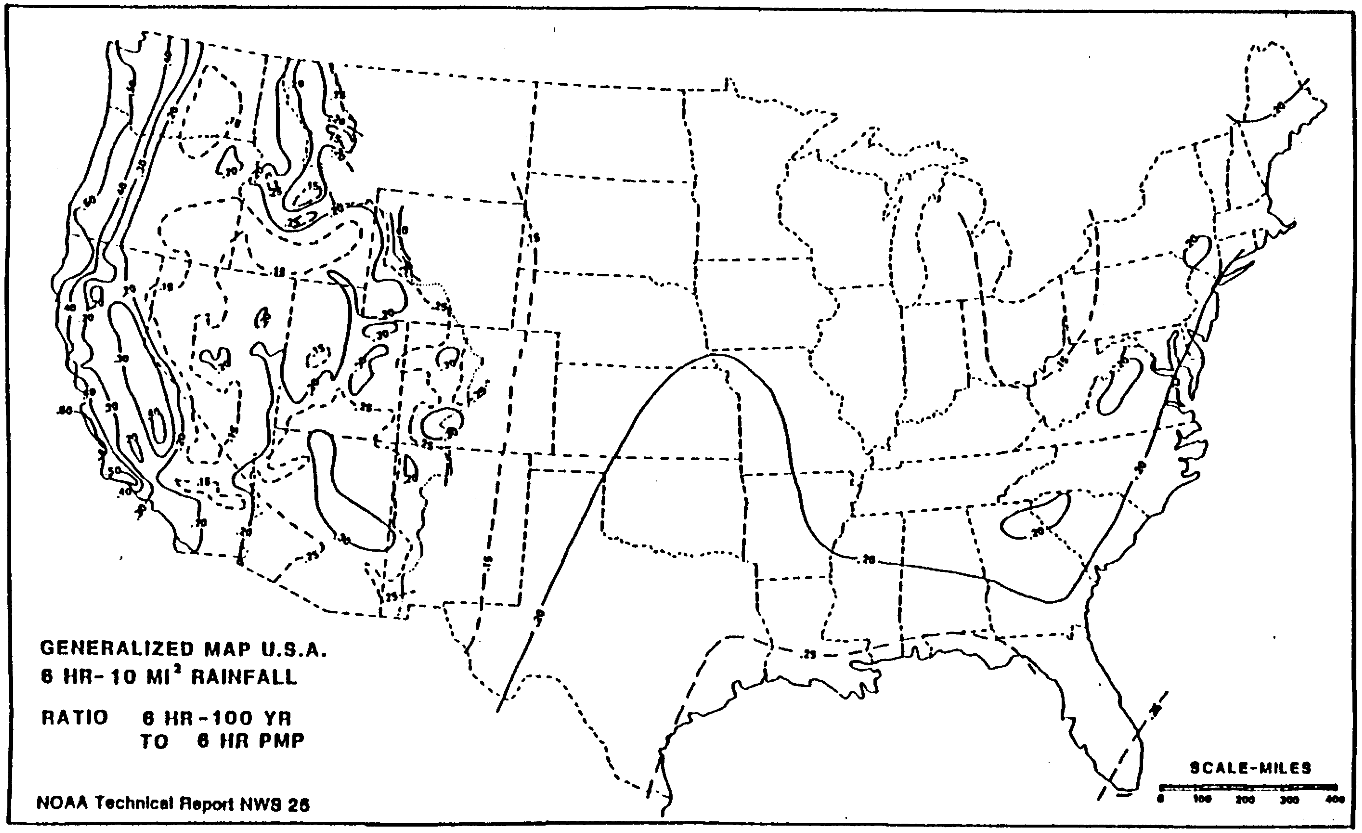
There are two primary objectives of this study. The first is to define suitable regions for analyzing extreme precipitation in Wyoming. Accompanying this is a the development of regression models to aid in the prediction of precipitation characteristics at a location where no data is available. The second primary objective is the choice of a distribution to describe the random nature of extreme precipitations for a region. This choice is critical in frequency analysis of hydrological extremes.

The generalized extreme value (GEV) distribution (a reparameterized reverse Weibull distribution) has been suggested for use in the State of Washington (Schaefer, 1990) and also for general use (Vogel et al., 1993). However, due to the large amount of local variation in precipitation amounts within the Rocky Mountains region, it is not certain that results from other areas can be correctly applied to the Wyoming/Rocky Mountain region. Figure 1.1 illustrates how 6 hour maximum precipitation changes in the vicinity of the rocky mountain zone. This study considers several different distributions for modeling the regional precipitation data, and will address the choice of an appropriate distribution (See Chapter 3).

In reality the annual extreme precipitation amounts could be generated by some mixture of distributions representing different types of events. However, the problem of selecting a mixture of distributions for a region is much more difficult than the case of a single distribution.

1.6 Organization of the Thesis

This thesis is organized into four chapters. The first is a discussion of the problem and an introduction to the tools that will be applied in the second and third chapters. Chapter two deals with regionalization of the Wyoming precipitation data and with proposed regional models. Chapter three addresses the issue of how to choose a regional distribution to best represent extreme precipitation data. Several methods are proposed and compared by numerical simulation. Chapter four contains an overall summary and suggestions for further research.



Comparison of Generalized Estimates of Probable Maximum Precipitation With Greatest Observed Rainfalls

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Figure 1.1

CHAPTER 2

REGIONAL ANALYSIS OF EXTREME PRECIPITATION EVENTS IN WYOMING

Hydrologic and regional analysis techniques and L-moment estimation as outlined in Sections 1.2 and 1.3 were used in combination with cluster analysis to define homogenous regions in Wyoming. Regional equations for the average annual maximum precipitation, τ_2 (L-cv) and τ_3 (L-skewness) were developed using regression analysis. Also presented here is a description of the seasonality and elevation effects in the data.

2.1 Precipitation Data

In this study, statistical characteristics of annual maximum precipitation depths for 2, 6, and 24-hour records are examined using data from 45 recording-gauge stations. Also included are daily observational data from 150 non-recording stations. There are only 180 separate data sites as 15 of these stations have both types of data available. Most of the recording-gage data record were furnished by the National Climatic Data Center (NCDC) for the period from about 1948 through 1992. Some of the data were flawed and could not be used. To this was added National Weather Bureau data for periods between 1940-1963. Latitude, longitude, and elevation data were obtained from the Water Resources Data System (WRDS) database at the Wyoming Water Research Center (WWRC). There were several cases where precipitation gauges had been moved from their original location to nearby locations. In these cases the latitude, longitude, and elevation used is the location for which the gauge was located for the longest period of time.

2.1.1 Data Quality Checking

The 180 gauge sites used in this study have been screened by a two step process, checking for validity of data values and removal of data containing too many missing values. The observational (daily) data were examined for errors by the WRDS staff. Maxima at each site were cross-checked with stream-flow data. Sites with a high correspondence between precipitation maxima and stream-flow peaks were deemed to be of good quality. Gauges found to have unreliable data were removed from consideration. The hourly data contained a number of unexpected errors. Many of the total annual precipitation values were zero or much less than the totals available from other sources (apparently due to a database error). Data years which were obviously flawed were eliminated or replaced by National Weather Bureau data where available. Tabulated annual maxima from the NCDC were compared with the precipitation per hour records for several sites. It was found that missing data often resulted in the tabulated maximum being less than the true recorded maximum. Missing values were not indicated in the NCDC data unless 30 or more days were missing out of the entire year. A computer routine was developed to scan for and remove all data years with more than 30 missing days indicated.

2.1.2 Missing Data

The non-recording gauge data, of which much more is available, was obtained from the WRDS database. A computer routine was written to extract the annual daily maxima from the available non-recording stations. Data years with more than 24 days missing, or more than 2 missing days in any single month, were not used in the analysis. Allowing two missing days per month increases the usable data years by 658 station-years, an increase of up to 14% in usable data years with minimal effect on the average annual maximum (see Table 2.1). Gauges with less than 12 years of data were discarded from the analysis. The computer routine also tabulates the annual maxima by elevation, month, and region. The WWRC also provided data on latitude, longitude, elevation, and mean annual precipitation (MAP) for the 180 gauged sites.

The method of this analysis allows that missing data in these datasets does not always imply flawed data. A data year is only incorrect if the largest storm occurred in an interval of missing data. All things being equal, the hourly (recording-gauge) data should be expected to be of better quality than the observational data. Hourly data is originally recorded as precipitation per hour and gives a good measure of the intensity and duration of a storm. However, an unknown number of missing days (up to 30) in a year means that the accuracy of this data is not exactly known. The daily (observational) data is subject to imprecise measurements and irregular intervals due to the human observer. The daily data includes a record of missing days and accumulated values, which allowed a more careful screening of the dataset. See Table 2.2 for number and of sites, type of gauge, and gauge density within the state of Wyoming.

Table 2.1 - Effect of Missing Data on
Mean Annual Daily Maximum Precipitation

# of missing days allowed	Mean Annual Daily Maxima*	Total # of station-years
0	1.3925	4418
2	1.3957	5132
7	1.3954	5317

* For 145 stations.

Table 2.2 - Number of Gauges and Gauge Density by Duration

Duration	Number of Stations	Station Density (sq. mi./sta.)
2 hours	40	2425
6,24 hours	45	2155
Daily	150	646
Daily & 24 hours	180	539

2.1.3 Considerations for Data Validity

Examination of the location of the precipitation gauges in Wyoming shows that there are large regions for which there is little or no data available. The available data come primarily from towns and agricultural areas. Very little data is available for high mountain areas. Therefore, conclusions from this analysis may not apply for all areas of the state. Since most mountain precipitation is in the form of snow, it would be more useful to concentrate on the snowmelt and stream-flow characteristics in these areas. Data for the Great Divide Basin and nearby areas is also scarce, but there is good reason to believe that this area will be fairly homogenous.

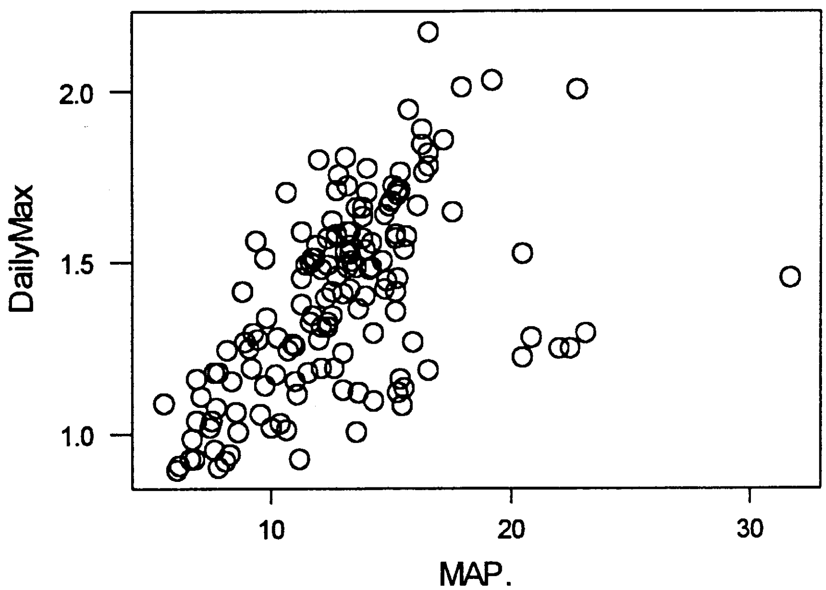
2.2 Delineation of Homogeneous Regions by Cluster Analysis

In this regionalization study, the first step is to divide the study area into regions that are relatively homogeneous with respect to certain precipitation characteristics. Delineation of homogeneous regions allows for the development of better prediction equations than would be possible had no regionalization been done. Application of cluster analysis techniques to the data will result in homogenous groups, which are the basis for determining climatically homogenous geographical regions. The precipitation region-map for the State of Wyoming will partially describe precipitation characteristics and help in establishing precipitation-frequency relation. Statistical characteristics of annual maximum precipitation used in the cluster analysis were the mean (mean annual maximum or XBAR), L-cv, L-skew. Elevation data was also used in the cluster analysis to represent geographical factors.

2.2.1 Methodology and Combining Information

In the cluster analyses, it was unknown how many groups were present in the data. Graphical comparison of the data shows at least two apparent groups (see Figure 2.1). Several trials with different numbers of clusters were used in the cluster analysis to determine how many regions are representative. The number of clusters was varied from 2 to 9, and the results compared for constancy between separate analyses and geographic correspondence. It was determined that four clusters are sufficient to represent climatically relatively homogeneous regions in Wyoming. The four regions correspond fairly well to definable geographic areas of Wyoming.

Cluster analysis was performed for the daily data because it is the largest single dataset (150 stations). The cluster analysis with four regions were also made for 2, 6, and 24 hour data for the purpose of checking the consistency or inconsistency of region classification among different durations. The results indicated that region classifications by the cluster analysis are generally consistent over the datasets. By consistency, it is meant that the assignment of a station to a region by the cluster analysis does not change with the duration of the data considered. Such consistency is expected because storm characteristics of different durations should be correlated as indicated in Figure 2.2, especially for the lower order moments. For this reason, and due to the fact that the data for 2, 6, and 24 hour durations are very sparse (see Table 2.2), it is felt that it would be advantageous to combine data with different durations together in region delineation. Combining storm characteristics of different durations in the cluster analysis has three advantages: (1) it reduces some of the inconsistency in region assignment, (2) it expands the size of the usable database leading to



Plot of Daily Mean Annual Maximum vs. Mean Annual Precipitation

Figure 2.1

(a) Mean:

```
                                     INDIVIDUAL 95% CONFIDENCE INTERVALS
                                    FOR     MEAN ANNUAL MAXIMUM BASED ON
                                    POOLED STDEV
DURATION     N      MEAN     STDEV  --------+---------+---------+--------
2-hour      40    0.6469    0.1975   (--*--)
6-hour      45    0.8440    0.2227           (--*--)
24-hour     45    1.2217    0.2886                          (--*--)
Daily      150    1.3969    0.2764                                  (-*-)
                                    --------+---------+---------+--------
                                          0.75      1.00      1.25
```

(b) LCV:

```
                                     INDIVIDUAL 95% CONFIDENCE INTERVALS
                                    FOR    MEAN LCV BASED ON POOLED STDEV

DURATION     N      MEAN     STDEV  ---------+---------+---------+-------
2-hour      40   0.25529   0.04266                       (-----*-----)
6-hour      45   0.22318   0.03909       (-----*----)
24-hour     45   0.22015   0.04315     (-----*-----)
Daily      150   0.20877   0.03702  (--*---)
                                    ---------+---------+---------+-------
 POOLED STDEV =  0.03921                   0.220     0.240     0.260
```

(c) LSKEW

```
                                     INDIVIDUAL 95% CONFIDENCE INTERVALS
                                    FOR      MEAN LSKEW BASED ON POOLED
                                    STDEV
DURATION     N      MEAN     STDEV  ---------+---------+---------+-------
2-hour      40    0.2563    0.1434               (----------*-----------)
6-hour      45    0.2300    0.1075       (----------*----------)
24-hour     45    0.2177    0.1009  (-----------*----------)
Daily      150    0.2125    0.1101      (-----*-----)
                                    ---------+---------+---------+-------
 POOLED STDEV =   0.1137                   0.210     0.240     0.270
```

Figure 2.2 - Summary of Statistical Characteristics of Annual Maximum Precipitation by Duration

a more reliable region delineation, and (3) from a practical viewpoint one region-map for all durations is preferable to four region-maps, one for each duration.

To combine data, stations with data available for different durations (2, 6, 24 hour, and daily) are treated as four separate records. Since the L-moment estimates from different durations are not directly comparable, some standardization was necessary. The L-moments ratio estimates of annual maximum storms were first standardized (separately by duration) by subtracting the respective means, then dividing by the standard deviations, and then the data were combined. Because elevation is a constant for each station, the standardization procedure was done for the overall data set. Standardization of involved variables in the cluster analysis is necessary to remove any scale effect among the variables. Initial region designations were found using a partitioning algorithm, FASTCLUS, in the SAS (Statistical Analysis System) statistical package (ref???) based on estimates of mean annual maximum, τ_2, τ_3, and elevation. Record lengths for each station (and duration) were used as weighting factors in the cluster analysis.

2.2.2 Results of Region Classification

Table 2.3 gives the regional values of the cluster analysis factors, with the exception of regional elevation, which is found in Table 2.4. The means of these factors represent the 'center' of each region. Standard deviations, range of data, and number of sites within each region are also given.

After the cluster analysis, there remained several cases of stations which were assigned to a region that does not 'fit' well with its neighboring stations. Although data combination

Table 2.3: Summary of Factors by Region							
	Region	N	Mean	Median	Standard Deviation	Minimum	Maximum
MAP	1	49	10	10.24	2.97	4.92	15.14
	2	89	13.93	13.38	1.858	11.42	22.78
	3	50	10.88	10.66	3.252	5.46	23.15
	4	92	12.56	11.33	4.822	5.96	31.67
XBAR	1	49	1.043	1.116	0.332	0.391	1.512
	2	89	1.468	1.533	0.33	0.727	2.169
	3	50	1.069	1.098	0.404	0.42	1.764
	4	92	1.011	1.044	0.294	0.399	1.641
L-CV	1	49	0.213	0.217	0.032	0.127	0.289
	2	89	0.216	0.215	0.031	0.131	0.289
	3	50	0.275	0.271	0.033	0.215	0.355
	4	92	0.195	0.196	0.03	0.098	0.276
L-SKEW	1	49	0.244	0.256	0.083	0.045	0.402
	2	89	0.196	0.201	0.092	-0.08	0.392
	3	50	0.354	0.342	0.077	0.215	0.533
	4	92	0.163	0.177	0.104	-0.1	0.379
L-KURT	1	49	0.176	0.176	0.084	-0.02	0.361
	2	89	0.144	0.149	0.075	-0.02	0.316
	3	50	0.23	0.218	0.095	0.056	0.447
	4	92	0.141	0.139	0.074	-0.12	0.281

Table 2.4 - Summary Statistics of Elevation By Region

Region Number	Number of Stations	Mean Elevation	Standard Deviation	Range Min	Max
1	49	4592.9	490.0	3840.0	5700.
2	89	4612.7	571.6	3530.0	6120.
3	50	6080.0	952.0	3830.0	7390.
4	92	6731.1	710.2	5280.0	9060.

removes some of the inconsistency in region designation in the cluster analysis, the inconsistency that a station having data with more than one duration can be assigned to more than one region cannot be totally eliminated. In practical application, the presence of 'misfitted' stations and inconsistency in region classification due to durations could create difficulties. Therefore, it is necessary to adjust and reassign some stations so that the regions can have better geographic definition. The issue of regional reclassification is described in the next section (Section 2.3).

2.3 Region Adjustment

As mentioned previously, two problems in the initial region classification from the cluster analysis are: (1) some stations do not 'fit' well with their neighboring stations in an otherwise homogeneous area, and (2) the same station may be assigned to a different region for different durations. Comparison of record length to identify less reliable values can often provide an easy solution to both problems, as the disagreeing value is often associated with a very short record length. To solve other conflicts, the following procedure was used to resolve the final region assignment.

For a station in question, it was determined whether the change of region assignment was acceptable. Because the determination of region assignment for a station depends on the relative magnitude of its statistical properties, the effect of moving a station to a different region must be examined with respect to the general relation of attributes within the region. This exercise, to a large extent, is subjective. To facilitate the task, a computer routine has been developed to show graphically the relation between the first three L-moment estimators and MAP. In those graphs the data points of different durations and region assignments are indicated by different colors and symbols. The station under consideration for changing region assignment is selected and highlighted on the monitor. This visual display allows one to judge how well the statistical properties of the station under question fit with those of the remaining stations in the region to which it is to be reassigned.

Note that the objective of hydrologic regionalization is to delineate climatically homogenous regions within which a more accurate precipitation frequency relationship can be defined. The purpose of region adjustment is to obtaining a better defined geographical region. For some stations this may be in conflict with the regionalization objective indicated above because not all attributes will simultaneously fit well with those in the 'new' region. In view of uncertainty in sample data, more weight is given to lower order L-moments than higher-order moments. In addition, from the viewpoint of engineering design under uncertainty, conservatism was used as a justification if a station is to be reassigned to a different region. Conservatism ensures that the reassignment of a station to a different region will not lead to an under-estimation of precipitation potential. The proposed region delineation for Wyoming is shown in Figure 2.3.

See Appendix D for a listing of stations and the adjusted region numbers, as well as the original region assignments from cluster analysis.

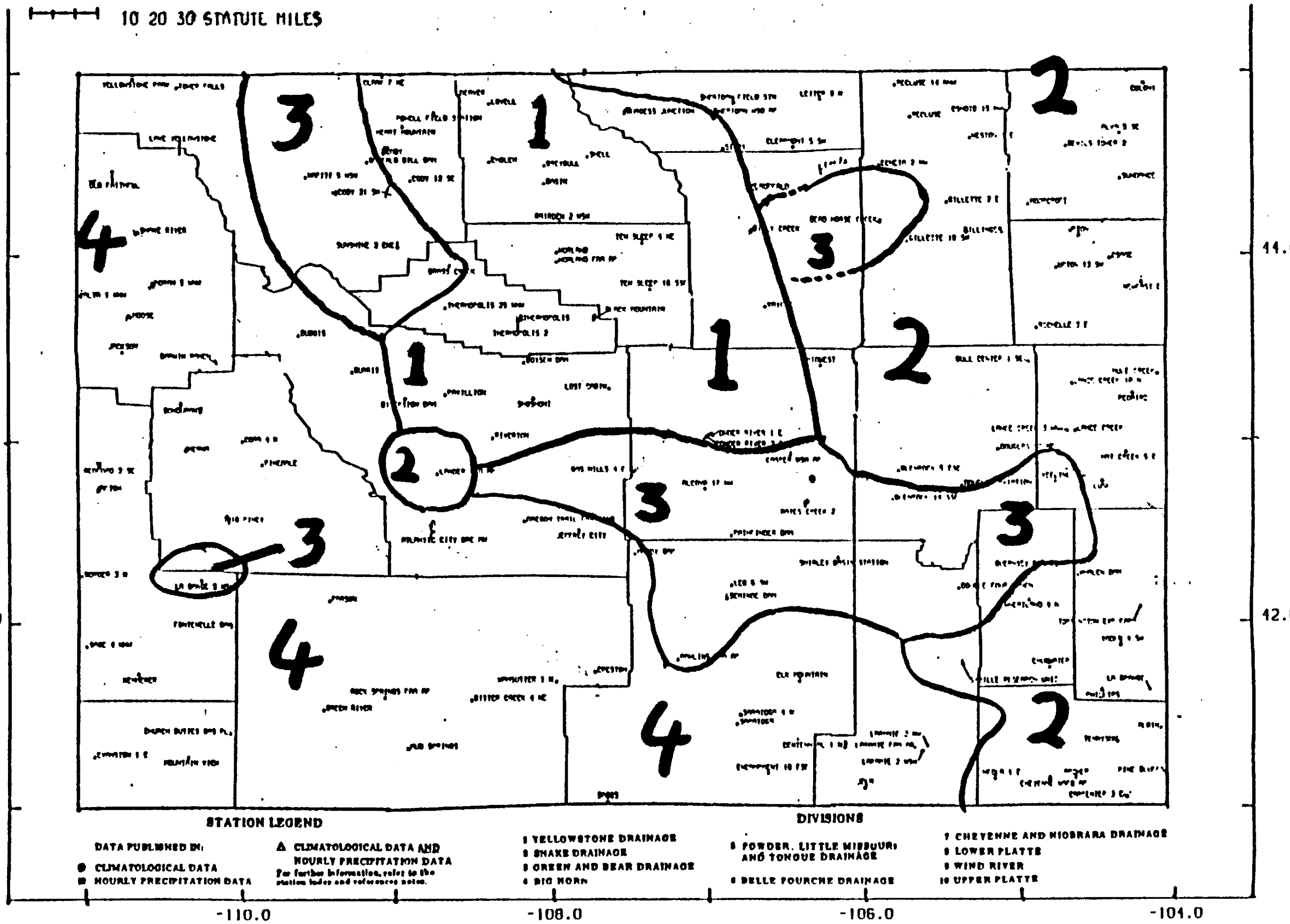


Figure 2.1

Figure 2.3: Proposed Region Map

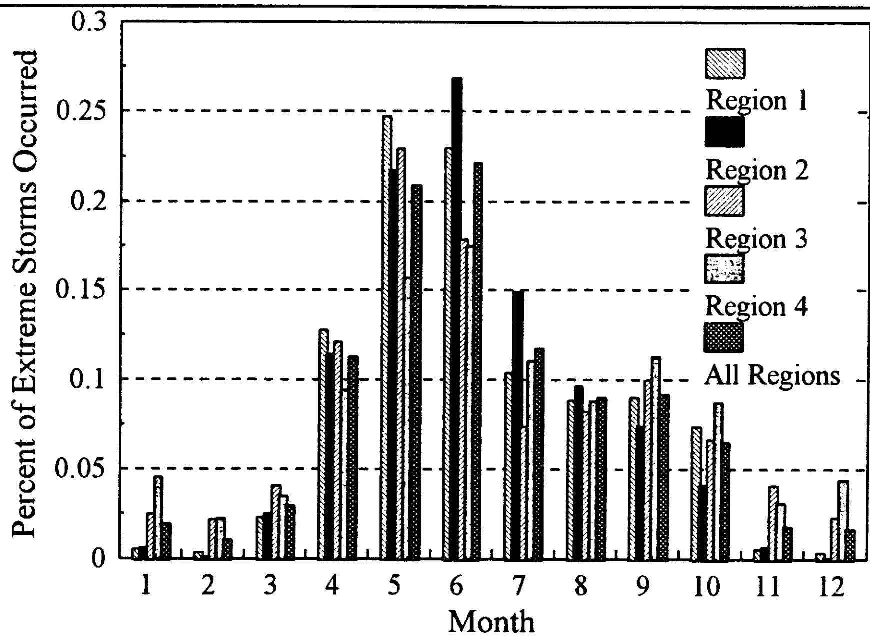
2.4 Seasonal and Elevation Effects on Extreme Storm Occurrences

The seasonal, elevation, and regional effects on the occurrences of annual maximum precipitation are useful descriptors of the results thus far. Figures 2.4 and 2.5, respectively, illustrate the frequency and relative percentage of annual maximum storm occurrences by region and month. From Figure 2.5 it is clear that the vast majority of annual maximum storm events occur between April and October. For a given region, May and June are the two months during which the annual maximum precipitation occurs most frequently. Figure 2.5 shows the relative percentage of storm occurrences by region and month. During the months of May and June, a great majority of the annual maximum storm events occur in region 2, whereas during the winter months region 4 has most of the annual maximum storms.

Figures 2.6 and 2.7 show the seasonal variation of the occurrence of annual maximum storm events as affected by elevation. A large proportion of the annual maximum storm events comes from the 3500'-5000' elevation range, which is not surprising. The highest frequency of storm occurrence for this range occurs in June. The 5000'-6000' elevation range is similar in percentages to the lower range, but has the highest peak in May. It should be noted (see Table 2.4) that there are some systematic differences in elevation among regions.

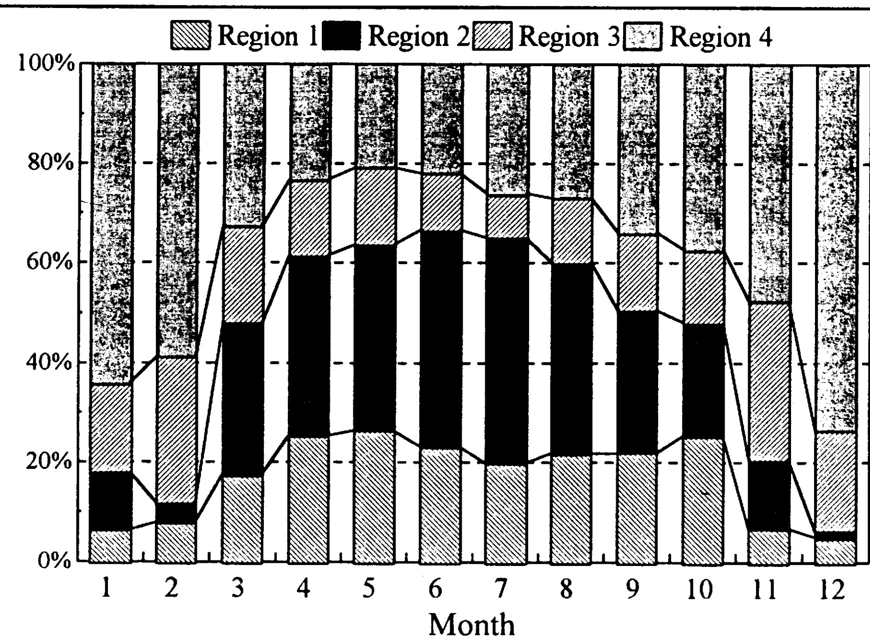
2.5 Development of Regional Equations

Regression models for each region were explored for relationships which would be useful in prediction of storm characteristics at ungauged sites. Equations were developed to predict the average annual maximum precipitation (referred to as XBAR), L-cv, L-skew, and L-kurtosis based on independent variables of storm duration (D_i) and MAP. It was found



Percentage of Annual Maximum Precipitation Occurred by Month for Different Regions

Figure 2.4



Relative Percentage of Annual Maximum Precipitation Occurred by Month for Different Regions

Figure 2.5

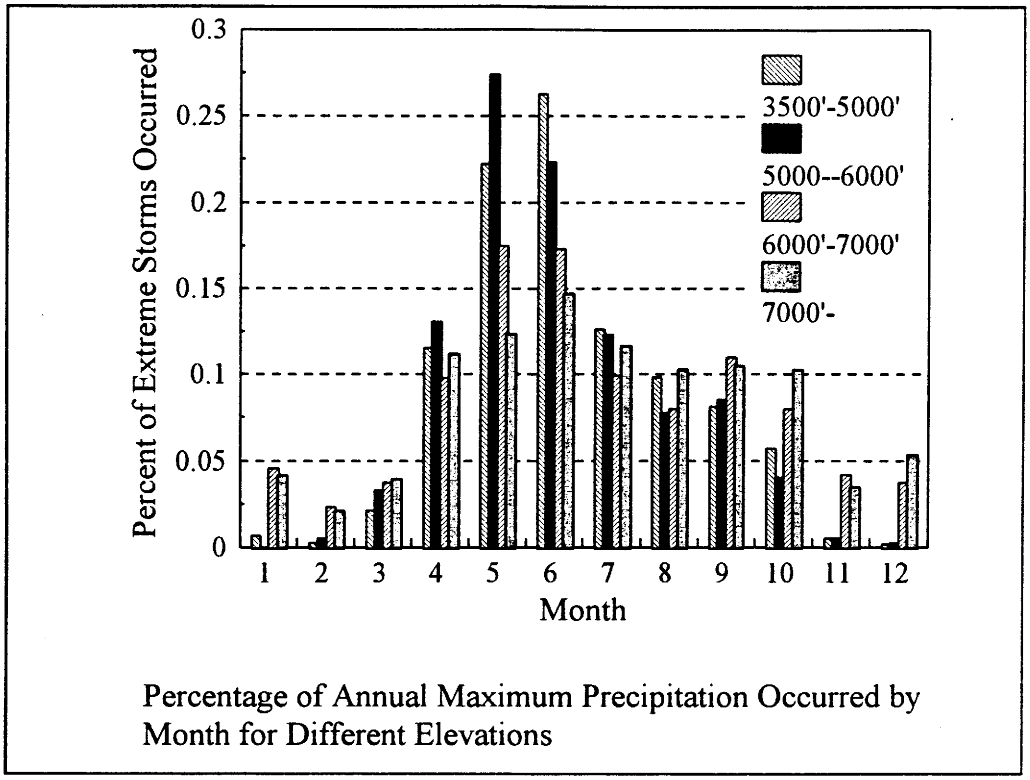


Figure 2.6

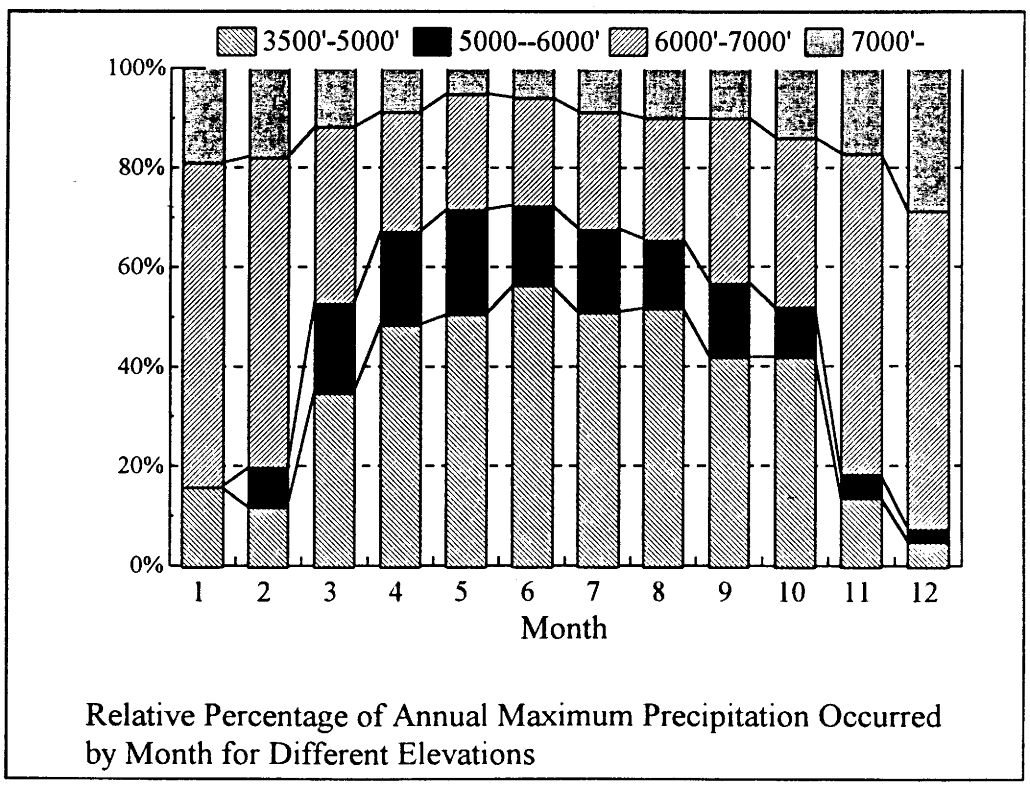


Figure 2.7

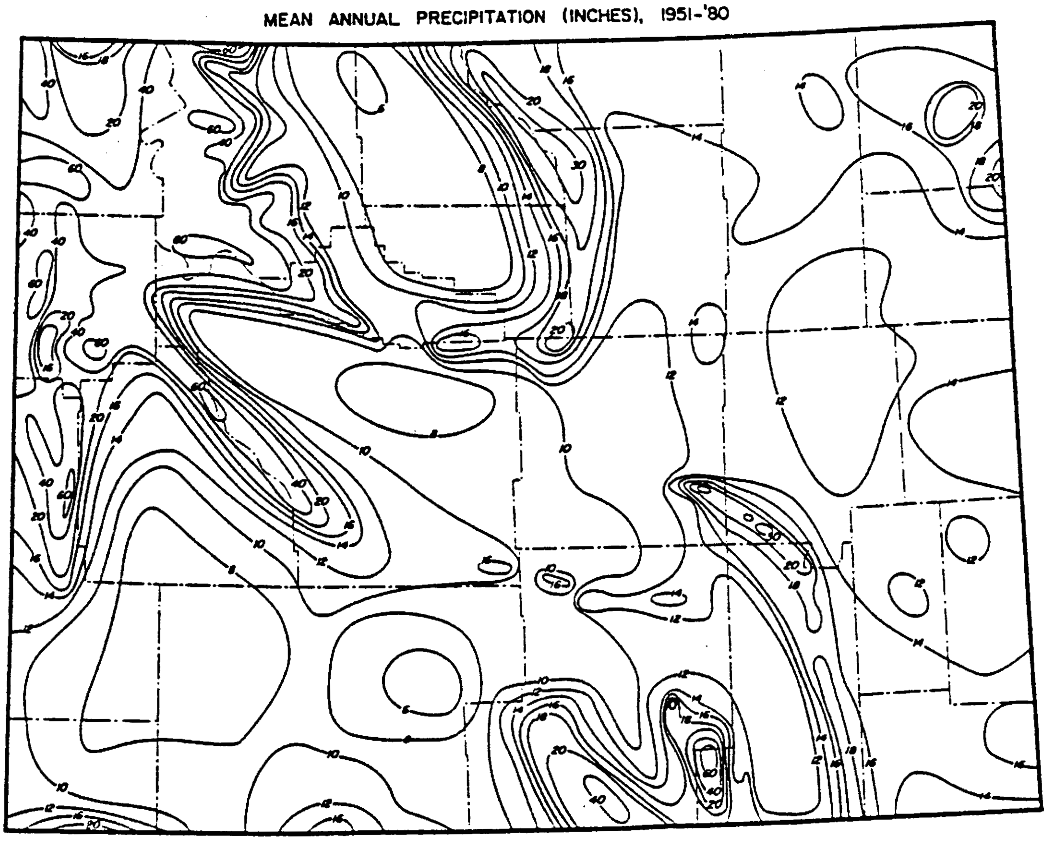
that inclusion of storm duration as a dummy variable (for 2, 6, 24 hours or Daily) preserved the overall relationships and generally improved the predictions for the hourly durations. Measured values of MAP are obviously not available for ungauged sites, but good estimates of MAP can be obtained from the Wyoming Climate Atlas (Martner 1986) as shown in Figure 2.8. Elevation was also considered as a predictor variable, but is not used because the effects of elevation appear to be highly dependent on local geography and are too complex for a simple model. Furthermore, elevation is important in the designation of regions by cluster analysis, and the effects of elevation are partially accounted for by the region designation.

It is assumed that an ungauged site can be classified into a region for prediction purposes. In practice there may be borderline cases which do not clearly belong to one region. Experience with the region adjustment procedure suggests that for borderline cases, the prediction error for a misclassified site will not be much greater than the error for a correct classification. Region was considered as a dummy variable in an overall model, but due to interaction effects it was decided that separate models for each region would be more useful. No interactions between MAP and storm duration should exist because storm duration is merely an indicator of the length (time) of measurement and has no relation to MAP. In any case, there is no meaningful interpretation to such an interaction, so it is not considered.

2.5.1 Methodology

This section deals with regression models and transforms used, as well as a discussion of how outliers and high-leverage observations were dealt with.

MEAN ANNUAL PRECIPITATION (INCHES), 1951-'80



Mean annual precipitation map for Wyoming.

Figure 2.8

Regression analysis was conducted using the Minitab (?REF?) statistical package. Two transforms were used in the analysis. The log-transform was used on some the dependent variables and on the independent variable MAP. The standardizing transform z(x) was also used on Log(MAP)

$$z(x) = \frac{x - \mu_{\log x}}{\sigma_{\log x}} \tag{2.1}$$

where $\mu_{\log x}$ is the mean of log(MAP) and $\sigma_{\log x}$ is the standard deviation of log(MAP).

Note that this standardization transform is not necessary for any of the final models presented here; it was utilized to reduce multicolinearity when testing for quadratic and interaction effects. The log transform is also used on the dependent variables XBAR and t_2. However, it is not appropriate to use this transform on variables t_3 and t_4 because they may take on negative values. The duration of the data is represented by dummy variable D_i (i = 1, 2, 3), with

$$D_1 = \begin{cases} 1, \textit{for} \;\; 2\textit{-hour} \;\; \textit{data} \\ 0, \textit{else} \end{cases}$$
$$D_2 = \begin{cases} 1, \textit{for} \;\; 6\textit{-hour} \;\; \textit{data} \\ 0, \textit{else} \end{cases} \tag{2.2}$$
$$D_3 = \begin{cases} 1, \textit{for} \;\; 24\textit{-hour} \;\; \textit{data} \\ 0, \textit{else} \end{cases}$$

The model form for XBAR and t_2 are then

$$Y = \beta_0 + \beta_1 X + \sum_{i=1}^{3} \beta_{2i} D_i + \xi \qquad (2.3)$$

where X = z(log MAP), Y = log(XBAR) for XBAR; Y = log(t_2) for L-cv; Y = t_3 for L-skewness; Y = t_4 for L-kurtosis. The β_0 term is the y-intercept for the daily data; β_1 is the linear coefficient; and β_{2i}, i = 1, 2, 3, is the change in intercept corresponding to the 2, 6, and 24 hour data, respectively.

Appendix A contains Tables describing the developed regression models and the mean/standard deviation of the MAP by region. Using an estimate of MAP from Figure 2.8 and a region designation from Figure 2.3, the regression equation may be used to obtain estimated values of XBAR, L-cv, L-skewness, and L-kurtosis of a specified duration for any location within Wyoming.

2.5.2 Regression Modelling Results

Several overall results are worth noting. The strength of the relationships between MAP and the L-moment ratios decreases as the order of the moment increases. The mean annual maximum (XBAR), not surprisingly, shows a strong relation to the MAP. L-cv decreases mildly as MAP increases. Relationships of L-skew and L-kurtosis to MAP are weak and vary by region. Some non-linear relationships with respect to MAP are suspected to exist. It is expected that as MAP increases the mean values of L-cv and L-skewness will decrease (similar to $y = 1/x$) to some constant and that the variance of these estimates will also decrease. Due to the very low-end range of MAP in Wyoming, the non-linear effects are negligible and cannot be reliably detected.

CHAPTER 3

SELECTION OF REGIONAL PROBABILITY MODELS FOR EXTREME STORMS

3.1 Problem Statement

To make accurate predictions about the return period for a storm event of a given depth, it is necessary to make certain assumptions. The first assumption is that a climatically homogeneous region can be defined. The advantages of this technique have been discussed previously (See Section 1.4). Within a climatically homogenous region all extreme storm events will have similar characteristics that can be described by a single probability model. Parameters may be adjusted for local variation within the scope of the model.

The available data record length is clearly not sufficient in itself to predict 10,000 year (or rarer) events. It will be necessary to make some distributional assumption if the model is to be used to make predictions about such extreme events. Using characteristics from the available data, a model which best fits these characteristics can be selected. The Generalized Extreme Value (GEV) distribution is of particular interest here because it has been proposed for use in the state of Washington (Schaefer 1980) and other locations. It is not known if the GEV model will be appropriate in an arid mountainous area such as Wyoming.

The probability models considered in this study are the GEV, Generalized Pareto (GPA), and Generalized Logistic (GLO) distributions. These are all three-parameter distributions with location, scale, and shape parameters ξ, α, and k. By first defining the variable y to be

$$y = \begin{cases} -k^{-1}\log(1-k(x-\xi)/\alpha)), & k \neq 0, \\ (x-\xi)/\alpha, & k = 0, \end{cases} \tag{3.1}$$

the GEV, GPA, and GLO cumulative density functions are then given, respectively, by

$$F_{GPA}(x) = 1 - e^{-y} \tag{3.2}$$

$$F_{GEV}(x) = e^{-e^{-y}} \tag{3.3}$$

$$F_{GLO}(x) = \frac{1}{(1 + e^{-y})} \tag{3.4}$$

respectively. Hosking (1986) gives details of L-moments for these distributions. The sample L-moments were compared to the theoretical relationships corresponding to the various probability models. Figure 3.1 shows an L-moment ratio diagram (See Section 1.3) of daily annual maximum data in Wyoming and the three distributional relationships. If one of the probability models under consideration is correct, then the sample (t_3,t_4) points from the at-site estimates would be expected to fall about one of the curved lines representing the particular model. By visual inspection, no single probability model stands out as being very much better than the others due to the great amount of variation in the at-site estimates of (t_3,t_4).

Although the GEV model may indeed be the best choice in this case, a more rigorous and less subjective method involving L-moments and the regionalization procedure is

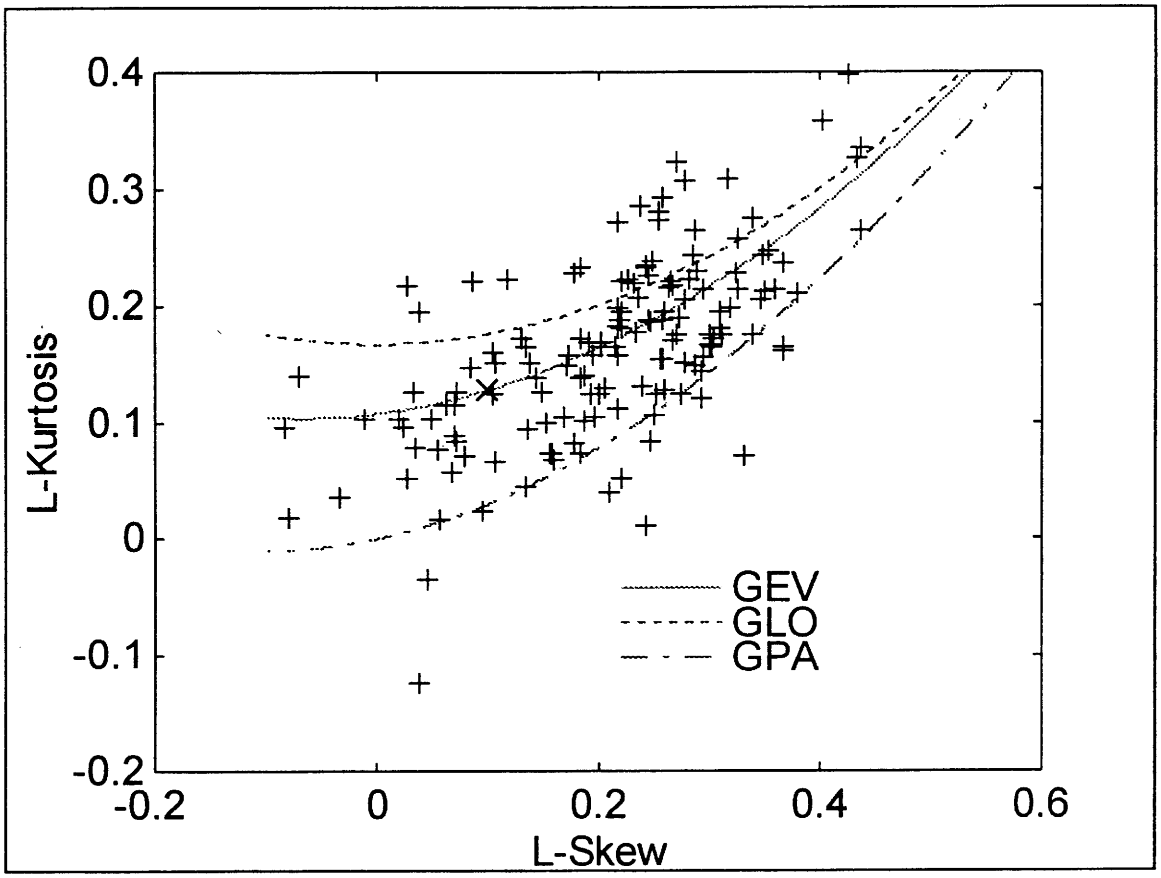


Figure 3.1: L-moment diagram showing Wyoming daily maximum data.

desirable. A 'good' method should select the correct model with high reliability. In practice only a limited amount of data is available, so an efficient selection method will also be useful. Refer to Figures 3.2 and 3.3, showing at-site estimates of L-skewness and L-kurtosis for simulated GEV data at 30 sites. Note that as L-skewness increases the GEV and GLO models rapidly converge, and variation and correlation for sample L-skewness and L-kurtosis of the at-site estimates greatly increases. It is apparent that the L-skewness of a distribution will have a significant effect on the accuracy of a model selection.

3.2 Methods of Model Selection

Ten methods for selecting regional probability models were considered. Each method selects a single probability model based on the sample L-moment ratios within a region. The model so selected will be referred to as the *estimated correct model* for the data. Selection errors are of course possible and worthy of consideration. The examination of these selection errors is in fact the main point of this study, and they will be more fully discussed in Section 3.3.

The methods consider herein may be classified into three general types. The first type utilizes linear programming and quadratic programming techniques to determine optimal weights for a 'mixture' model. The model with the greatest weight is then selected as the estimated correct model. Four different objective functions, namely the minimize the sum of absolute deviations (MSAD), minimize the largest absolute deviation (MLAD), minimize the largest range of error (MRNG), and minimize the sum of squared deviations (MSSD) are considered here. The second type of methods are based on two reliability indices suggested

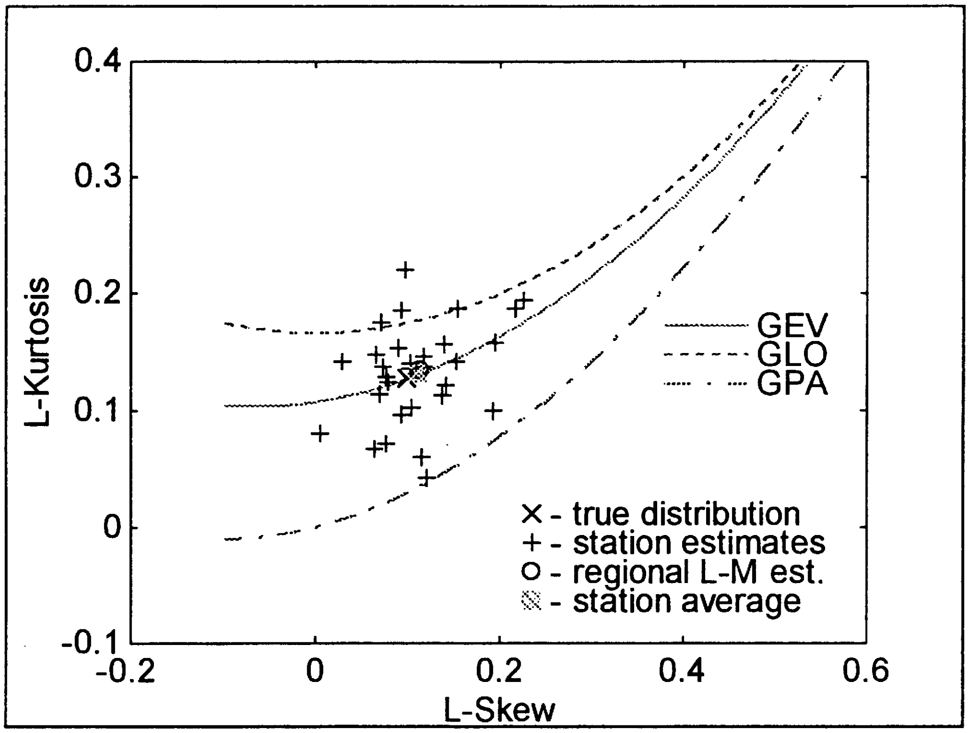


Figure 3.2: L-moment diagram for simulated GEV data, $\tau_2 = .2$, $\tau_3 = .1$, at 30 sites, 60 years/site.

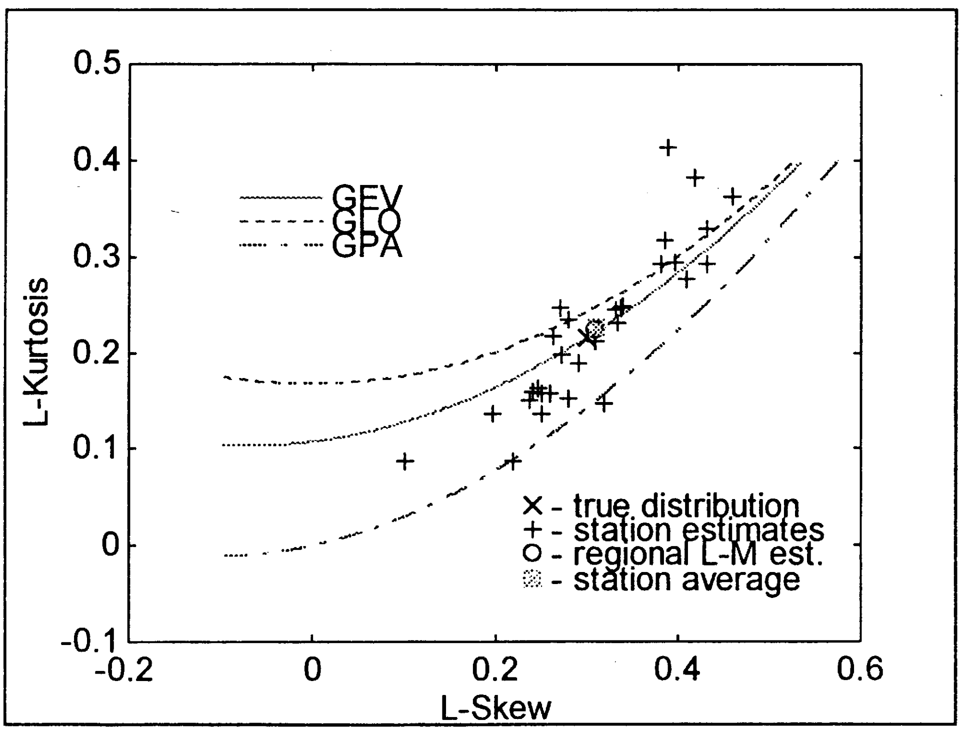


Figure 3.3: L-moment diagram for simulated GEV data, $\tau_2 = .2$, $\tau_3 = .3$, at 30 sites, 60 years/site.

by Leggett and Williams (1981). A probability model which is determined to be most reliable (smallest index value) by these methods is selected as the estimated correct model. The third type of method will be called 'region-based' methods because all the data are used to determine a single 'regional solution'. This regional solution is then used to select the estimated correct model.

Two important issues arise in the region-based methods: (1) how to arrive at a regional solution; and (2) how to choose the estimated correct model. These problems will be described in Sections 3.2.4 and 3.2.5. The various methods for regional probability model selection are described below, with some discussion of their advantages and disadvantages.

Since the observed values of (t_3,t_4) will be based on the amount of available data at each site, all methods presented here include a weight corresponding to the amount of data available. Methods presented below will have a weight factor r_i representing at site record length. In order to simplify the simulation procedure the same record length will be used for all sites (see Section 3.3), and so all weights r_i are equal.

3.2.1 Linear Programming Based Methods

Linear programming (LP) methods were used to determine the optimal weights for a mixture model. The mixture model $f_{mix}(x)$ is defined as

$$f_{mix}(x) = \sum_{j=1}^{d} w_j f_j(x) \;, \; subject \; to \sum_{j=1}^{d} w_j = 1 \; and \; 0 \le w_j \le 1 \tag{3.5}$$

where w_j is the weight corresponding to distribution f_j and d is the number of distributions considered. For each distribution the sample value of L-skewness (t_3) is used to calculate a predicted value of L-kurtosis (τ_4). Let $\tau_4 = g_j(\tau_3)$ represent the functional relation between τ_3 and τ_4 for distribution j ($g_j(\tau_3)$ is predicted τ_4 given τ_3 and distribution f_j.). Zhao and Tung (1994) formulate three different LP models which can be applied here.

Method 1 - *Minimization of the sum of absolute deviations (MSAD)*. The problem can be cast as

$$\min_{w} \sum_{i=1}^{m} r_i(\epsilon_i^- + \epsilon_i^+) \tag{3.6}$$

subject to

$$w_1 g_1(t_{3i}) + w_2 g_2(t_{3i}) + \dots + w_d g_d(t_{3i}) + \epsilon_i^- - \epsilon_i^+ = t_{4i}\ , \quad i=1,2,\dots,m \tag{3.7}$$

where $\boldsymbol{w} = (w_1, w_2, \dots, w_d)$; $\epsilon_i^- \geq 0$ and $\epsilon_i^+ \geq 0$ each represent negative and positive errors, respectively, for the i^{th} observation and m is the number of observed (t_3,t_4) pairs. Note that at least one of these values ϵ_i^- and ϵ_i^+ must equal zero. This method selects the best distribution j associated with the largest weight w_j. The method can be very computationally demanding when the number of observations (number of stations) is large.

Method 2 - *Minimization of the largest error (MLAD)*. This LP model has the following objective function

$$\min_{w} \left[r_t \epsilon_{max} \right] \tag{3.8}$$

subject to

$$\begin{aligned} w_1 g_1(t_{3i}) + w_2 g_2(t_{3i}) + \cdots + w_d g_d(t_{3i}) - \epsilon_{max} &\le t_{4i} \quad , i=1,\ldots,m \\ w_1 g_1(t_{3i}) + w_2 g_2(t_{3i}) + \cdots + w_d g_d(t_{3i}) + \epsilon_{max} &\ge t_{4i} \quad , i=1,\ldots,m \end{aligned} \tag{3.9}$$

where ϵ_{max} = MAX [$\epsilon_i^-, \epsilon_i^+$] will minimize the largest absolute deviation. The best distribution is chosen by the largest weight in the same manner as method 1.

Method 3 - *Minimization of the largest range of error (MRNG).* The objective function of the LP model is

$$\min_{w} r_t \epsilon_{rng} = \min_{w} r_t \left[\epsilon_{max}^- + \epsilon_{max}^+ \right] \tag{3.10}$$

subject to

$$\begin{aligned} w_1 g_1(t_{3i}) + w_2 g_2(t_{3i}) + \cdots + w_d g_d(t_{3i}) - \epsilon_{max}^+ &\le t_{4i}, \quad i=1,2,\ldots,m \\ w_1 g_1(t_{3i}) + w_2 g_2(t_{3i}) + \cdots + w_d g_d(t_{3i}) + \epsilon_{max}^- &\ge t_{4i}, \quad i=1,2,\ldots,m. \end{aligned} \tag{3.11}$$

where $\epsilon_{rng} = \epsilon_{max}^- + \epsilon_{max}^+$, $\epsilon_{max}^- = MAX(\epsilon_{max,i}^-)$, $\epsilon_{max}^+ = MAX(\epsilon_{max,i}^+)$ will minimize the largest range of error (MRNG). The best distribution is choosen by the largest weight in the same manner as method 1.

The optimal solutions described in this study were calculated using the *lp.m* function in the MATLAB Optimization Toolbox (Grace, 1990).

3.2.2 Quadratic Programming Method

The forth method is very similar to method 1 (MSAD). The constraint (Eq. 3.7) are again used, with the objective function

$$\min_{w} \sum_{i=1}^{m} r_i[(\epsilon_i^-)^2 + (\epsilon_i^+)^2] \tag{3.12}$$

defines the minimum sum of squared deviations (MSSD). The best regional probability model is selected in the same manner as the LP methods described in Section 3.2.1. This method shares many of the characteristics of method 1 (MSAD) and gives similar results. The *qp.m* function in the MATLAB Optimization Toolbox (Grace, 1990) was used for this method.

3.2.3 Reliability Indices

Leggett and Williams (1981) proposed two reliability indices for the purpose of environmental model selection. Brief descriptions of these two indices are given below.

Consider a graph of observed versus predicted values. A perfect model would result in a 45 degree line because the predictions exactly match the observations. Imperfect predictions will plot points somewhere off of the 45 degree line. A prediction error may be measured by a function of the tangent of the angle of the line to the plotted point. Let x_i be the theoretical value of L-kurtosis predicted from L-skewness for a particular distribution, and y_i be values of observed L-kurtosis (note: $\tan(\theta_i) = y_i/x_i$). Let the reliability index k_g be defined as

$$t = \sqrt{\frac{1}{m}\sum_{i=1}^{m} r_i \left[\frac{1-(y_i/x_i)}{1+(y_i/x_i)}\right]^2}$$

$$k_g = \frac{1+t}{1-t} \quad , \quad k_g \geq 1 \tag{3.13}$$

and statistical reliability index k_s be defined as

$$k_s = \exp\sqrt{\frac{1}{m}\sum_{i=1}^{m} r_i \log\frac{y_i}{x_i}} \tag{3.14}$$

where m is the number of stations in the region. For both k_g and k_s an index value of one indicates a perfect model fit, and larger values are the result of errors in the model. The values of k_g and k_s are calculated for each of the distributions under consideration, and the model with the smallest value of k_g or k_s is then selected as the correct model. The fifth and sixth selection methods are k_g and k_s, respectively.

The k_s index gives unusual results when the values of an observation and the corresponding prediction differ in sign. Using log(-1) = i a complex value for k_s may be determined. The smallest k_s value is then found by using the absolute value of k_s, defined using the real and imaginary components of k_s as

$$|k_s| = \sqrt{real\,(k_s)^2 + imaginary\,(k_s)^2} \tag{3.15}$$

Unfortunately, $|k_s|$ may now be less than one. The interpretation of k_s in the presence of differing signs for y_i and x_i is uncertain. Calculations using k_s were carried through for completeness.

3.2.4 Regional Estimates Based on Averaged Station Estimates

The L-moment estimates of (t_3,t_4) at all sites in the region is first calculated. The regional estimate (T_3,T_4) is then determined by taking a weighted average of the at-site (t_3,t_4) values as

$$T_3 = \frac{1}{m}\sum_{j=1}^{m} r_t t_{3_j} \;,\quad T_4 = \frac{1}{m}\sum_{j=1}^{m} r_t t_{4_j} \tag{3.16}$$

Two distance criteria for identifying the correct model are considered, as shown in Figure 3.4. The first way is to choose the model with the smallest vertical distance from (T_3,T_4) to the theoretical (τ_3,τ_4) curve associated with each distribution. The second uses the shortest distance from (T_3,T_4) to the theoretical (τ_3,τ_4) curve. Method 7 selects the model

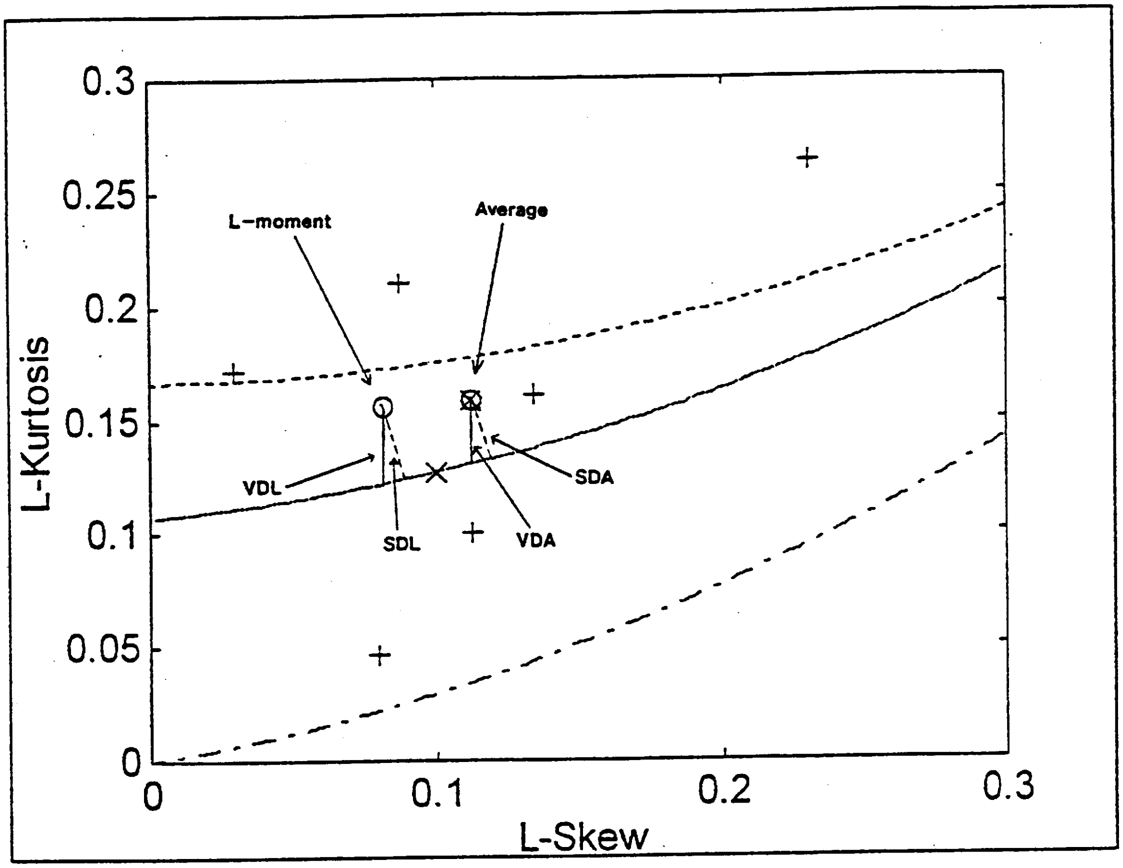


Figure 3.4: Example L-moment diagram showing selection methods.

with the smallest vertical distance as the estimated correct model, and method 8 selects the model with the shortest distance as the estimated model. These methods will be referred to as vertical distance to average (**VDA**)and shortest distance to average (**SDA**), respectively.

3.2.5 Regional Estimate by L-Moments

The regional estimates for (T_3, T_4) are calculated using L-moment solutions for the combined data from all stations in the region. This can be a very computationally intensive calculation. Fortunately, a method exists which allows for efficient computation and extensive simulation.

Let **x** be the column vector of *n* order statistics and **C** be a 4 by *n* matrix containing the appropriate coefficients in each row for the first four L-moments estimators. The L-moment estimators are now easily found in vector form.

$$\boldsymbol{L} = \begin{bmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \\ \ell_4 \end{bmatrix} = \boldsymbol{C}\boldsymbol{x} \tag{3.17}$$

The matrix **C** needs only to be calculated once for a region of a give size, thus greatly reducing time required for repeated simulations.

The estimated correct model can then be selected based on the two distance criteria described in the previous section. Method 9 selects the model with the smallest vertical distance to the regional estimate (**VDL**). Method 10 selects the model with the shortest distance to the regional solution (**SDL**).

3.3 Evaluation of Regional Probability Model Selection Methods

3.3.1 Factors Influencing the Performance of Selection Methods

Monte Carlo simulation was conducted in order to study how each method will work. Four factors are considered that have potential effect on the accuracy of selection. These factors are: 1) The number of sites within a region (NS); 2) The number of years of data available at each site (NY); 3) L-cv or τ_2; 4) L-skewness or τ_3.

Note that the factors NS and NY both deal with the amount of data available, and that it is the methods which cause them to be considered separately. The methods using the regional L-moment estimate will combine NS and NY into a single factor of 'total years' or sample size.

3.3.2 Experiment by Simulation

Computer simulation techniques were used to evaluate the performance of the 10 selection methods for regional probability models. Simulation routines were written in MATLAB. Development and testing was done on a 486 computer. The Technical Information Processing System (TIPS) lab at the University of Wyoming provided use of a Silicon Graphics IRIX/Indigo computer for the final simulation runs.

The simulation procedure may be briefly described as the following steps:

1) One probability distribution among GEV, GLO, and GPA is chosen as the 'true'

model.

2) Parameters for the distribution are determined from the given values of τ_2 and τ_3.

3) Random data was generated , given NS and NY, with no spatial correlation from the 'true' distribution. Each selection method was then applied to the data, resulting in a single model choice for each method. Selection results are recorded for later analysis.

4) Repeat steps 2 and 3 a large number of times in order to estimate the probability that each method correctly selects the 'true' model under a given set of conditions.

5) Repeat steps 2 through 4 for various combinations of with NS, NY, τ_2, and τ_3.

6) Repeat steps 1 through 5 for each of the distributions under consideration.

7) Results for each method are then tabulated and analyzed (see Section 3.3.3).

The objective of this experiment is to understand the behavior of the selection methods, and to determine which are useful enough to be worthy of further study. Once a successful method(s) is found a more extensive simulation study can be conducted. Determining the precise behavior of each selection method under a wide variety of conditions could be an extremely laborious process and it will not be attempted here.

The consideration of four factors leads to a practical problem in implementing the intended numerical simulation. In order to study the effects of a factor at different levels in combination with other factors, a large number of different simulations is necessary. Also, 10,000 repetitions of a simulation are generally suggested. If 4 levels of L-skewness and NY are desired, and 3 levels of L-cv and NS are desired, then there will be 144 combinations each to be simulated 10,000 times. Due to the computational intensive nature of the optimization algorithms for methods 1 and 4, 10 seconds or longer for a single simulation on a fast 486 computer is not unusual. All together this would require approximately 4000 hours of computer time. Clearly some compromise is necessary, at least so that an initial study of these methods can be made.

Three things were done in order to make this numerical simulation investigation feasible. First, a faster computer was used to reduce the time required by a factor of 2 to 3, which was useful but not sufficient. Considering only two levels of each factor resulted in a large reduction in the number of combinations, but non-linear effects caused by a factor now cannot be observed. Finally, statistical experimental design techniques were applied to reduce the total number of required simulation runs. The findings presented in this study are based on the result of 420 simulations.

The two levels of each factor are as follows:

(1) NS is considered at region sizes of 15 and 30 data-sites to represent regions with small to moderate amounts of data available. This factor is useful in determining how many stations are needed to provide good regional estimates. Large regions were not considered due to extremely long computation times for methods 1 and 4.

(2) NY is considered for record lengths of 20 and 60 years. 20 years may be considered a minimum length for reliable at-site results whereas 60 years represents a fairly long record (In fact, double the average record length available in Wyoming.).

(3) τ_2 is considered at .2 and .4, representing low-average and high end values of L-cv.

(4) τ_3 is considered at .1 and .3, representing low-average and high-average values of L-skewness.

3.3.3 Factorial Design for Binomial Proportions

Consider a set of simulated of data from a given model. Define a correct selection to be the event that a selection method indicates the true model, and an incorrect selection to be the indication of any other model. Let this simulation be repeated n times and Y be the total number of times the correct model is selected. The variable Y will follow a binomial distribution with a parameter n and p, where p indicates the probability of a correct selection. Point estimates and inference on p is easily done. Note that for moderately large sample sizes the normal approximation to the binomial may be used.

Using the four factors, a 2^4 ($2\times2\times2\times2$) factorial design with a sample size of $n = 420$ in each cell is used. Detailed description of experimental design methods can be found in many textbooks on the subject (see Montgomery, 1985). This design allows tests of significance on the effects of the four factors on the probability of correct selection p. A more useful aspect of this analysis is that it enables tests on interaction effects between factors to be conducted. This is useful because the effect of a factor may potentially depend on anther factor. Utilizing this design aids in determining which selection method(s) will work best in a variety of situations.

Recall that for methods 8 and 9 which use the regional L-moment solution, NY and NS are not separate factors and the Total Years of record should be used here instead. The 2^4 factorial design is therefore not appropriate for these two methods. The 2^3 ($2\times2\times2$) design could be used here, but this is not useful. Such a design would conduct a statistical test to determine if the estimates become more accurate as the sample size increases. The effect of τ_2 and τ_3 on these methods can be more clearly and effectively shown by the use of graphs.

To allow comparison of methods these results are also presented in the form of the 2^4 design, even though this is not the best way to interpret this data.

The experimental design may be viewed as 16 cells representing the combinations of factors. Table 3.1 illustrates the design layout used in this study. For each experiment 15 separate tests will be conducted. There will be four main effects tests, six 2-way interaction tests, four 3-way interaction tests, and one 4-way interaction test. A Bonferroni adjustment will be used to maintain overall type I error level for each experiment.

Table 3-1: 2^4 Factorial Design Layout

	NY = 20 $\tau_3=.1$,	$\tau_3=.3$		NY = 60 $\tau_3=.1$,	$\tau_3=.3$
NS=15 $\tau_2=.2$	1	2	NS=15 $\tau_2=.2$	4	6
$\tau_2=.4$	3	4	$\tau_2=.4$	7	8
	NY = 20 $\tau3=.1$,	$\tau3=.3$		NY = 60 $\tau3=.1$,	$\tau3=.3$
NS=30 $\tau_2=.2$	9	10	NS=30 $\tau_2=.2$	13	14
$\tau_2=.4$	11	12	$\tau_2=.4$	15	16

3.3.4 Interpretation of Experimental Results

Results are presented in appendix B with Tables B.1(a,b,c) through B.10(a,b,c), each table represents a separate test using the 2^4 factorial design. Table B.11 is included as key to aid in reading the test results. This section will discuss how to read and interpret the results in general. Specific results will be given in the next section.

The top line of each table (Tables B.1 through B.10) gives the method being tested and the probability distribution it is being tested for. The shaded column (containing two sub-columns) lists all the information about the method correctly selecting the true probability model. Located at the top of this column is a number p which is an estimate of overall average probability of selecting the true probability model by that method. The unshaded columns list the information about incorrect selection of the other distributions. Some useful information about bias in the selection methods is contained here. The results for incorrect selections are presented in full and may be considered as a separate statistical tests. The results will of course be strongly negatively correlated to the results in the true distribution column.

The left-hand column contains a four digit combination of 0's and 1's representing a test of a particular combination of factors. The digits represent NS, NY, τ_2, and τ_3 in that order. A "1" indicates that the factor is present at different levels in the test, and a "0" means that factor is averaged over the levels in the test. A line with more than a single '1' is an interaction tests between those factors. A line with a single one is a main effects test.

The 'Δ =' sub-column of the true distribution column is an average percentage change between levels in the probability of correct selection as the factor increases. This is most easily interpreted in the case of a main effect as the average percentage increase in p between levels of that factor. It is important to note that it may not be possible to directly interpret main effects in the presence of significant interactions involving that factor. However, note that for a large main effect a very small interaction effect may not be practically meaningful.

The "Z =" sub-column gives the value of the test statistic

$$Z = \frac{\sum_{i=1}^{16} h_i p_i}{\sqrt{\sum_{i=1}^{16} h_i^2 \frac{p_i(1-p_i)}{n_i}}} \tag{3.18}$$

which has a standard normal distribution since the normal approximation to the binomial will be very accurate with a sample size of $n_i = 420$. The values h_i are the contrasts for the tests, and are given in Table 3.2. The statistics are marked with one or more asterisk if the value exceeds a certain critical value. These critical values are calculated for a two-tailed test with a Bonferroni adjustment for 15 simultaneous tests (critical value = $Z_{\alpha/30}$). One, two, and three asterisk correspond to type I error levels of $\alpha = 0.15$, $\alpha = 0.05$, and $\alpha = 0.01$ respectively. For the discussion a type I error lever of $\alpha = 0.05$ will be assumed.

Interaction effects between two factors occur when the effect caused by one factor changes in response to another factor. Interactions can occur in any combination of two or more variables. A main effect is the change in response due to the change in the level of one factor. However, if this factor interacts significantly with any other factors, then the main

Table 3-2: Coefficients h_i for Tests

Test* : i =	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1111:	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1
0111:	-1	1	1	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1
1011:	-1	1	-1	1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1
1101:	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1	1	-1	-1	1
1110:	-1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	-1	1	1
0011:	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
0101:	1	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1
1001:	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1
0110:	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1
1010:	1	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1
1100:	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
0001:	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1
0010:	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1
0100:	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1	1	1
1000:	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1
p :	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

* Refer to Table B.11 for meaning of tests.

effect is not so easily interpreted. It is possible to interpret two-way interactions given that no higher order interactions exist. Similarly, higher-order interactions may be interpreted, but it becomes increasingly difficult to do so in a meaningful manner.

Several two-way and three-way interactions were anticipated in this study. NS and NY both represent the amount of information available in regional analysis. A two-way interaction can be expected here, because doubling the amount of data generally does not double the accuracy of a statistic. Many of the methods considered do not respond in the same way to each factor. NS and NY may both interact with L-skewness (τ_3) to cause a two-way or three-way interaction. This may be due to the increasing variability of the L-moment ratio estimates as τ_2 and τ_3 increase. Initial testing gave little evidence to suggest any interactions between τ_2 and any other factor, and the experimental results tend to support this. Sample values of t_2 and t_3 may be correlated, but this does not seem to significantly affect the results of most selection methods. No significant four-way interactions were anticipated or found in the experimental results.

Marginal plots are a useful for displaying the effects and interactions of factors. The marginal plots for each experiment are given in Figures C.1(a,b,c) through C.10(a,b,c) (see Appendix C). Methods 7 through 10 have additional marginal plots (d, e, f). The selection method and the true regional probability model are displayed is given at the top. The percentage each model selected is shown on the vertical axis and NY is on the horizontal axis. These Figures show the selection of all three probability models on each plot, with the point type indicating the probability model. For the 'true' model the plot shows the probability of a correct selection, for the other models it is the probability of selecting that model incorrectly. Line type show the levels of NS and τ_3. Since τ_2 had almost no effect on the results it has been averaged in with the other factors, and is not shown on these plots. Interaction effects can be seen within the lines for the 'true' probability model. If no interaction exists then the lines can be expected to be parallel, as a factor will have equal response at all levels of other factors. Interactions will show as non-parallel, converging, or crossing lines. Note that lines resulting from different models cannot be interpreted this way.

3.3.5 Performance Assessment Based on Simulations

Method 1 (LP/MSAD): The overall accuracy is 59% for GEV data, and 57% for GLO data. Strong interactions exist between NS and τ_3, and between NY and τ_3. This method exhibits a strong bias towards the GEV model at the high level of τ_3, especially in the case of GLO data. This method is more reliable with GPA data. See Table B.1(a,b,c) and Figure C.1(a,b,c).

Method 2 (LP/MLAD): This method does well when the true model os GPA, but it cannot reliably distinguish between GEV and GLO models. See Table B.2(a,b,c) and Figure C.2(a,b,c).

Method 3 (LP/MRNG): This selection method gives very poor results under all circumstances. For GEV and GLO models the probability of correct selection is less than 50%. See Table B.3(a,b,c) and Figure C.3(a,b,c).

Method 4 (QP/MSSD): Results here are generally similar to method 2, with greater sensitivity to τ_3. See Table B.4(a,b,c) and Figure C.4(a,b,c).

Method 5 (k_g): This method is extremely sensitive to NY and τ_3. Interactions between

NS, NY and τ_3 are strong, and it seems to be biased against the GLO model. This method can only give good results when all at-site estimates are very accurate and τ_3 is low. See Table B.5(a,b,c) and Figure C.5(a,b,c).

Method 6 (k_o): Exhibits similar behavior to method 5, but even these results may be suspect (see Section 3.2.3). See Table B.6(a,b,c) and Figure C.6(a,b,c).

Method 7 (VDA): The overall accuracy is 67% for the GLO model, 84% for GEV, and 98% for GPA. This method is not at high τ_3, but does not appear to be biased towards the GEV model as occurs with some other methods. Two and three way interactions exist between NS, NY, and τ_3. This method is the worst of all the region-based methods considered, but it is still considerably better than the earlier methods. See Tables B.7(a,b,c) and Figures C.7(a,b,c,d,e,f).

Method 8 (SDA): The overall accuracy is 82% for the GLO model, 90% for GEV, and 99.5% for GPA. The use of the shortest distance rather than the vertical distance greatly reduces the sensitivity to τ_3. Some interactions exist like those of method 7, but they tend to be much smaller. See Tables B.8(a,b,c) and Figures C.8(a,b,c,d,e,f).

Method 9 (VDL): Results are somewhat similar to method 8, but is very sensitive to τ_3. The use of the regional L-moment solution gives better results at shorter record lengths. This indicates that this method may converge more quickly to 100% probability of correct model selection. See Tables B.9(a,b,c) and Figures C.9(a,b,c,d,e,f).

Method 10 (SDL): This method exhibits much less sensitivity to τ_3 and more rapid convergence always selecting the true model than all other methods. It is near perfect with GPA data, and is much more accurate even in the worse case (of those examined) for GEV and GLO data. See Tables B.10(a,b,c) and Figures C.10(a,b,c,d,e,f).

3.3.6 Summary and Conclusions

L-skewness is the single most important factor determining the accuracy of the selection methods. All methods considered show significant decreases in accuracy or at high τ_3. L-skewness also interacts very strongly with other factors. The GEV and GLO distribution can be very hard to distinguish at high levels of L-skewness. It should be noted that these two distributions converge quickly as L-skewness increases, and selection errors may become less meaningful in this case. The effects of τ_3 are likely to be non-linear, but this could not be examined due to the limited scope of this study.

L-cv has no effect on model selection by these methods. Future examinations of selection methods may ignore τ_2 or only consider reduced number of interaction effects (by use of partial factorial design).

The GPA distribution is easy to distinguish from the GEV and GLO. Several methods perform very well in this case (LP/MSAD, QP/MSSD, and k_g). This is an 'easy' problem in comparison to distinguishing between GEV and GLO data.

The linear programming based methods do poorly with GEV and GLO data. These method (perhaps modified) may still be useful for estimating mixture distributions. Also, this approach may have some advantage in the case that two probability models have the same regional values of τ_3 and τ_4. It may be possible to improve upon the results of these methods by use of discriminant analysis, but this would be a topic for further research.

The k_g reliability is highly sensitive to the accuracy of at-site estimates and L-skewness. This method is not suggested because better methods are available. A multivariate version of this index can be defined using τ_3 and τ_4. This might improve the results greatly. The k_s index is not appropriate for this type of data encountered in this problem.

The method(s) based on the regional L-moment solution and shortest distance are superior to those based on at-site averages and vertical errors. The use of shortest distance in selection greatly reduces sensitivity of model selection to τ_3. Use of the regional L-moment solution results in faster convergence to the true regional values of (τ_3, τ_4). When combined there is little interaction effects between factors. The SDL method is the best of all methods examined.

In the real world of spatially correlated data none of these methods would actually perform as well as is shown in these simulation experiments. These results are useful though as an indication of which methods are worth pursuing for further study.

CHAPTER 4

SUMMARY AND PROPOSED FUTURE RESEARCH

4.1 Comments on Extreme Precipitation Studies

4.1.1 Other Studies

This study has emphasized the use of statistical properties in the analysis of extreme precipitation data. There are concurrent studies in the regionalization of extreme precipitation also sponsored by the Association of State Dam Safety Officials, such as the recently concluded Montana study (Parrett, 1995). In that, regionalization of the State of Montana was done primarily on the basis of geography. The regions in the Montana study determined on a physical basis match up well with the regions in Wyoming determined by cluster analysis (Schaefer, 1995). This common result supports the use of physical and statistical properties in the regionalization process. Physical properties should be used when there are clearly understood patterns of precipitation. Statistical properties can then be examined to help find further patterns in the data that might otherwise be overlooked.

4.1.2 Usefulness of this Study

The regionalization procedure as described in Section 1.4 clearly leads to improved parameter estimation. The results from Chapter 3 indicate that regionalization also greatly improves the model selection process as well. The regions (see Figure 2.3) and regional prediction equations determined by this study (see Section 2.5.2) can be used for parameter estimation within Wyoming. However, it should be noted that for the higher-order L-

L-moments ratios such as L-skewness and L-kurtosis, the prediction equations may be of little value due to large prediction errors. In this case it is recommended that the regional values of the L-moment ratios (see Table 2.3) be used in place of the prediction equations .

Examination of the question of probability models for extreme precipitation has led to new methods for model selection. These methods tend to support the use of the GEV distribution in Wyoming. Exactly how well these methods perform has yet to be determined, but results to this point are promising. Further study of these methods and application to model selection in other situations would be a logical next step.

4.2 Regionalization Revisited

Regionalization is a difficult and subjective process which results in improved overall estimation at the cost of possible large errors at a given location. This is not intended as a criticism of regionalization, but rather to point out that some compromises are necessary to reach a solution to the difficult problem of predicting extreme precipitation events. There are several potential ways that the regionalization process might be improved, such as including dew-point and seasonal information. This is a topic which needs combined knowledge in hydrology, meteorology, and statistics in order for improvements and refinements to be made.

4.3 Model Selection Revisited

4.3.1 Unsuccessful Methods

Model selection methods based on linear and quadratic programming techniques performed poorly. Theses selection methods fail because it is very difficult to distinguish between a single distribution and a 'similar looking' mixture of two (or more) other distributions. It is possible that the use of discriminant analysis methods might offer a solution to this difficulty.

Likewise, the two reliability indices did not perform well. It is possible that these methods were poorly formulated. These methods utilize the tangent of predicted over observed values, and choosing a different point of origin might change the accuracy of these methods. It is not apparent, however, that a different form of selection these selection methods work any better. A multivariate form of these indices can be defined which should not suffer from the difficulties found in the univariate form, and this is possibly a topic of further research.

4.3.2 Successful Methods

The so called *region based* methods work well for model selection. The strength and weakness here is the reliance on the assumption of a climatically homogenous region. The accuracy of the region based methods clearly depends on good results from the regionalization process. Since regionalization results in better overall estimates, the region based model selection methods should also give better overall results.

4.3.3 Further Research in Model Selection

Further research possibilities in model selection methods include study of L-moments and conventional product moments. It is planned to examine the small sample properties of L-moments ratio estimators, it is of interest to know in what situation L-moment based estimation will outperform product moments. A comparison to maximum-likelihood estimation, in statistical efficiency and computational requirements, would be useful. Selection methods, whether based on L-moments or product moments can be applied to the problem of model selection in general. In this last case, and extensive study of the power of selection methods and the probability of a model selection error could be conducted.

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APPENDIX A: REGRESSION MODELS

Table A.1: Mean and Standard Deviatiion of MAP by Region*								
	Region 1		Region 2		Region 3		Region 4	
	μ	σ	μ	σ	μ	σ	μ	σ
MAP	12.1	2.9	12.0	3.06	12.6	4.63	10.9	2.85
log(MAP)	2.46	.281	2.45	.295	2.47	.340	2.36	.274
z(log MAP)	.048	.932	.015	.978	.091	1.13	-.29	.908

* Based on regression dataset with outliers removed

Table A.2: Model Parameters for Region 1								
Y =	ln XBAR		ln τ_2		τ_3		τ_4	
	β-hat	σ_β	β-hat	σ_β	β-hat	σ_β	β-hat	σ_β
β_0	.346*	.0142	-1.5*	.0136	.241*	.241	.183*	.0085
β_1	.205*	.0120	-.005	.0115	.003	.003	.0053	.0072
D_1	.689*	.0415	.173*	.04	.039	.039	.0106	.0249
D_2	.339*	.0343	.079*	.0329	.010	.010	-.018	.0206
D_3	.0098	.0316	.072*	.0304	.008	.008	-.03	.019
F	160.*		6.12*		.90		.98	
MSE	62.6		2.21		.1037		.1377	

* Significant at α = 0.05

Table A.3: Model Parameters for Region 2								
Y =	ln XBAR		ln τ_2		τ_3		τ_4	
	β-hat	σ_β	β-hat	σ_β	β-hat	σ_β	β-hat	σ_β
β_0	.324*	.0284	-1.7*	.0206	.083*	.0120	.096*	.0109
β_1	.206*	.0219	-.017	.0159	.027*	.0092	.021*	.0084
D_1	-.68*	.0559	.112*	.0406	.058*	.0237	.018	.0216
D_2	-.47*	.0530	.0618	.0385	.0330	.0224	.05*	.0205
D_3	-.078	.0620	.0174	.0450	.0387	.0262	.018	.0239
F	87.4*		.855*		3.14*		2.52	
MSE	49.7		2.85		0.321		.214	

* Significant at α = 0.05

Table A.4: Model Parameters for Region 3								
Y =	ln XBAR		ln τ_2		τ_3		τ_4	
	β-hat	σ_β	β-hat	σ_β	β-hat	σ_β	β-hat	σ_β
β_0	.112*	.02	-1.6*	.0173	.205*	.0125	.166*	.0118
β_1	.075*	.0139	-.08*	.0120	-.0035	.0087	-.012	.0082
D_1	-.67*	.0515	.217*	.0445	.056*	.0321	-.029	.0305
D_2	-.43*	.0424	-.0004	.0366	.0166	.0264	-.015	.025
D_3	-.069	.0406	.0215	.0351	.0029	.0253	-.017	.024
F	36.9*		19.6*		.96		.69	
MSE	77.4		6.99		.1771		.1152	

* Significant at α = 0.05

Table A.5: Model Parameters for Region 4								
Y =	ln XBAR		ln τ_2		τ_3		τ_4	
	β-hat	σ_β	β-hat	σ_β	β-hat	σ_β	β-hat	σ_β
β_0	.369*	.027	-1.3*	-1.33	.338*	.0146	.235*	.0187
β_1	.137*	.0227	-.04*	-.039	.0017	.0124	.014	.0160
D_1	-.83*	.0587	.161*	.161	.062*	.0322	.027	.0413
D_2	-.65*	.0532	-.05	-.047	.0092	.0292	.016	.0374
D_3	-.22*	.0676	-.04	-.043	.0160	.0371	.064	.0476
F	43.6*		8.02*		.92		.65	
MSE	99.1		1.53		.1224		.1429	

* Significant at α = 0.05

APPENDIX B: Results of Simulation Experiments

Table B.1a: Experimental Results for Method 1 on GEV Data

Method 1 - MSAD, for simulated GEV data						
	GEV, overall p = 0.5929		GPA, overall p = 0.1119		GLO, overall p = 0.2952	
TEST	Δ =	z =	Δ =	z =	Δ =	z =
1111	-0.00416	-0.3682	0.00773	1.074	-0.00357	-0.3429
1110	-0.01845	-1.631	0.00714	0.991	0.01131	1.086
1011	0.00357	0.3156	-0.00595	-0.8258	0.00238	0.2286
1101	0.00535	0.4735	-0.00595	-0.8258	0.00059	0.0571
1110	-0.00654	-0.5787	0.00297	0.4129	0.00357	0.3429
0011	0.01905	1.683	-0.00297	-0.4129	-0.01607	-1.543
0101	0.1065	9.417***	-0.06726	-9.332	-0.03929	-3.772
1001	0.03929	3.472***	0.00654	0.9084	-0.04583	-4.401
0110	0.00892	0.7891	0.00476	0.6607	-0.01369	-1.315
1010	-0.01429	-1.263	-0.00119	-0.1652	0.01548	1.486
1100	-0.01726	-1.526	0.02857	3.964	-0.01131	-1.086
0001	0.2417	21.36***	0.07024	9.745	-0.3119	-29.95
0010	0.00952	0.8417	-0.00535	-0.7432	-0.00416	-0.4001
0100	0.1613	14.26***	-0.1875	-26.01	0.02619	2.515
1000	0.07976	7.049***	-0.05179	-7.185	-0.02798	-2.686

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$

Table B.1b: Experimental Results for Method 1 on GPA Data

Method 1 - MSAD, for simulated GPA data						
	GEV, overall p = 0.0016		GPA, overall p = 0.9978		GLO, overall p = 0.0006	
TEST	Δ =	z =	Δ =	z =	Δ =	z =
1111	0.00089	0.9084	-0.00208	-1.821	0.00119	2.01
0111	-0.00148	-1.514	0.00267	2.342	-0.00119	-2.01
1011	-0.00089	-0.9084	0.00208	1.821	-0.00119	-2.01
1101	0.00148	1.514	-0.00267	-2.342	0.00119	2.01
1110	-0.00029	-0.3028	-0.00089	-0.7805	0.00119	2.01
0011	0.00148	1.514	-0.00267	-2.342	0.00119	2.01
0101	-0.00208	-2.12	0.00327	2.862*	-0.00119	-2.01
1001	-0.00148	-1.514	0.00267	2.342	-0.00119	-2.01
0110	-0.00029	-0.3028	0.00148	1.301	-0.00119	-2.01
1010	0.00029	0.3028	0.00089	0.7805	-0.00119	-2.01
1100	0.00267	2.725	-0.00386	-3.382**	0.00119	2.01
0001	0.00208	2.12	-0.00327	-2.862*	0.00119	2.01
0010	0.00029	0.3028	-0.00148	-1.301	0.00119	2.01
0100	-0.00327	-3.331	0.00446	3.903***	-0.00119	-2.01
1000	-0.00267	-2.725	0.00386	3.382***	-0.00119	-2.01

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$

Table B.1c: Experimental Results for Method 1 on GLO Data

Method 1 - MSAD, for simulated GLO data						
	GEV, overall p = 0.4180		GPA, overall p = 0.0171		GLO, overall p = 0.5649	
TEST	Δ =	z =	Δ =	z =	Δ =	z =
1111	0.00803	0.7824	-0.00089	-0.2907	-0.00714	-0.706
0111	-0.00922	-0.8984	0.00148	0.4845	0.00773	0.765
1011	0.00089	0.0869	0.00148	0.4845	-0.00238	-0.235
1101	0.00446	0.4347	0.0128	4.167	-0.01726	-1.708
1110	-0.00506	-0.4926	0.00029	0.0969	0.00476	0.471
0011	0.0122	1.188	-0.00208	-0.6783	-0.01012	-1.001
0101	0.09077	8.839	-0.03006	-9.787	-0.06071	-6.006***
1001	0.06518	6.346	-0.01339	-4.36	-0.05179	-5.123***
0110	-0.00684	-0.6665	0.00029	0.0969	0.00654	0.647
1010	0.00327	0.3188	0.00029	0.0969	-0.00357	-0.353
1100	-0.0372	-3.622	0.01637	5.329	0.02083	2.061
0001	0.4729	46.05	0.03065	9.98	-0.5036	-49.81***
0010	0.01101	1.072	-0.00089	-0.2907	-0.01012	-1.001
0100	-0.1473	-14.34	-0.03363	-10.95	0.181	17.9***
1000	0.06756	6.578	-0.01696	-5.523	-0.0506	-5.005***

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.2a: Experimental Results for Method 2 on GEV Data

Method 2 - MLAD, for simulated GEV data						
	GEV, overall p = 0.4948		GPA, overall p = 0.0927		GLO, overall p = 0.4125	
TEST	Δ =	z =	Δ =	z =	Δ =	z =
1111	0.00089	0.0755	-0.00625	-0.914	-0.00625	-0.914
0111	-0.01875	-1.587	0.00684	1.001	0.00684	1.001
1011	0.00506	0.4283	-0.00565	-0.827	-0.00565	-0.827
1101	0.02946	2.494	0.00148	0.217	0.00148	0.217
1110	-0.00684	-0.5795	-0.00982	-1.437	-0.00982	-1.437
0011	0.01756	1.486	0.00386	0.566	0.00386	0.566
0101	0.0622	5.266***	-0.0378	-5.53	-0.0378	-5.53
1001	0.02887	2.444	-0.00982	-1.437	-0.00982	-1.437
0110	0.01042	0.8818	0.00327	0.478	0.00327	0.478
1010	0.00208	0.1764	0.00148	0.217	0.00148	0.217
1100	-0.00208	-0.1764	0.0122	1.785	0.0122	1.785
0001	0.1485	12.57***	0.0497	7.271	0.0497	7.271
0010	0.00863	0.7306	0.00386	0.566	0.00386	0.566
0100	0.1771	14.99***	-0.1295	-18.94	-0.1295	-18.94
1000	0.04732	4.006***	-0.03601	-5.268	-0.03601	-5.268

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.2b: Experimental Results for Method 2 on GPA Data

Method 2 - MLAD, for simulated GPA data						
	GEV, overall p = 0.0280		**GPA, overall p = 0.9637**		GLO, overall p = 0.0083	
TEST	**Δ** =	z =	**Δ =**	**z =**	**Δ** =	z =
1111	0.00773	1.96	**-0.01012**	**-2.272**	0.00238	1.08
0111	-0.00595	-1.508	**0.00595**	**1.336**	0	0
1011	-0.00714	-1.81	**0.00952**	**2.138**	-0.00238	-1.08
1101	-0.00833	-2.111	**0.00833**	**1.871**	0	0
1110	0.00297	0.754	**-0.00773**	**-1.737**	0.00476	2.159
0011	0.00773	1.96	**-0.00773**	**-1.737**	0	0
0101	-0.02083	-5.278	**0.0244**	**5.479*****	-0.00357	1.619
1001	0.00892	2.262	**-0.00892**	**-2.004**	0	0
0110	0.00476	1.206	**-0.00714**	**-1.604**	0.00238	1.08
1010	-0.00238	-0.603	**0.00714**	**1.604**	-0.00476	-2.159
1100	-0.00476	-1.206	**0.00595**	**1.336**	-0.00119	-0.539
0001	0.02738	6.937	**-0.03095**	**-6.949*****	0.00357	1.619
0010	-0.00297	-0.754	**0.00535**	**1.203**	-0.00238	-1.08
0100	-0.0494	-12.52	**0.06607**	**14.83*****	-0.01667	-7.558
1000	0.00535	1.357	**-0.00654**	**-1.47**	0.00119	0.539

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.2c: Experimental Results for Method 2 on GLO Data

Method 2 - MLAD, for simulated GLO data						
	GEV, overall p = 0.2280		GPA, overall p = 0.0176		**GLO, overall p = 0.7545**	
TEST	Δ =	z =	Δ =	z =	**Δ =**	**z =**
1111	0.01369	1.409	-0.00119	-0.380	**-0.0125**	**-1.272**
0111	0.00357	0.3676	0.00595	1.903	**-0.0095**	**-0.969**
1011	0.00119	0.1225	0.00238	0.761	**-0.0035**	**-0.363**
1101	-0.03631	-3.737	0.00892	2.855	**0.0273**	**2.786***
1110	-0.01071	-1.103	-0.00119	-0.380	**0.0119**	**1.211**
0011	-0.00416	-0.4289	-0.00595	-1.903	**0.0101**	**1.03**
0101	-0.00238	-0.2451	-0.02798	-8.944	**0.0303**	**3.089****
1001	0.01667	1.716	-0.00892	-2.855	**-0.0077**	**-0.787**
0110	0.00059	0.0612	0.00357	1.142	**-0.0041**	**-0.423**
1010	0.00892	0.919	0.00238	0.761	**-0.0113**	**-1.151**
1100	-0.02262	-2.328	0.01012	3.235	**0.0125**	**1.272**
0001	0.2458	25.3	0.02917	9.325	**-0.275**	**-27.98*****
0010	0.00595	0.6127	-0.00357	-1.142	**-0.0023**	**-0.242**
0100	-0.08155	-8.394	-0.03393	-10.85	**0.1155**	**11.75*****
1000	-0.00892	-0.919	-0.01012	-3.235	**0.0190**	**1.938**

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.3a: Experimental Results for Method 3 on GEV Data

Method 3 - MRNG, for simulated GEV data						
	GEV, overall p = 0.0993		GPA, overall p = 0.5531		GLO, overall p = 0.3476	
TEST	Δ =	z =	Δ =	z =	Δ =	z =
1111	-0.00327	-0.4507	0.00744	0.644	-0.00416	-0.374
0111	-0.00386	-0.5326	-0.02173	-1.882	0.0256	2.298
1011	0.00208	0.2868	0.01161	1.005	-0.01369	-1.229
1101	0.01042	1.434	0.02589	2.243	-0.03631	-3.259
1110	-0.01042	-1.434	0.00922	0.799	0.00119	0.1069
0011	-0.01042	-1.434	0.0253	2.191	-0.01488	-1.336
0101	-0.02113	-2.909*	0.0753	6.522	-0.05417	-4.862
1001	-0.00684	-0.9423	0.00148	0.128	0.00535	0.4809
0110	0.00565	0.7785	0.00267	0.232	-0.00833	-0.7481
1010	-0.00029	-0.0409	0.00029	0.025	0.0	0.0
1100	-0.02173	-2.991*	0.01696	1.469	0.00476	0.4275
0001	-0.03363	-4.63***	0.289	25.03	-0.2554	-22.92
0010	0.01339	1.844	-0.00386	-0.335	-0.00952	-0.8549
0100	0.01577	2.171	0.03185	2.758	-0.04762	-4.275
1000	-0.01042	-1.434	0.02232	1.933	-0.0119	-1.069

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.3b: Experimental Results for Method 3 on GPA Data

Method 3 - MRNG, for simulated GPA data						
	GEV, overall p =0.0604		GPA, overall p = 0.6052		GLO, overall p = 0.3344	
TEST	Δ =	z =	Δ =	z =	Δ =	z =
1111	0.00654	1.136	0.00863	0.757	-0.01518	-1.368
0111	0.0125	2.169	-0.02232	-1.958	0.00982	0.885
1011	-0.00595	-1.033	-0.01518	-1.332	0.02113	1.905
1101	0.01071	1.859	-0.01458	-1.28	0.00386	0.348
1110	-0.00059	-0.103	-0.00089	-0.078	0.00148	0.134
0011	-0.01071	-1.859	0.00625	0.548	0.00446	0.402
0101	0.0119	2.065	-0.1372	-12.04***	0.1253	11.29
1001	0.00297	0.516	0.03304	2.899*	-0.03601	-3.246
0110	0.00297	0.516	-0.01637	-1.436	0.01339	1.207
1010	-0.00714	-1.239	0.00863	0.757	-0.00148	-0.134
1100	-0.00119	-0.206	-0.01815	-1.593	0.01935	1.744
0001	0.02202	3.821	0.08423	7.39***	-0.1062	-9.578
0010	-0.00714	-1.239	0.00029	0.026	0.00684	0.617
0100	-0.04286	-7.436	0.2199	19.3***	-0.1771	-15.96
1000	0.02798	4.854	-0.0753	-6.607***	0.04732	4.266

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.3c: Experimental Results for Method 3 on GLO Data

Method 3 - MRNG, for simulated GLO data						
	GEV, overall p = 0.1863		GPA, overall p = 0.4967		**GLO, overall p = 0.3170**	
TEST	Δ =	z =	Δ =	z =	**Δ =**	**z =**
1111	-0.0053	-0.611	0.02202	1.988	**-0.01667**	**-1.513**
0111	-0.0113	-1.29	0.01012	0.913	**0.00119**	**0.108**
1011	-0.0065	-0.746	-0.01667	-1.504	**0.02321**	**2.108**
1101	-0.0101	-1.154	0.00952	0.859	**0.00059**	**0.054**
1110	-0.0047	-0.543	0.00178	0.161	**0.00297**	**0.270**
0011	-0.0017	-0.203	0.00119	0.107	**0.00059**	**0.054**
0101	-0.1315	-15.0	0.1643	14.83	**-0.03274**	**-2.972****
1001	-0.0005	-0.068	0.01369	1.236	**-0.0131**	**-1.189**
0110	0.0154	1.765	-0.00535	-0.483	**-0.01012**	**-0.918**
1010	0.0131	1.494	-0.00952	-0.859	**-0.00357**	**-0.324**
1100	-0.05	-5.703	0.06548	5.909	**-0.01548**	**-1.405**
0001	-0.1589	-18.13	0.3554	32.07	**-0.1964**	**-17.83*****
0010	0.0107	1.222	-0.00119	-0.107	**-0.00952**	**-0.864**
0100	0.2048	23.36	-0.1083	-9.777	**-0.09643**	**-8.755*****
1000	-0.0452	-5.16	0.06607	5.963	**-0.02083**	**-1.892**

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.4a: Experimental Results for Method 4 on GEV Data

Method 4 - MSSD, for simulated GEV data						
	GEV, overall p = 0.6421		GPA, overall p = 0.0656		GLO, overall p = 0.2923	
TEST	**Δ =**	**z =**	Δ =	z =	Δ =	z =
1111	**0.00327**	**0.305**	-0.00386	-0.668	0.00059	0.059
0111	**-0.02351**	**-2.191**	-0.00029	-0.051	0.02381	2.377
1011	**0.00089**	**0.083**	-0.00327	-0.565	0.00238	0.237
1101	**-0.00327**	**-0.305**	0.00625	1.079	-0.00297	-0.297
1110	**0.00446**	**0.416**	-0.00327	-0.565	-0.00119	-0.118
0011	**0.02411**	**2.246**	0.00148	0.256	-0.0256	-2.555
0101	**0.08542**	**7.958*** **	-0.04137	-7.143	-0.04405	-4.398
1001	**0.04315**	**4.021*** **	-0.00506	-0.873	-0.0381	-3.803
0110	**0.01577**	**1.47**	0.00386	0.668	-0.01964	-1.961
1010	**-0.02173**	**-2.024**	-0.00148	-0.256	0.02321	2.318
1100	**-0.02946**	**-2.745* **	0.03185	5.498	-0.00238	-0.237
0001	**0.3414**	**31.8*** **	0.04137	7.143	-0.3827	-38.21
0010	**-0.00327**	**-0.305**	-0.00029	-0.051	0.00357	0.356
0100	**0.1068**	**9.954*** **	-0.117	-20.19	0.01012	1.01
1000	**0.07173**	**6.683*** **	-0.04018	-6.937	-0.03155	-3.15

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.4b: Experimental Results for Method 4 on GPA Data

Method 4 - MSSD, for simulated GPA data			
	GEV, overall p = 0.0007	**GPA, overall p = 0.9991**	GLO, overall p = 0.0001
TEST	Probabilities very close to zero and one, test is not useful.		

Table B.4c: Experimental Results for Method 4 on GLO Data

Method 4 - MSSD, for simulated GLO data						
	GEV, overall p = 0.4262		GPA, overall p = 0.0048		**GLO, overall p = 0.5690**	
TEST	**Δ** =	z =	**Δ** =	z =	**Δ =**	**z =**
1111	0.00773	0.7968	0.0006	0.358	**-0.0083**	**-0.862**
0111	0.00238	0.2452	-0.0012	-0.717	**-0.0011**	**-0.123**
1011	0.00238	0.2452	-0.0006	-0.358	**-0.0017**	**-0.184**
1101	-0.00773	-0.7968	0.0065	3.946	**0.0011**	**0.123**
1110	-0.00714	-0.7355	0.00178	1.076	**0.0053**	**0.554**
0011	-0.00178	-0.1839	0.0012	0.717	**0.0005**	**0.061**
0101	0.06548	6.743	-0.0083	-5.022	**-0.0571**	**-5.917*****
1001	0.06429	6.62	-0.0065	-3.946	**-0.0577**	**-5.978*****
0110	-0.01012	-1.042	-0.0024	-1.435	**0.0125**	**1.294**
1010	0.01012	1.042	-0.0017	-1.076	**-0.0083**	**-0.862**
1100	-0.02619	-2.697	0.0077	4.664	**0.0184**	**1.911**
0001	0.5482	56.45	0.0083	5.022	**-0.5565**	**-57.63*****
0010	-0.00357	-0.3678	0.0024	1.435	**0.0011**	**0.123**
0100	-0.1768	-18.2	-0.0095	-5.74	**0.1863**	**19.29*****
1000	0.05655	5.823	-0.0077	-4.664	**-0.0488**	**-5.054*****

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.5a: Experimental Results for Method 5 on GEV Data

Method 5 - Geometric Reliability Index k_g, for simulated GEV data						
	GEV, overall p = 0.7936		GPA, overall p = 0.0789		GLO, overall p = 0.1275	
TEST	**Δ =**	**z =**	**Δ =**	z =	**Δ =**	z =
1111	**-0.0110**	**-1.314**	0.00178	0.284	0.00922	1.255
0111	**0.0056**	**0.674**	-0.00357	-0.568	-0.00208	-0.283
1011	**0.0080**	**0.959**	-0.00119	-0.189	-0.00684	-0.931
1101	**-0.0306**	**-3.658*****	0.02024	3.219	0.01042	1.417
1110	**-0.0014**	**-0.177**	0.00833	1.325	-0.00684	-0.931
0011	**-0.0014**	**-0.177**	0.00178	0.284	-0.00029	-0.040
0101	**-0.0711**	**-8.489**	-0.00654	-1.041	0.07768	10.57
1001	**0.0336**	**4.013*****	-0.02321	-3.692	-0.01042	-1.417
0110	**-0.0062**	**-0.746**	0.00059	0.094	0.00565	0.769
1010	**0.0032**	**0.390**	-0.00773	-1.231	0.00446	0.607
1100	**0.0681**	**8.133*****	-0.00773	-1.231	-0.06042	-8.219
0001	**0.0669**	**7.991*****	0.01071	1.704	-0.07768	-10.57
0010	**0.0056**	**0.675**	-0.00238	-0.378	-0.00327	-0.445
0100	**0.4039**	**48.2*****	-0.1536	-24.42	-0.2503	-34.05
1000	**-0.0604**	**-7.21*****	0.00476	0.757	0.05565	7.571

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.5b: Experimental Results for Method 5 on GPA Data

Method 5 - Geometric Reliability Index k_g, for simulated GPA data						
	GEV, overall p = 0.2061		**GPA, overall p = 0.7253**		GLO, overall p = 0.0686	
TEST	Δ =	z =	**Δ =**	**z =**	Δ =	z =
1111	0.00863	0.925	**-0.00654**	**-0.653**	-0.00208	-0.352
0111	0.00684	0.733	**-0.01548**	**-1.544**	0.00863	1.462
1011	-0.00922	-0.988	**0.00476**	**0.475**	0.00446	0.756
1101	-0.00089	-0.095	**0.01964**	**1.96**	-0.01875	-3.176
1110	0.00803	0.861	**-0.01131**	**-1.129**	0.00327	0.554
0011	0.00565	0.606	**-0.00416**	**-0.415**	-0.00148	-0.252
0101	-0.03006	-3.222	**0.1238**	**12.36*** **	-0.09375	-15.88
1001	0.01935	2.074	**-0.05**	**-4.99*** **	0.03065	5.192
0110	-0.0128	-1.372	**0.0119**	**1.188**	0.00089	0.151
1010	-0.00267	-0.287	**0.00833**	**0.831**	-0.00565	-0.957
1100	-0.01458	-1.563	**0.03036**	**3.029** **	-0.01577	-2.672
0001	-0.1527	-16.36	**0.1042**	**10.4*** **	0.04851	8.217
0010	-0.00208	-0.223	**0.01012**	**1.01**	-0.00803	-1.361
0100	-0.2104	-22.55	**0.3024**	**30.18*** **	-0.09196	-15.58
1000	-0.00148	-0.159	**-0.00238**	**-0.237**	0.00386	0.655

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.5c: Experimental Results for Method 5 on GLO Data

Method 5 - Geometric Reliability Index k_g, for simulated GLO data						
	GEV, overall p = 0.4982		GPA, overall p = 0.0365		**GLO, overall p = 0.4653**	
TEST	**Δ** =	z =	**Δ** =	z =	**Δ =**	**z =**
1111	0.01012	1.027	-0.00268	-0.598	**-0.00744**	**-0.770**
0111	-0.01488	-1.51	0.00327	0.731	**0.01161**	**1.202**
1011	0.00059	0.060	0.00268	0.598	**-0.00327**	**-0.339**
1101	0.03929	3.987	0.00684	1.53	**-0.04613**	**-4.778*****
1110	-0.00535	-0.544	0.00327	0.731	**0.00208**	**0.215**
0011	0.00297	0.302	-0.00327	-0.731	**0.00029**	**0.030**
0101	0.2083	21.15	-0.01577	-3.525	**-0.1926**	**-19.95*****
1001	0.03214	3.262	-0.00684	-1.53	**-0.0253**	**-2.62***
0110	-0.01488	-1.51	0.00208	0.465	**0.0128**	**1.326**
1010	0.01131	1.148	-0.00327	-0.731	**-0.00803**	**-0.832**
1100	0.03452	3.504	-0.00268	-0.598	**-0.03185**	**-3.299****
0001	0.5452	55.34	0.01577	3.525	**-0.561**	**-58.11*****
0010	0.00535	0.545	-0.00208	-0.465	**-0.00327**	**-0.339**
0100	-0.03571	-3.625	-0.07292	-16.3	**0.1086**	**11.25*****
1000	-0.03452	-3.504	0.00268	0.598	**0.03185**	**3.299****

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.6a: Experimental Results for Method 6 on GEV Data

Method 6 - Statistical Reliability Index k_s, for simulated GEV data						
	GEV, overall p = 0.8170		GPA, overall p = 0.1204		GLO, overall p = 0.0626	
TEST	**Δ =**	**z =**	Δ =	z =	Δ =	z =
1111	**-0.00952**	**-1.114**	0.01101	1.479	-0.00148	-0.260
0111	**0.01369**	**1.601**	-0.01101	-1.479	-0.00267	-0.469
1011	**0.01429**	**1.671**	-0.01815	-2.438	0.00386	0.677
1101	**0.00833**	**0.974**	-0.01637	-2.198	0.00803	1.407
1110	**-0.01667**	**-1.949**	0.00625	0.839	0.01042	1.824
0011	**-0.0125**	**-1.462**	0.0122	1.638	0.00029	0.052
0101	**0.04345**	**5.082*** **	-0.07054	-9.47	0.02708	4.743
1001	**-0.00238**	**-0.278**	0.01042	1.399	-0.00803	-1.407
0110	**-0.00059**	**-0.069**	-0.00268	-0.359	0.00327	0.573
1010	**0.0131**	**1.532**	-0.00029	-0.039	-0.0128	-2.241
1100	**-0.03452**	**-4.038*** **	0.02768	3.716	0.00684	1.199
0001	**-0.01726**	**-2.019**	0.04911	6.593	-0.03185	-5.577
0010	**-0.00178**	**-0.208**	0.00268	0.359	-0.00089	-0.156
0100	**0.3173**	**37.11*** **	-0.2015	-27.05	-0.1158	-20.27
1000	**0.04167**	**4.874*** **	-0.03006	-4.036	-0.01161	-2.033

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.6b: Experimental Results for Method 6 on GPA Data

Method 6 - Statistical Reliability Index k_e, for simulated GPA data						
	GEV, overall p = 0.1650		**GPA, overall p = 0.7592**		GLO, overall p = 0.0757	
TEST	Δ =	z =	**Δ =**	**z =**	Δ =	z =
1111	0.01518	1.824	**-0.02202**	**-2.384**	0.00684	1.09
0111	-0.00267	-0.321	**0.00178**	**0.193**	0.00089	0.142
1011	-0.00565	-0.679	**0.00714**	**0.773**	-0.00148	-0.237
1101	-0.02113	-2.539	**0.03333**	**3.608*****	-0.0122	-1.943
1110	-0.0128	-1.538	**0.01726**	**1.868**	-0.00446	-0.711
0011	0.00029	0.035	**0.00357**	**0.386**	-0.00386	-0.616
0101	0.07054	8.475	**-0.05595**	**-6.056*****	-0.01458	-2.322
1001	0.0247	2.968	**-0.02917**	**-3.157****	0.00446	0.711
0110	-0.00684	-0.822	**0.01131**	**1.224**	-0.00446	-0.711
1010	-0.00267	-0.321	**0.0**	**0.0**	0.00267	0.426
1100	0.00208	0.250	**0.00714**	**0.773**	-0.00922	-1.469
0001	-0.1574	-18.92	**0.1637**	**17.72*****	-0.00625	-0.995
0010	0.01518	1.824	**-0.01905**	**-2.061**	0.00386	0.616
0100	-0.2324	-27.93	**0.3524**	**38.14*****	-0.1199	-19.1
1000	-0.01161	-1.395	**0.00535**	**0.579**	0.00625	0.995

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.6c: Experimental Results for Method 6 on GLO Data

Method 6 - Statistical Reliability Index k_s, for simulated GLO data						
	GEV, overall p = 0.5560		GPA, overall p = 0.0845		**GLO, overall p = 0.3595**	
TEST	Δ =	z =	Δ =	z =	**Δ =**	**z =**
1111	0.00178	0.175	0.00357	0.562	**-0.00535**	**-0.586**
0111	-0.00357	-0.351	-0.00059	-0.093	**0.00416**	**0.456**
1011	0.00238	0.234	-0.00178	-0.281	**-0.00059**	**-0.065**
1101	0.05179	5.087	0.00833	1.312	**-0.06012**	**-6.582*****
1110	0.00357	0.351	0.00357	0.562	**-0.00714**	**-0.782**
0011	-0.00773	-0.760	0.0	0.0	**0.00773**	**0.847**
0101	0.3357	32.98	-0.08988	-14.16	**-0.2458**	**-26.91*****
1001	0.01786	1.754	-0.00535	-0.843	**-0.0125**	**-1.369**
0110	-0.01607	-1.579	0.00178	0.281	**0.01429**	**1.564**
1010	0.00178	0.175	-0.00416	-0.656	**0.00238**	**0.260**
1100	-0.06429	-6.314	0.01786	2.812	**0.04643**	**5.083*****
0001	0.3899	38.3	0.07619	12.	**-0.4661**	**-51.03*****
0010	0.00595	0.587	-0.00238	-0.375	**-0.00357**	**-0.391**
0100	-0.1185	-11.63	-0.1542	-24.28	**0.2726**	**29.85*****
1000	0.06845	6.724	-0.02202	-3.469	**-0.04643**	**-5.083*****

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.7a: Experimental Results for Method 7 on GEV Data

Method 7 - Vertical Distance to Station Average for simulated GEV data						
	GEV, overall p = 0.8385		GPA, overall p = 0.0632		GLO, overall p = 0.0982	
TEST	Δ =	z =	Δ =	z =	Δ =	z =
1111	0.00744	0.895	-0.00029	-0.054	-0.00714	-1.006
0111	-0.00922	-1.11	0.00684	1.244	0.00238	0.335
1011	-0.01161	-1.396	-0.00089	-0.162	0.0125	1.761
1101	0.00565	0.680	0.00506	0.919	-0.01071	-1.509
1110	0.00863	1.038	-0.00208	-0.378	-0.00654	-0.922
0011	0.01815	2.184	-0.00684	-1.244	-0.01131	-1.593
0101	0.05089	6.122***	-0.08899	-16.18	0.0381	5.366
1001	0.04494	5.406***	-0.02649	-4.815	-0.01845	-2.599
0110	-0.00327	-0.393	0.00863	1.569	-0.00535	-0.754
1010	0.00029	0.035	0.00089	0.162	-0.00119	-0.167
1100	-0.02887	-3.473***	0.01042	1.893	0.01845	2.599
0001	-0.1896	-22.81***	0.1211	22.02	0.06845	9.642
0010	0.00863	1.038	-0.00863	-1.569	0.0	0.0
0100	0.164	19.73***	-0.09435	-17.15	-0.06964	-9.81
1000	0.09018	10.85***	-0.03185	-5.789	-0.05833	-8.217

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.

** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.

*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.7b: Experimental Results for Method 7 on GPA Data

Method 7 - Vertical Distance to Station Average for simulated GPA data						
	GEV, overall p = 0.0132		GPA, overall p = 0.9868		GLO, overall p = 0	
TEST	Δ =	z =	Δ =	z =	Δ =	z =
1111	0.00208	0.760	-0.00208	-0.760	p = 0, no test conducted	
0111	-0.00148	-0.543	0.00148	0.543		
1011	-0.00148	-0.543	0.00148	0.543		
1101	0.00803	2.933	-0.00803	-2.933*		
1110	0.00148	0.543	-0.00148	-0.543		
0011	0.00089	0.325	-0.00089	-0.325		
0101	-0.01696	-6.192	0.01696	6.192***		
1001	-0.00982	-3.585	0.00982	3.585***		
0110	0.00148	0.543	-0.00148	-0.543		
1010	-0.00089	-0.325	0.00089	0.325		
1100	0.01339	4.889	-0.01339	-4.889***		
0001	0.01875	6.844	-0.01875	-6.844***		
0010	-0.00208	-0.760	0.00208	0.760		
0100	-0.0247	-9.017	0.0247	9.017***		
1000	-0.01518	-5.541	0.01518	5.541***		

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.7c: Experimental Results for Method 7 on GLO Data

Method 7 - Vertical Distance to Station Average for simulated GLO data						
	GEV, overall p = 0.3082		GPA, overall p = 0.0220		**GLO, overall p = 0.6698**	
TEST	Δ =	z =	Δ =	z =	**Δ =**	**z =**
1111	-0.01161	-1.186	0.00297	0.874	**0.00863**	**0.889**
0111	0.00982	1.003	-0.00535	-1.574	**-0.00446**	**-0.459**
1011	0.00684	0.699	-0.00238	-0.699	**-0.00446**	**-0.459**
1101	-0.05506	-5.624	0.0244	7.169	**0.03065**	**3.158****
1110	-0.00863	-0.881	0.00297	0.874	**0.00565**	**0.582**
0011	-0.01458	-1.49	0.00476	1.399	**0.00982**	**1.012**
0101	0.02708	2.767	-0.04345	-12.77	**0.01637**	**1.686**
1001	0.07173	7.327	-0.025	-7.344	**-0.04673**	**-4.814*****
0110	0.00089	0.091	-0.00535	-1.574	**0.00446**	**0.459**
1010	0.00148	0.152	-0.00238	-0.699	**0.00089**	**0.091**
1100	-0.04613	-4.712	0.0244	7.169	**0.02173**	**2.238**
0001	0.4301	43.93	0.04405	12.94	**-0.4741**	**-48.84*****
0010	-0.00327	-0.334	0.00476	1.399	**-0.00148**	**-0.153**
0100	-0.1068	-10.91	-0.04345	-12.77	**0.1503**	**15.48*****
1000	0.02946	3.01	-0.025	-7.344	**-0.00446**	**-0.459**

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.8a: Experimental Results for Method 8 on GEV Data

Method 8 - Shortest Distance to station Average for simulated GEV data						
	GEV, overall p = 0.9006		GPA, overall p = 0.0068		GLO, overall p = 0.0926	
TEST	**Δ =**	**z =**	Δ =	z =	Δ =	z =
1111	**0.00178**	**0.254**	0.00119	0.6	-0.00297	-0.434
0111	**0.00238**	**0.339**	-0.00059	-0.3	-0.00178	-0.260
1011	**-0.02083**	**-2.972****	-0.00119	-0.6	0.02202	3.217
1101	**0.00178**	**0.254**	0.00714	3.6	-0.00892	-1.304
1110	**-0.01131**	**-1.613**	-0.00119	-0.6	0.0125	1.826
0011	**0.00595**	**0.849**	0.00059	0.3	-0.00654	-0.956
0101	**0.01905**	**2.717***	-0.01012	-5.1	-0.00892	-1.304
1001	**0.01845**	**2.632***	-0.00714	-3.6	-0.01131	-1.652
0110	**0.00595**	**0.849**	0.00178	0.9	-0.00773	-1.13
1010	**-0.00059**	**-0.084**	0.00119	0.6	-0.00059	-0.086
1100	**-0.03512**	**-5.009*****	0.01071	5.4	0.0244	3.564
0001	**-0.08095**	**-11.55*****	0.01012	5.1	0.07083	10.35
0010	**0.00238**	**0.339**	-0.00178	-0.9	-0.00059	-0.086
0100	**0.1202**	**17.15*****	-0.01369	-6.9	-0.1065	-15.56
1000	**0.06488**	**9.255*****	-0.01071	-5.4	-0.05417	-7.911

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.8b: Experimental Results for Method 8 on GPA Data

Method 8 - Shortest Distance to station Average for simulated GPA data			
	GEV, overall p = 0.0052	**GPA, overall p = 0.9948**	GLO, overall p = 0
TEST	Probabilities very close to zero and one, test is not useful.		

Table B.8c: Experimental Results for Method 8 on GLO Data

Method 8 - Shortest Distance to station Average for simulated GLO data						
	GEV, overall p = 0.1774		GPA, overall p = 0.003		**GLO, overall p = 0.8223**	
TEST	Δ =	z =	Δ =	z =	**Δ =**	**z =**
1111	0.00952	1.084	0	0	**-0.00952**	**-1.084**
0111	0.01607	1.829	0	0	**-0.01607**	**-1.828**
1011	-0.00833	-0.948	0	0	**0.00833**	**0.948**
1101	-0.01131	-1.287	0.00059	1.416	**0.01071**	**1.219**
1110	0.00476	0.542	0	0	**-0.00476**	**-0.541**
0011	-0.01369	-1.558	0	0	**0.01369**	**1.558**
0101	-0.02143	-2.439	-0.000595	-1.416	**0.02202**	**2.506**
1001	-0.00892	-1.016	-0.000595	-1.416	**0.00952**	**1.084**
0110	0.00892	1.016	0	0	**-0.00892**	**-1.016**
1010	-0.00714	-0.813	0	0	**0.00714**	**0.812**
1100	-0.00297	-0.338	0.00059	1.416	**0.00238**	**0.270**
0001	0.2119	24.11	0.00059	1.416	**-0.2125**	**-24.18*****
0010	-0.00059	-0.067	0	0	**0.00059**	**0.067**
0100	-0.1274	-14.5	-0.00059	-1.416	**0.128**	**14.56*****
1000	-0.04702	-5.351	-0.00059	-1.416	**0.04762**	**5.418*****

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.9a: Experimental Results for Method 9 on GEV Data

Method 9 - Vertical Distance to L-moment solution, simulated GEV data						
	GEV, overall p = 0.8213		GPA, overall p = 0.0323		GLO, overall p = 0.1464	
TEST	Δ =	z =	Δ =	z =	Δ =	z =
1111	-0.00089	-0.1063	-0.00089	-0.2161	0.00178	0.2211
0111	0.00267	0.3189	0.00327	0.7925	-0.00595	-0.7368
1011	-0.01994	-2.374	0.00089	0.2161	0.01905	2.358
1101	0.01577	1.878	0.01339	3.242	-0.02917	-3.61
1110	-0.00625	-0.744	0.00029	0.0721	0.00595	0.7368
0011	0.00625	0.744	-0.00565	-1.369	-0.00059	-0.0736
0101	0.06935	8.255***	-0.04435	-10.74	-0.025	-3.095
1001	0.03958	4.712***	-0.02887	-6.989	-0.01071	-1.326
0110	0.00922	1.098	0.00208	0.5043	-0.01131	-1.4
1010	-0.00506	-0.6023	-0.00029	-0.0721	0.00535	0.6632
1100	-0.0253	-3.012**	0.01577	3.819	0.00952	1.179
0001	-0.2818	-33.55**	0.0622	15.06	0.2196	27.19
0010	-0.00506	-0.6023	-0.00446	-1.081	0.00952	1.179
0100	0.1354	16.12***	-0.04673	-11.31	-0.08869	-10.98
1000	0.08542	10.17***	-0.03125	-7.565	-0.05417	-6.705

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.9b: Experimental Results for Method 9 on GPA Data

Method 9 - Vertical Distance to L-moment solution, simulated GPA data						
	GEV, overall p = 0.0088		GPA, overall p = 0.9912		GLO, overall p = 0.0	
TEST	Δ =	z =	Δ =	z =	Δ =	z =
1111	-0.00149	-0.6647	0.00148	0.6647	p = 0, no test conducted.	
0111	0.00268	1.196	-0.00267	-1.196		
1011	0.00030	0.1329	-0.00029	-0.1329		
1101	0.00805	3.589	-0.00803	-3.589***		
1110	-0.00268	-1.196	0.00267	1.196		
0011	-0.00149	-0.6647	0.00148	0.6647		
0101	-0.01399	-6.248	0.01399	6.248***		
1001	-0.01042	-4.653	0.01042	4.653***		
0110	0.00387	1.728	-0.00386	-1.728		
1010	0.00149	0.6647	-0.00148	-0.6647		
1100	0.00923	4.121	-0.00922	-4.121***		
0001	0.01637	7.312	-0.01637	-7.312***		
0010	-0.00268	-1.196	0.00267	1.196		
0100	-0.01518	-6.78	0.01518	6.78***		
1000	-0.01161	-5.185	0.01161	5.185***		

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.

** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.

*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.9c: Experimental Results for Method 9 on GLO Data

Method 9 - Vertical Distance to L-moment solution, simulated GLO data						
	GEV, overall p = 0.1908		GPA, overall p = 0.0107		GLO, overall p = 0.7985	
TEST	Δ =	z =	Δ =	z =	Δ =	z =
1111	-0.00535	-0.6007	0.00178	0.7304	0.00357	0.3966
0111	0.00476	0.534	-0.00357	-1.461	-0.00119	-0.1322
1011	-0.00714	-0.801	-0.00119	-0.4869	0.00833	0.9255
1101	-0.02917	-3.271	0.01429	5.843	0.01488	1.653
1110	0.00297	0.3337	0.00178	0.7304	-0.00476	-0.5288
0011	0.00416	0.4672	0.00297	1.217	-0.00714	-0.7933
0101	0.01548	1.735	-0.02083	-8.521	0.00535	0.5949
1001	0	0.0	-0.01488	-6.087	0.01488	1.653
0110	0.00119	0.1335	-0.00357	-1.461	0.00238	0.2644
1010	-0.0131	-1.468	-0.00119	-0.4869	0.01429	1.587
1100	-0.01964	-2.203	0.01429	5.843	0.00535	0.5949
0001	0.2768	31.04	0.02143	8.765	-0.2982	-33.12***
0010	0.00535	0.6007	0.00297	1.217	-0.00833	-0.9255
0100	-0.05833	-6.541	-0.02083	-8.521	0.07917	8.792***
1000	-0.03571	-4.005	-0.01488	-6.087	0.0506	5.619***

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.10a: Experimental Results for Method 10 on GEV Data

Method 10 - Shortest Distance to L-moment solution, simulated GEV data						
	GEV, overall p = 0.9088		GPA, overall p = 0.0027		GLO, overall p = 0.0885	
TEST	**Δ =**	**z =**	Δ =	z =	Δ =	z =
1111	**-0.00803**	**-1.194**	0.00119	0.9504	0.00684	1.027
0111	**0.00149**	**0.2211**	-0.00059	-0.4752	-0.00089	-0.134
1011	**-0.01815**	**-2.697***	-0.00119	-0.9504	0.01935	2.903
1101	**0.00804**	**1.194**	0.00357	2.851	-0.01161	-1.742
1110	**-0.00625**	**-0.9286**	0.00119	0.9504	0.00506	0.7592
0011	**0.00565**	**0.8402**	0.00059	0.4752	-0.00625	-0.9379
0101	**0.03065**	**4.555*****	-0.00416	-3.326	-0.02649	-3.975
1001	**0.01458**	**2.167**	-0.00357	-2.851	-0.01101	-1.652
0110	**0.00446**	**0.6633**	-0.00059	-0.4752	-0.00386	-0.5806
1010	**-0.01161**	**-1.725**	-0.00119	-0.9504	0.0128	1.92
1100	**-0.02708**	**-4.024*****	0.00476	3.802	0.02232	3.35
0001	**-0.1223**	**-18.17*****	0.00416	3.326	0.1182	17.73
0010	**-0.00089**	**-0.1327**	0.00059	0.4752	0.00029	0.0446
0100	**0.08244**	**12.25*****	-0.00535	-4.277	-0.07708	-11.57
1000	**0.05327**	**7.915*****	-0.00476	-3.802	-0.04851	-7.28

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.10b: Experimental Results for Method 10 on GPA Data

Method 10 - Shortest Distance to L-moment solution, simulated GPA data			
	GEV, overall p = 0.0016	**GPA, overall p = 0.9984**	GLO, overall p = 0
TEST	Probabilities very close to zero and one, test is not useful.		

Table B.10c: Experimental Results for Method 10 on GLO Data

Method 10 - Shortest Distance to L-moment solution, simulated GLO data						
	GEV, overall p = 0.1330		GPA, overall p = 0		**GLO, overall p = 0.8670**	
TEST	Δ =	z =	Δ =	z =	**Δ =**	**TEST**
1111	0.00059	0.0761	p = 0, no test conducted.		**-0.00059**	**-0.0760**
0111	0.01071	1.369			**-0.01071**	**-1.369**
1011	-0.00178	-0.2282			**0.00178**	**0.2282**
1101	-0.01071	-1.369			**0.01071**	**1.369**
1110	-0.00416	-0.5325			**0.00416**	**0.5325**
0011	0.00357	0.4564			**-0.00357**	**-0.4564**
0101	-0.04464	-5.705			**0.04464**	**5.705*** **
1001	-0.02143	-2.738			**0.02143**	**2.738* **
0110	0.0131	1.673			**-0.0131**	**-1.673**
1010	-0.00416	-0.5325			**0.00416**	**0.5325**
1100	0.00357	0.4564			**-0.00357**	**-0.4564**
0001	0.1804	23.05			**-0.1804**	**-23.05*** **
0010	0.00833	1.065			**-0.00833**	**-1.065**
0100	-0.1065	-13.62			**0.1065**	**13.62*** **
1000	-0.05476	-6.998			**0.05476**	**6.998*** **

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$.
** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$.
*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$.

Table B.11: Key for reading tests

Method # - Method Name, for Simulated GEV Data						
	GEV, overall p = % Correct Selection, This column is Highlighted		GPA, overall p = % Wrong selection 1, p's Sum to 1.0		GLO, overall p = % Wrong selection 2	
TEST	**Δ = %Change**	**z = Test Statistic**	Δ =	z =	Δ =	z =
1111	**4 way interaction: NS, NY, L-cv, L-skewness**					
0111	**3 way interaction: NY, L-cv, L-skewness**					
1011	**3 way interaction: NS, L-cv, L-skewness**					
1101	**3 way interaction: NS, NY, L-skewness**					
1110	**3 way interaction: NS, NY, L-cv**					
0011	**2 way interaction: L-cv, L-skewness**					
0101	**2 way interaction: NY, L-skewness**					
1001	**2 way interaction: NS, L-skewness**					
0110	**2 way interaction: NY, L-cv**					
1010	**2 way interaction: NS, L-cv**					
1100	**2 way interaction: NS, NY**					
0001	**Main effect of L-skewness**					
0010	**Main effect of L-cv**					
0100	**Main effect of At-Site Record Length (NY)**					
1000	**Main effect of Number of Sites (NS)**					

* Significant at α = .15 with Bonferroni adjustment, $|z| > 2.58$

** Significant at α = .05 with Bonferroni adjustment, $|z| > 2.94$

*** Significant at α = .01 with Bonferroni adjustment, $|z| > 3.4$

These are two-tailed tests, critical value = $Z_{\alpha/30}$.

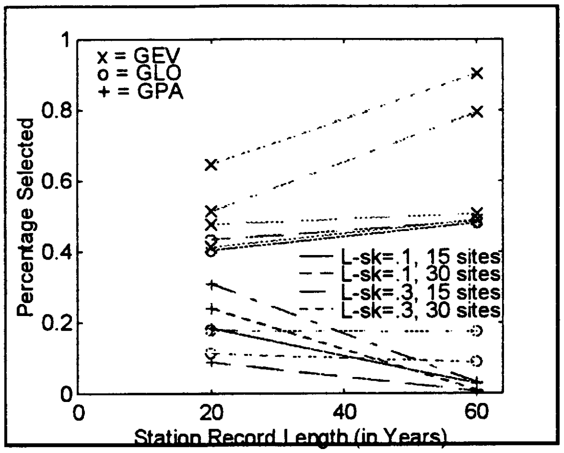

Figure C.1a: Method 1 - Selection by MSAD (LP) for GEV Data

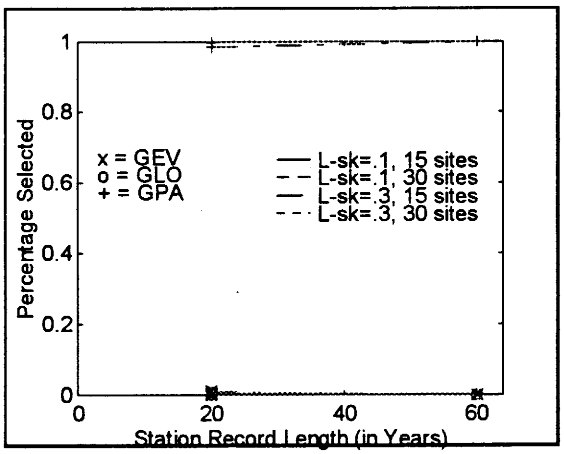

Figure C.1b: Method 1 - Selection by MSAD (LP) for GPA Data

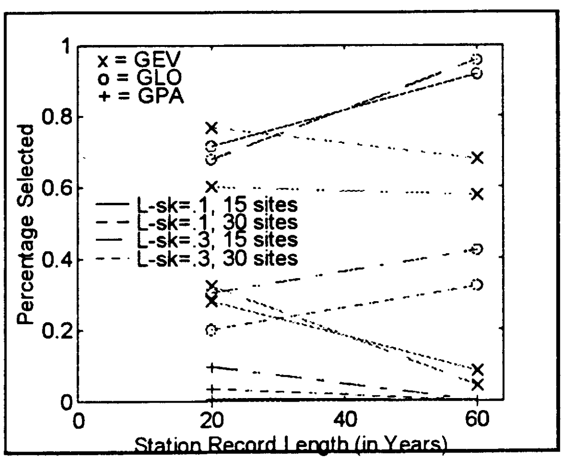

Figure C.1c: Method 1 - Selection by MSAD (LP) for GLO Data

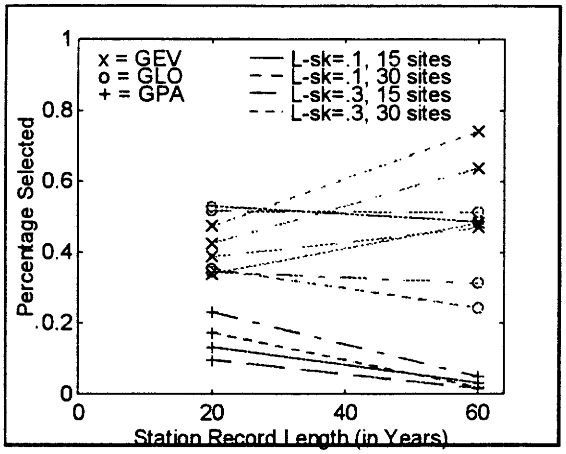

Figure C.2a: Method 2 - Selection by MLAD (LP) for GEV Data

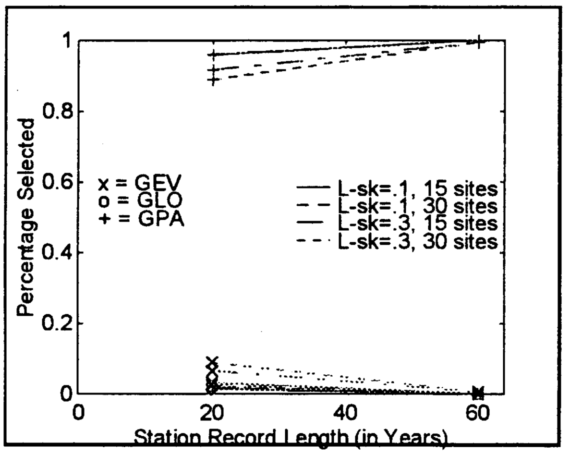

Figure C.2b: Method 2 - Selection by MLAD (LP) for GPA Data

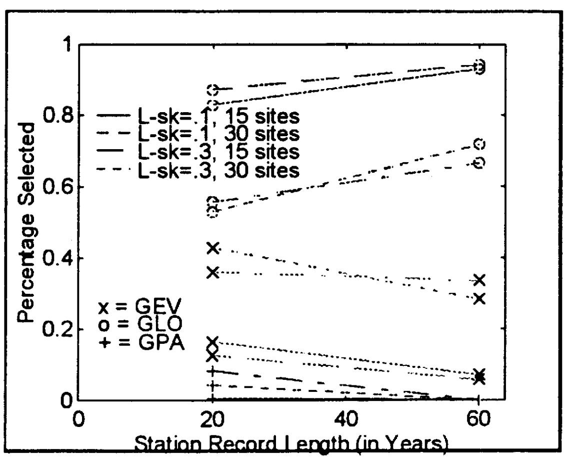

Figure C.2c: Method 2 - Selection by MLAD (LP) for GLO Data

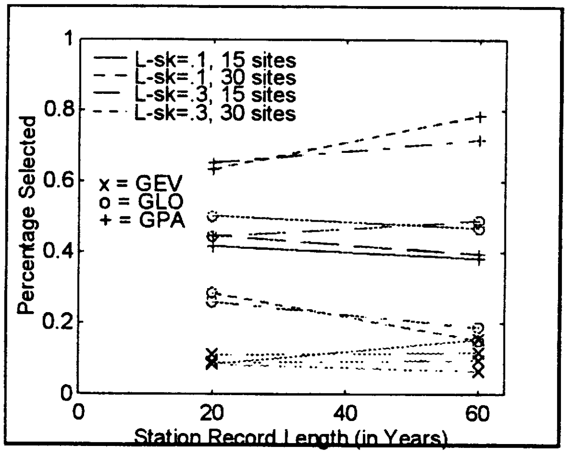

Figure C.3a - Selection by MRNG (LP) for GEV Data

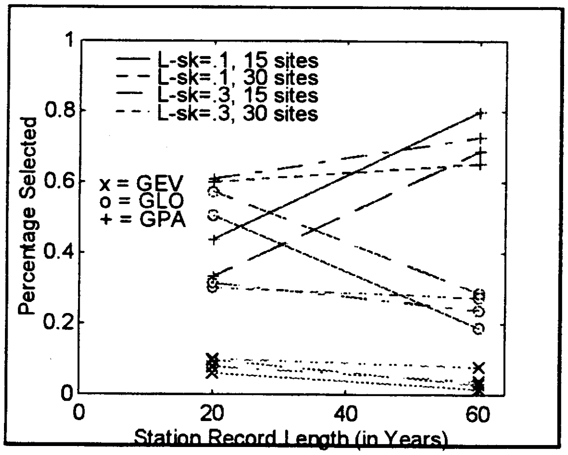

Figure C.3b - Selection by MRNG (LP) for GPA Data

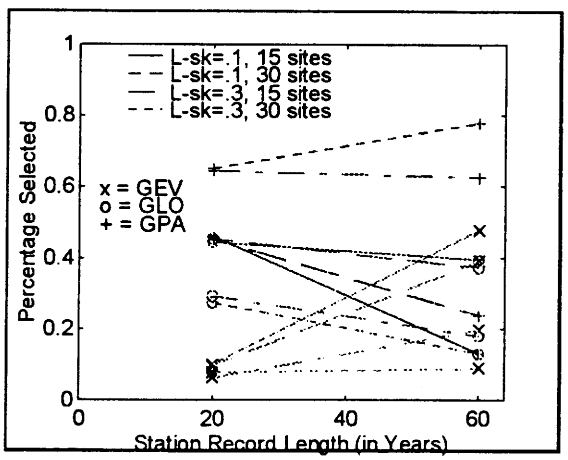

Figure C.3c - Selection by MRNG (LP) for GLO Data

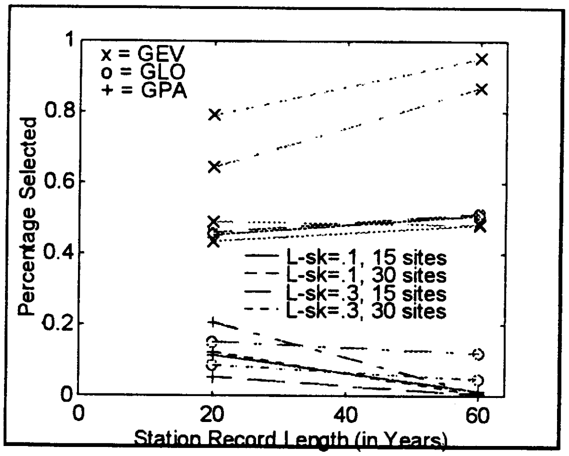

Figure C.4a: Method 4 - Selection by MSSD (QP) for GEV Data

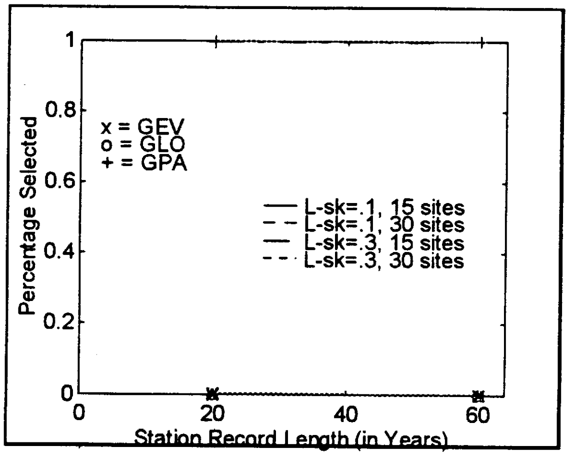

Figure C.4b: Method 4 - Selection by MSSD (QP) for GPA Data

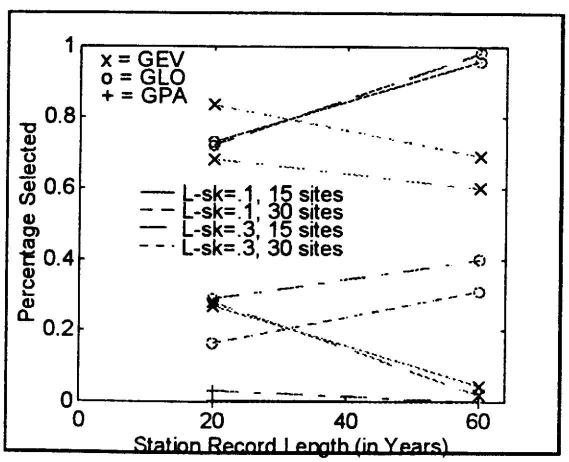

Figure C.4c: Method 4 - Selection by MSSD (QP) for GLO Data

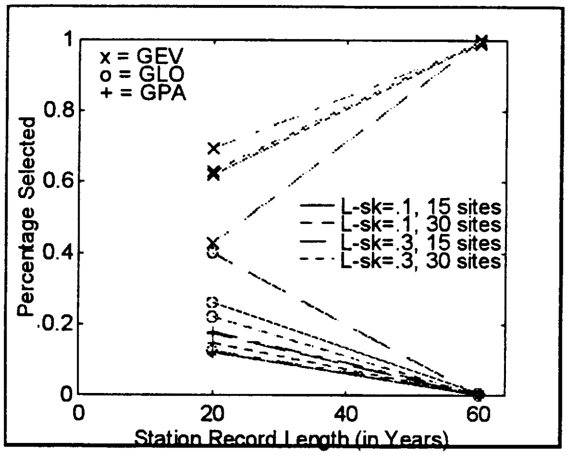

Figure C.5a - Selection by Geometric Reliability Index k_g for GEV Data

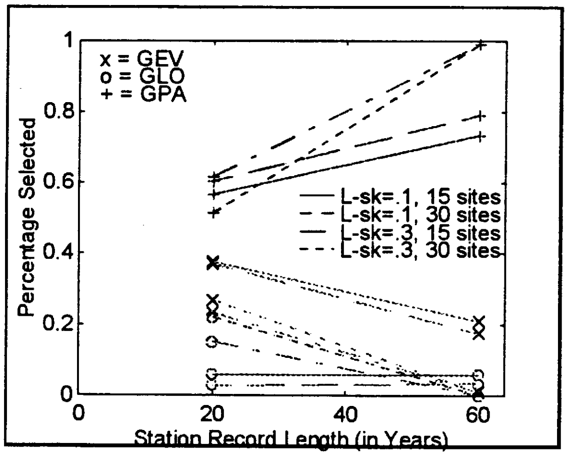

Figure C.5b - Selection by Geometric Reliability Index k_g for GPA Data

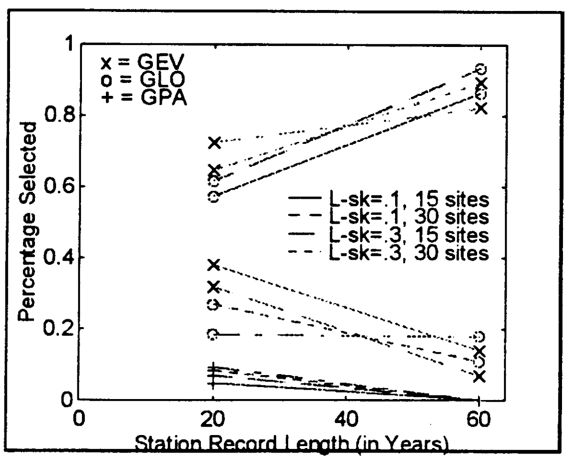

Figure C.5c - Selection by Geometric Reliability Index k_g for GLO Data

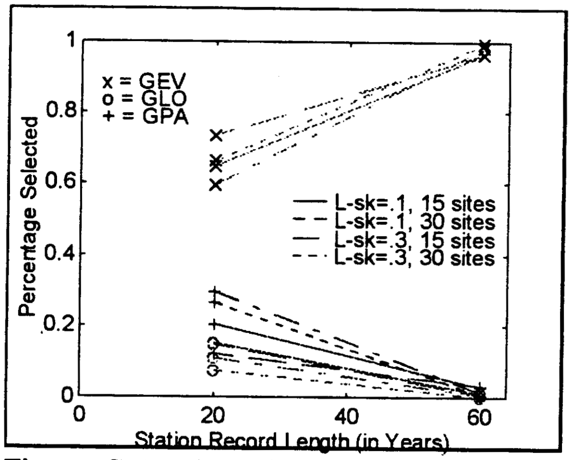

Figure C.6a - Selection by Statistical Reliability Index k_s for GEV Data

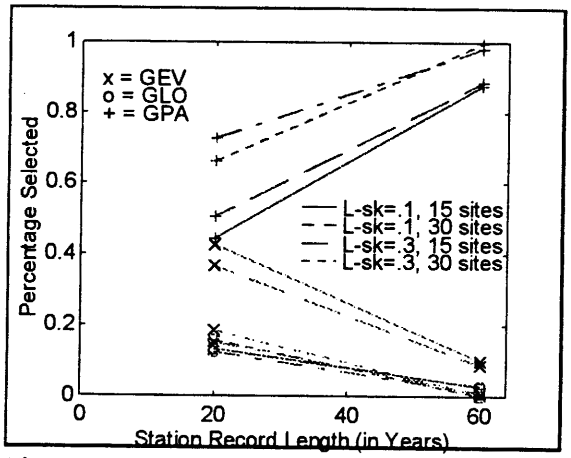

Figure C.6b - Selection by Statistical Reliability Index k_s for GPA Data

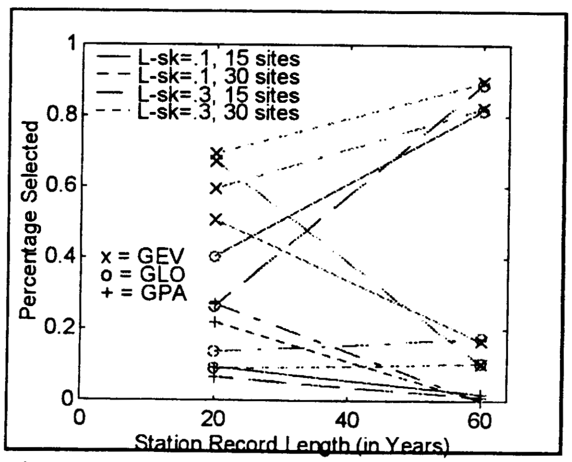

Figure C.6c - Selection by Statistical Reliability Index k_s for GLO Data

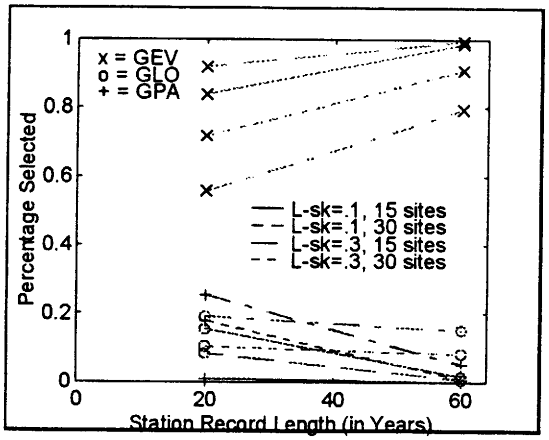

Figure C.7a: Method 7 - Selection by VDA for GEV Data

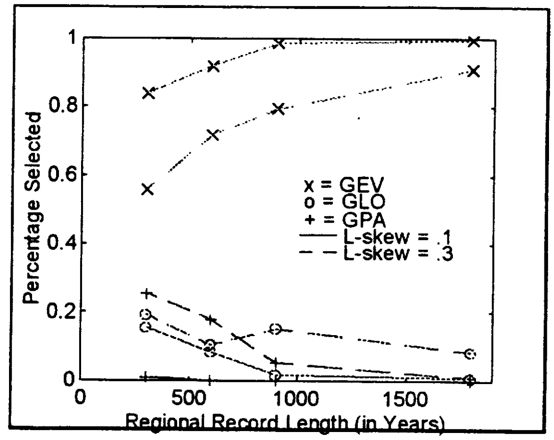

Figure C.7d: Method 7 - Selection by VDA for GEV Data

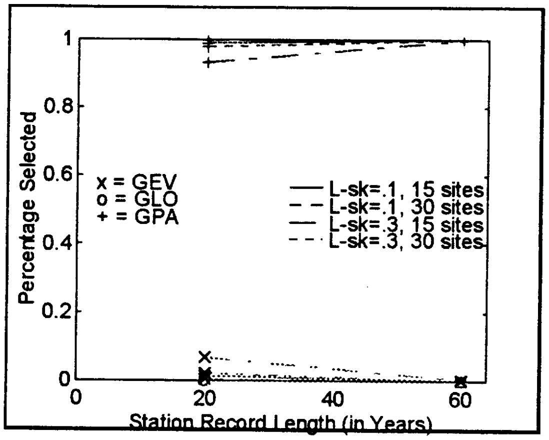

Figure C.7b: Method 7 - Selection by VDA for GPA Data

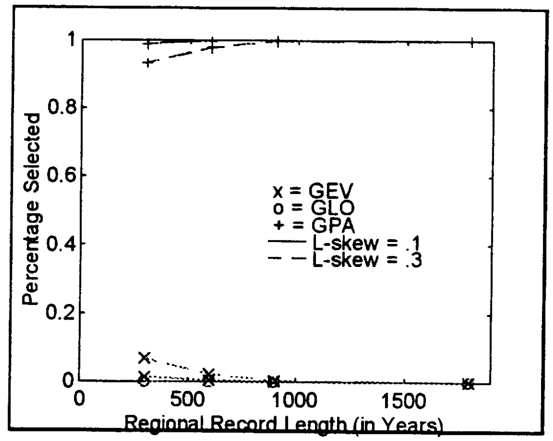

Figure C.7e: Method 7 - Selection by VDA for GPA Data

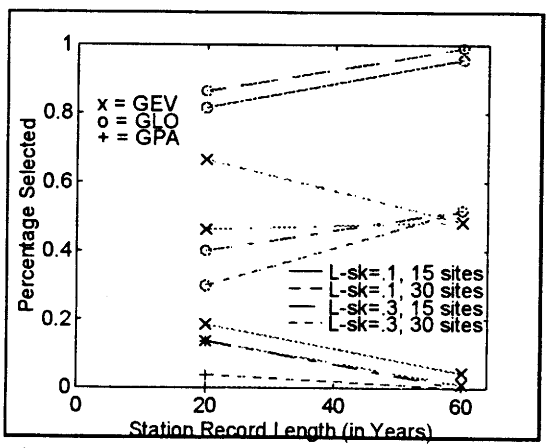

Figure C.7c: Method 7 - Selection by VDA for GLO Data

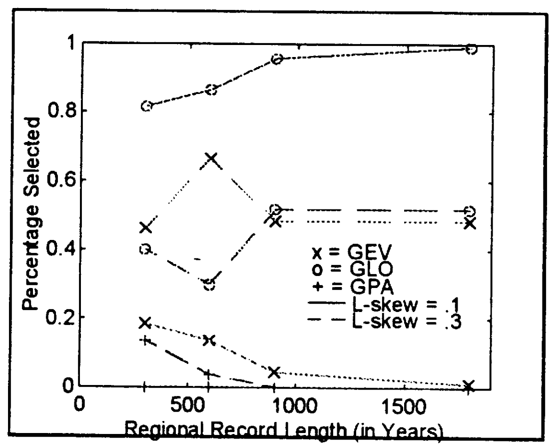

Figure C.7f: Method 7 - Selection by VDA for GLO Data

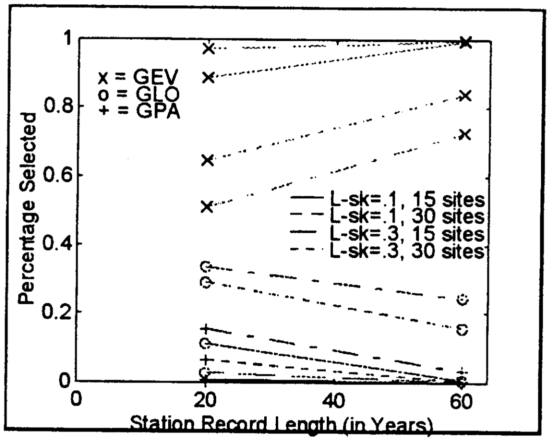

Figure C.8a: Method 8 - Selection by VDR for GEV Data

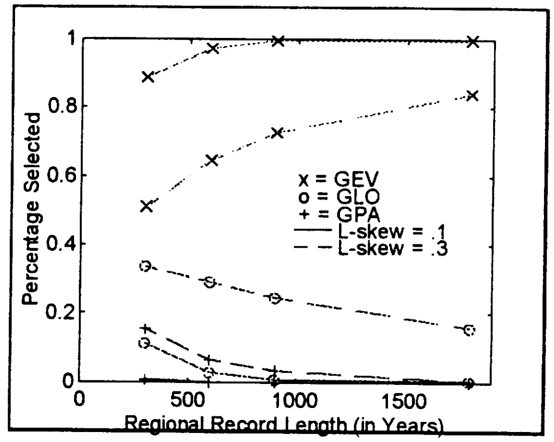

Figure C.8d: Method 8 - Selection by VDR for GEV Data

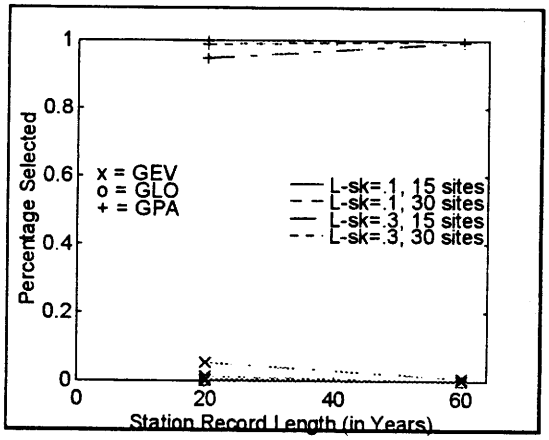

Figure C.8b: Method 8 - Selection by VDR for GPA Data

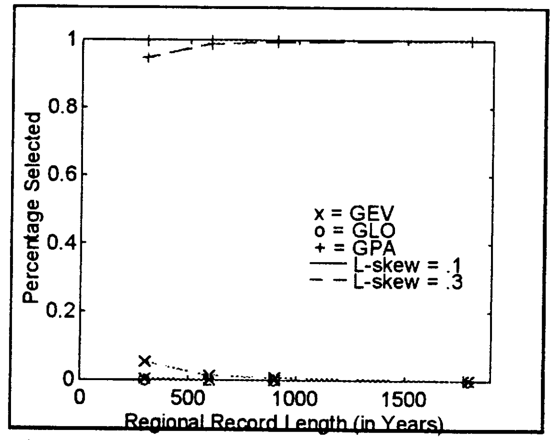

Figure C.8e: Method 8 - Selection by VDR for GPA Data

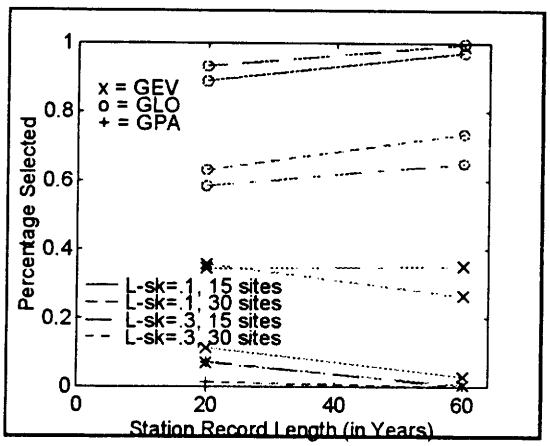

Figure C.8c: Method 8 - Selection by VDR for GLO Data

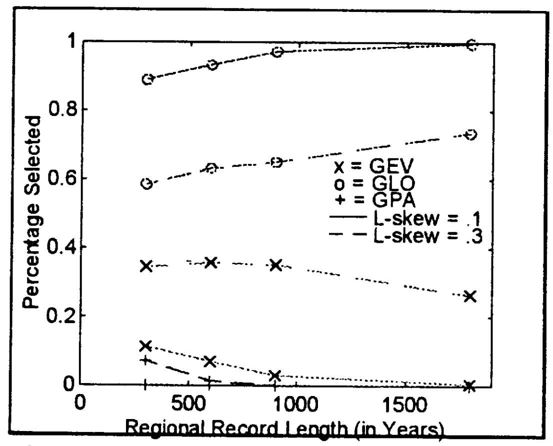

Figure C.8f: Method 8 - Selection by VDR for GLO Data

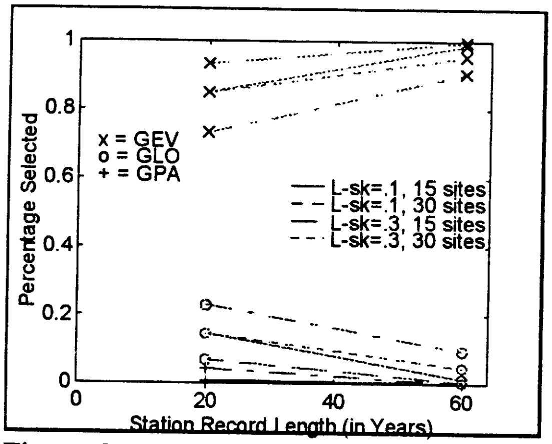

Figure C.9a: Method 9 - Selection by CDA for GEV Data

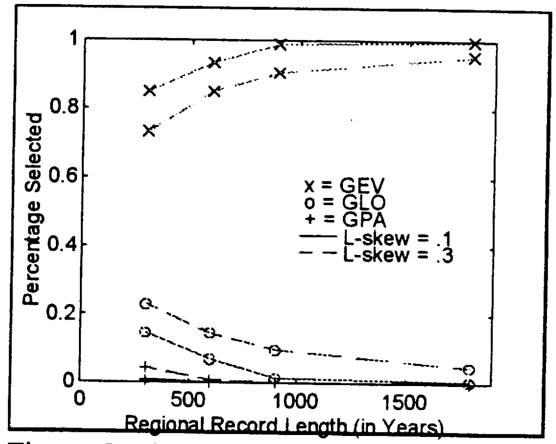

Figure C.9d: Method 9 - Selection by CDA for GEV Data

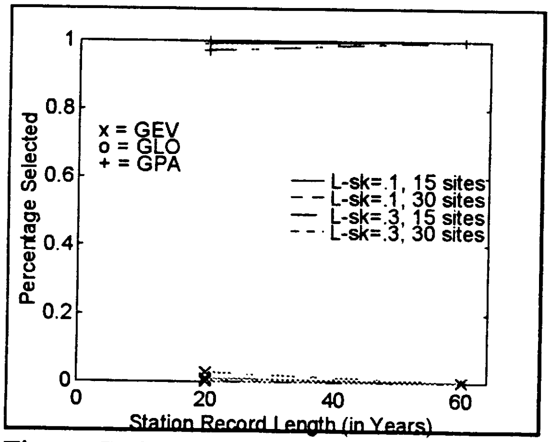

Figure C.9b: Method 9 - Selection by CDA for GPA Data

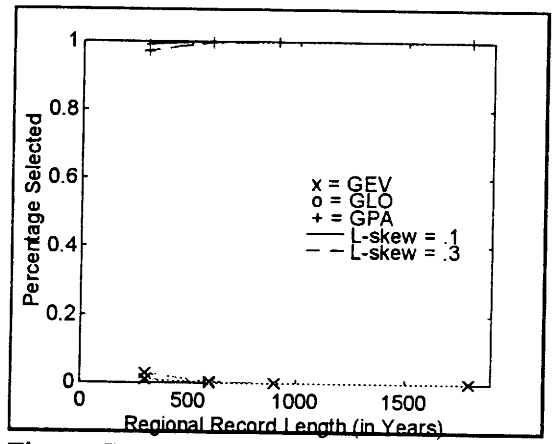

Figure C.9e: Method 9 - Selection by CDA for GPA Data

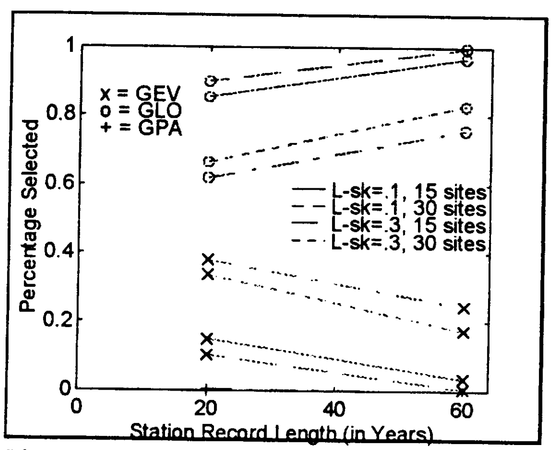

Figure C.9c: Method 9 - Selection by CDA for GLO Data

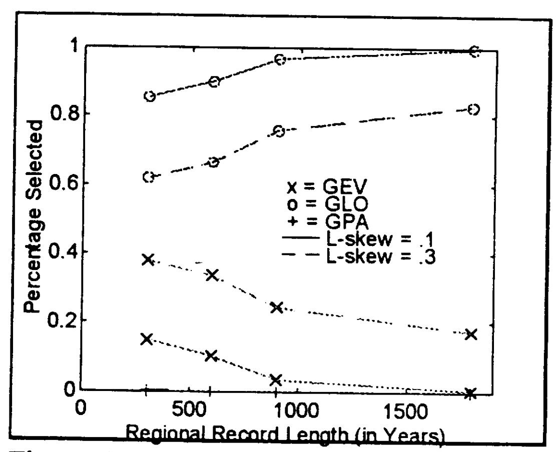

Figure C.9f: Method 9 - Selection by CDA for GLO Data

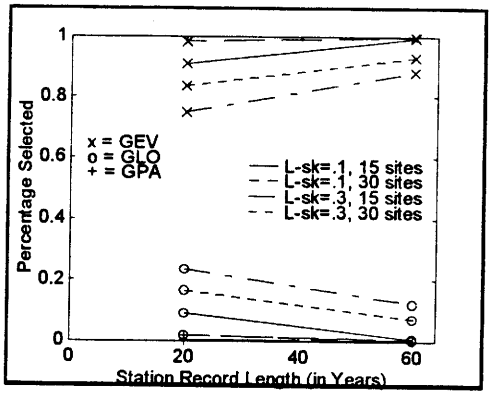

Figure C.10a: Method 10 - Selection by CDL for GEV Data

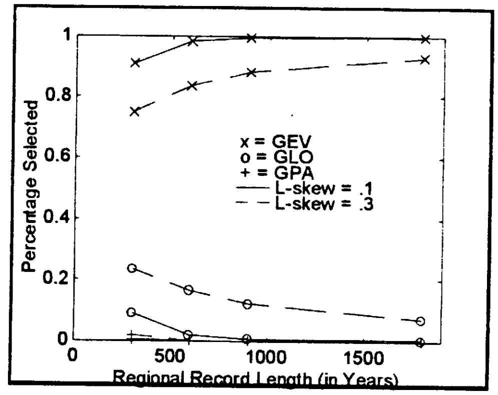

Figure C.10d: Method 10 - Selection by CDL for GEV Data

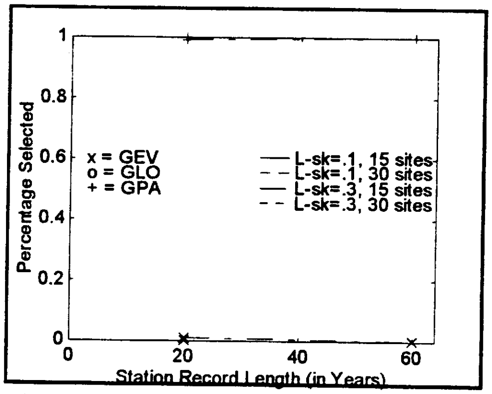

Figure C.10b: Method 10 - Selection by CDL for GPA Data

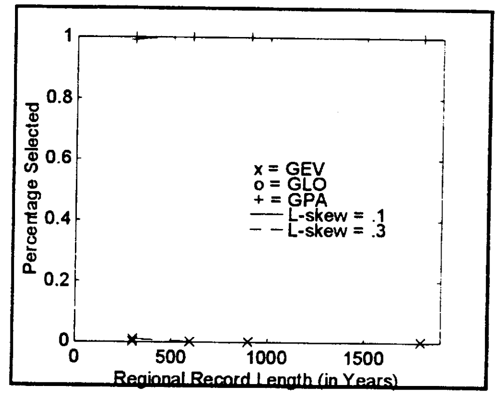

Figure C.10e: Method 10 - Selection by CDL for GPA Data

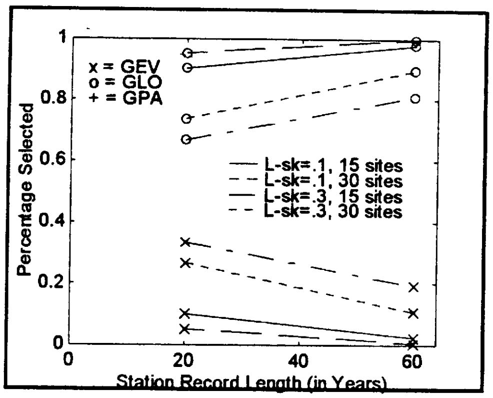

Figure C.10c: Method 10 - Selection by CDL for GLO Data

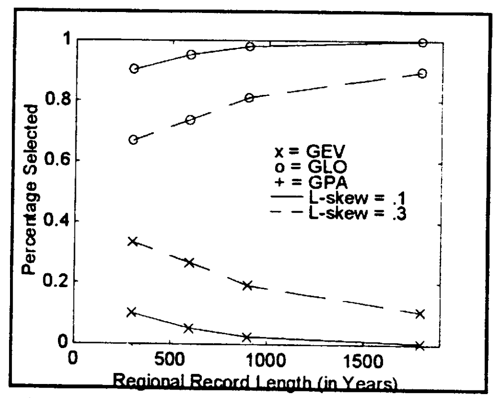

Figure C.10f: Method 10 - Selection by CDL for GLO Data

Appendix D: Station Data, At-Site Values, and Region Numbers

OBS NUM	48-#### NWSID	DURA TION	ORIGINAL REGION NUMBER	ADJ RGN NUM	NYRS	LAT	LON	ELEV ATION	MAP	XBAR	L-CV	L-SKEW	L-KURT
1	27	4	4	4	27	42.73	-110.93	6210	18.55	1.193	0.1717	0.2111	0.0396
2	80	4	2	2	36	41.41	-104.10	5350	17.89	2.012	0.1649	0.1165	0.2224
3	140	4	4	4	43	43.78	-111.03	6430	22.08	1.249	0.1392	0.1427	0.1382
4	200	4	2	2	37	44.65	-104.35	4390	22.78	2.003	0.2464	0.2872	0.1491
5	270	4	4	2	42	41.15	-104.65	6010	15.62	1.579	0.1665	0.0281	0.2166
6	380	4	2	2	21	44.69	-106.10	3680	11.59	1.497	0.2572	0.2528	0.1234
7	443	4	4	4	13	42.53	-108.76	8200	15.54	1.540	0.1943	0.0457	-0.0343
8	470	2	4	1	18	43.35	-107.41	6260	7.90	0.711	0.2199	0.1138	0.1550
9	470	3	4	1	18	43.35	-107.41	6260	7.90	1.009	0.1894	0.0266	0.0654
10	540	4	1	1	37	44.38	-108.05	3840	6.62	0.984	0.1928	0.1881	0.1476
11	552	4	3	3	22	42.63	-106.38	6010	12.35	1.493	0.2541	0.2484	0.0831
12	605	4	4	4	38	42.86	-110.90	6330	20.49	1.224	0.1602	0.2027	0.1642
13	695	4	3	4	16	42.55	-110.11	6880	8.24	0.941	0.2447	0.3323	0.0714
14	695	1	4	4	21	42.55	-110.11	6880	8.24	0.441	0.2006	0.3790	0.2535
15	695	2	4	4	31	42.55	-110.11	6880	8.24	0.616	0.1992	0.2835	0.1011
16	695	3	4	4	31	42.55	-110.11	6880	8.24	0.922	0.2057	0.1050	0.0573
17	725	4	2	2	18	43.18	-105.25	4750	11.42	1.494	0.1961	0.1044	0.1593
18	740	4	1	1	29	44.13	-106.73	4950	12.30	1.314	0.1998	0.1603	0.0679
19	761	4	4	4	13	41.58	-108.51	6720	6.87	1.160	0.2208	0.1300	0.1717
20	778	4	1	1	20	43.65	-107.73	5640	13.57	1.364	0.1758	0.2904	0.2292
21	865	4	4	4	28	43.23	-110.43	6500	22.49	1.247	0.1478	0.2223	0.1804
22	915	4	4	4	81	42.25	-111.03	6120	13.59	1.124	0.1956	0.2356	0.2063
23	1000	1	1	1	24	43.41	-108.18	4640	9.16	0.460	0.2109	0.3767	0.2804
24	1000	2	1	1	29	43.41	-108.18	4640	9.16	0.683	0.1932	0.3569	0.1808
25	1000	3	1	1	29	43.41	-108.18	4640	9.16	1.116	0.2577	0.3036	0.2054
26	1000	4	1	1	37	43.41	-108.18	4640	9.16	1.190	0.2321	0.3508	0.2763
27	1160	4	1	1	13	44.36	-106.80	5240	13.23	1.421	0.2211	0.2169	0.1972

28	1165	1	1	2	21	44.35	-106.68	4670	12.71	0.664	0.2617	0.2554	0.1721
29	1165	2	2	2	32	44.35	-106.68	4670	12.71	0.881	0.1746	0.0281	0.0761
30	1165	3	2	2	32	44.35	-106.68	4670	12.71	1.367	0.1947	0.0996	0.1513
31	1165	4	2	2	21	44.35	-106.68	4670	12.71	1.581	0.2441	0.2582	0.1866
32	1175	4	3	3	36	44.50	-109.18	5160	11.23	1.377	0.2831	0.3015	0.1750
33	1220	4	4	3	18	44.76	-107.53	8040	20.45	1.522	0.2146	0.2432	0.0115
34	1284	4	4	4	19	43.36	-109.28	6140	9.07	1.248	0.2004	0.3115	0.1752
35	1547	4	2	2	40	41.05	-104.35	5390	13.96	1.702	0.1983	0.2005	0.1242
36	1565	4	2	2	26	42.85	-106.26	5200	12.50	1.624	0.2158	0.3389	0.2749
37	1570	4	1	3	35	42.91	-106.46	5340	11.92	1.276	0.2382	0.2212	0.0516
38	1570	3	2	3	38	42.91	-106.46	5340	11.92	1.314	0.2702	0.1690	0.0235
39	1570	1	3	3	28	42.91	-106.46	5340	11.92	0.760	0.3222	0.2834	0.2237
40	1570	2	3	3	38	42.91	-106.46	5340	11.92	0.947	0.2532	0.2777	0.1506
41	1610	4	4	4	31	41.30	-106.13	8070	14.30	1.095	0.1871	0.2201	0.1868
42	1675	1	2	2	13	41.15	-104.81	6120	15.18	0.867	0.2234	0.1248	0.0064
43	1675	2	2	2	24	41.15	-104.81	6120	15.18	1.112	0.2028	0.2705	0.0780
44	1675	3	2	2	24	41.15	-104.81	6120	15.18	1.503	0.1688	0.1943	0.0548
45	1675	4	3	2	78	41.15	-104.81	6120	15.18	1.567	0.2294	0.3170	0.3081
46	1730	4	2	2	70	41.75	-104.81	5280	16.53	1.820	0.2370	0.2714	0.1736
47	1736	4	4	4	25	41.40	-110.08	7080	8.49	1.061	0.2081	0.0845	0.1457
48	1775	4	1	1	21	44.98	-109.08	4030	7.52	1.182	0.2331	0.1928	0.1234
49	1816	4	2	2	43	44.58	-106.45	4060	13.88	1.538	0.2130	0.3489	0.2428
50	1840	4	1	1	71	44.55	-109.06	4990	9.69	1.143	0.2098	0.2469	0.1847
51	1850	4	1	1	27	44.40	-108.90	5250	11.22	1.457	0.2222	0.2881	0.2640
52	1855	4	3	3	24	44.33	-109.38	5840	12.93	1.409	0.2333	0.2779	0.1496
53	1905	4	2	2	74	44.93	-104.20	3550	15.22	1.713	0.1945	0.1854	0.1383
54	2135	4	3	3	25	44.90	-109.66	6600	15.18	1.356	0.2636	0.4328	0.3260
55	2175	1	3	4	14	41.73	-108.73	7040	6.32	0.493	0.3098	0.3280	0.1924
56	2175	2	3	4	25	41.73	-108.73	7040	6.32	0.475	0.2418	0.4078	0.1712
57	2175	3	4	4	26	41.73	-108.73	7040	6.32	0.713	0.1544	0.0404	0.1012
58	2410	4	3	3	17	44.18	-105.90	4440	11.19	1.590	0.2724	0.3799	0.2105
59	2415	4	3	3	35	44.88	-108.60	4100	5.46	1.089	0.2929	0.3680	0.2373
60	2466	4	2	2	32	44.58	-104.70	3860	17.17	1.858	0.1906	0.1976	0.1046
61	2580	4	2	2	39	44.11	-105.11	4310	13.49	1.487	0.1745	0.1064	0.0660

62	2595	4	3	1	30	43.23	-108.93	5580	9.17	1.296	0.2376	0.3466	0.2044
63	2610	4	4	4	42	41.03	-107.53	6360	12.03	1.193	0.2071	0.2730	0.1876
64	2680	4	3	3	35	42.18	-105.39	6200	14.08	1.481	0.2672	0.3195	0.1979
65	2685	4	2	2	38	42.76	-105.38	4820	13.89	1.404	0.1956	0.0485	0.1018
66	2693	4	1	2	15	42.75	-105.38	4810	11.60	1.324	0.2039	0.2380	0.2851
67	2693	1	2	2	12	42.75	-105.38	4810	11.60	0.789	0.2048	0.1766	0.1285
68	2693	2	2	2	12	42.75	-105.38	4810	11.60	1.010	0.2333	0.2532	-0.0152
69	2693	3	3	2	12	42.75	-105.38	4810	11.60	1.633	0.3432	0.3870	0.2054
70	2715	4	4	4	28	43.55	-109.61	6920	8.60	1.004	0.1983	0.0243	0.0949
71	2725	4	1	2	28	43.41	-104.95	4420	11.75	1.512	0.1967	0.3622	0.2106
72	2725	1	2	2	27	43.41	-104.95	4420	11.75	0.894	0.2354	0.2709	0.2216
73	2725	2	2	2	40	43.41	-104.95	4420	11.75	1.074	0.2137	0.1997	0.0924
74	2725	3	2	2	40	43.41	-104.95	4420	11.75	1.417	0.2390	0.2718	0.1769
75	2881	4	2	2	14	44.48	-105.90	4000	13.78	1.631	0.2117	0.2178	0.1123
76	2995	4	4	4	35	41.68	-106.41	7270	12.29	1.325	0.2227	0.2437	0.2343
77	3031	4	1	1	38	44.50	-108.39	4450	7.38	1.020	0.2224	0.2268	0.2224
78	3045	4	3	4	38	41.18	-106.61	7390	14.28	1.297	0.2154	0.3255	0.2131
79	3050	1	4	4	13	41.19	-106.78	7360	11.79	0.573	0.2370	0.1307	0.0204
80	3050	2	4	4	26	41.19	-106.78	7360	11.79	0.708	0.2152	0.2655	0.1543
81	3050	3	4	4	26	41.19	-106.78	7360	11.79	0.943	0.2286	0.2933	0.2168
82	3170	4	4	4	45	42.11	-109.45	6590	7.70	0.906	0.2146	0.2023	0.1669
83	3396	4	4	4	24	41.98	-110.06	6480	7.52	0.958	0.1931	0.2552	0.2729
84	3430	2	3	4	20	41.40	-110.41	7020	10.14	0.590	0.2721	0.4418	0.4466
85	3430	3	4	4	20	41.40	-110.41	7020	10.14	0.959	0.2394	0.1976	0.1214
86	3490	4	3	3	28	42.38	-104.53	4760	11.65	1.513	0.2419	0.2929	0.1209
87	3570	4	2	2	24	42.98	-108.86	5580	11.97	1.802	0.1839	-0.0114	0.1022
88	3630	4	4	4	24	41.08	-106.14	9060	15.89	1.269	0.1955	0.0388	-0.1245
89	3770	1	1	1	12	44.78	-108.66	4250	4.92	0.391	0.2191	0.1448	-0.0248
90	3770	2	1	1	24	44.78	-108.66	4250	4.92	0.566	0.1870	0.0458	0.0618
91	3770	3	1	1	24	44.78	-108.66	4250	4.92	0.781	0.2037	0.2042	0.1077
92	3801	4	3	3	15	42.83	-107.48	6470	9.29	1.564	0.3168	0.2752	0.1228
93	3855	4	2	2	62	44.28	-105.46	4560	15.24	1.695	0.2240	0.2455	0.1872
94	3860	2	2	2	24	44.21	-105.63	4850	13.38	1.058	0.2886	0.1856	0.0690
95	3860	3	2	2	24	44.21	-105.63	4850	13.38	1.501	0.2866	0.2509	0.1377

96	3860	1	3	2	13	44.21	-105.63	4850	13.38	0.882	0.3282	0.3398	0.2750
97	3865	4	2	2	31	44.08	-105.71	4900	16.23	1.848	0.1956	0.0789	0.0712
98	3950	4	2	2	26	42.83	-105.78	4950	13.48	1.657	0.2464	0.2965	0.2139
99	3960	4	3	3	32	42.66	-105.81	6430	14.98	1.677	0.2797	0.4368	0.2650
100	4036	4	3	3	19	43.95	-108.65	5580	10.56	1.702	0.2825	0.2782	0.3058
101	4065	4	4	4	67	41.53	-109.48	6090	7.98	0.920	0.2052	0.1779	0.0818
102	4080	4	3	3	33	44.48	-108.05	3830	6.99	1.110	0.3418	0.4264	0.3983
103	4125	4	1	2	13	42.30	-104.76	4500	13.37	1.506	0.1822	0.3392	0.1747
104	4126	4	2	2	26	42.30	-104.76	4360	13.11	1.724	0.2291	0.2218	0.2201
105	4300	4	2	2	17	42.93	-104.36	4500	13.03	1.533	0.1806	0.0557	0.0171
106	4303	4	2	2	14	42.93	-104.31	4380	16.55	2.169	0.2228	0.0347	0.0784
107	4411	4	1	1	38	44.68	-108.95	4790	8.26	1.152	0.1787	0.2320	0.2602
108	4440	4	4	2	28	41.15	-105.18	6800	15.13	1.580	0.1945	0.0333	0.1249
109	4442	4	3	2	12	41.15	-105.16	6690	16.34	1.764	0.2369	0.3676	0.1600
110	4700	4	4	4	20	41.45	-105.23	6310	15.23	1.456	0.1475	0.0670	0.0571
111	4760	4	2	2	23	44.68	-104.60	3760	16.59	1.782	0.2627	0.3026	0.1640
112	4910	1	3	4	22	43.48	-110.76	6230	15.47	0.501	0.2863	0.5334	0.4060
113	4910	2	4	4	33	43.48	-110.76	6230	15.47	0.632	0.2057	0.3094	0.2266
114	4910	3	4	4	33	43.48	-110.76	6230	15.47	0.957	0.1764	0.1508	0.1240
115	4910	4	4	4	33	43.48	-110.76	6230	15.47	1.084	0.1527	0.0627	0.1147
116	4920	4	2	2	18	42.46	-104.36	4610	13.22	1.528	0.1442	0.0952	0.0237
117	4930	1	4	4	21	41.10	-106.00	7640	12.27	0.564	0.2497	0.2571	0.1232
118	4930	2	4	4	27	41.10	-106.00	7640	12.27	0.720	0.1771	0.1589	0.1414
119	4930	3	4	4	27	41.10	-106.00	7640	12.27	1.064	0.1621	0.0870	0.0944
120	5055	4	1	1	41	43.71	-106.63	4660	12.05	1.311	0.2314	0.2598	0.1941
121	5065	4	1	1	16	43.35	-106.76	5440	10.13	1.173	0.2300	0.2848	0.2431
122	5085	4	3	3	29	42.75	-104.74	5280	14.19	1.556	0.2615	0.3509	0.2109
123	5105	4	4	4	35	41.80	-110.53	6950	10.33	1.032	0.1536	0.0858	0.2210
124	5115	4	4	4	31	43.19	-109.98	7670	15.26	1.121	0.1895	0.2212	0.1952
125	5170	4	1	2	15	42.83	-104.11	5070	15.14	1.413	0.1278	0.2511	0.1055
126	5252	4	3	4	13	42.28	-110.25	6830	8.12	1.243	0.2671	0.3548	0.2470
127	5260	4	2	2	37	41.63	-104.16	4590	15.74	1.947	0.1974	0.1848	0.0719
128	5345	1	4	4	20	44.55	-110.39	7760	19.47	0.399	0.1913	-0.0325	0.0744
129	5345	2	4	4	32	44.55	-110.39	7760	19.47	0.651	0.1511	0.1284	0.0723

130	5345	3	4	4	32	44.55	-110.39	7760	19.47	1.010	0.1311	0.2256	0.2184
131	5355	4	4	4	16	44.90	-110.23	6470	13.54	1.007	0.1678	0.1337	0.1644
132	5377	4	2	2	19	43.30	-104.66	4120	13.19	1.489	0.1578	0.1391	0.1512
133	5390	1	1	2	29	42.81	-108.73	5370	13.26	0.582	0.2892	0.2860	0.1590
134	5390	2	1	2	39	42.81	-108.73	5370	13.26	0.876	0.1711	0.2771	0.2404
135	5390	3	2	2	39	42.81	-108.73	5370	13.26	1.646	0.1796	0.2239	0.1779
136	5390	4	2	2	32	42.81	-108.73	5370	13.26	1.552	0.1895	0.1532	0.0985
137	5410	4	4	4	47	41.31	-105.58	7200	11.53	1.176	0.2064	0.2638	0.2160
138	5411	4	4	4	13	41.31	-105.58	7170	10.92	1.155	0.1746	0.0545	0.0768
139	5415	4	4	4	41	41.31	-105.68	7270	10.90	1.261	0.2078	0.2657	0.2208
140	5420	2	3	4	32	41.30	-105.63	7180	10.17	0.958	0.2678	0.2493	0.1201
141	5420	1	4	4	20	41.30	-105.63	7180	10.17	0.827	0.2761	0.0111	0.0310
142	5420	3	4	4	32	41.30	-105.63	7180	10.17	1.148	0.2344	0.2120	0.1631
143	5467	4	2	2	20	43.76	-105.38	4900	11.85	1.549	0.2096	0.2680	0.2165
144	5506	4	2	2	24	44.85	-106.28	4200	14.80	1.449	0.1691	-0.0842	0.0949
145	5525	4	4	3	37	42.19	-106.85	6040	10.69	1.243	0.2001	0.2395	0.1310
146	5612	4	2	2	27	42.10	-104.35	4150	13.02	1.807	0.2867	0.2824	0.2224
147	5685	2	3	3	21	42.43	-106.03	7340	10.76	0.991	0.3547	0.5091	0.3574
148	5685	3	3	3	21	42.43	-106.03	7340	10.76	1.200	0.2812	0.4766	0.3561
149	5734	4	4	1	13	43.28	-107.63	5420	8.76	1.418	0.1950	-0.0715	0.1387
150	5770	4	1	3	38	44.83	-108.39	3840	6.72	0.929	0.2172	0.3098	0.1799
151	5830	4	2	2	64	42.76	-104.43	5010	15.36	1.711	0.2155	0.1714	0.1486
152	6120	1	3	4	19	41.90	-106.20	6570	9.95	0.589	0.3442	0.5088	0.3718
153	6120	2	3	4	30	41.90	-106.20	6570	9.95	0.654	0.2477	0.3450	0.2747
154	6120	3	4	4	30	41.90	-106.20	6570	9.95	0.876	0.2273	0.2515	0.2386
155	6120	4	4	4	23	41.90	-106.20	6570	9.95	1.022	0.2271	0.1779	0.2267
156	6140	1	4	1	12	44.15	-108.85	5830	10.82	0.636	0.2034	-0.0977	0.1782
157	6140	2	4	1	24	44.15	-108.85	5830	10.82	0.835	0.2179	0.1392	0.1097
158	6140	3	4	1	24	44.15	-108.85	5830	10.82	1.238	0.2091	0.1865	0.0792
159	6165	4	4	4	24	42.95	-110.36	7700	15.32	1.157	0.1704	0.0711	0.1248
160	6175	4	4	1	13	44.21	-106.74	5280	11.09	1.118	0.1779	-0.0344	0.0346
161	6195	4	2	2	33	43.40	-106.28	4820	13.22	1.589	0.2250	0.1495	0.1249
162	6395	4	1	2	24	44.26	-104.95	4280	12.50	1.348	0.2256	0.2964	0.1560
163	6395	2	2	2	21	44.26	-104.95	4280	12.50	1.086	0.2067	0.1431	0.0115

164	6395	3	2	2	21	44.26	-104.95	4280	12.50	1.526	0.2488	0.2006	0.0491
165	6428	4	4	4	24	43.66	-110.71	6470	20.86	1.283	0.1854	0.2604	0.1270
166	6440	1	3	4	12	43.85	-110.58	6790	23.15	0.420	0.3162	0.5191	0.2784
167	6440	2	4	4	21	43.85	-110.58	6790	23.15	0.619	0.1659	0.2034	0.2756
168	6440	3	4	4	22	43.85	-110.58	6790	23.15	1.075	0.0982	-0.0401	0.1069
169	6440	4	4	4	78	43.85	-110.58	6790	23.15	1.292	0.1833	0.3239	0.2272
170	6450	4	1	2	20	43.51	-104.33	4100	12.05	1.477	0.2046	0.2665	0.1697
171	6470	4	4	1	18	43.21	-108.80	5460	8.87	1.269	0.1760	0.0690	0.1154
172	6555	1	4	4	23	41.26	-110.35	6800	8.55	0.549	0.2483	0.1727	0.0423
173	6555	2	4	4	35	41.26	-110.35	6800	8.55	0.796	0.2271	0.2458	0.0854
174	6555	3	4	4	36	41.26	-110.35	6800	8.55	1.093	0.2377	0.2199	0.0691
175	6595	4	4	4	29	42.35	-107.46	6310	9.76	1.337	0.2091	0.0199	0.1027
176	6597	1	3	4	29	41.31	-108.91	6740	8.06	0.498	0.2828	0.3067	0.1415
177	6597	2	4	4	29	41.31	-108.91	6740	8.06	0.615	0.2262	0.1337	0.1018
178	6597	3	4	4	29	41.31	-108.91	6740	8.06	0.856	0.1994	0.0180	0.0498
179	6600	1	2	2	24	43.35	-104.11	4130	14.59	1.096	0.2295	0.1496	0.1632
180	6600	2	2	2	24	43.35	-104.11	4130	14.59	1.500	0.2124	0.2190	0.2215
181	6600	3	2	2	24	43.35	-104.11	4130	14.59	1.966	0.2353	0.2505	0.1187
182	6660	1	2	2	12	43.85	-104.20	4480	14.65	0.727	0.2541	0.3924	0.3131
183	6660	2	2	2	12	43.85	-104.20	4480	14.65	0.975	0.2181	0.1097	0.0838
184	6660	3	2	2	12	43.85	-104.20	4480	14.65	1.327	0.2190	0.1453	0.1333
185	6660	4	2	2	65	43.85	-104.20	4480	14.65	1.507	0.1956	0.2063	0.1291
186	6875	1	4	4	26	42.55	-108.18	6540	7.55	0.506	0.2071	0.0775	0.0510
187	6875	2	4	4	34	42.55	-108.18	6540	7.55	0.652	0.1943	0.2417	0.2671
188	6875	3	4	4	34	42.55	-108.18	6540	7.55	1.000	0.2240	0.2328	0.2646
189	6935	1	2	2	35	43.98	-104.41	4320	12.31	0.891	0.2644	0.2429	0.1966
190	6935	2	2	2	35	43.98	-104.41	4320	12.31	1.041	0.2166	0.1383	0.2169
191	6935	3	2	2	35	43.98	-104.41	4320	12.31	1.382	0.2049	0.2013	0.1714
192	7079	4	2	2	20	44.98	-107.43	4200	19.23	2.030	0.2291	0.2441	0.2318
193	7105	3	3	3	50	42.46	-106.85	5930	9.40	1.020	0.2391	0.2716	0.2471
194	7105	4	3	3	28	42.46	-106.85	5930	9.40	1.278	0.2778	0.3600	0.2127
195	7105	1	4	3	41	42.46	-106.85	5930	9.40	0.441	0.2117	0.1512	0.1241
196	7105	2	4	3	50	42.46	-106.85	5930	9.40	0.647	0.2039	0.1015	0.1516
197	7115	4	4	1	29	43.25	-108.68	5440	7.79	1.178	0.2020	0.1069	0.1512

198	7200	1	2	2	39	41.63	-104.48	4980	14.89	1.038	0.2679	0.3746	0.2504
199	7200	2	2	2	47	41.63	-104.48	4980	14.89	1.230	0.2335	0.2836	0.2370
200	7200	3	2	2	47	41.63	-104.48	4980	14.89	1.633	0.2299	0.3009	0.2118
201	7200	4	2	2	32	41.63	-104.48	4980	14.89	1.664	0.2055	0.2930	0.1430
202	7235	4	2	2	53	41.18	-104.06	5050	15.04	1.726	0.2410	0.3268	0.2584
203	7260	4	4	4	32	42.86	-109.86	7180	11.13	0.928	0.1874	0.2345	0.1770
204	7270	1	2	2	30	43.71	-105.63	5110	12.31	1.034	0.2329	0.0974	0.0830
205	7270	2	2	2	30	43.71	-105.63	5110	12.31	1.241	0.2242	0.1964	0.1435
206	7270	3	2	2	30	43.71	-105.63	5110	12.31	1.662	0.2504	0.2554	0.2293
207	7376	4	4	1	12	43.01	-107.00	5960	12.63	1.193	0.1927	0.0394	0.1950
208	7380	4	1	1	62	44.75	-108.76	4380	6.07	0.913	0.2257	0.1374	0.0934
209	7473	4	1	1	36	44.18	-107.95	4020	6.80	1.035	0.2408	0.2563	0.1532
210	7533	2	3	4	33	41.80	-107.20	6740	8.25	0.585	0.2568	0.3010	0.2546
211	7533	3	3	4	33	41.80	-107.20	6740	8.25	0.855	0.2405	0.3177	0.2944
212	7533	1	4	4	33	41.80	-107.20	6740	8.25	0.456	0.2664	0.3631	0.2727
213	7545	1	2	2	36	44.75	-105.70	4150	13.25	0.874	0.2023	0.2199	0.1505
214	7545	2	2	2	45	44.75	-105.70	4150	13.25	1.053	0.1822	0.2012	0.2306
215	7545	3	2	2	45	44.75	-105.70	4150	13.25	1.362	0.1847	0.2118	0.1751
216	7760	1	1	1	26	43.01	-108.38	4950	7.68	0.517	0.2849	0.2916	0.3614
217	7760	2	1	1	26	43.01	-108.38	4950	7.68	0.743	0.2537	0.1280	0.0866
218	7760	3	1	1	26	43.01	-108.38	4950	7.68	1.194	0.2212	0.1161	0.0032
219	7810	4	2	2	40	43.60	-104.90	4500	12.72	1.462	0.2061	0.1340	0.0447
220	7840	1	3	4	20	41.58	-109.21	6270	7.91	0.486	0.2854	0.3115	0.1216
221	7840	2	3	4	20	41.58	-109.21	6270	7.91	0.689	0.2569	0.3113	0.0605
222	7840	3	3	4	20	41.58	-109.21	6270	7.91	1.107	0.2468	0.2288	0.0563
223	7955	4	4	4	21	41.86	-111.00	6210	10.62	1.014	0.1448	0.2177	0.1820
224	7980	4	3	3	21	42.76	-108.18	6060	9.70	1.514	0.2474	0.2150	0.1641
225	7990	4	4	4	39	41.45	-106.81	6790	9.51	1.056	0.2125	0.2997	0.1658
226	7995	1	3	4	27	41.50	-106.80	6800	8.56	0.552	0.3242	0.4654	0.3424
227	7995	2	3	4	36	41.50	-106.80	6800	8.56	0.678	0.2700	0.3768	0.3213
228	7995	3	4	4	36	41.50	-106.80	6800	8.56	0.872	0.2278	0.2844	0.2510
229	8070	2	3	3	43	42.13	-106.88	6840	12.49	0.820	0.2571	0.3633	0.2655
230	8070	3	3	3	44	42.13	-106.88	6840	12.49	1.229	0.2615	0.3883	0.3253
231	8070	4	3	3	19	42.13	-106.88	6840	12.49	1.417	0.2743	0.4368	0.3352

232	8070	1	4	3	35	42.13	-106.88	6840	12.49	0.582	0.2466	0.2918	0.1126
233	8124	4	1	3	17	44.55	-107.80	4230	10.24	1.282	0.1523	0.3671	0.1652
234	8155	1	1	2	41	44.76	-106.96	3940	14.67	0.665	0.1732	0.1818	0.1545
235	8155	4	1	2	34	44.76	-106.96	3940	14.67	1.425	0.2305	0.1955	0.1571
236	8155	2	2	2	50	44.76	-106.96	3940	14.67	0.992	0.2043	0.2872	0.2035
237	8155	3	2	2	50	44.76	-106.96	3940	14.67	1.626	0.2198	0.2844	0.1690
238	8160	4	2	2	68	44.85	-106.86	3800	15.32	1.759	0.2333	0.2181	0.1570
239	8209	4	1	1	37	43.23	-108.11	4830	6.53	0.931	0.1897	0.0688	0.0876
240	8315	4	4	4	19	44.13	-110.66	6880	31.67	1.452	0.1731	0.1559	0.0719
241	8385	4	4	4	57	42.46	-108.80	7880	12.96	1.126	0.1793	0.1907	0.1684
242	8475	4	2	2	18	43.43	-104.16	3800	13.78	1.567	0.1313	-0.0793	0.0185
243	8705	4	2	2	66	44.40	-104.35	4750	17.54	1.645	0.1913	0.1699	0.1047
244	8758	4	4	3	17	44.05	-108.98	6440	14.67	1.641	0.1296	0.1041	0.1227
245	8808	4	2	2	26	41.76	-105.38	6100	16.11	1.665	0.1795	0.0268	0.0509
246	8820	2	4	4	15	43.71	-109.63	7840	17.63	0.815	0.1699	0.1245	0.1397
247	8820	3	4	4	15	43.71	-109.63	7840	17.63	1.371	0.1697	0.2898	0.1957
248	8845	4	2	2	26	41.35	-104.38	5620	16.28	1.887	0.2316	0.2590	0.1539
249	8852	2	2	3	21	44.06	-107.41	4800	13.14	0.974	0.2574	0.2490	0.0705
250	8852	3	2	3	21	44.06	-107.41	4800	13.14	1.414	0.2395	0.2559	0.0308
251	8852	4	2	3	21	44.06	-107.41	4800	13.14	1.591	0.2016	0.1741	0.1573
252	8852	1	3	3	21	44.06	-107.41	4800	13.14	0.695	0.3143	0.3950	0.1101
253	8858	4	1	1	33	43.81	-107.36	4680	13.01	1.239	0.1723	0.4023	0.3574
254	8875	1	1	1	27	43.65	-108.20	4310	10.93	0.520	0.2594	0.1609	0.0745
255	8875	2	1	1	27	43.65	-108.20	4310	10.93	0.731	0.2036	0.0544	0.2210
256	8875	4	1	1	13	43.65	-108.20	4310	10.93	1.253	0.1822	0.2776	0.2045
257	8875	3	3	1	27	43.65	-108.20	4310	10.93	1.173	0.2751	0.3161	0.2756
258	8880	4	1	1	23	43.65	-108.21	4400	12.26	1.396	0.1846	0.2717	0.3219
259	8888	1	1	1	26	43.71	-108.68	5700	11.72	0.615	0.2366	0.3001	0.1828
260	8888	2	4	1	26	43.71	-108.68	5700	11.72	0.766	0.1884	0.1062	0.1084
261	8888	3	4	1	26	43.71	-108.68	5700	11.72	1.220	0.1882	0.1449	0.1083
262	8888	4	4	1	22	43.71	-108.68	5700	11.72	1.346	0.1756	0.1877	0.1006
263	8995	4	2	2	66	42.08	-104.21	4100	13.79	1.662	0.2242	0.2448	0.2250
264	9000	1	2	2	28	42.05	-104.18	4090	12.85	0.962	0.2504	0.1889	0.1943
265	9000	2	2	2	36	42.05	-104.18	4090	12.85	1.218	0.2436	0.2690	0.3162

266	9000	3	2	2	36	42.05	-104.18	4090	12.85	1.684	0.2294	0.3060	0.2008
267	9025	4	4	4	25	44.91	-110.41	6270	16.55	1.186	0.1807	0.2538	0.2807
268	9205	4	1	2	38	44.10	-104.61	4260	14.19	1.485	0.1832	0.2579	0.2932
269	9207	4	2	2	44	43.93	-104.76	4780	12.71	1.576	0.2054	0.0708	0.0828
270	9459	4	4	4	32	41.68	-107.98	6800	5.96	0.900	0.2038	0.2494	0.2391
271	9580	4	2	2	34	44.63	-105.31	3530	12.30	1.569	0.2134	0.2182	0.2711
272	9604	4	2	2	38	42.25	-104.63	4290	12.68	1.711	0.2451	0.2328	0.2182
273	9615	4	2	2	71	42.11	-104.95	4640	12.82	1.756	0.2024	0.1837	0.2330
274	9770	4	1	1	72	44.01	-107.96	4060	7.62	1.078	0.2407	0.1847	0.1715
275	9775	1	1	1	15	43.95	-108.03	4150	5.65	0.439	0.2598	0.3218	0.0727
276	9775	3	1	1	24	43.95	-108.03	4150	5.65	1.014	0.2331	0.2391	0.1762
277	9775	2	3	1	24	43.95	-108.03	4150	5.65	0.768	0.2828	0.3135	0.1800
278	9785	4	1	1	30	43.96	-107.96	4170	7.49	1.039	0.1974	0.3104	0.1949
279	9905	4	4	4	29	44.96	-110.70	6200	15.55	1.133	0.2082	0.3045	0.1717
280	9925	4	2	2	35	41.93	-104.30	4230	14.02	1.773	0.1819	0.1576	0.0717