Analysis of Wyoming Extreme Precipitation Patterns and Their Uncertainty for Safety Evaluation of Hydraulic Structures

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by

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ABSTRACT

This study contains two major tasks. The first task is concerned with the identification of representative significant storm patterns in Wyoming. Significant storms refer to those events with the total depth equal to or greater than that of a 10-year storm. The method employed was cluster analysis. Contingency tests were performed to examine the dependency of the occurrence of storm patterns on the climatic region, storm duration, and seasonality. Parametric models were used to fit storm patterns. The second task is the development of procedures to stochastically generate storm patterns. Three methods based on the multivariate Johnson system are proposed. An additional issue addressed under this task is the selection of alternative normal transformations with favorable small sample qualities. Four such procedures are proposed and their performance is examined.

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CHAPTER 1

INTRODUCTION

1.1 PROBLEM STATEMENT

To design a hydraulic structure or to evaluate the performance of a hydraulic structure, information about the storm event under consideration is often required. For example, in dam safety evaluation, the inflow design flood of a specified return period is often computed from a synthetic (design) storm in conjunction with the use of an appropriate rainfall-runoff model. In this process, the complete description of a design storm involves the specification of storm depth, duration, and its temporal pattern. To use an appropriate temporal pattern for a design storm is of great importance in the design and evaluation of hydrologic safety for hydraulic structures.

To assess the reliability of hydraulic structures, the general engineering practice is to pre-select the duration and the return period of the design storm from which the corresponding storm depth is determined by frequency analysis. For each storm type, there exists intrinsic randomness in the amount of precipitation within the design storm. Therefore, the actual time distribution of precipitation amount within a design storm is subject to uncertainty. One should evaluate the performance of hydraulic structures for a given design storm under several possible storm distributions. Procedures to generate temporal distributions for design storms are essential in this process.

Generating temporal distributions for design storms requires Monte Carlo simulation involving non-normal random variables. In such an exercise, a common practice to derive the distribution representation for a given sample is to fit a parametric distribution by matching their moments. In general, the distribution will be better described if the number of statistical moments used increases. However, the higher order moments such as skewness and kurtosis generally are associated with large standard errors, especially when the sample size is not large, and the extent of sample error increases rapidly as the order of moments gets large. Therefore, it is desirable to find normal transformations with favorable small sample qualities.

1.2 OBJECTIVES AND SCOPE OF THE RESEARCH

There are two primary objectives in this study. The first is to identify representative significant storm patterns in Wyoming. Accompanying this is the selection of parametric models to describe the representative storm patterns. The second primary objective is the development of statistical procedures to generate temporal distributions for a given storm pattern. Efforts have also been made to find appropriate alternative normal transformations for generating storm patterns.

Various methods have been developed to identify storm patterns for general storms, while this study concentrates on the identification of storm patterns for significant storms having a total depth no less than that of a 10-year event. The storm pattern identified in this study is the average storm patterns for complete storms but not intense bursts of individual rainfall. The Monte Carlo simulation procedure developed in this study to generate storm patterns (Chapter 3) is not a full multivariate simulation procedure but preserves partial information such as the marginal density functions, moments and correlation structure of a given storm pattern.

1.3 ORGANIZATION OF THE THESIS

The remainder of this thesis consists of four chapters. Chapter 2 describes the procedure for the identification of representative storm patterns in Wyoming. Parametric models are selected to describe representative storm patterns. In Chapter 3, the procedure for generating the representative storm patterns are developed based on the multivariate Johnson system. Chapter 4 focuses on finding the appropriate alternatives for normal transformation with favorable small sample properties. Performance evaluation of several alternative methods are made. A summary and some recommendations for future research are given in Chapter 5.

CHAPTER 2

IDENTIFICATION OF STORM PATTERNS IN WYOMING

2.1 INTRODUCTION

To design a hydraulic structure or to evaluate its performance, information about the storm event under consideration is often required. The essential characteristics of a storm event include its total depth, duration, and the temporal variation of precipitation amount within the storm duration. For example, in dam safety evaluation, the inflow design flood of a specified return period is often computed from a synthetic (design) storm in conjunction with the use of an appropriate rainfall-runoff model. In this process, the depth, duration, and temporal pattern of the design storm are required.

Various methods have been developed to generate synthetic (design) storms. Chen (1976) categorized these methods into four general types. One typical way to formulate the synthetic storm is to derive it from the intensity-duration-frequency (IDF) relationship (e.g. Bandyopadhyay, 1972; Chen, 1976; Raudkivi and Lawgun, 1970). As Pilgrim and Cordery (1975) pointed out, the rainfall represented by the IDF formulas only depicts intense bursts within storms. It is important to develop a complete storm pattern rather than intense bursts of individual rainfall. The approach taken in this study follows those which develop the average storm patterns for complete storms, rather than intense bursts of individual rainfall, using observed precipitation records.

In general, temporal patterns for complete storms were sought empirically. In the study of storm patterns in Illinois urban areas, Huff (1967) divided all storms into four groups called the first-quartile, second quartile, third quartile, and fourth quartile storms, according to whether the heaviest rainfall occurs in the first, second, third, or fourth quarter of the storm. Pilgrim and Cordery (1975) developed a procedure based on the rank of the rainfall depth in particular time periods. In this paper, cluster analysis is applied for identifying storm patterns.

The main concern of this chapter is the identification of representative storm patterns in Wyoming for design and safety evaluation of major hydraulic structures such as dams and bridges. Tyrrell (1982) developed design storm patterns in Wyoming for use in predicting floods and design storm patterns were constructed from all the observed rainfalls. However, the focus of this study is placed on the significant storms which can be region and location specific. A significant storm event in a dry region may be considered as insignificant in a wet region. Therefore, a significant storm in this study was defined as a storm event having the total depth equal to or greater than the storm depth of a 10-year event at a particular location.

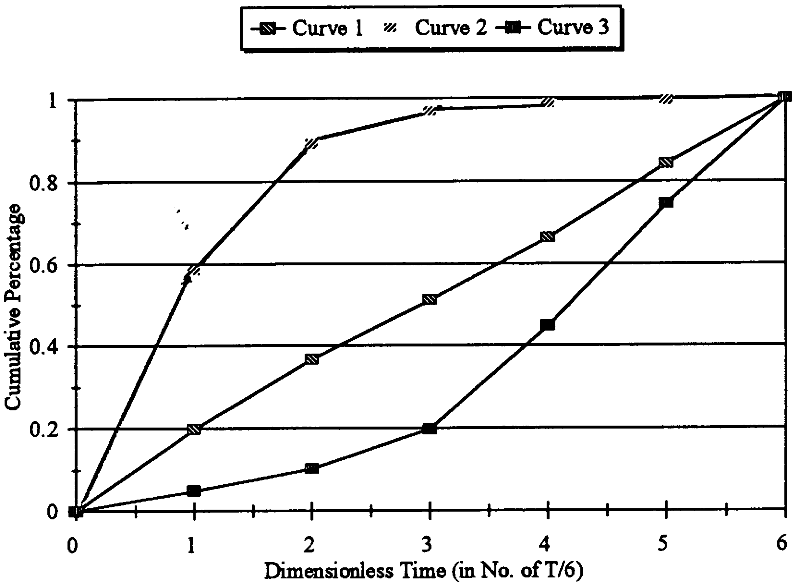
2.2 CHARACTERIZATION OF STORM TEMPORAL PATTERN

During a storm event, the precipitation amounts vary with respect to time at a given location. To identify the distinctly different storm patterns for the observed storm events, temporal patterns about storm events should be characterized. Due to the variation of storm duration and storm depth from one event to another, characterization of similarity or dissimilarity of different storm temporal patterns can best be achieved by using a dimensionless scale.

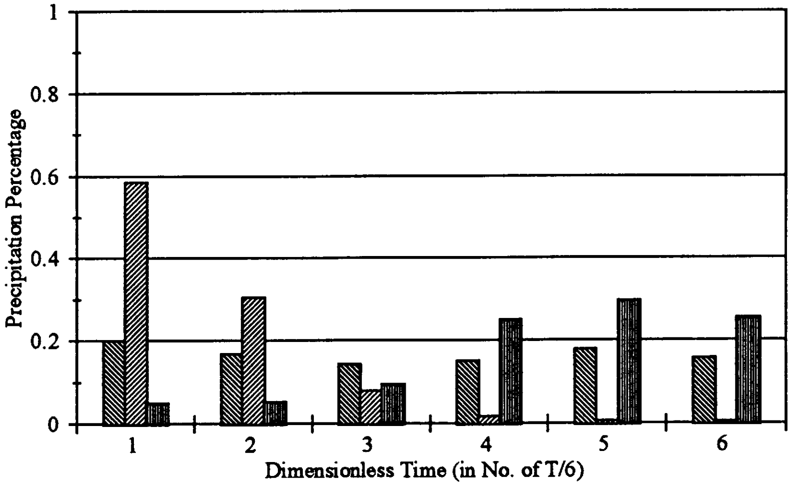
The dimensionless storm pattern can be obtained by dividing the storm depths at different times (d_t) by the total storm depth (D) as $P_t=d_t/D$ and the time by the storm duration (T) as $t'=t/T$. Three example dimensionless rainfall mass curves and their corresponding dimensionless rainfall hyetographs using six time increments are shown in Figs. 2.1(a) and 2.1(b), respectively. Using the dimensionless measures of time and rainfall depth, storm events with different depths and durations can be combined to identify the typical family of storm patterns.

The dimensionless mass curve shows the cumulative percentages of storm depth, $\Sigma_{t'}P_{t'}$, over the non-dimensionalized time (t'). To characterize the storm temporal pattern using the rainfall mass curve, the selection of the number of time points over the storm duration is subjective. Too few points may not accurately describe the underlying variation of storm pattern. On the other hand, too many points would capture unwanted sample noises masking the essential feature of the storm pattern. For this study, the entire storm duration was divided into six equal intervals in this study and the cumulated percentages at T/6, 2T/6, 3T/6, 4T/6, and 5T/6 were used in the statistical cluster analysis for identifying representative storm patterns.

Since the vertical axis of the dimensionless rainfall mass curve ranges from 0 to 1 and it is non-decreasing, these storm patterns can be viewed as a cumulative distribution function for the dimensionless time. As an alternative to the rainfall mass curve, statistical moments



(a) Dimensionless Rainfall Mass Curves



(b) Dimensionless Rainfall Hyetograph

Fig. 2.1. Illustrations of Dimesionless Rainfall Mass Curves and the Corresponding Dimensionless Rainfall Hyetographs

of the dimensionless time for each storm event can be calculated and used to characterize the temporal pattern of a storm event. In this study, the first four central moments of the dimensionless time (t/T) were adopted to characterize the storm temporal pattern.

Note that the original data are the hourly precipitations and the continuous variation of precipitation within one hour is not known. As a result, the continuous rainfall hyetograph can be discretely approximated as Fig. 2.1(b) from which the statistical moments of dimensionless time can be calculated as

$$\mu = \sum_{t=1}^{T} \frac{t}{T} \times P_t \tag{2.1}$$

$$\sigma = \sqrt{\sum_{t=1}^{T} \left(\frac{t}{T} - \mu \right)^2 \times P_t} \tag{2.2}$$

$$\gamma = \frac{\sum_{t=1}^{T} \left(\frac{t}{T} - \mu \right)^3 \times P_t}{\sigma^3} \tag{2.3}$$

$$\kappa = \frac{\sum_{t=1}^{T} \left(\frac{t}{T} - \mu \right)^4 \times P_t}{\sigma^4} \tag{2.4}$$

where μ, σ, γ, and κ are the mean, standard deviation, skew coefficient, and kurtosis of the dimensionless time, respectively.

2.3 DATABASE FOR WYOMING STORMS

Significant storms were extracted from NCDC HOURLY AND 15 MINUTE PRECIPITATION which is commercially available from EarthInfo, Inc.. A total of 379 storm events were extracted based on the threshold value approximately equal to the depth of a 10-year event for each precipitation gage station. The threshold precipitation depths were determined from the at-site frequency analysis on the annual maximum depth of different durations (Eastwood, 1995). The L-moments method (Hosking, 1986) was used in the frequency analysis. According to the regional analysis of annual maximum storm statistics (Eastwood, et al., 1994), the entire State of Wyoming was divided into four climatically homogeneous regions. This information was combined with state code, station identification, date of storm occurrence, storm duration, and hourly precipitation depth to generate the data file of significant storms. Table 2.1 lists the information for Wyoming stations analyzed in the study.

Based on the compiled hourly precipitation data file, storm patterns for two database types were generated: duration-based database and event-based database. The former is needed for establishing storm patterns whose storm duration is specified whereas the latter is useful for generating storm patterns with variable storm duration and storm depth. Duration-based storm patterns are useful in design practice where the duration of a design storm is specified in advance whereas the event-based storm patterns are useful for synthesizing and simulating storm events.

Table 2.1 Information about Precipitation Stations Analyzed in Wyoming

Station Name	Station ID	Begin Date	End Date	Rec Years	Latitude	Longitude	Elev.	Threshold (inch)	Region	#. of event
ALADDIN	50	04/01/1951	04/30/1952	2	N44:38:00	W104:11:00	3740	2.5	2	0
ALADDIN 6 S	56	04/01/1952	08/31/1952	1	N44:33:00	W104:12:00	3850	2.5	2	0
ANTELOPE SPRINGS	237	08/01/1950	09/30/1950	1	N43:26:00	W106:35:00	5180	1.9	2	0
ARAPAHOE RANCH	250	08/01/1948	04/07/1950	3	N43:43:00	W108:32:00	---	1.4	1	0
BADWATER 2 N	470	08/01/1948	03/31/1960	13	N43:21:00	W107:25:00	6260	1.7	1	0
BARNUM 1 N	528	11/01/1949	06/24/1958	10	N43:42:00	W106:55:00	5150	1.7	1	0
BASIN	540	08/01/1949	06/23/1958	10	N44:23:00	W108:03:00	3840	1.2	1	0
BEULAH 8 WSW	640	08/01/1952	06/22/1958	7	N44:31:00	W104:16:00	4170	2.5	2	0
BIG PINEY	695	08/01/1948	08/31/1992	45	N42:33:00	W110:07:00	6820	1.2	4	5
	697	11/01/1992	12/15/1992	1	N--:--:--	W---:--:--	---	---	-	---
BILL 15 SW	735	08/01/1950	10/31/1972	21	N43:07:00	W105:30:00	5410	1.9	2	5
BOYSEN DAM	1000	08/01/1948	12/15/1992	45	N43:25:00	W108:11:00	4640	1.5	1	7
BUFFALO	1165	08/01/1948	12/29/1992	45	N44:21:00	W106:41:00	4670	1.8	3	4
BURLINGTON	1240	02/01/1950	11/30/1957	8	N44:27:00	W108:25:00	4430	1.2	1	0
BURRIS	1284	11/01/1951	04/30/1952	2	N43:22:00	W109:17:00	6120	1.5	4	0
CASPER WSO AP	1570	08/01/1948	12/30/1992	45	N42:55:00	W106:28:00	5340	2.0	3	9
CHEYENNE WSFO AP	1675	08/01/1948	12/18/1992	45	N41:09:00	W104:49:00	6120	1.9	2	10
CIRCLE H RANCH	1743	03/01/1951	10/31/1957	7	N44:30:00	W109:32:00	6310	1.3	3	2
CLARETON 3 NE	1750	11/01/1949	07/31/1955	7	N43:44:00	W104:40:00	4220	1.9	2	2
CODY	1840	09/01/1948	09/25/1951	4	N44:31:00	W109:04:00	5050	1.3	1	0
CODY 1 SSE	1845	08/01/1949	12/31/1959	11	N44:31:00	W109:03:00	5050	1.3	1	2
CRESTON	2175	08/01/1948	02/29/1984	35	N41:44:00	W107:44:00	7040	1.1	4	1
CROWHEART	2223	09/01/1949	02/11/1950	2	N43:18:00	W109:11:00	6070	1.5	4	0
CROWHEART 2 E	2228	06/01/1950	11/30/1951	2	N43:18:00	W109:08:00	5960	1.5	4	2
DIVERSION DAM	2595	08/01/1949	03/31/1955	7	N43:14:00	W108:56:00	5580	1.7	1	1
DIVERSION DAM 1 E	2596	04/01/1955	06/25/1958	4	N43:14:00	W108:56:00	5560	1.7	1	0
DOUGLAS	2685	02/01/1960	08/31/1962	3	N42:46:00	W105:23:00	4800	1.7	2	1
DOUGLAS AIRPORT CAA 1 S	2690	08/01/1948	02/29/1960	13	N42:45:00	W105:22:00	4880	1.7	2	0
DOUGLAS AVIATION	2693	08/01/1962	12/15/1992	31	N42:45:00	W105:23:00	4810	1.7	2	8
DOUGLAS 17 NE	2696	09/01/1957	12/31/1992	36	N42:57:00	W105:09:00	4930	1.7	2	14
DUBOIS	2715	08/01/1949	12/29/1992	33	N43:34:00	W109:38:00	6960	1.2	4	4
DULL CENTER 1 SE	2725	08/01/1948	12/30/1992	45	N43:25:00	W104:57:00	4420	1.9	2	5
EMBLEM	3031	11/01/1957	06/24/1958	2	N44:30:00	W108:24:00	4450	1.3	1	0
ENCAMPMENT	3050	08/01/1948	12/30/1992	42	N41:12:00	W106:47:00	7290	1.6	4	3
EVANSTON 1 E	3100	08/01/1948	12/30/1992	45	N41:16:00	W110:57:00	6810	1.1	4	6
FORT BRIDGER CAA AIRPOR	3430	08/01/1948	03/31/1966	16	N41:24:00	W110:25:00	7020	1.1	4	1
GARLAND	3770	08/01/1948	08/31/1966	19	N44:47:00	W108:40:00	4250	1.2	1	2
GARRETT	3784	09/01/1951	04/30/1956	6	N42:07:00	W105:36:00	6820	1.3	3	0
GARRETT 3 WNW	3787	05/01/1956	06/25/1958	3	N42:08:00	W105:39:00	7120	1.3	3	0

Table 2.1 (continued)

Station Name	Station ID	Begin Date	End Date	Rec Years	Latitude	Longitude	Elev.	Threshold (inch)	Region	#. of event
GILLETTE 9 SW	3860	08/01/1948	05/31/1960	13	N44:13:00	W105:38:00	4850	2.2	2	2
GILLETTE 18 SW	3865	05/01/1960	05/31/1986	27	N44:05:00	W105:43:00	4910	2.2	2	5
HAMILTON DOME	4205	06/01/1950	06/25/1958	9	N43:47:00	W108:33:00	5600	2.4	1	0
HAT CREEK 14 N	4310	08/01/1950	01/31/1984	35	N43:08:00	W104:22:00	4320	1.8	2	20
HILAND	4546	06/01/1950	06/24/1958	9	N43:07:00	W107:20:00	6000	1.5	1	1
HORSE CREEK RANGER STN	4710	08/01/1968	04/30/1983	12	N43:40:00	W109:38:00	7810	1.7	4	0
HYATTVILLE	4796	08/01/1949	06/24/1958	10	N44:15:00	W107:35:00	4470	1.2	1	0
JACKSON	4910	08/01/1948	12/31/1992	45	N43:29:00	W110:46:00	6230	1.2	4	6
JELM 2 S	4930	08/01/1948	12/13/1992	45	N41:05:00	W106:00:00	7580	1.6	4	1
KANE 1 N	5021	08/01/1949	06/24/1958	4	N44:51:00	W108:12:00	3640	2.0	1	0
KANE 2 SW	5026	11/01/1951	03/31/1958	8	N44:50:00	W108:14:00	3770	2.0	1	0
KEYHOLE DAM	5137	10/01/1949	12/31/1958	10	N44:23:00	W104:46:00	4190	2.0	2	0
KIRWIN	5186	01/01/1969	08/31/1975	5	N43:54:00	W109:17:00	9190	1.2	3	1
LAKE YELLOWSTONE	5345	08/01/1948	12/30/1992	45	N44:33:00	W110:24:00	7770	1.4	4	0
LANCE CREEK	5371	08/01/1950	12/31/1992	43	N43:03:00	W104:39:00	4410	1.7	2	9
LANDER WSO AP	5390	08/01/1948	12/15/1992	45	N42:49:00	W108:44:00	5560	1.8	2	20
LARAMIE FAA AIRPORT	5415	09/01/1948	09/21/1951	4	N41:19:00	W105:41:00	7270	1.5	4	0
LARAMIE 2 WSW	5420	08/01/1948	12/30/1992	45	N41:18:00	W105:38:00	7180	1.5	4	7
LITTLE MEDICINE 4 NNW	5685	08/01/1948	06/30/1961	14	N42:26:00	W106:02:00	7350	1.3	3	3
LYMAN	5836	10/01/1956	03/31/1960	5	N41:20:00	W110:18:00	6710	1.1	4	0
LYSITE 12 NNW	5850	08/01/1950	10/31/1954	5	N43:26:00	W107:43:00	5600	1.7	1	0
MAIL CAMP	5878	06/01/1950	03/31/1954	5	N43:35:00	W108:51:00	6460	1.4	4	1
MAYOWORTH	6075	11/01/1949	06/24/1958	10	N43:50:00	W106:47:00	5220	1.7	1	0
MEDICINE BOW	6120	08/01/1948	12/31/1992	44	N41:54:00	W106:12:00	6570	1.3	4	11
MEETEETSE	6140	08/01/1948	11/30/1976	29	N44:09:00	W108:51:00	5830	2.4	3	1
MEETEETSE 2	6145	10/01/1976	01/31/1982	7	N44:09:00	W108:53:00	5750	2.4	3	0
MIDDLE FORK	6185	08/01/1948	12/31/1955	8	N42:45:00	W108:48:00	6280	1.9	2	2
MIDWEST 6 N	6200	08/01/1948	10/31/1958	11	N43:30:00	W106:16:00	---	2.1	2	1
	6210	03/01/1987	03/20/1987	1	N--:--:--	W---:--:--	---	---	-	---
MONETA 2 NNE	6381	06/01/1950	07/29/1950	1	N43:10:00	W107:42:00	5520	1.7	1	0
MONETA 21 SSE	6382	08/01/1953	09/30/1955	3	N42:52:00	W107:36:00	---	1.7	1	0
MONETA 25 S	6383	08/01/1950	08/31/1953	4	N42:48:00	W107:43:00	---	1.7	1	0
MOORCROFT CAA	6395	11/01/1948	12/31/1992	39	N44:16:00	W104:57:00	4210	1.6	2	11
MOORCROFT 2	6410	08/01/1948	10/29/1948	1	N44:16:00	W104:56:00	4220	1.6	2	0
MOORCROFT 16 S	6415	12/01/1949	04/29/1950	2	N44:02:00	W104:58:00	4350	1.6	2	0
MORAN 5 WNW	6440	08/01/1948	12/31/1992	45	N43:51:00	W110:35:00	6790	1.6	4	7
MORTON 28 NW	6465	06/01/1950	06/24/1958	9	N43:31:00	W109:04:00	7150	1.5	4	0
MOUNTAIN VIEW	6555	03/01/1966	12/30/1992	27	N41:16:00	W110:21:00	6800	1.1	4	3
MUD SPRINGS	6597	05/01/1953	12/30/1992	40	N41:19:00	W108:55:00	6740	1.1	4	6

Table 2.1 (continued)

Station Name	Station ID	Begin Date	End Date	Rec Years	Latitude	Longitude	Elev.	Threshold (inch)	Region	#. of event
MULE CREEK	6600	12/01/1949	02/29/1984	34	N43:21:00	W104:07:00	4120	1.7	2	14
NEWCASTLE	6660	08/01/1948	12/30/1992	45	N43:51:00	W104:13:00	4410	1.9	2	5
OREGON TRAIL CROSSING	6875	08/01/1948	12/31/1992	45	N42:33:00	W108:11:00	6540	1.9	4	0
OSAGE	6935	03/01/1950	12/28/1992	43	N43:59:00	W104:25:00	4320	1.9	2	8
PATHFINDER DAM	7105	08/01/1948	12/28/1992	45	N42:28:00	W106:51:00	5930	1.8	3	4
PAVILLION	7115	07/01/1949	06/25/1958	10	N43:15:00	W108:41:00	5440	1.4	1	2
PHILLIPS	7200	08/01/1948	12/16/1992	45	N41:38:00	W104:29:00	4980	2.1	2	4
PINE BLUFFS	7235	05/01/1979	03/31/1988	10	N41:11:00	W104:04:00	5070	2.3	2	2
PINE BLUFFS 5 W	7240	08/01/1948	12/05/1992	37	N41:10:00	W104:09:00	5180	2.3	2	5
PINE TREE 9 NE	7270	12/01/1949	03/31/1984	36	N43:43:00	W105:38:00	5110	1.6	2	15
POWDER RIVER SCHOOL	7375	10/01/1958	12/30/1992	35	N43:02:00	W106:59:00	5690	1.5	1	5
POWELL FIELD STN	7388	08/01/1966	12/12/1992	27	N44:47:00	W108:45:00	4370	1.2	1	6
RAVEN	7518	10/17/1950	06/22/1958	9	N43:55:00	W104:55:00	4950	1.9	2	2
RAWLINS FAA AIRPORT	7533	03/01/1951	12/16/1992	42	N41:48:00	W107:12:00	6740	1.5	3	3
RECLUSE	7545	08/01/1948	12/31/1992	45	N44:45:00	W105:42:00	4150	2.2	2	1
RIVERTON USBR	7760	09/01/1955	12/15/1992	38	N43:01:00	W108:23:00	4950	1.4	1	9
RIVERTON USBR	7765	08/01/1949	09/30/1955	7	N43:02:00	W108:22:00	4940	1.4	1	0
RIVERTON 19 N	7780	10/01/1951	11/30/1953	3	N43:18:00	W108:22:00	5000	1.4	1	0
ROCK SPRINGS	7840	04/01/1954	05/31/1979	26	N41:35:00	W109:13:00	6370	1.1	4	4
ROCK SPRINGS FAA AP	7845	08/01/1948	12/30/1992	21	N41:36:00	W109:04:00	6740	1.1	4	3
SAND DRAW 21 ENE	7982	07/01/1949	11/30/1949	1	N42:47:00	W107:51:00	---	2.0	3	0
SARATOGA 4 N	7995	08/01/1948	12/30/1992	45	N41:30:00	W106:48:00	6800	1.5	4	3
	7997	10/01/1967	10/29/1967	1	N--:--:--	W---:--:--	---	---	-	---
	7998	06/01/1967	06/28/1967	1	N--:--:--	W---:--:--	---	---	-	---
SEMINOE DAM	8070	08/01/1948	12/30/1992	45	N42:08:00	W106:53:00	6840	1.9	3	3
SHAWNEE 14 N	8099	11/01/1949	09/30/1957	9	N42:56:00	W105:01:00	4890	1.7	2	4
SHELL	8124	08/01/1953	06/24/1958	6	N44:32:00	W107:46:00	4280	1.6	1	0
SHELL 5 N	8127	09/01/1949	08/31/1953	5	N44:36:00	W107:46:00	4210	1.6	1	0
SHERIDAN WSO AP	8155	08/01/1948	12/30/1992	45	N44:46:00	W106:58:00	3960	1.9	1	8
SHIRLEY BASIN STN	8192	06/01/1961	12/28/1992	32	N42:21:00	W106:10:00	7170	1.3	3	3
SINCLAIR AIRPORT CAA 2	8285	08/01/1948	02/19/1951	4	N41:48:00	W107:03:00	6560	1.5	3	0
STORY	8626	04/01/1950	12/31/1992	43	N44:34:00	W106:54:00	5080	1.9	1	15
SUSSEX 3 W	8801	11/01/1949	06/24/1958	10	N43:42:00	W106:21:00	4500	1.7	2	0
T CROSS RANCH	8820	08/01/1948	09/30/1968	20	N43:43:00	W109:38:00	7840	1.2	3	8
TECKLA 3 E	8830	08/01/1950	04/30/1957	8	N43:35:00	W105:17:00	4780	1.9	2	2
TEN SLEEP 4 NE	8852	05/01/1964	12/31/1992	29	N44:04:00	W107:25:00	4800	2.0	1	3
TEN SLEEP	8855	08/01/1948	05/31/1964	17	N44:02:00	W107:27:00	4510	2.0	1	0
TEN SLEEP 16 SSE	8858	09/01/1979	12/31/1992	14	N43:49:00	W107:22:00	4680	2.0	1	1
THERMOPOLIS	8875	08/01/1949	12/12/1992	44	N43:39:00	W108:12:00	4310	1.5	1	6

Table 2.1 (concluded)

Station Name	Station ID	Begin Date	End Date	Rec Years	Latitude	Longitude	Elev.	Threshold (inch)	Region	#. of event
THERMOPOLIS 25 WNW	8888	05/01/1951	12/30/1992	42	N43:43:00	W108:41:00	5700	1.5	1	7
THERMOPOLIS 27 WNW	8889	09/01/1949	05/31/1951	3	N43:43:00	W108:45:00	5890	1.5	1	0
TOLTEC	8983	05/01/1950	09/13/1951	2	N42:18:00	W105:39:00	---	1.7	3	0
TORRINGTON EXP FARM	8995	08/01/1979	12/16/1992	14	N42:05:00	W104:13:00	4100	2.2	2	0
TORRINGTON 1 S	9000	08/01/1948	07/31/1979	32	N42:03:00	W104:11:00	4090	2.2	2	3
VALLEY 3 NE	9231	08/01/1949	06/25/1958	10	N44:12:00	W109:33:00	6200	1.3	3	1
WAMSUTTER 8 W	9460	08/01/1948	06/20/1951	4	N41:39:00	W108:07:00	---	1.1	4	0
WAPITI 2 E	9467	06/01/1950	10/31/1950	1	N44:28:00	W109:25:00	5560	1.3	3	0
WAPITI 9 W	9471	08/01/1949	02/28/1950	2	N44:29:00	W109:36:00	---	1.3	3	0
WHEATLAND 4 N	9615	08/01/1948	12/13/1992	43	N42:07:00	W104:57:00	4640	2.0	2	3
WHEATLAND 2	9621	03/01/1967	06/30/1970	4	N42:03:00	W104:57:00	4760	2.0	2	1
WORLAND	9770	03/01/1964	08/31/1979	16	N44:01:00	W107:58:00	4060	1.3	1	4
WORLAND 5 SW	9775	08/01/1948	03/31/1964	17	N43:57:00	W108:02:00	4150	1.3	1	0
WORLAND 18 W	9780	11/01/1950	12/08/1950	1	N44:00:00	W108:19:00	4750	1.3	1	0
WORLAND 21 W	9782	08/01/1949	09/30/1950	2	N44:00:00	W108:24:00	---	1.3	1	0
WRIGHT	9800	07/01/1982	09/30/1984	3	N43:42:00	W105:25:00	5010	1.6	2	0

2.3.1 Duration-Based Data Set

For duration-based storms, the maximum storm depths for a specified storm duration were searched from the compiled significant storms. Specifically, the maximum depth for storm durations of 6-hr, 12-hr, 18-hr, and 24-hr were obtained. Through the data compilation, there are 439 6-hr cases, 410 12-hr cases, 394 18-hr cases, and 368 24-hr cases obtained in the duration-based database. Non-dimensionalization of all storm cases for all four durations results in a total of 1611 dimensionless rainfall mass curves.

Based on the maximum depth for the four durations mentioned above, the cumulative percentage of rainfall in the mass curve at T/6, 2T/6, 3T/6, 4T/6, and 5T/6 were used as the attributes in cluster analysis. These cumulative percentages can be easily calculated for the durations considered herein. To compute the statistical moments of the dimensionless time, the dimensionless rainfall hyetograph representing the storm depth percentages at t' = 1/6, 2/6, 3/6, 4/6, 5/6, 6/6 were used, rather than the hourly percentages.

2.3.2 Event-Based Data Set

The first four statistical moments of the dimensionless time for event-based storm cases were calculated using Eqs. (2.1)-(2.4) based on hourly storm depth percentages. As for the cumulative percentages at T/6, 2T/6, 3T/6, 4T/6, and 5T/6, these values generally are not directly available because the storm duration may not necessarily be a multiple of 6. Therefore, to compute these cumulative percentages, the assumption of uniform rainfall intensity within each hour was made. Then, the calculation of cumulative percentages can be easily made by dividing storm duration into 6 equal parts and summing up all the percentages sequentially. Alternatively, an interpolation method can be applied to calculate the cumulative percentages. The assumption of uniform intensity within each hour is appropriate when the storm duration is long enough. However, if the storm duration is very short, say, less than 3 hours, this assumption may not be appropriate. Hence, storm events with duration less than 3 hours were dropped from the database. Originally, there were 379 events in the event-based database extracted from the compiled significant storms. After removing 4 events with storm duration equal to 1-hr and 13 events with storm duration equal to 2-hr, a total of 362 cases were used in the cluster analysis.

2.4 CLUSTER ANALYSIS

Statistical Analysis Systems (SAS) procedure, CLUSTER, was used to perform the cluster analysis to identify representative temporal patterns for both duration-based and event-based storms. To remove the scale effects in the cluster analysis, all attribute variables were standardized resulting in zero mean and unit standard deviation for all attributes. There are alternative ways to remove scale effect. However, the difference between the classification results using different standardization procedures was found negligible. In SAS, several methods can be used to conduct cluster analysis and different methods define similarity matrix and the distance measurement differently. A preliminary investigation indicated that their effects on the final classification result for the data in hand were nil and, therefore, the average linkage method was adopted.

In the cluster analysis, two different characterizations of temporal distribution of storm were used: (1) cumulative percentages of storm depth and (2) statistical moments of the dimensionless time. The use of the two characterizations resulted in somewhat different classifications of storm patterns. Since the number of representative storm patterns is not known in advance and, therefore, they can only be determined subjectively through a trial-and-error process.

When the statistical moments of dimensionless time were used, the concerns were how many and which moments should be used in the cluster analysis. Because there is no guideline as to how many moments should be used in the cluster analysis, selection was made by trial-and-error. It was found that the results of cluster analysis are extremely sensitive to the skew coefficient and kurtosis. It appears that the use of the first two moments, that is, mean and standard deviation, of dimensionless time would yield reasonable storm classification.

Once the storm patterns were obtained using the two methods, the patterns were compared so as to describe storm patterns typically occurring in Wyoming for both duration-based and event-based storms. It turns out using cumulative percentages leads to better clusters than those obtained using the first two central moments. The suggested storm patterns are shown in Figs. 2.2 and 2.3, respectively.

It should be noticed that some patterns only have a few cases (see Table 2.2). For example, for duration-based storm patterns, patterns 5-8 have 12, 7, 4 and 7 cases, respectively, while pattern 1 has 1000 cases. Generally, while performing cluster analysis, if some clusters have much fewer cases than others, these clusters should be combined into

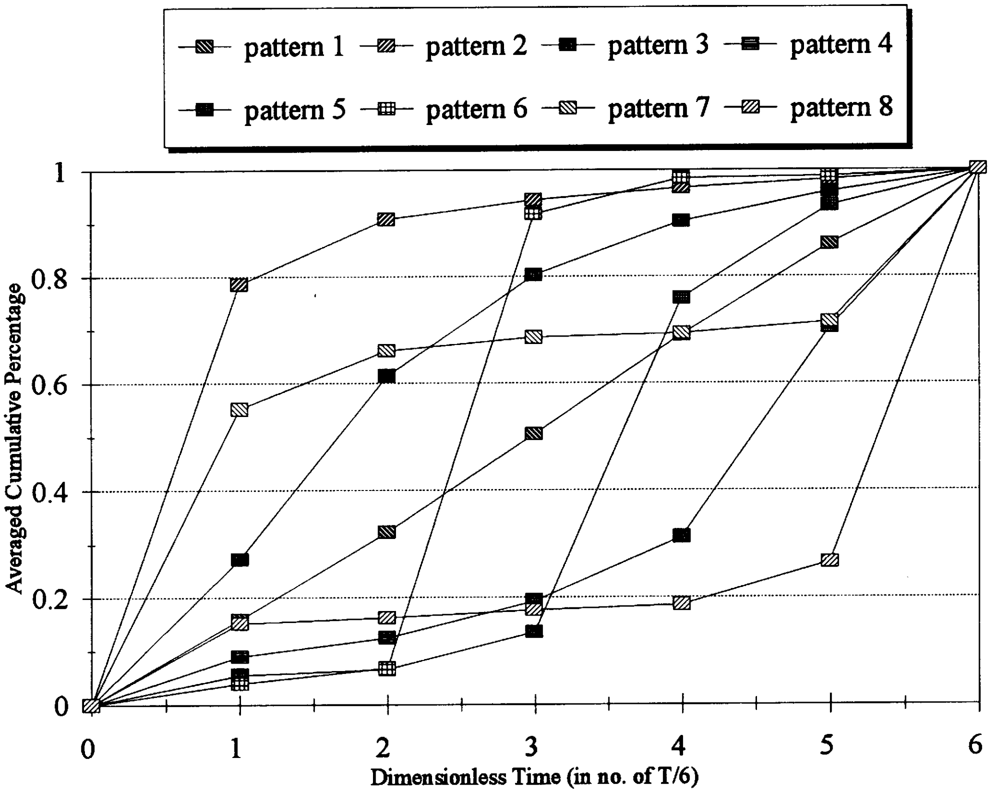


Fig. 2.2. Eight Representative Duration-Based Storm Patterns in Wyoming

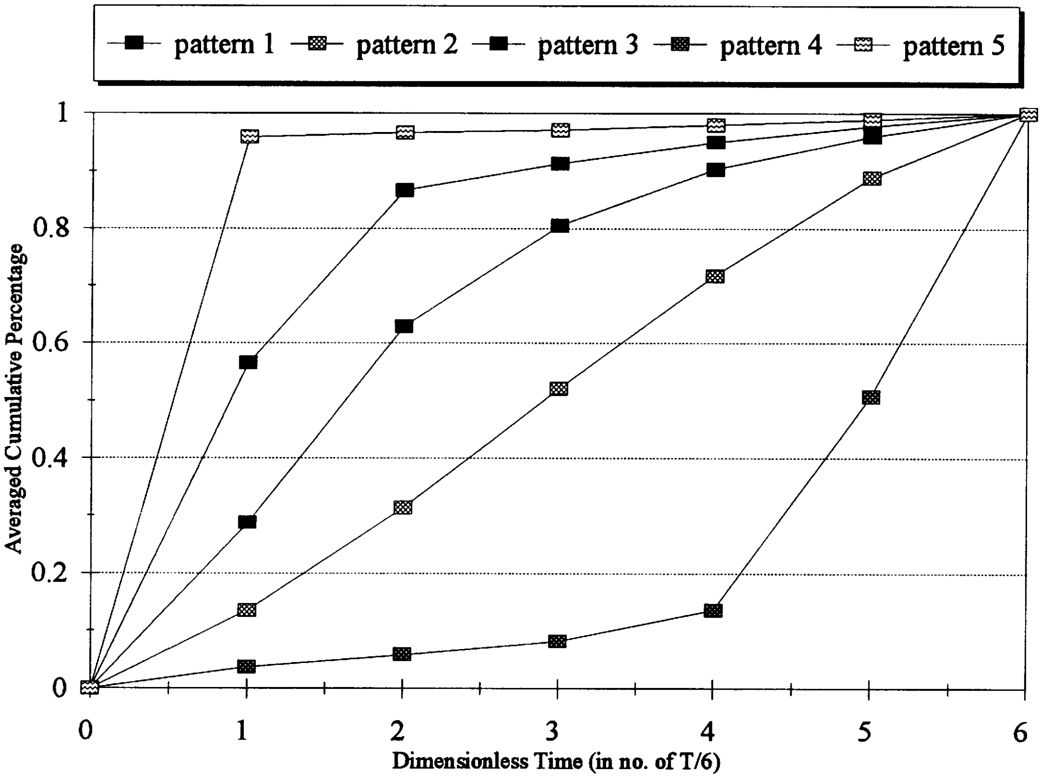


Fig. 2.3. Five Representative Event-Based Storm Patterns in Wyoming

others or cluster analysis should be performed again so that there will not be significant differences in the frequency of each cluster. The reason these clusters were not combined in the study is that each of these clusters presents a distinctly unique storm pattern which is of interest.

Tables 2.2 and 2.3 show the statistical moments (in upper portion) and the correlations between dimensionless rainfall hyetograph ordinates (the lower portion) for the duration-based and event-based storm patterns, respectively. Furthermore, Tables 2.4 and 2.5 show the sample properties of mean, standard deviation, skewness and kurtiosis for the duration-based and event-based storm patterns, respectively. Information presented in these tables can be used to perform uncertainty analysis and Monte Carlo simulation.

2.5 FACTORS AFFECTING THE OCCURRENCE OF STORM PATTERNS

It should be noted that the suggested patterns presented in Figs 2.2 and 2.3 are derived from combining rainfall mass curves and hyetographs of different durations that have occurred in different months over the entire State of Wyoming. It is important to examine whether a particular storm pattern would occur in a certain season or climatic region, or whether it is dependent on storm duration. Such information is essential to avoid erroneous selection of storm patterns in the design and performance evaluation of hydraulic structures.

To test the existence of possible relation or dependence between storm pattern and climatic region, storm duration, or season, chi-square tests were performed using SAS

Table 2.2 Sample Properties of Dimensionless Rainfall Hyetograph Ordinates for Duration-Based Storm Patterns

(a) Pattern 1 (1000 Cases)

	P1	P2	P3	P4	P5	P6
Mean	0.15891	0.16380	0.18268	0.18560	0.16873	0.14029
Stdev	0.07720	0.07149	0.09110	0.08548	0.07633	0.07764
Skewness	0.65027	0.32196	1.19350	0.83840	0.41000	0.37661
Kurtosis	3.72483	3.54807	5.92435	4.58673	3.6146	3.24666
Minimum	0.00327	0.00000	0.00000	0.00000	0.00000	0.00000
Maximum	0.50000	0.46087	0.65625	0.56250	0.47619	0.45455
P1	1.00000	-0.14052	-0.40608	-0.29759	-0.21022	0.14590
P2	-0.14052	1.00000	-0.06842	-0.29414	-0.24383	-0.13720
P3	-0.40608	-0.06842	1.00000	0.01874	-0.30635	-0.42604
P4	-0.29759	-0.29414	0.01874	1.00000	-0.12957	-0.42887
P5	-0.21022	-0.24383	-0.30635	-0.12957	1.00000	-0.04748
P6	0.14590	-0.13720	-0.42604	-0.42887	-0.04748	1.00000

(b) Pattern 2 (234 Cases)

	P1	P2	P3	P4	P5	P6
Mean	0.78651	0.12099	0.03599	0.02318	0.01495	0.01837
Stdev	0.16531	0.11856	0.05320	0.04585	0.03353	0.03897
Skewness	-0.31510	0.96983	1.88161	2.72222	3.32915	2.51301
Kurtosis	1.76890	3.26556	6.85497	10.8720	16.4610	9.16871
Minimum	0.46667	0.00000	0.00000	0.00000	0.00000	0.00000
Maximum	1.00000	0.48837	0.27273	0.25806	0.22222	0.21586
P1	1.00000	-0.78978	-0.49540	-0.37907	-0.38247	-0.38770
P2	-0.78978	1.00000	0.15683	-0.01416	0.06031	0.05855
P3	-0.49540	0.15683	1.00000	0.15858	0.00892	0.06477
P4	-0.37907	-0.01416	0.15858	1.00000	0.20376	0.08254
P5	-0.38247	0.06031	0.00892	0.20376	1.00000	0.32649
P6	-0.38770	0.05855	0.06477	0.08254	0.32649	1.00000

(c) Pattern 3 (312 Cases)

	P1	P2	P3	P4	P5	P6
Mean	0.27244	0.34217	0.18809	0.10093	0.05525	0.04112
Stdev	0.11982	0.18609	0.11601	0.07592	0.05408	0.04755
Skewness	-0.20223	0.92652	0.72358	0.49444	1.05751	1.14574
Kurtosis	2.25020	3.74566	4.20065	2.83959	3.8973	3.64115
Minimum	0.00459	0.00000	0.00000	0.00000	0.00000	0.00000
Maximum	0.54472	0.91781	0.65728	0.38000	0.28571	0.21121
P1	1.00000	-0.58518	-0.24033	-0.00591	0.11171	0.23906
P2	-0.58518	1.00000	-0.38447	-0.49986	-0.34521	-0.31032
P3	-0.24033	-0.38447	1.00000	0.10841	-0.22726	-0.24411
P4	-0.00591	-0.49986	0.10841	1.00000	0.14363	-0.05326
P5	0.11171	-0.34521	-0.22726	0.14363	1.00000	0.25728
P6	0.23906	-0.31032	-0.24411	-0.05326	0.25728	1.00000

Table 2.2 (continued)

(d) Pattern 4 (35 Cases)

	P1	P2	P3	P4	P5	P6
Mean	0.08905	0.03574	0.06910	0.12008	0.39072	0.29532
Stdev	0.05518	0.04753	0.05747	0.10606	0.25797	0.15449
Skewness	1.20110	1.28648	0.03695	1.07608	0.63363	-0.45040
Kurtosis	5.39192	3.80568	4.59169	4.79366	2.61060	2.25763
Minimum	0.00697	0.00000	0.00000	0.00000	0.05000	0.00000
Maximum	0.26667	0.15789	0.15789	0.46182	0.98266	0.55833
P1	1.00000	0.02820	0.04855	-0.26675	-0.14499	0.04135
P2	0.02820	1.00000	0.43316	-0.03172	-0.58846	0.52553
P3	0.04855	0.43316	1.00000	0.12460	-0.65960	0.49328
P4	-0.26675	-0.03172	0.12460	1.00000	-0.46272	0.14482
P5	-0.14499	-0.58846	-0.65960	-0.46272	1.00000	-0.87397
P6	0.04135	0.52553	0.49328	0.14482	-0.87397	1.00000

(e) Pattern 5 (12 Cases)

	P1	P2	P3	P4	P5	P6
Mean	0.05530	0.01067	0.06895	0.62493	0.17323	0.06693
Stdev	0.02504	0.02208	0.06303	0.18865	0.13135	0.07096
Skewness	-0.08185	1.95680	0.38498	0.82074	-0.00196	0.56396
Kurtosis	2.09089	5.46921	4.02956	2.45352	1.54660	1.88735
Minimum	0.01168	0.00000	0.00000	0.41818	0.00000	0.00000
Maximum	0.09091	0.06135	0.17290	0.95082	0.36970	0.19626
P1	1.00000	-0.30999	-0.24833	0.00147	0.19588	-0.40238
P2	-0.30999	1.00000	0.27904	-0.28870	-0.07158	0.45035
P3	-0.24833	0.27904	1.00000	-0.68684	0.31714	0.35147
P4	0.00147	-0.28870	-0.68684	1.00000	-0.80598	-0.46717
P5	0.19588	-0.07158	0.31714	-0.80598	1.00000	-0.03694
P6	-0.40238	0.45035	0.35147	-0.46717	-0.03694	1.00000

(f) Pattern 6 (7 Cases)

	P1	P2	P3	P4	P5	P6
Mean	0.03908	0.02842	0.85065	0.06624	0.00416	0.01145
Stdev	0.02592	0.03823	0.09577	0.08835	0.00546	0.01904
Skewness	0.72680	0.95586	-0.43525	1.17866	0.74256	2.19780
Kurtosis	2.74339	2.58520	4.19136	2.61770	1.53860	8.00390
Minimum	0.00952	0.00000	0.70205	0.00000	0.00000	0.00000
Maximum	0.08333	0.09496	0.96667	0.21575	0.01274	0.05263
P1	1.00000	-0.26724	-0.02530	-0.12788	-0.66489	0.08663
P2	-0.26724	1.00000	-0.37432	0.03327	-0.37975	0.19310
P3	-0.02530	-0.37432	1.00000	-0.88487	0.32388	-0.23069
P4	-0.12788	0.03327	-0.88487	1.00000	-0.04026	-0.07062
P5	-0.66489	-0.37975	0.32388	-0.04026	1.00000	-0.06139
P6	0.08663	0.19310	-0.23069	-0.07062	-0.06139	1.00000

Table 2.2 (concluded)

(g) Pattern 7 (4 Cases)

	P1	P2	P3	P4	P5	P6
Mean	0.55210	0.10910	0.02526	0.00725	0.02132	0.28497
Stdev	0.17159	0.16343	0.02967	0.01449	0.02402	0.02555
Skewness	-1.20293	1.66397	1.45878	2.00000	0.69370	-0.77118
Kurtosis	3.58836	5.64049	5.28694	7.00000	1.1250	3.42907
Minimum	0.31556	0.00000	0.00000	0.00000	0.00000	0.25121
Maximum	0.68994	0.34667	0.06763	0.02899	0.05185	0.31111
P1	1.00000	-0.98650	-0.31357	-0.06165	0.35828	-0.34320
P2	-0.98650	1.00000	0.17827	-0.09034	-0.44530	0.49136
P3	-0.31357	0.17827	1.00000	0.95211	-0.04436	-0.69395
P4	-0.06165	-0.09034	0.95211	1.00000	0.21278	-0.88086
P5	0.35828	-0.44530	-0.04436	0.21278	1.00000	-0.56679
P6	-0.34320	0.49136	-0.69395	-0.88086	-0.56679	1.00000

(h) Pattern 8 (7 Cases)

	P1	P2	P3	P4	P5	P6
Mean	0.15109	0.01202	0.01335	0.01068	0.07952	0.73335
Stdev	0.11393	0.03179	0.03532	0.02826	0.11559	0.11576
Skewness	0.73517	2.64575	2.64575	2.64575	1.29237	1.34272
Kurtosis	1.73058	10.0000	10.0000	10.0000	2.92060	4.55754
Minimum	0.04545	0.00000	0.00000	0.00000	0.00000	0.61624
Maximum	0.32895	0.08411	0.09346	0.07477	0.28044	0.95455
P1	1.00000	-0.36776	-0.36776	-0.36776	-0.39868	-0.28307
P2	-0.36776	1.00000	1.00000	1.00000	-0.16073	-0.30146
P3	-0.36776	1.00000	1.00000	1.00000	-0.16073	-0.30146
P4	-0.36776	1.00000	1.00000	1.00000	-0.16073	-0.30146
P5	-0.39868	-0.16073	-0.16073	-0.16073	1.00000	-0.47375
P6	-0.28307	-0.30146	-0.30146	-0.30146	-0.47375	1.00000

Table 2.3 Sample Properties of Dimensionless Rainfall Hyetograph Ordinates for Event-Based Storm Patterns

(a) Pattern 1 (20 Cases)

	P1	P2	P3	P4	P5	P6
Mean	0.56493	0.30213	0.04621	0.03766	0.02743	0.02165
Stdev	0.12254	0.15697	0.04115	0.039019	0.026941	0.025426
Skewness	-0.01237	0.42394	1.74489	2.16138	1.20082	2.89517
Kurtosis	3.12578	2.60604	7.74625	9.05658	3.16847	13.2654
Minimum	0.2890	0.0355	0.0000	0.0000	0.0020	0.0021
Maximun	0.7972	0.6438	0.1785	0.1684	0.0869	0.1159
P1	1.00000	-0.86691	0.19867	0.14112	0.07639	-0.08632
P2	-0.86691	1.00000	-0.45572	-0.47658	-0.37371	-0.13090
P3	0.19867	-0.45572	1.00000	0.34893	-0.04688	-0.24813
P4	0.14112	-0.47658	0.34893	1.00000	0.19569	-0.04451
P5	0.07639	-0.37371	-0.04688	0.19569	1.00000	0.65500
P6	-0.08632	-0.13090	-0.24813	-0.04451	0.65500	1.00000

(b) Pattern 2 (279 Cases)

	P1	P2	P3	P4	P5	P6
Mean	0.13578	0.17851	0.20742	0.19640	0.17133	0.11056
Stdev	0.08672	0.08777	0.10285	0.087786	0.083845	0.073142
Skewness	1.23863	0.67008	0.87836	0.56786	0.42606	0.98966
Kurtosis	5.35083	3.52969	4.81377	3.48551	3.29527	4.1294
Minimum	0.0043	0.0000	0.0184	0.0160	0.0000	0.0014
Maximun	0.5315	0.4697	0.7020	0.5101	0.4935	0.3939
P1	1.00000	-0.04580	-0.32750	-0.37773	-0.17672	-0.01418
P2	-0.04580	1.00000	-0.11001	-0.38976	-0.27246	-0.21083
P3	-0.32750	-0.11001	1.00000	0.17198	-0.56140	-0.44879
P4	-0.37773	-0.38976	0.17198	1.00000	-0.11883	-0.39030
P5	-0.17672	-0.27246	-0.56140	-0.11883	1.00000	0.32219
P6	-0.01418	-0.21083	-0.44879	-0.39030	0.32219	1.00000

(c) Pattern 3 (60 Cases)

	P1	P2	P3	P4	P5	P6
Mean	0.28764	0.34115	0.17689	0.09749	0.05713	0.03971
Stdev	0.11810	0.16059	0.08436	0.055440	0.040917	0.027855
Skewness	-0.84471	1.51805	0.27345	0.59713	1.12658	0.93793
Kurtosis	3.50826	5.04292	3.73628	3.32028	4.67760	3.2905
Minimum	0.0038	0.1182	0.0154	0.0032	0.0000	0.0023
Maximun	0.4902	0.8227	0.3544	0.2442	0.2000	0.1218
P1	1.00000	-0.82883	-0.18864	0.28921	0.19269	0.25092
P2	-0.82883	1.00000	-0.23674	-0.54647	-0.13153	-0.25304
P3	-0.18864	-0.23674	1.00000	0.09701	-0.51400	-0.30187
P4	0.28921	-0.54647	0.09701	1.00000	-0.16808	-0.11302
P5	0.19269	-0.13153	-0.51400	-0.16808	1.00000	0.36354
P6	0.25092	-0.25304	-0.30187	-0.11302	0.36354	1.00000

Table 2.3 (concluded)

(d) Pattern 4 (2 Cases)

	P1	P2	P3	P4	P5	P6
Mean	0.03700	0.02180	0.02375	0.05405	0.37105	0.49235
Stdev	0.03338	0.01202	0.03359	0.009263	0.096237	0.075165
Skewness	.	.	.	.	.	.
Kurtosis	.	.	.	.	.	.
Minimum	0.0134	0.0133	0.0000	0.0475	0.3030	0.4392
Maximun	0.0606	0.0303	0.0475	0.0606	0.4391	0.5455
P1	1.00000	1.00000	-1.00000	1.00000	-1.00000	1.00000
P2	1.00000	1.00000	-1.00000	1.00000	-1.00000	1.00000
P3	-1.00000	-1.00000	1.00000	-1.00000	1.00000	-1.00000
P4	1.00000	1.00000	-1.00000	1.00000	-1.00000	1.00000
P5	-1.00000	-1.00000	1.00000	-1.00000	1.00000	-1.00000
P6	1.00000	1.00000	-1.00000	1.00000	-1.00000	1.00000

(e) Pattern 5 (1 Case)

	P1	P2	P3	P4	P5	P6
Mean	0.95900	0.00820	0.00490	0.00820	0.00990	0.00980
Stdev	.	.	.	.	.	.
Skewness	.	.	.	.	.	.
Kurtosis	.	.	.	.	.	.
Minimum	0.9590	0.0082	0.0049	0.0082	0.0099	0.0098
Maximun	0.9590	0.0082	0.0049	0.0082	0.0099	0.0098
P1	.	.	.	.	.	.
P2	.	.	.	.	.	.
P3	.	.	.	.	.	.
P4	.	.	.	.	.	.
P5	.	.	.	.	.	.
P6	.	.	.	.	.	.

Table 2.4 Sample Properties of Statistical Moments for Duration-Based Storm Patterns

(a) Pattern 1 (1000 Cases)

	MEAN	STDEV	SKEW	KURT
Mean	0.57705	0.26626	-0.01908	1.9590
Stdev	0.054679	0.036976	0.31146	0.4107
Skewness	0.17866	-0.59779	-0.14946	1.77971
Kurtosis	-2.42668	3.79799	3.12001	8.4008
Minimum	0.44414	0.13878	-1.29837	1.23769
Maximum	0.72827	0.37602	1.0765	4.480
MEAN	1.00000	0.07587	-0.72854	-0.01149
STDEV	0.07587	1.00000	0.14208	-0.82320
SKEW	-0.72854	0.14208	1.00000	-0.10573
KURT	-0.01149	-0.82320	-0.10573	1.00000

(b) Pattern 2 (234 Cases)

	MEAN	STDEV	SKEW	KURT
Mean	0.23570	0.12743	4.23433	42.0917
Stdev	0.063458	0.087048	4.71162	84.4262
Skewness	0.99821	0.48078	1.93793	2.38442
Kurtosis	3.27128	2.41723	5.51261	7.0029
Minimum	0.16667	0.00000	0.19768	1.03908
Maximum	0.42424	0.34285	16.8227	284.004
MEAN	1.00000	0.91985	-0.57754	-0.46472
STDEV	0.91985	1.00000	-0.62268	-0.56335
SKEW	-0.57754	-0.62268	1.00000	0.97627
KURT	-0.46472	-0.56335	0.97627	1.00000

(c) Pattern 3 (312 Cases)

	MEAN	STDEV	SKEW	KURT
Mean	0.40796	0.20901	0.68893	3.2699
Stdev	0.055127	0.061823	0.63204	2.5051
Skewness	-0.54604	-0.43880	0.95708	3.93602
Kurtosis	2.40771	2.55564	8.43210	23.0126
Minimum	0.26923	0.04690	-1.86734	1.35099
Maximum	0.50407	0.32457	4.3070	23.263
MEAN	1.00000	0.69284	-0.15383	-0.39848
STDEV	0.69284	1.00000	0.10856	-0.44215
SKEW	-0.15383	0.10856	1.00000	0.64248
KURT	-0.39848	-0.44215	0.64248	1.00000

Table 2.4 (continued)

(d) Pattern 4 (35 Cases)

	MEAN	STDEV	SKEW	KURT
Mean	0.76227	0.23932	-1.54164	6.4713
Stdev	0.046296	0.058490	1.41015	10.3372
Skewness	-0.21892	-0.75946	-2.97586	3.94545
Kurtosis	2.72160	3.51287	12.80108	19.5816
Minimum	0.64444	0.08703	-7.39488	1.51958
Maximum	0.84375	0.32646	-0.3930	55.684
MEAN	1.00000	-0.69069	-0.56293	0.46550
STDEV	-0.69069	1.00000	0.80948	-0.75003
SKEW	-0.56293	0.80948	1.00000	-0.97288
KURT	0.46550	-0.75003	-0.97288	1.00000

(e) Pattern 5 (12 Cases)

	MEAN	STDEV	SKEW	KURT
Mean	0.67515	0.16453	-1.64648	8.3641
Stdev	0.030318	0.036024	1.38092	7.9096
Skewness	-0.72510	-1.47604	-1.16709	2.04950
Kurtosis	4.56000	4.17932	3.56410	6.1809
Minimum	0.60606	0.08760	-4.39406	2.86498
Maximum	0.71729	0.19849	0.0149	27.830
MEAN	1.00000	0.49392	0.59880	-0.39164
STDEV	0.49392	1.00000	0.94100	-0.95162
SKEW	0.59880	0.94100	1.00000	-0.92384
KURT	-0.39164	-0.95162	-0.92384	1.00000

(f) Pattern 6 (7 Cases)

	MEAN	STDEV	SKEW	KURT
Mean	0.50039	0.09631	-0.21800	14.8512
Stdev	0.019770	0.028299	2.29034	12.5256
Skewness	-0.51342	0.73657	-0.14673	2.20576
Kurtosis	2.40126	4.36486	1.30581	7.9995
Minimum	0.47222	0.05963	-3.01511	6.93485
Maximum	0.52740	0.14783	2.8154	41.946
MEAN	1.00000	0.40460	0.67233	0.00221
STDEV	0.40460	1.00000	0.10347	-0.63734
SKEW	0.67233	0.10347	1.00000	0.61578
KURT	0.00221	-0.63734	0.61578	1.00000

Table 2.4 (concluded)

(g) Pattern 7 (4 Cases)

	MEAN	STDEV	SKEW	KURT
Mean	0.44858	0.36681	0.74479	1.6700
Stdev	0.032639	0.016322	0.09263	0.1018
Skewness	1.19444	-0.03630	0.94059	-0.04790
Kurtosis	4.86485	-5.50742	4.99697	-4.2730
Minimum	0.41760	0.35061	0.64842	1.56107
Maximum	0.49407	0.38221	0.8713	1.776
MEAN	1.00000	-0.81623	-0.89288	-0.48233
STDEV	-0.81623	1.00000	0.69183	-0.02318
SKEW	-0.89288	0.69183	1.00000	0.69821
KURT	-0.48233	-0.02318	0.69821	1.00000

(h) Pattern 8 (7 Cases)

	MEAN	STDEV	SKEW	KURT
Mean	0.84259	0.28432	-2.00991	6.7981
Stdev	0.081159	0.081805	1.29577	6.7107
Skewness	0.01637	0.00817	-1.09530	1.5838
Kurtosis	2.39073	1.52930	3.54376	5.1438
Minimum	0.72588	0.17358	-4.36436	1.53020
Maximum	0.96212	0.39153	-0.7281	20.048
MEAN	1.00000	-0.95967	-0.94895	0.89762
STDEV	-0.95967	1.00000	0.90873	-0.86897
SKEW	-0.94895	0.90873	1.00000	-0.98704
KURT	0.89762	-0.86897	-0.98704	1.00000

Table 2.5 Sample Properties of Statistical Moments for Event-Based Storm Patterns

(a) Pattern 1 (20 Cases)

	MEAN	STDEV	SKEW	KURT
Mean	0.29735	0.16761	3.10274	16.9518
Stdev	0.054786	0.059110	2.09915	20.9944
Skewness	0.68563	-0.01909	1.62745	2.14652
Kurtosis	2.49417	3.00826	5.00027	6.7679
Minimum	0.2189	0.0654	1.0840	2.8614
Maximum	0.4048	0.2925	8.5520	76.3027
MEAN	1.00000	0.00306	0.31801	0.30265
STDEV	0.00306	1.00000	-0.84073	-0.79998
SKEW	0.31801	-0.84073	1.00000	0.98030
KURT	0.30265	-0.79998	0.98030	1.00000

(b) Pattern 2 (279 Cases)

	MEAN	STDEV	SKEW	KURT
Mean	0.52457	0.24517	0.01443	2.3165
Stdev	0.077808	0.043023	0.41913	0.9115
Skewness	0.35766	-0.53805	-0.38058	4.21279
Kurtosis	2.67511	3.50386	3.39458	29.9452
Minimum	0.3683	0.1032	-1.5307	1.3232
Maximum	0.7424	0.3499	1.0432	10.2557
MEAN	1.00000	-0.19982	-0.75502	0.20466
STDEV	-0.19982	1.00000	0.07760	-0.75273
SKEW	-0.75502	0.07760	1.00000	-0.10134
KURT	0.20466	-0.75273	-0.10134	1.00000

(c) Pattern 3 (60 Cases)

	MEAN	STDEV	SKEW	KURT
Mean	0.38729	0.21364	1.14061	4.2796
Stdev	0.057627	0.044941	0.68187	3.8131
Skewness	0.56961	-0.79310	2.35609	3.53906
Kurtosis	2.49536	3.20065	10.8362	16.6867
Minimum	0.2874	0.0992	0.2416	1.7521
Maximum	0.5168	0.2878	4.3070	23.2626
MEAN	1.00000	-0.00034	-0.06693	-0.10527
STDEV	-0.00034	1.00000	-0.64458	-0.74620
SKEW	-0.06693	-0.64458	1.00000	0.92430
KURT	-0.10527	-0.74620	0.92430	1.00000

Table 2.5 (concluded)

(d) Pattern 4 (2 Cases)

	MEAN	STDEV	SKEW	KURT
Mean	0.92980	0.18080	-2.68290	9.2991
Stdev	0.029274	0.055861	0.38650	2.7914
Skewness	.	.	.	.
Kurtosis	.	.	.	.
Minimum	0.9091	0.1413	-2.9562	7.3253
Maximum	0.9505	0.2203	-2.4096	11.2729
MEAN	1.00000	-1.00000	-1.00000	1.00000
STDEV	-1.00000	1.00000	1.00000	-1.00000
SKEW	-1.00000	1.00000	1.00000	-1.00000
KURT	1.00000	-1.00000	-1.00000	1.00000

(e) Pattern 5 (1 Case)

	MEAN	STDEV	SKEW	KURT
Mean	0.13440	0.12110	5.60300	33.9496
Stdev	.	.	.	.
Skewness	.	.	.	.
Kurtosis	.	.	.	.
Minimum	0.1344	0.1211	5.6030	33.9496
Maximum	0.1344	0.1211	5.6030	33.9496
MEAN	.	.	.	.
STDEV	.	.	.	.
SKEW	.	.	.	.
KURT	.	.	.	.

procedure FREQ. The null hypothesis that two variables are independent can be tested by the statistic, χ^2, given by

$$\chi^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{\left(N_{ij} - \frac{N_{i\cdot} N_{\cdot j}}{n} \right)^2}{\frac{N_{i\cdot} N_{\cdot j}}{n}} \tag{2.5}$$

where r = number of rows in the contingency table; c = number of columns in the contingency table; N_{ij} = number of occurrences in the ith row and jth column; $N_{i.}$ = number of occurrences in the ith row; $N_{.j}$ = number of occurrences in the jth column; and n = total number of occurrence. The statistic χ^2 has a chi-square distribution with $(r\text{-}1) \times (c\text{-}1)$ degrees of freedom, provided that the two variables are independent. Acceptance or rejection of the hypothesis is based on the probability of the obtained χ^2 value (See Everitt, 1992, for a more detailed discussion of tests about contingency table).

The contingency tables and the corresponding tests for the duration-based storm patterns are shown in Tables 2.6 - 2.9. Tables 2.10 - 2.13 are the results for the event-based storm patterns. It should be noticed that in these tables, some cells have expected counts less than 5. It might be argued that the chi-square test is not appropriate for contingency tables. However, Cochran (1952) pointed out that 5, sometimes even 2, is too conservative. Although literature exists for finding the exact p-value for the contingency table (Mehta and Patel, 1983), its calculation is not implemented herein.

Table 2.6 Contingency Tables for Duration-Based Storm Patterns (All 8 Patterns Included)

(a) By Region

```
CLUSTER          REGION

Frequency       |
Expected        |
Cell Chi-Square |
Row Pct         |
Col Pct         |       1|       2|       3|       4|  Total
----------------+--------+--------+--------+--------+
              1 |   267  |   444  |   115  |   174  |   1000
                | 232.77 | 471.14 | 109.87 | 186.22 |
                | 5.0322 | 1.5629 | 0.2396 | 0.8019 |
                |  26.70 |  44.40 |  11.50 |  17.40 |
                |  71.20 |  58.50 |  64.97 |  58.00 |
----------------+--------+--------+--------+--------+
              2 |    28  |   134  |    17  |    55  |    234
                | 54.469 | 110.25 | 25.709 | 43.575 |
                | 12.863 | 5.1182 | 2.9505 | 2.9953 |
                |  11.97 |  57.26 |   7.26 |  23.50 |
                |   7.47 |  17.65 |   9.60 |  18.33 |
----------------+--------+--------+--------+--------+
              3 |    65  |   152  |    40  |    55  |    312
                | 72.626 | 146.99 | 34.279 | 58.101 |
                | 0.8007 | 0.1705 | 0.9547 | 0.1655 |
                |  20.83 |  48.72 |  12.82 |  17.63 |
                |  17.33 |  20.03 |  22.60 |  18.33 |
----------------+--------+--------+--------+--------+
              4 |     9  |    14  |     3  |     9  |     35
                | 8.1471 |  16.49 | 3.8454 | 6.5177 |
                | 0.0893 | 0.3759 | 0.1859 | 0.9454 |
                |  25.71 |  40.00 |   8.57 |  25.71 |
                |   2.40 |   1.84 |   1.69 |   3.00 |
----------------+--------+--------+--------+--------+
              5 |     4  |     4  |     1  |     3  |     12
                | 2.7933 | 5.6536 | 1.3184 | 2.2346 |
                | 0.5213 | 0.4837 | 0.0769 | 0.2621 |
                |  33.33 |  33.33 |   8.33 |  25.00 |
                |   1.07 |   0.53 |   0.56 |   1.00 |
----------------+--------+--------+--------+--------+
              6 |     1  |     5  |     0  |     1  |      7
                | 1.6294 |  3.298 | 0.7691 | 1.3035 |
                | 0.2431 | 0.8784 | 0.7691 | 0.0707 |
                |  14.29 |  71.43 |   0.00 |  14.29 |
                |   0.27 |   0.66 |   0.00 |   0.33 |
----------------+--------+--------+--------+--------+
              7 |     1  |     2  |     0  |     1  |      4
                | 0.9311 | 1.8845 | 0.4395 | 0.7449 |
                | 0.0051 | 0.0071 | 0.4395 | 0.0874 |
                |  25.00 |  50.00 |   0.00 |  25.00 |
                |   0.27 |   0.26 |   0.00 |   0.33 |
----------------+--------+--------+--------+--------+
              8 |     0  |     4  |     1  |     2  |      7
                | 1.6294 |  3.298 | 0.7691 | 1.3035 |
                | 1.6294 | 0.1494 | 0.0693 | 0.3721 |
                |   0.00 |  57.14 |  14.29 |  28.57 |
                |   0.00 |   0.53 |   0.56 |   0.67 |
----------------+--------+--------+--------+--------+
Total               375      759      177      300     1611
```

Table 2.6 (continued)

(b) By Duration

```
CLUSTER        DURATION

Frequency      |
Expected       |
Cell Chi-Square|
Row Pct        |
Col Pct        |       6|      12|      18|      24|  Total
---------------+--------+--------+--------+--------+
             1 |    311 |    271 |    237 |    181 |   1000
               |  272.5 |  254.5 | 244.57 | 228.43 |
               |  5.439 | 1.0697 | 0.2342 | 9.8479 |
               |  31.10 |  27.10 |  23.70 |  18.10 |
               |  70.84 |  66.10 |  60.15 |  49.18 |
---------------+--------+--------+--------+--------+
             2 |     50 |     56 |     62 |     66 |    234
               | 63.765 | 59.553 | 57.229 | 53.453 |
               | 2.9716 |  0.212 | 0.3977 | 2.9454 |
               |  21.37 |  23.93 |  26.50 |  28.21 |
               |  11.39 |  13.66 |  15.74 |  17.93 |
---------------+--------+--------+--------+--------+
             3 |     60 |     67 |     82 |    103 |    312
               |  85.02 | 79.404 | 76.305 |  71.27 |
               | 7.3632 | 1.9377 |  0.425 | 14.126 |
               |  19.23 |  21.47 |  26.28 |  33.01 |
               |  13.67 |  16.34 |  20.81 |  27.99 |
---------------+--------+--------+--------+--------+
             4 |      9 |     10 |      9 |      7 |     35
               | 9.5376 | 8.9075 | 8.5599 |  7.995 |
               | 0.0303 |  0.134 | 0.0226 | 0.1238 |
               |  25.71 |  28.57 |  25.71 |  20.00 |
               |   2.05 |   2.44 |   2.28 |   1.90 |
---------------+--------+--------+--------+--------+
             5 |      5 |      3 |      0 |      4 |     12
               |   3.27 |  3.054 | 2.9348 | 2.7412 |
               | 0.9152 |  0.001 | 2.9348 | 0.5781 |
               |  41.67 |  25.00 |   0.00 |  33.33 |
               |   1.14 |   0.73 |   0.00 |   1.09 |
---------------+--------+--------+--------+--------+
             6 |      2 |      1 |      4 |      0 |      7
               | 1.9075 | 1.7815 |  1.712 |  1.599 |
               | 0.0045 | 0.3428 | 3.0579 |  1.599 |
               |  28.57 |  14.29 |  57.14 |   0.00 |
               |   0.46 |   0.24 |   1.02 |   0.00 |
---------------+--------+--------+--------+--------+
             7 |      1 |      2 |      0 |      1 |      4
               |   1.09 |  1.018 | 0.9783 | 0.9137 |
               | 0.0074 | 0.9473 | 0.9783 | 0.0081 |
               |  25.00 |  50.00 |   0.00 |  25.00 |
               |   0.23 |   0.49 |   0.00 |   0.27 |
---------------+--------+--------+--------+--------+
             8 |      1 |      0 |      0 |      6 |      7
               | 1.9075 | 1.7815 |  1.712 |  1.599 |
               | 0.4318 | 1.7815 |  1.712 | 12.113 |
               |  14.29 |   0.00 |   0.00 |  85.71 |
               |   0.23 |   0.00 |   0.00 |   1.63 |
---------------+--------+--------+--------+--------+
Total               439      410      394      368     1611
```

Table 2.6 (concluded)

(c) By Season

```
CLUSTER         SEASON

Frequency      |
Expected       |
Cell Chi-Square|
Row Pct        | Spring | Summer | Fall   | Winter |
Col Pct        | (3-5)  | (6-8)  | (9-11) | (12-2) |  Total
---------------+--------+--------+--------+--------+
             1 |    505 |    308 |    145 |     42 |   1000
               | 453.13 | 394.79 | 126.01 | 26.071 |
               | 5.9364 | 19.078 | 2.8623 | 9.7328 |
               |  50.50 |  30.80 |  14.50 |   4.20 |
               |  69.18 |  48.43 |  71.43 | 100.00 |
---------------+--------+--------+--------+--------+
             2 |     59 |    163 |     12 |      0 |    234
               | 106.03 |  92.38 | 29.486 | 6.1006 |
               | 20.863 | 53.986 |  10.37 | 6.1006 |
               |  25.21 |  69.66 |   5.13 |   0.00 |
               |   8.08 |  25.63 |   5.91 |   0.00 |
---------------+--------+--------+--------+--------+
             3 |    135 |    138 |     39 |      0 |    312
               | 141.38 | 123.17 | 39.315 | 8.1341 |
               | 0.2877 | 1.7848 | 0.0025 | 8.1341 |
               |  43.27 |  44.23 |  12.50 |   0.00 |
               |  18.49 |  21.70 |  19.21 |   0.00 |
---------------+--------+--------+--------+--------+
             4 |     18 |     12 |      5 |      0 |     35
               |  15.86 | 13.818 | 4.4103 | 0.9125 |
               | 0.2888 | 0.2391 | 0.0788 | 0.9125 |
               |  51.43 |  34.29 |  14.29 |   0.00 |
               |   2.47 |   1.89 |   2.46 |   0.00 |
---------------+--------+--------+--------+--------+
             5 |      5 |      5 |      2 |      0 |     12
               | 5.4376 | 4.7374 | 1.5121 | 0.3128 |
               | 0.0352 | 0.0146 | 0.1574 | 0.3128 |
               |  41.67 |  41.67 |  16.67 |   0.00 |
               |   0.68 |   0.79 |   0.99 |   0.00 |
---------------+--------+--------+--------+--------+
             6 |      2 |      5 |      0 |      0 |      7
               | 3.1719 | 2.7635 | 0.8821 | 0.1825 |
               |  0.433 |   1.81 | 0.8821 | 0.1825 |
               |  28.57 |  71.43 |   0.00 |   0.00 |
               |   0.27 |   0.79 |   0.00 |   0.00 |
---------------+--------+--------+--------+--------+
             7 |      1 |      3 |      0 |      0 |      4
               | 1.8125 | 1.5791 |  0.504 | 0.1043 |
               | 0.3643 | 1.2784 |  0.504 | 0.1043 |
               |  25.00 |  75.00 |   0.00 |   0.00 |
               |   0.14 |   0.47 |   0.00 |   0.00 |
---------------+--------+--------+--------+--------+
             8 |      5 |      2 |      0 |      0 |      7
               | 3.1719 | 2.7635 | 0.8821 | 0.1825 |
               | 1.0535 | 0.2109 | 0.8821 | 0.1825 |
               |  71.43 |  28.57 |   0.00 |   0.00 |
               |   0.68 |   0.31 |   0.00 |   0.00 |
---------------+--------+--------+--------+--------+
Total               730      636      203       42     1611
```

Table 2.7 Statistical Tests of Contingency Table for Duration-Based Storm Patterns (All 8 Patterns Included)

(a) By Region

```
Statistic                     DF      Value         Prob
--------------------------------------------------------
Chi-Square                    21     41.316        0.005
Likelihood Ratio Chi-Square   21     46.356        0.001
Mantel-Haenszel Chi-Square     1      3.651        0.056
Phi Coefficient                       0.160
Contingency Coefficient               0.158
Cramer's V                            0.092

Sample Size = 1611
WARNING:  50% of the cells have expected counts less
           than 5. Chi-Square may not be a valid test.
```

(b) By Duration

```
Statistic                     DF      Value         Prob
--------------------------------------------------------
Chi-Square                    21     74.693        0.000
Likelihood Ratio Chi-Square   21     77.310        0.000
Mantel-Haenszel Chi-Square     1     30.503        0.000
Phi Coefficient                       0.215
Contingency Coefficient               0.210
Cramer's V                            0.124

Sample Size = 1611
WARNING:  50% of the cells have expected counts less
           than 5. Chi-Square may not be a valid test.
```

(c) By Season

```
Statistic                     DF      Value         Prob
--------------------------------------------------------
Chi-Square                    21    149.064        0.000
Likelihood Ratio Chi-Square   21    163.197        0.000
Mantel-Haenszel Chi-Square     1      6.521        0.011
Phi Coefficient                       0.304
Contingency Coefficient               0.291
Cramer's V                            0.176

Sample Size = 1611
WARNING:  53% of the cells have expected counts less
           than 5. Chi-Square may not be a valid test.
```

Table 2.8 Contingency Tables for Duration-Based Storm Patterns (Without Patterns 6, 7 and 8)

(a) By Region

```
CLUSTER         REGION

Frequency       |
Expected        |
Cell Chi-Square |
Percent         |
Row Pct         |
Col Pct         |       1|       2|       3|       4|  Total
----------------+--------+--------+--------+--------+
              1 |    267 |    444 |    115 |    174 |   1000
                | 234.15 | 469.55 | 110.48 | 185.81 |
                | 4.6089 | 1.3907 | 0.1846 |  0.751 |
                |  16.76 |  27.87 |   7.22 |  10.92 |  62.77
                |  26.70 |  44.40 |  11.50 |  17.40 |
                |  71.58 |  59.36 |  65.34 |  58.78 |
----------------+--------+--------+--------+--------+
              2 |     28 |    134 |     17 |     55 |    234
                | 54.791 | 109.88 | 25.853 |  43.48 |
                |   13.1 | 5.2967 | 3.0316 | 3.0521 |
                |   1.76 |   8.41 |   1.07 |   3.45 |  14.69
                |  11.97 |  57.26 |   7.26 |  23.50 |
                |   7.51 |  17.91 |   9.66 |  18.58 |
----------------+--------+--------+--------+--------+
              3 |     65 |    152 |     40 |     55 |    312
                | 73.055 |  146.5 | 34.471 | 57.974 |
                | 0.8881 | 0.2064 | 0.8869 | 0.1525 |
                |   4.08 |   9.54 |   2.51 |   3.45 |  19.59
                |  20.83 |  48.72 |  12.82 |  17.63 |
                |  17.43 |  20.32 |  22.73 |  18.58 |
----------------+--------+--------+--------+--------+
              4 |      9 |     14 |      3 |      9 |     35
                | 8.1952 | 16.434 | 3.8669 | 6.5035 |
                |  0.079 | 0.3606 | 0.1944 | 0.9584 |
                |   0.56 |   0.88 |   0.19 |   0.56 |   2.20
                |  25.71 |  40.00 |   8.57 |  25.71 |
                |   2.41 |   1.87 |   1.70 |   3.04 |
----------------+--------+--------+--------+--------+
              5 |      4 |      4 |      1 |      3 |     12
                | 2.8098 | 5.6347 | 1.3258 | 2.2298 |
                | 0.5042 | 0.4742 | 0.0801 | 0.2661 |
                |   0.25 |   0.25 |   0.06 |   0.19 |   0.75
                |  33.33 |  33.33 |   8.33 |  25.00 |
                |   1.07 |   0.53 |   0.57 |   1.01 |
----------------+--------+--------+--------+--------+
Total                373      748      176      296     1593
                   23.41    46.96    11.05    18.58   100.00
```

Table 2.8 (continued)

(b) By Duration

```
CLUSTER         DURATION

Frequency      |
Expected       |
Cell Chi-Square|
Row Pct        |
Col Pct        |       6|      12|      18|      24|  Total
---------------+--------+--------+--------+--------+
             1 |   311  |   271  |   237  |   181  |   1000
               | 273.07 | 255.49 | 244.82 | 226.62 |
               | 5.2687 | 0.9412 | 0.2499 | 9.1823 |
               |  31.10 |  27.10 |  23.70 |  18.10 |
               |  71.49 |  66.58 |  60.77 |  50.14 |
---------------+--------+--------+--------+--------+
             2 |    50  |    56  |    62  |    66  |    234
               | 63.898 | 59.785 | 57.288 | 53.028 |
               |  3.023 | 0.2397 | 0.3875 | 3.1731 |
               |  21.37 |  23.93 |  26.50 |  28.21 |
               |  11.49 |  13.76 |  15.90 |  18.28 |
---------------+--------+--------+--------+--------+
             3 |    60  |    67  |    82  |   103  |    312
               | 85.198 | 79.714 | 76.384 | 70.704 |
               | 7.4524 | 2.0277 | 0.4129 | 14.752 |
               |  19.23 |  21.47 |  26.28 |  33.01 |
               |  13.79 |  16.46 |  21.03 |  28.53 |
---------------+--------+--------+--------+--------+
             4 |     9  |    10  |     9  |     7  |     35
               | 9.5574 | 8.9422 | 8.5687 | 7.9316 |
               | 0.0325 | 0.1251 | 0.0217 | 0.1094 |
               |  25.71 |  28.57 |  25.71 |  20.00 |
               |   2.07 |   2.46 |   2.31 |   1.94 |
---------------+--------+--------+--------+--------+
             5 |     5  |     3  |     0  |     4  |     12
               | 3.2768 | 3.0659 | 2.9379 | 2.7194 |
               | 0.9061 | 0.0014 | 2.9379 | 0.6031 |
               |  41.67 |  25.00 |   0.00 |  33.33 |
               |   1.15 |   0.74 |   0.00 |   1.11 |
---------------+--------+--------+--------+--------+
Total               435      407      390      361     1593
```

Table 2.8 (concluded)

(c) By Season

```
CLUSTER          SEASON

Frequency       |
Expected        |
Cell Chi-Square |
Percent         |
Row Pct         | Spring | Summer |  Fall  | Winter |
Col Pct         |  (3-5) |  (6-8) | (9-11) | (12-2) |  Total
----------------+--------+--------+--------+--------+
              1 |    505 |    308 |    145 |     42 |   1000
                | 453.23 | 392.97 | 127.43 | 26.365 |
                | 5.9127 | 18.372 | 2.4218 | 9.2713 |
                |  31.70 |  19.33 |   9.10 |   2.64 |  62.77
                |  50.50 |  30.80 |  14.50 |   4.20 |
                |  69.94 |  49.20 |  71.43 | 100.00 |
----------------+--------+--------+--------+--------+
              2 |     59 |    163 |     12 |      0 |    234
                | 106.06 | 91.955 | 29.819 | 6.1695 |
                | 20.879 |  54.89 | 10.648 | 6.1695 |
                |   3.70 |  10.23 |   0.75 |   0.00 |  14.69
                |  25.21 |  69.66 |   5.13 |   0.00 |
                |   8.17 |  26.04 |   5.91 |   0.00 |
----------------+--------+--------+--------+--------+
              3 |    135 |    138 |     39 |      0 |    312
                | 141.41 | 122.61 | 39.759 |  8.226 |
                | 0.2904 | 1.9327 | 0.0145 |  8.226 |
                |   8.47 |   8.66 |   2.45 |   0.00 |  19.59
                |  43.27 |  44.23 |  12.50 |   0.00 |
                |  18.70 |  22.04 |  19.21 |   0.00 |
----------------+--------+--------+--------+--------+
              4 |     18 |     12 |      5 |      0 |     35
                | 15.863 | 13.754 | 4.4601 | 0.9228 |
                | 0.2878 | 0.2237 | 0.0653 | 0.9228 |
                |   1.13 |   0.75 |   0.31 |   0.00 |   2.20
                |  51.43 |  34.29 |  14.29 |   0.00 |
                |   2.49 |   1.92 |   2.46 |   0.00 |
----------------+--------+--------+--------+--------+
              5 |      5 |      5 |      2 |      0 |     12
                | 5.4388 | 4.7156 | 1.5292 | 0.3164 |
                | 0.0354 | 0.0171 |  0.145 | 0.3164 |
                |   0.31 |   0.31 |   0.13 |   0.00 |   0.75
                |  41.67 |  41.67 |  16.67 |   0.00 |
                |   0.69 |   0.80 |   0.99 |   0.00 |
----------------+--------+--------+--------+--------+
Total                722      626      203       42     1593
                   45.32    39.30    12.74     2.64   100.00
```

Table 2.9 Statistical Tests of Contingency Table for Duration-Based Storm Patterns (Without Patterns 6, 7 and 8)

(a) By Region

```
Statistic                      DF     Value          Prob
------------------------------------------------------
Chi-Square                     12    36.466         0.000
Likelihood Ratio Chi-Square    12    38.922         0.000
Mantel-Haenszel Chi-Square      1     3.178         0.075
Phi Coefficient                       0.151
Contingency Coefficient               0.150
Cramer's V                            0.087

Sample Size = 1593
```

(b) By Duration

```
Statistic                      DF     Value          Prob
------------------------------------------------------
Chi-Square                     12    51.847         0.000
Likelihood Ratio Chi-Square    12    54.168         0.000
Mantel-Haenszel Chi-Square      1    30.014         0.000
Phi Coefficient                       0.180
Contingency Coefficient               0.178
Cramer's V                            0.104

Sample Size = 1593
```

(c) By Season

```
Statistic                      DF     Value          Prob
------------------------------------------------------
Chi-Square                     12   141.042         0.000
Likelihood Ratio Chi-Square    12   153.150         0.000
Mantel-Haenszel Chi-Square      1     5.130         0.024
Phi Coefficient                       0.298
Contingency Coefficient               0.285
Cramer's V                            0.172

Sample Size = 1593
WARNING:  25% of the cells have expected counts less
           than 5. Chi-Square may not be a valid test.
```

Table 2.10 Contingency Tables for Event-Based Storm Patterns (All 5 Patterns Included)

(a) By Region

```
CLUSTER          REGION

Frequency       |
Expected        |
Cell Chi-Square |
Percent         |
Row Pct         |
Col Pct         |       1|       2|       3|       4|  Total
----------------+--------+--------+--------+--------+
              1 |      1 |     14 |      1 |      4 |     20
                | 4.2541 | 9.6685 | 2.2652 | 3.8122 |
                | 2.4892 | 1.9405 | 0.7067 | 0.0093 |
                |   0.28 |   3.87 |   0.28 |   1.10 |   5.52
                |   5.00 |  70.00 |   5.00 |  20.00 |
                |   1.30 |   8.00 |   2.44 |   5.80 |
----------------+--------+--------+--------+--------+
              2 |     67 |    131 |     31 |     50 |    279
                | 59.345 | 134.88 | 31.599 |  53.18 |
                | 0.9873 | 0.1114 | 0.0114 | 0.1901 |
                |  18.51 |  36.19 |   8.56 |  13.81 |  77.07
                |  24.01 |  46.95 |  11.11 |  17.92 |
                |  87.01 |  74.86 |  75.61 |  72.46 |
----------------+--------+--------+--------+--------+
              3 |      9 |     28 |      9 |     14 |     60
                | 12.762 | 29.006 | 6.7956 | 11.436 |
                | 1.1092 | 0.0349 | 0.7151 | 0.5746 |
                |   2.49 |   7.73 |   2.49 |   3.87 |  16.57
                |  15.00 |  46.67 |  15.00 |  23.33 |
                |  11.69 |  16.00 |  21.95 |  20.29 |
----------------+--------+--------+--------+--------+
              4 |      0 |      1 |      0 |      1 |      2
                | 0.4254 | 0.9669 | 0.2265 | 0.3812 |
                | 0.4254 | 0.0011 | 0.2265 | 1.0044 |
                |   0.00 |   0.28 |   0.00 |   0.28 |   0.55
                |   0.00 |  50.00 |   0.00 |  50.00 |
                |   0.00 |   0.57 |   0.00 |   1.45 |
----------------+--------+--------+--------+--------+
              5 |      0 |      1 |      0 |      0 |      1
                | 0.2127 | 0.4834 | 0.1133 | 0.1906 |
                | 0.2127 |  0.552 | 0.1133 | 0.1906 |
                |   0.00 |   0.28 |   0.00 |   0.00 |   0.28
                |   0.00 | 100.00 |   0.00 |   0.00 |
                |   0.00 |   0.57 |   0.00 |   0.00 |
----------------+--------+--------+--------+--------+
Total                 77      175       41       69      362
                   21.27    48.34    11.33    19.06   100.00
```

Table 2.10 (continued)

(b) By Duration

```
CLUSTER        DURATION

Frequency      |
Expected       |
Cell Chi-Square|
Percent        |
Row Pct        |
Col Pct        |<= 6 hr |7-12 hr |13-18 h |19-24 h |> 24 hr |  Total
---------------+--------+--------+--------+--------+--------+
             1 |     11 |      6 |      1 |      2 |      0 |     20
               | 3.6464 | 3.3702 | 3.7569 | 4.1436 | 5.0829 |
               |  14.83 | 2.0521 | 2.0231 |  1.109 | 5.0829 |
               |   3.04 |   1.66 |   0.28 |   0.55 |   0.00 |   5.52
               |  55.00 |  30.00 |   5.00 |  10.00 |   0.00 |
               |  16.67 |   9.84 |   1.47 |   2.67 |   0.00 |
---------------+--------+--------+--------+--------+--------+
             2 |     29 |     43 |     61 |     63 |     83 |    279
               | 50.867 | 47.014 | 52.409 | 57.804 | 70.906 |
               | 9.4006 | 0.3427 | 1.4083 | 0.4671 | 2.0628 |
               |   8.01 |  11.88 |  16.85 |  17.40 |  22.93 |  77.07
               |  10.39 |  15.41 |  21.86 |  22.58 |  29.75 |
               |  43.94 |  70.49 |  89.71 |  84.00 |  90.22 |
---------------+--------+--------+--------+--------+--------+
             3 |     24 |     11 |      6 |     10 |      9 |     60
               | 10.939 |  10.11 | 11.271 | 12.431 | 15.249 |
               | 15.594 | 0.0783 | 2.4648 | 0.4754 | 2.5606 |
               |   6.63 |   3.04 |   1.66 |   2.76 |   2.49 |  16.57
               |  40.00 |  18.33 |  10.00 |  16.67 |  15.00 |
               |  36.36 |  18.03 |   8.82 |  13.33 |   9.78 |
---------------+--------+--------+--------+--------+--------+
             4 |      2 |      0 |      0 |      0 |      0 |      2
               | 0.3646 |  0.337 | 0.3757 | 0.4144 | 0.5083 |
               | 7.3343 |  0.337 | 0.3757 | 0.4144 | 0.5083 |
               |   0.55 |   0.00 |   0.00 |   0.00 |   0.00 |   0.55
               | 100.00 |   0.00 |   0.00 |   0.00 |   0.00 |
               |   3.03 |   0.00 |   0.00 |   0.00 |   0.00 |
---------------+--------+--------+--------+--------+--------+
             5 |      0 |      1 |      0 |      0 |      0 |      1
               | 0.1823 | 0.1685 | 0.1878 | 0.2072 | 0.2541 |
               | 0.1823 | 4.1029 | 0.1878 | 0.2072 | 0.2541 |
               |   0.00 |   0.28 |   0.00 |   0.00 |   0.00 |   0.28
               |   0.00 | 100.00 |   0.00 |   0.00 |   0.00 |
               |   0.00 |   1.64 |   0.00 |   0.00 |   0.00 |
---------------+--------+--------+--------+--------+--------+
Total                66       61       68       75       92      362
                  18.23    16.85    18.78    20.72    25.41   100.00
```

Table 2.10 (concluded)

(c) By Season

```
CLUSTER          SEASON

Frequency       |
Expected        |
Cell Chi-Square|
Percent         |
Row Pct         | Spring | Summer |  Fall  | Winter |
Col Pct         |  (3-5) |  (6-8) | (9-11) | (12-2) |  Total
----------------+--------+--------+--------+--------+
              1 |      4 |     14 |      2 |      0 |     20
                | 8.3978 | 8.7845 | 2.3757 |  0.442 |
                | 2.3031 | 3.0965 | 0.0594 |  0.442 |
                |   1.10 |   3.87 |   0.55 |   0.00 |   5.52
                |  20.00 |  70.00 |  10.00 |   0.00 |
                |   2.63 |   8.81 |   4.65 |   0.00 |
----------------+--------+--------+--------+--------+
              2 |    129 |    107 |     35 |      8 |    279
                | 117.15 | 122.54 | 33.141 | 6.1657 |
                | 1.1988 | 1.9717 | 0.1043 | 0.5457 |
                |  35.64 |  29.56 |   9.67 |   2.21 |  77.07
                |  46.24 |  38.35 |  12.54 |   2.87 |
                |  84.87 |  67.30 |  81.40 | 100.00 |
----------------+--------+--------+--------+--------+
              3 |     18 |     36 |      6 |      0 |     60
                | 25.193 | 26.354 | 7.1271 |  1.326 |
                | 2.0539 | 3.5309 | 0.1782 |  1.326 |
                |   4.97 |   9.94 |   1.66 |   0.00 |  16.57
                |  30.00 |  60.00 |  10.00 |   0.00 |
                |  11.84 |  22.64 |  13.95 |   0.00 |
----------------+--------+--------+--------+--------+
              4 |      0 |      2 |      0 |      0 |      2
                | 0.8398 | 0.8785 | 0.2376 | 0.0442 |
                | 0.8398 | 1.4319 | 0.2376 | 0.0442 |
                |   0.00 |   0.55 |   0.00 |   0.00 |   0.55
                |   0.00 | 100.00 |   0.00 |   0.00 |
                |   0.00 |   1.26 |   0.00 |   0.00 |
----------------+--------+--------+--------+--------+
              5 |      1 |      0 |      0 |      0 |      1
                | 0.4199 | 0.4392 | 0.1188 | 0.0221 |
                | 0.8015 | 0.4392 | 0.1188 | 0.0221 |
                |   0.28 |   0.00 |   0.00 |   0.00 |   0.28
                | 100.00 |   0.00 |   0.00 |   0.00 |
                |   0.66 |   0.00 |   0.00 |   0.00 |
----------------+--------+--------+--------+--------+
Total                152      159       43        8      362
                   41.99    43.92    11.88     2.21   100.00
```

Table 2.11 Statistical Tests of Contingency Table for Event-Based Storm Patterns (All 5 Patterns Included)

(a) By Region

```
Statistic                     DF      Value         Prob
------------------------------------------------------
Chi-Square                    12     11.606        0.478
Likelihood Ratio Chi-Square   12     13.391        0.341
Mantel-Haenszel Chi-Square     1      1.258        0.262
Phi Coefficient                       0.179
Contingency Coefficient               0.176
Cramer's V                            0.103

Sample Size = 362
WARNING:  55% of the cells have expected counts less
           than 5. Chi-Square may not be a valid test.
```

(b) By Duration

```
Statistic                     DF      Value         Prob
------------------------------------------------------
Chi-Square                    16     73.855        0.000
Likelihood Ratio Chi-Square   16     69.423        0.000
Mantel-Haenszel Chi-Square     1      0.728        0.393
Phi Coefficient                       0.452
Contingency Coefficient               0.412
Cramer's V                            0.226

Sample Size = 362
WARNING:  56% of the cells have expected counts less
           than 5. Chi-Square may not be a valid test.
```

(c) By Season

```
Statistic                     DF      Value         Prob
------------------------------------------------------
Chi-Square                    12     20.746        0.054
Likelihood Ratio Chi-Square   12     23.565        0.023
Mantel-Haenszel Chi-Square     1      0.505        0.477
Phi Coefficient                       0.239
Contingency Coefficient               0.233
Cramer's V                            0.138

Sample Size = 362
WARNING:  55% of the cells have expected counts less
           than 5. Chi-Square may not be a valid test.
```

Table 2.12 Contigency Tables for Event-Based Storm Patterns (Without Pattern 4 and Pattern 5)

(a) By Region

```
CLUSTER          REGION

Frequency        |
Expected         |
Cell Chi-Square  |
Percent          |
Row Pct          |
Col Pct          |       1|       2|       3|       4|  Total
-----------------+--------+--------+--------+--------+
               1 |      1 |     14 |      1 |      4 |     20
                 | 4.2897 | 9.6379 | 2.2841 | 3.7883 |
                 | 2.5228 | 1.9743 | 0.7219 | 0.0118 |
                 |   0.28 |   3.90 |   0.28 |   1.11 |   5.57
                 |   5.00 |  70.00 |   5.00 |  20.00 |
                 |   1.30 |   8.09 |   2.44 |   5.88 |
-----------------+--------+--------+--------+--------+
               2 |     67 |    131 |     31 |     50 |    279
                 | 59.841 | 134.45 | 31.864 | 52.847 |
                 | 0.8564 | 0.0884 | 0.0234 | 0.1534 |
                 |  18.66 |  36.49 |   8.64 |  13.93 |  77.72
                 |  24.01 |  46.95 |  11.11 |  17.92 |
                 |  87.01 |  75.72 |  75.61 |  73.53 |
-----------------+--------+--------+--------+--------+
               3 |      9 |     28 |      9 |     14 |     60
                 | 12.869 | 28.914 | 6.8524 | 11.365 |
                 | 1.1632 | 0.0289 | 0.6731 |  0.611 |
                 |   2.51 |   7.80 |   2.51 |   3.90 |  16.71
                 |  15.00 |  46.67 |  15.00 |  23.33 |
                 |  11.69 |  16.18 |  21.95 |  20.59 |
-----------------+--------+--------+--------+--------+
Total                  77      173       41       68      359
                    21.45    48.19    11.42    18.94   100.00
```

Table 2.12 (continued)

(b) By Duration

```
CLUSTER        DURATION

Frequency      |
Expected       |
Cell Chi-Square|
Percent        |
Row Pct        |
Col Pct        |<= 6 hr |7-12 hr |13-18 h |19-24 h |> 24 hr |  Total
---------------+--------+--------+--------+--------+--------+
             1 |     11 |      6 |      1 |      2 |      0 |     20
               | 3.5655 | 3.3426 | 3.7883 | 4.1783 | 5.1253 |
               | 15.502 | 2.1126 | 2.0523 | 1.1356 | 5.1253 |
               |   3.06 |   1.67 |   0.28 |   0.56 |   0.00 |   5.57
               |  55.00 |  30.00 |   5.00 |  10.00 |   0.00 |
               |  17.19 |  10.00 |   1.47 |   2.67 |   0.00 |
---------------+--------+--------+--------+--------+--------+
             2 |     29 |     43 |     61 |     63 |     83 |    279
               | 49.738 |  46.63 | 52.847 | 58.287 | 71.499 |
               | 8.6467 | 0.2825 | 1.2579 | 0.3811 | 1.8501 |
               |   8.08 |  11.98 |  16.99 |  17.55 |  23.12 |  77.72
               |  10.39 |  15.41 |  21.86 |  22.58 |  29.75 |
               |  45.31 |  71.67 |  89.71 |  84.00 |  90.22 |
---------------+--------+--------+--------+--------+--------+
             3 |     24 |     11 |      6 |     10 |      9 |     60
               | 10.696 | 10.028 | 11.365 | 12.535 | 15.376 |
               | 16.546 | 0.0942 | 2.5325 | 0.5126 |  2.644 |
               |   6.69 |   3.06 |   1.67 |   2.79 |   2.51 |  16.71
               |  40.00 |  18.33 |  10.00 |  16.67 |  15.00 |
               |  37.50 |  18.33 |   8.82 |  13.33 |   9.78 |
---------------+--------+--------+--------+--------+--------+
Total                64       60       68       75       92      359
                  17.83    16.71    18.94    20.89    25.63   100.00
```

Table 2.12 (concluded)

(c) By Season

```
CLUSTER           SEASON

Frequency        |
Expected         |
Cell Chi-Square  |
Percent          |
Row Pct          | Spring | Summer |  Fall  | Winter |
Col Pct          |  (3-5) |  (6-8) | (9-11) | (12-2) |  Total
-----------------+--------+--------+--------+--------+
               1 |      4 |     14 |      2 |      0 |     20
                 | 8.4123 | 8.7465 | 2.3955 | 0.4457 |
                 | 2.3142 | 3.1554 | 0.0653 | 0.4457 |
                 |   1.11 |   3.90 |   0.56 |   0.00 |   5.57
                 |  20.00 |  70.00 |  10.00 |   0.00 |
                 |   2.65 |   8.92 |   4.65 |   0.00 |
-----------------+--------+--------+--------+--------+
               2 |    129 |    107 |     35 |      8 |    279
                 | 117.35 | 122.01 | 33.418 | 6.2173 |
                 | 1.1564 | 1.8475 | 0.0749 | 0.5112 |
                 |  35.93 |  29.81 |   9.75 |   2.23 |  77.72
                 |  46.24 |  38.35 |  12.54 |   2.87 |
                 |  85.43 |  68.15 |  81.40 | 100.00 |
-----------------+--------+--------+--------+--------+
               3 |     18 |     36 |      6 |      0 |     60
                 | 25.237 |  26.24 | 7.1866 |  1.337 |
                 | 2.0752 | 3.6306 | 0.1959 |  1.337 |
                 |   5.01 |  10.03 |   1.67 |   0.00 |  16.71
                 |  30.00 |  60.00 |  10.00 |   0.00 |
                 |  11.92 |  22.93 |  13.95 |   0.00 |
-----------------+--------+--------+--------+--------+
Total                 151      157       43        8      359
                    42.06    43.73    11.98     2.23   100.00
```

Table 2.13 Statistical Tests of Contingency Table for Event-Based Storm Patterns (Without Pattern 4 and Pattern 5)

(a) By Region

```
Statistic                       DF      Value         Prob
------------------------------------------------------
Chi-Square                       6      8.829        0.183
Likelihood Ratio Chi-Square      6      9.929        0.128
Mantel-Haenszel Chi-Square       1      1.027        0.311
Phi Coefficient                         0.157
Contingency Coefficient                 0.155
Cramer's V                              0.111

Sample Size = 359
WARNING:  25% of the cells have expected counts less
           than 5. Chi-Square may not be a valid test.
```

(b) By Duration

```
Statistic                       DF      Value         Prob
------------------------------------------------------
Chi-Square                       8     60.676        0.000
Likelihood Ratio Chi-Square      8     59.001        0.000
Mantel-Haenszel Chi-Square       1      0.482        0.488
Phi Coefficient                         0.411
Contingency Coefficient                 0.380
Cramer's V                              0.291

Sample Size = 359
WARNING:  27% of the cells have expected counts less
           than 5. Chi-Square may not be a valid test.
```

(c) By Season

```
Statistic                       DF      Value         Prob
------------------------------------------------------
Chi-Square                       6     16.809        0.010
Likelihood Ratio Chi-Square      6     18.532        0.005
Mantel-Haenszel Chi-Square       1      0.317        0.574
Phi Coefficient                         0.216
Contingency Coefficient                 0.211
Cramer's V                              0.153

Sample Size = 359
WARNING:  25% of the cells have expected counts less
           than 5. Chi-Square may not be a valid test.
```

2.5.1 Duration-Based Storm Patterns

Figures 2.4 and 2.5 show the relative frequency of the occurrence of the duration-based storm patterns in four different climatic regions in Wyoming. Note that in regions 2 and 4 all storm patterns occur. On the other hand, storms of pattern 8 were not found in region 1 and storm patterns 6 and 7 do not occur in region 3.

Figures 2.6 and 2.7 show the relationship between the duration-based storms and the four selected storm durations. Pattern 8 does not contain storms with 12-hr duration and 18-hr duration. Pattern 7 and pattern 5 do not contain storms with 18-hr duration. Pattern 6 does not have 24-hr duration. All storm patterns contain 6-hr duration.

As shown in Figs 2.8 and 2.9, the duration-based storms of different patterns all happen in Spring (March-May) and Summer (June-August). In Fall (September-November), storms with patterns 6, 7 and 8 are not observed. Only storms with pattern 1 occur in Winter (December-February) indicating that Winter storms have uniform intensity.

Tables 2.7 and 2.9 suggest that, for the duration-based storms, storm patterns are dependent on climatic region, storm duration, and seasonality. Storm patterns should be adopted according to different regions, durations and seasons.

2.5.2 Event-Based Storm Patterns

The relationship between the event-based storm patterns and climatic regions is shown in Figs. 2.10 and 2.11. Only region 2 was observed with all five types of storms. Pattern 5 only occurred in region 2 and pattern 4 only occurred in regions 2 and 4. This is most likely due to the scarcity of storm cases found in these two storm patterns. As can be seen in Fig.

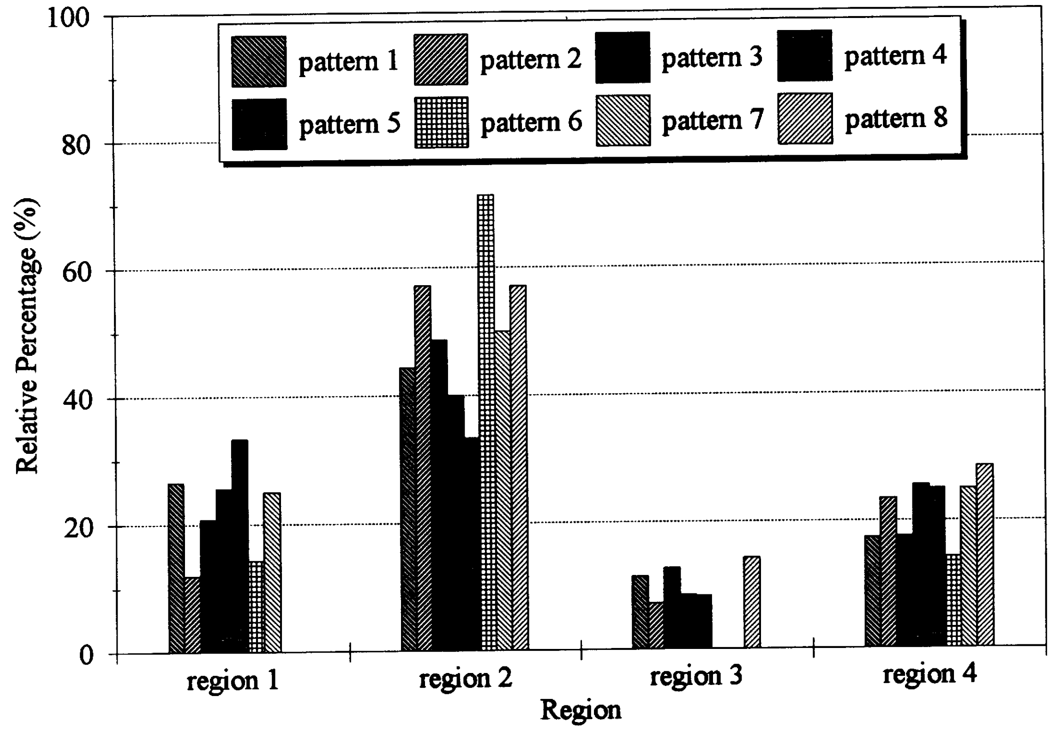


Fig. 2.4 Relation of Storm Pattern by Region, for Duration-Based Strom Patterns, Shown in Relative Percentage

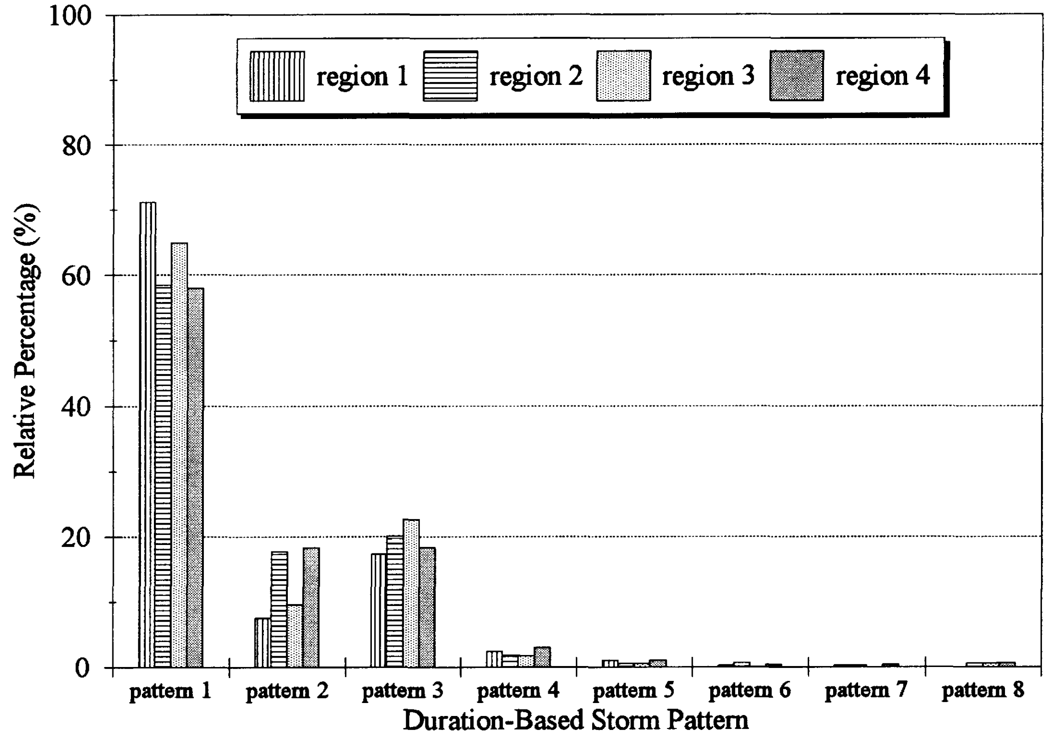


Fig. 2.5 Relation of Region by Storm Pattern, for Duration-Based Strom Patterns, Shown in Relative Percentage

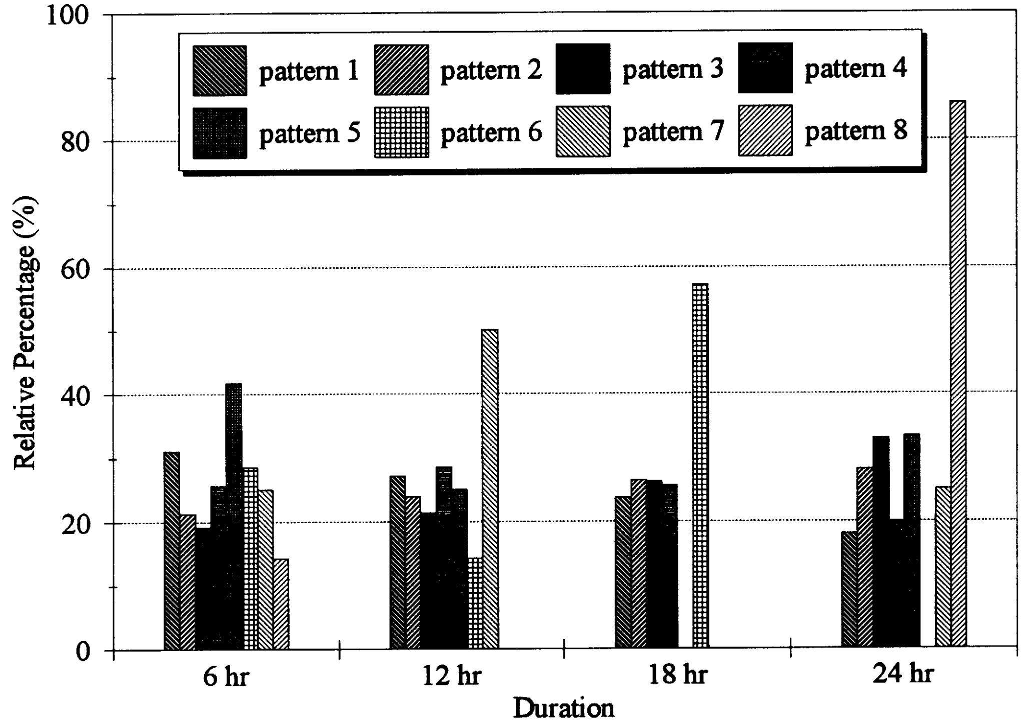


Fig. 2.6 Relation of Storm Pattern by Duration, for Duration-Based Storm Patterns, Shown in Relative Percentage

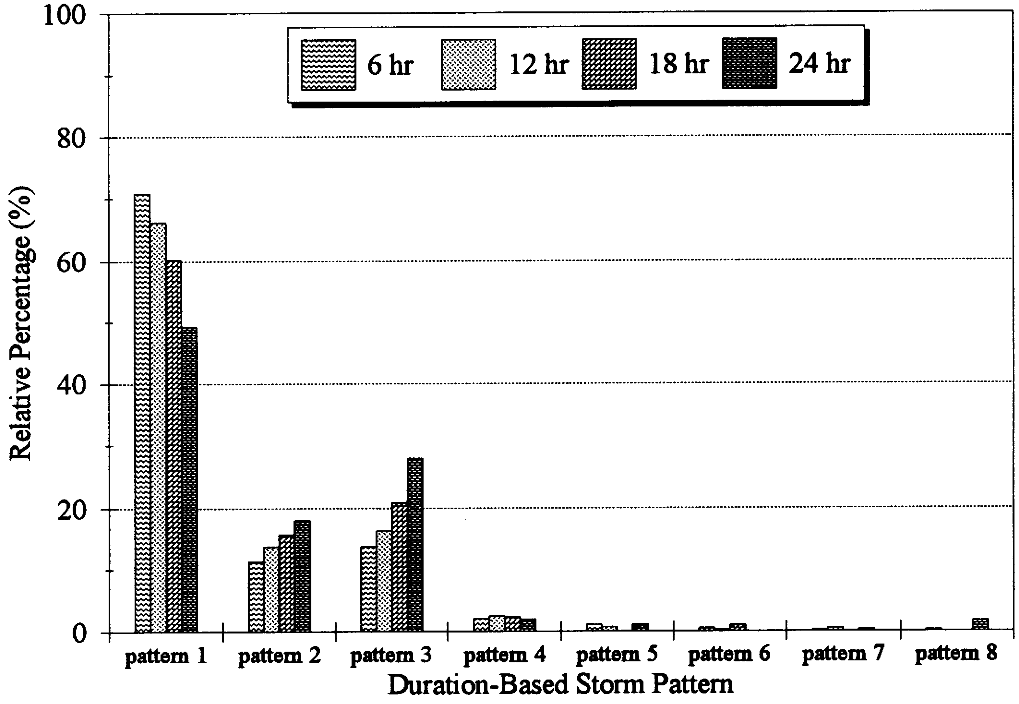


Fig. 2.7 Relation of Duration by Storm Pattern, for Duration-Based Storm Patterns, Shown in Relative Percentage

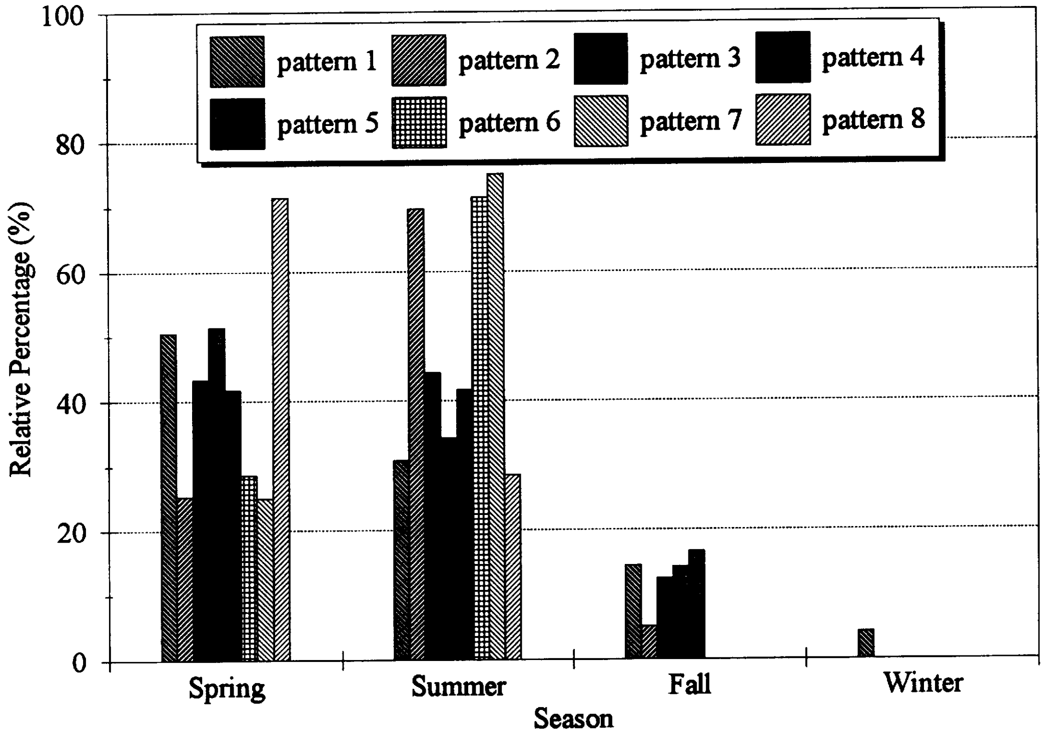


Fig. 2.8 Relation of Storm Pattern by Season, for Duration-Based Storm Patterns, Shown in Relative Percentage

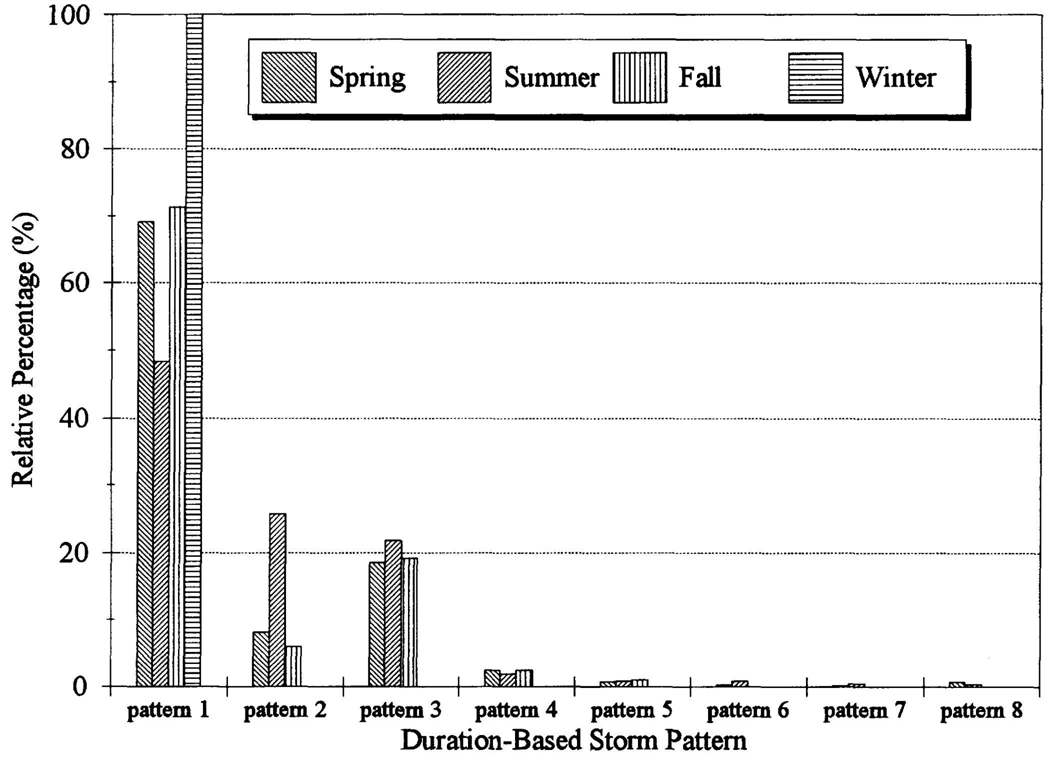


Fig. 2.9 Relation of Season by Storm Pattern, for Duration-Based Storm Patterns, Shown in Relative Percentage

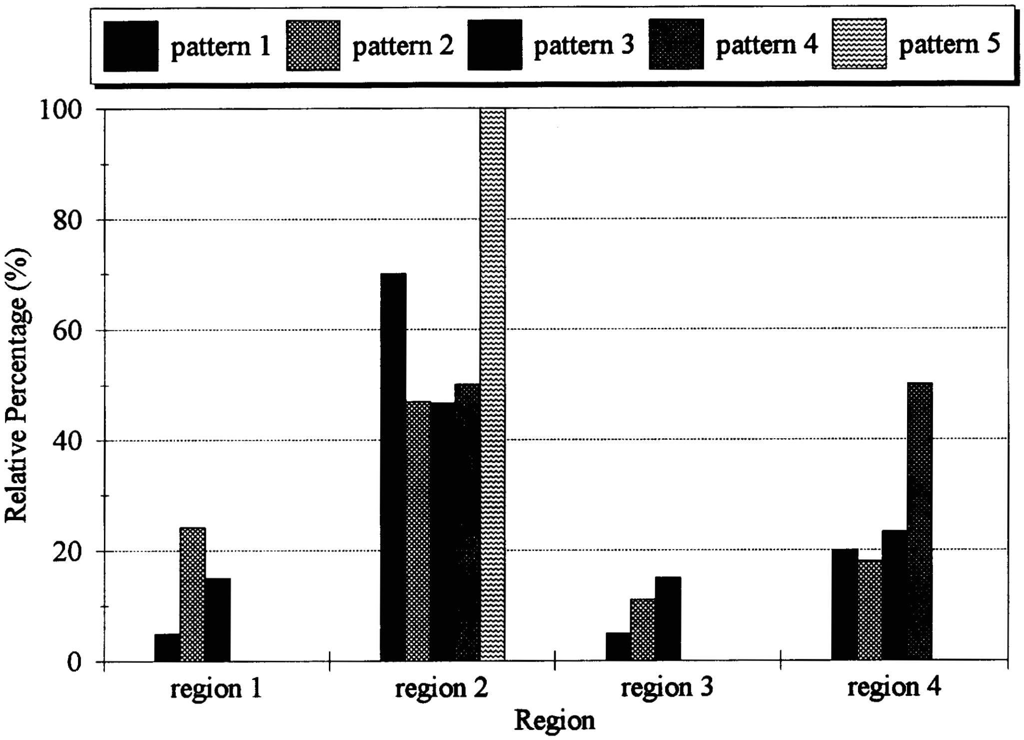


Fig. 2.10 Relation of Storm Pattern by Region, for Event-Based Storm Patterns, Shown in Relative Percentage

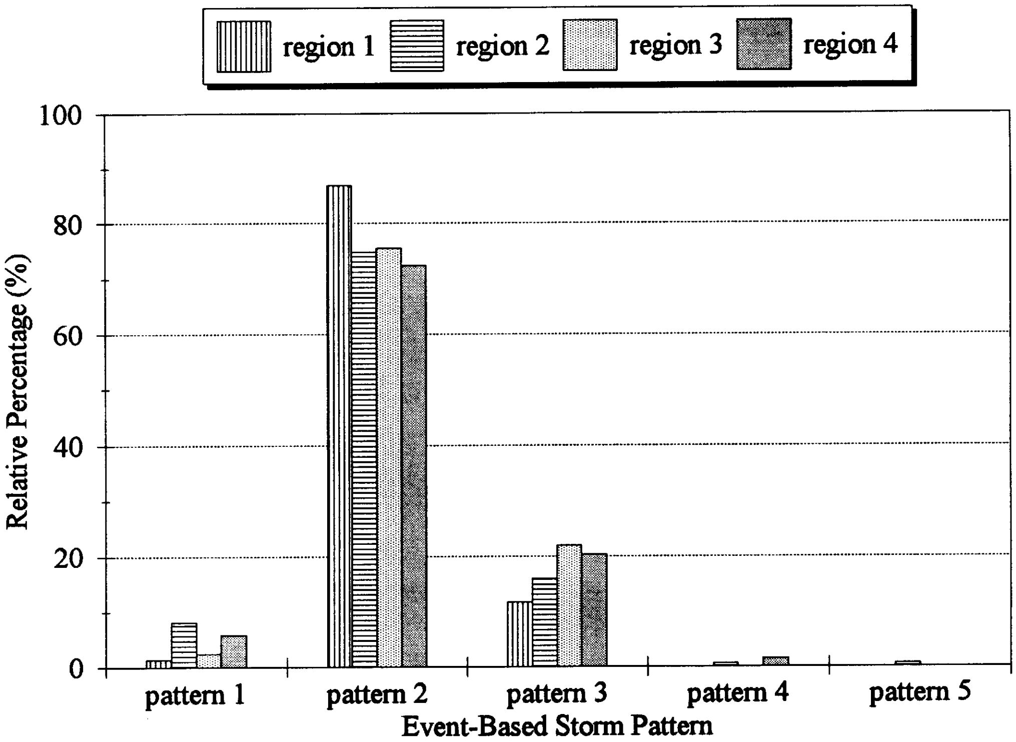


Fig. 2.11 Relation of Region by Storm Pattern, for Event-Based Storm Patterns, Shown in Relative Percentage

2.11, the heights of relative percentage for the different regions are almost the same in each pattern. This suggests that there is not much dependence between the event-based storm patterns and climatic regions.

The storm duration was artificially divided into five categories for the event-based storm patterns. The relationship between event-based storm patterns and durations are shown in Figs. 2.12 and 2.13. The storm durations for pattern 4 and pattern 5 are relatively short. Pattern 4 only contains storms with a duration less than or equal to 6-hr. Pattern 5 only contains storms with duration between 6-hr and 12-hr. Pattern 1 does not have any storm longer than 24-hr. Pattern 2 and pattern 4 have storms with all durations.

From Figs. 2.14 and 2.15, the relationship between the event-based storm patterns and seasons of occurrence are shown and it is observed that storms with pattern 2 occur in all seasons. Storms of pattern 1 and pattern 3 occur in all seasons but Winter. Only storms of pattern 2, those with uniform intensity, occur in Winter.

Table 2.11 and 2.13 suggest that for the event-based storms, storm patterns are not dependent on climatic region and the relation between storm patterns and seasonality is not very strong. These five representative event-based storm patterns can be used for the entire State of Wyoming.

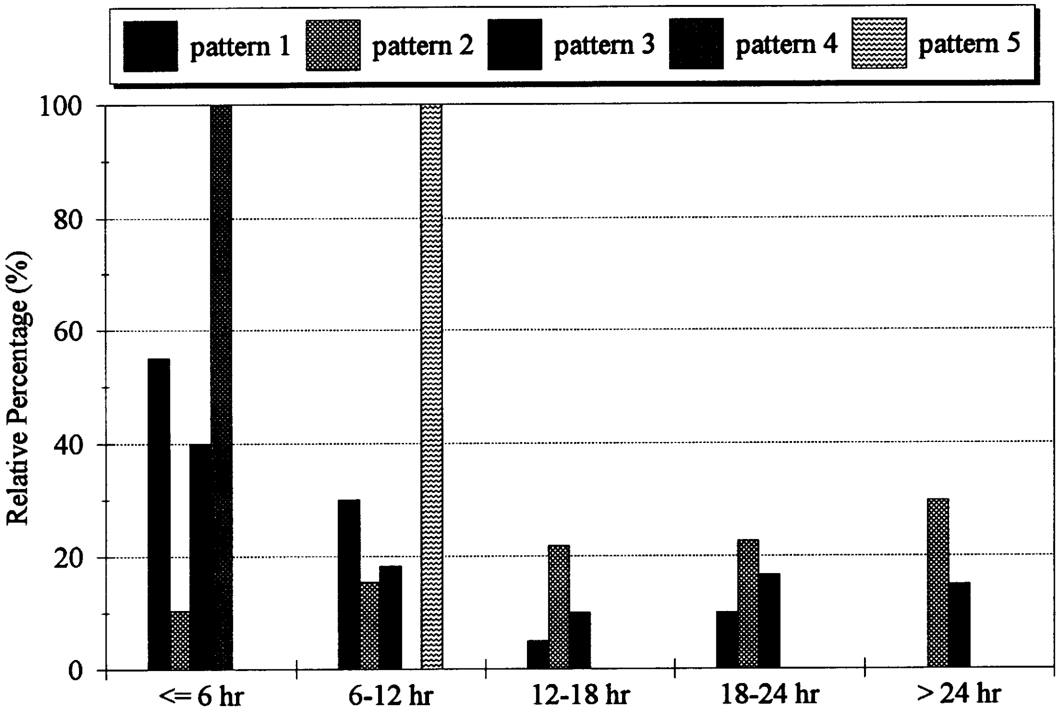


Fig. 2.12 Relation of Storm Pattern by Duration, for Event-Based Storm Patterns, Shown in Relative Percentage

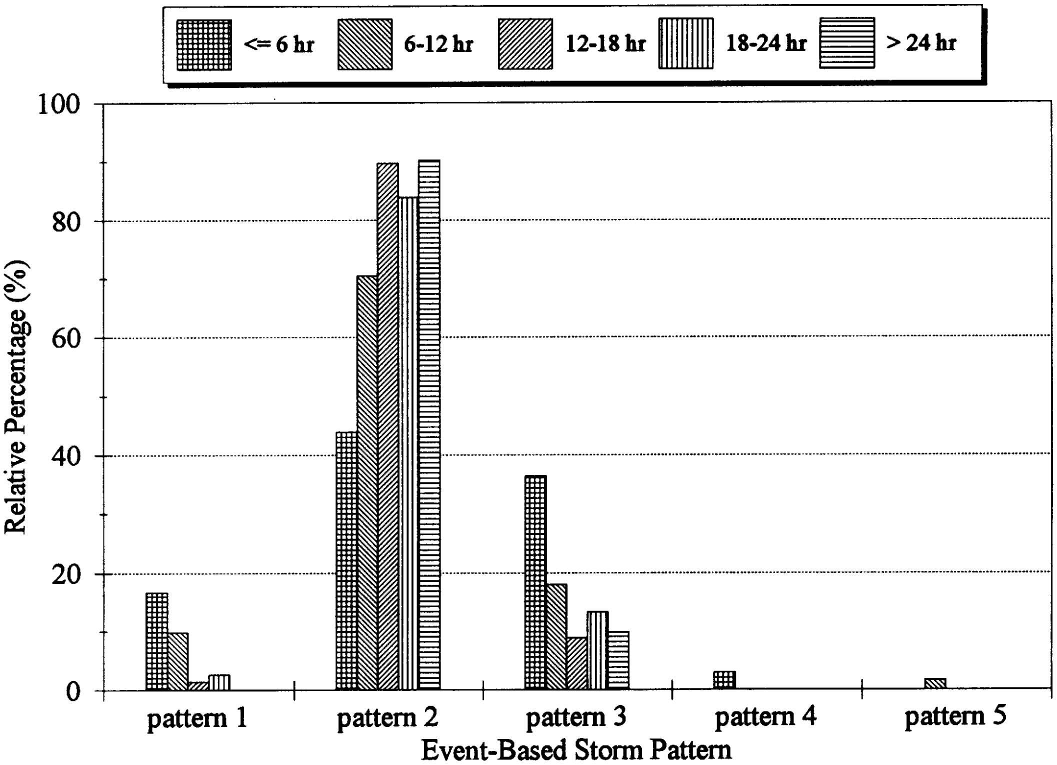


Fig. 2.13 Relation of Duration by Storm Pattern, for Event-Based Storm Patterns, Shown in Relative Percentage

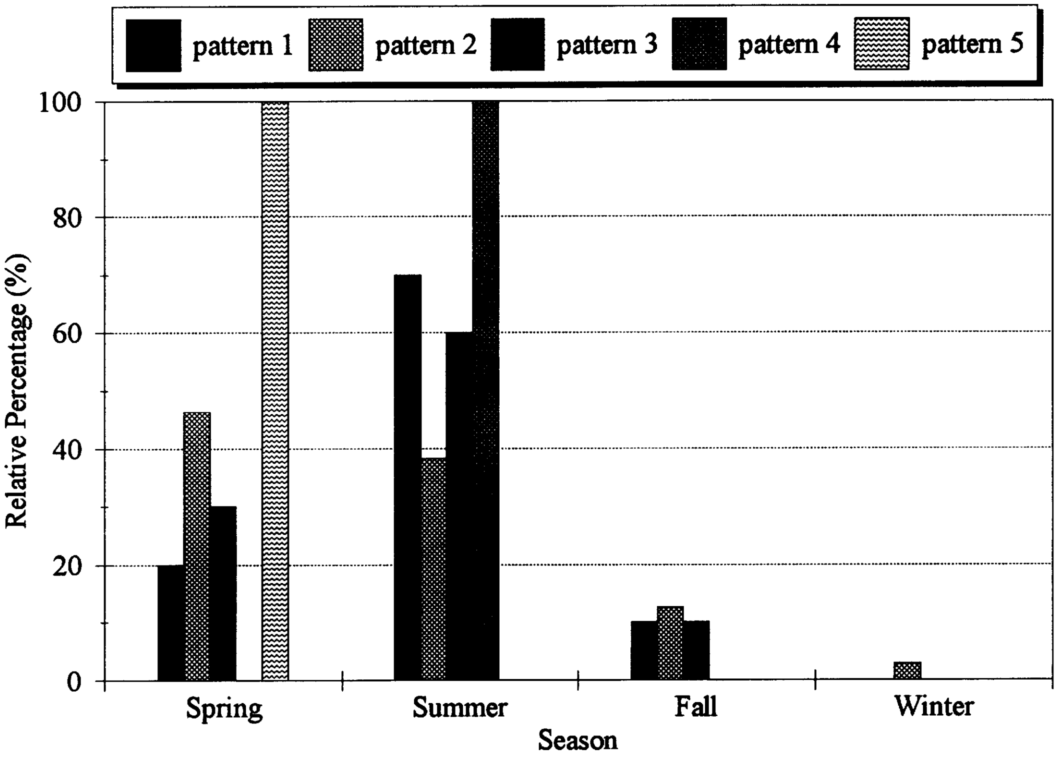


Fig. 2.14 Relation of Storm Pattern by Season, for Event-Based Storm Patterns, Shown in Relative Percentage

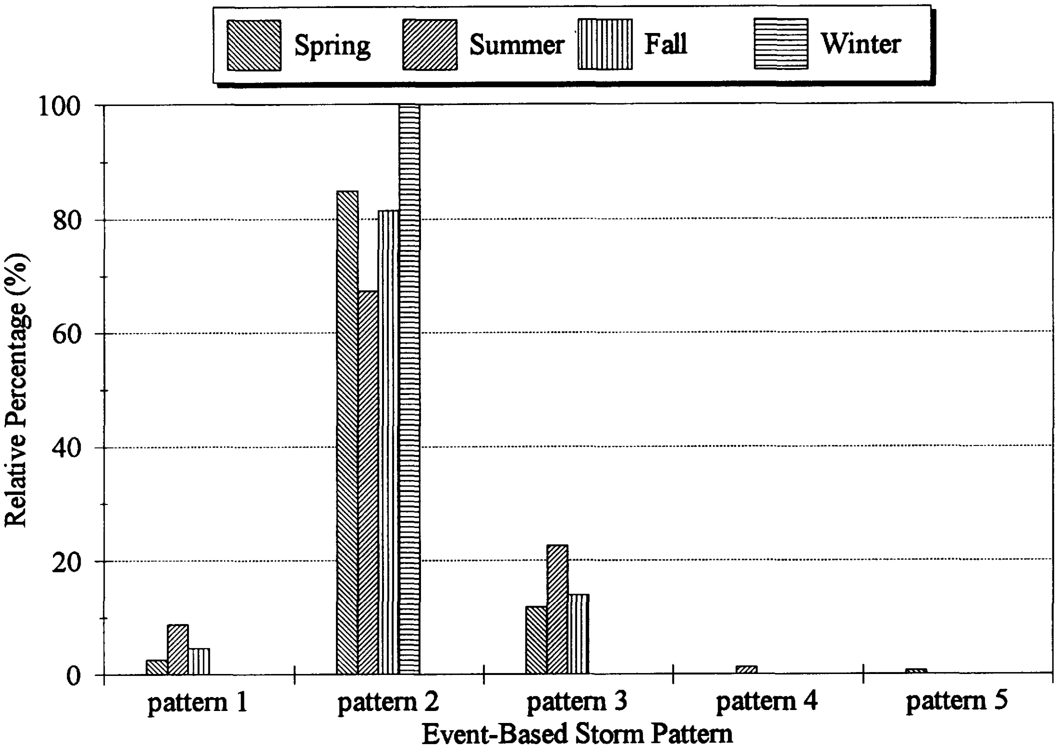


Fig. 2.15 Relation of Season by Storm Pattern, for Event-Based Storm Patterns, Shown in Relative Percentage

2.6 FITTING PARAMETRIC MODELS TO STORM PATTERNS

For modeling convenience, parametric models can be used to describe the temporal variation of storm patterns obtained from the cluster analysis. Since the dimensionless rainfall hyetographs are bounded between 0 and 1 for the dimensionless time, probability distributions for bounded random variables might be appropriate. Specifically, two distribution models, namely, the Beta distribution and Johnson's S_B distribution, were considered in this study. The following subsections describe the procedure for fitting the two models to the storm patterns.

2.6.1 Fitting Storm Patterns by Beta Distribution

The beta distribution is appropriate for describing a random variable having both lower and upper bounds. The non-standard Beta PDF is

$$f_{NB}(x|a,b,\alpha,\beta) = \frac{1}{B(\alpha,\beta)(b-a)}(x-a)^{\alpha-1}(b-x)^{\beta-1}, \quad a \leq x \leq b \tag{2.6}$$

in which a and b are the lower bound and upper bound, respectively. By using a standard random variable $Y=(X-a)/(b-a)$, the standard Beta PDF is obtained as

$$f_B(y|\alpha,\beta) = \frac{1}{B(\alpha,\beta)} y^{\alpha-1}(1-y)^{\beta-1}, \quad 0 \leq y \leq 1 \tag{2.7}$$

The mean and variance of the standard beta random variable Y are, respectively,

$$\mu_Y = \frac{\alpha}{\alpha + \beta} \tag{2.8a}$$

$$\sigma_Y^2 = \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2} \tag{2.8b}$$

From the above two equations, the parameters α and β in the standard Beta PDF can be easily obtained as

$$\alpha = \frac{\mu_Y(1-\mu_Y)-\sigma_Y^2}{\sigma_Y^2}\mu_Y \tag{2.9a}$$

$$\beta = \frac{\mu_Y(1-\mu_Y)-\sigma_Y^2}{\sigma_Y^2}(1-\mu_Y) \tag{2.9b}$$

In the present problem context, the random variable Y is the dimensionless time having a CDF represented by the dimensionless rainfall mass curve. The two parameters in the Beta PDF can be calculated using the first two moments of Y. The means of the first two moments for each storm pattern (see Tables 2.4 and 2.5) were used in this study.

Differences between the observed storm patterns (solid lines) and those fitted by the Beta distribution (dash lines) are shown in Figs. 2.16 and 2.17. These figures indicate that the Beta distribution cannot represent the storm patterns under consideration.

2.6.2 Fitting Storm Patterns by Johnson S_B Distribution

Johnson (1949) described a system of frequency curves consisting of a four-parameter bounded system S_B:

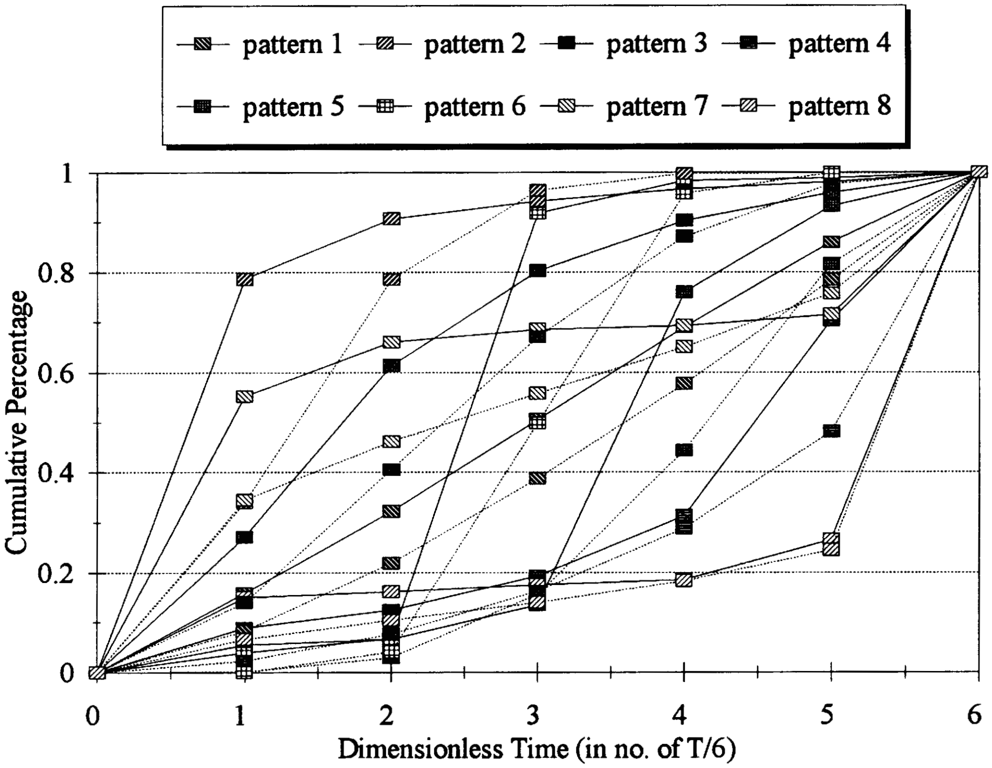


Fig. 2.16 Comparison of 8 Representative Duration-Based Storm Patterns with Those Fitted by the Beta Distribution (Dash Lines)

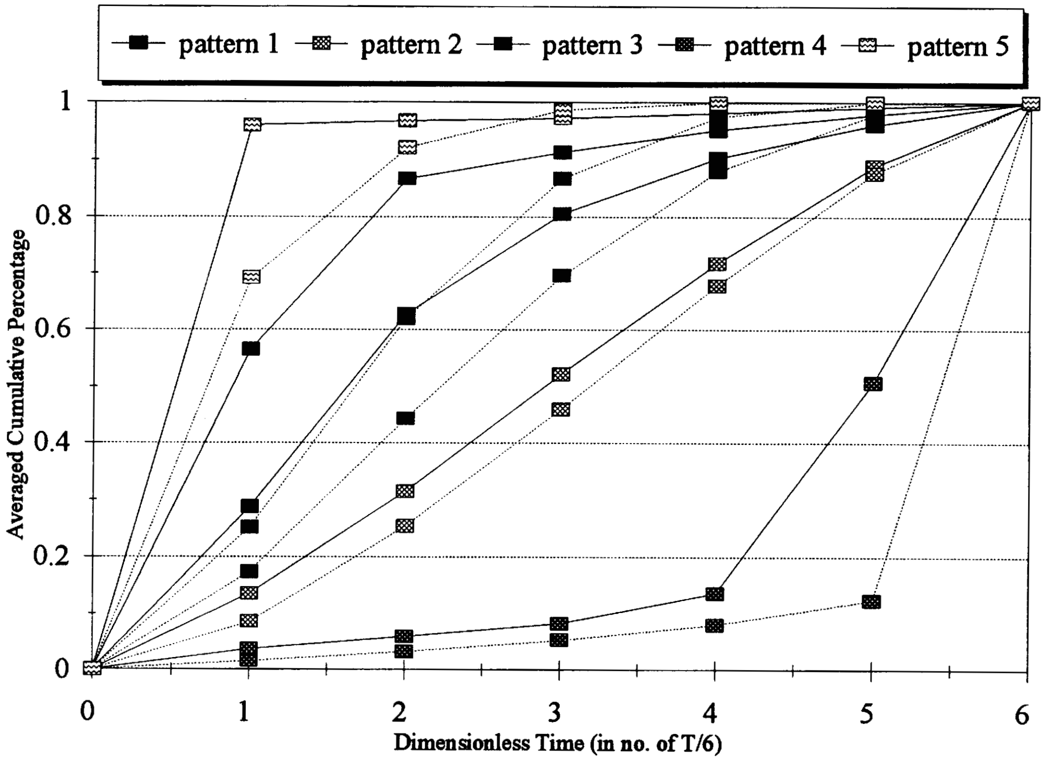


Fig. 2.17 Comparison of 5 Representative Event-Based Storm Patterns with Those Fitted by the Beta Distribution (Dash Lines)

$$Z = \gamma + \delta \ln[(X-\xi)/(\xi+\lambda-X)], \quad \xi<X<\xi+\lambda \tag{2.10}$$

where Z is the standardized normal variable.

Hill et al. (1976) provided an algorithm to estimate γ, δ, λ and ξ by matching the first four central moments of X. However, the algorithm they developed does not fit the bounded system S_B only. Instead, it chooses the best fitted Johnson's curves which also include the lognormal system S_L, the unbounded system S_U, and the normal system. This algorithm is not suitable for the problem at hand because, in our case, only bounded systems are desired.

Equation (2.10) for the S_B curve can be reduced to a 2-parameter S_B as

$$Z = \gamma + \delta \ln[Y/(1-Y)], \quad 0<Y<1 \tag{2.11}$$

in which Y is the reduced random variable defined as

$$Y = (X-\xi)/\lambda, \quad \xi<X<\xi+\lambda \tag{2.12}$$

Bacon-Shone (1985) generalizes the above two-parameter S_B model to a three-parameter S_B model through the introduction of a new parameter, α, as

$$Z = \gamma + \delta \ln[Y^{\alpha}/(1-Y^{\alpha})], \quad 0<Y<1 \tag{2.13}$$

where Y is defined as Eq. (2.12). Furthermore, he also developed an algorithm to compute γ, δ and α by matching the first three moments or to match the first two moments for the special case when $\alpha = 1$. Since the dimensionless storm patterns are bounded between 0 and 1,

$\lambda = 1$ and $\xi = 0$ in Eq. (2.12). In this study, both Bacon-Shone's three-parameter Johnson S_B model (2.13) and the two-parameter S_B model (2.11) were used to fit the various storm patterns in Figs. 2.2 and 2.3.

Model Fitting Using Moments - Using the means of the statistical moments with respect to dimensionless time, the parameters in Eqs. (2.11) and (2.13) were found. The results are shown in rows M-3 and M-2, respectively, in Tables 2.16 and 2.17, in which M-3 and M-2, respectively, stand for fitting the reduced 3-parameter and 2-parameter S_B curves by the moments.

While using the first three moments (M-3) to fit the Johnson S_B curve, there are some patterns, e.g., pattern 3 of the event-based storm, whose fitted parameters are out of bounds according to Bacon-Shone's algorithm. The lack of accurate skewness may be the cause of this problem.

Model Fitting Using CDF and Quantiles - Since the ordinates of a dimensionless mass curve represent the cumulative percentages at different time points, it is reasonable to treat these ordinates as the CDF values:

$$p_i = P\{Y \leq y_i\}, \qquad 1<i<5 \tag{2.14}$$

in which $y_i = i/6$ is the dimensionless time point; and p_i is the cumulative percentage at the corresponding time point.

Using the three-parameter Johnson S_B distribution to fit the dimensionless rainfall mass curve, the cumulative percentage p_i at y_i can be calculated as:

$$\begin{aligned} p_i = P\{Y \le y_i\} &= P\{Z \le \gamma + \delta \ln(y_i^\alpha/(1-y_i^\alpha)\} \\ &= P\{Z \le z_i\} \\ &= \Phi(z_i | \gamma, \delta, \alpha) \end{aligned} \tag{2.15}$$

where $\Phi(z_i)$ is the standard normal CDF, and $z_i = \gamma + \delta \ln[y_i^\alpha/(1-y_i^\alpha)]$ is the normal quantile corresponding to the dimensionless time, y_i, defined by Eq. (2.13).

Two approaches can be applied to fit the 3-parameter reduced Johnson S_B model. One approach focuses on the fitting of the CDF ordinates in which the least square method can be applied to find the three parameters by solving the following optimization problem,

$$Minimize \quad S_2(\gamma, \delta, \alpha) = \sum_{i=1}^{5} (p_i - \hat{p}_i)^2 = \sum_{i=1}^{5} [p_i - \Phi(\hat{z}_i | \gamma, \delta, \alpha)]^2 \tag{2.16}$$

Alternatively, the model parameters can be obtained by focusing on the quantiles in which the following optimization problem is solved,

$$Minimize \quad S_1(\gamma, \delta, \alpha) = \sum_{i=1}^{5} (z_i - \hat{z}_i)^2 = \sum_{i=1}^{5} [\Phi^{-1}(p_i) - \hat{z}_i(\gamma, \delta, \alpha)]^2 \tag{2.17}$$

where $\Phi^{-1}(p)$ is the inverse standard normal CDF.

According to the above two types of objective functions, four cases were considered in this study:

(a) LS-1a: fitting Eq. (2.11) using $S_1(\gamma, \delta, 1.0)$.

(b) LS-2a: fitting Eq. (2.11) using $S_2(\gamma, \delta, 1.0)$.

(c) LS-1b: fitting Eq. (2.13) using $S_1(\gamma, \delta, \alpha)$.

(d) LS-2b: fitting Eq. (2.13) using $S_2(\gamma,\delta,\alpha)$.

The optimization problem for Eq. (2.17) can be solved based on the necessary condition for the minimum of S_1 as

$$\frac{\partial S_1}{\partial \gamma} = \sum_{i=1}^{5}\left[\gamma+\delta\ln\left(\frac{y_i^{\alpha}}{1-y_i^{\alpha}}\right)-\Phi^{-1}(p_i)\right] = 0 \tag{2.18a}$$

$$\frac{\partial S_1}{\partial \delta} = \sum_{i=1}^{5}\ln\frac{y_i^{\alpha}}{1-y_i^{\alpha}}\times\left[\gamma+\delta\ln\left(\frac{y_i^{\alpha}}{1-y_i^{\alpha}}\right)-\Phi^{-1}(p_i)\right] = 0 \tag{2.18b}$$

$$\frac{\partial S_1}{\partial \alpha} = \sum_{i=1}^{5}\frac{\ln y_i}{1-y_i^{\alpha}}\times\left[\gamma+\delta\ln\left(\frac{y_i^{\alpha}}{1-y_i^{\alpha}}\right)-\Phi^{-1}(p_i)\right] = 0 \tag{2.18c}$$

For the special case with $\alpha = 1$, the above equations can be written as

$$5\gamma + \delta\sum_{i=1}^{5}\ln\left(\frac{y_i}{1-y_i}\right) = \sum_{i=1}^{5}\Phi^{-1}(p_i) \tag{2.19a}$$

$$\left[\sum_{i=1}^{5}\ln\left(\frac{y_i}{1-y_i}\right)\right]\gamma + \delta\sum_{i=1}^{5}\left[\ln\left(\frac{y_i}{1-y_i}\right)\right]^2 = \sum_{i=1}^{5}\left[\Phi^{-1}(p_i)\times\ln\left(\frac{y_i}{1-y_i}\right)\right] \tag{2.19b}$$

in which $\Phi(z)$ and $\Phi^{-1}(p)$ can be calculated accurately by the approximation methods suggested by Abramowitz and Stegun (1972).

Press et al. (1989) pointed out "there are no good, general methods for solving systems of more than one nonlinear equation". However, finding a minimum of a function is relatively easier and "there are efficient general techniques for finding a minimum of a function of many variables". The Newton-Ralphson method for solving the nonlinear system of equations, Eqs. (2.18), failed because of the problems with frequent divergence. Hence, the downhill multidimensional simplex method in multidimension was used (Press et al., 1989). The basic idea of this method is to evaluate the objective functions at the (N+1) vertices of a general simplex and to move towards the (at least local) optimum point. Reflection, contraction, and expansion are the three basic operations to achieve this goal. In this study, the corresponding objective functions for the 3-parameter Johnson S_B curve and 2-parameter Johnson S_B curve are $S_2(\gamma,\delta,\alpha)$ and $S_2(\gamma,\delta,1.0)$, respectively. The objective functions $S_1(\gamma,\delta,\alpha)$ and $S_1(\gamma,\delta,1.0)$ were also considered for the purpose of comparison.

The downhill simplex method requires (N+1) points to define an initial simplex. After making a guess of the optimum point X_0, one can generate the other N points as

$$X_i = X_0 + \lambda_i e_i \tag{2.20}$$

where the e_i's are N unit vectors and λ_i's are the characteristic length scales for each vector direction. Although the Taylor expansion can be applied to Eqs. (2.18) to find the starting point $X_0(\gamma,\delta,\alpha)$, a preliminary investigation shows that $(\gamma,\delta,1.0)$ is a good initial guess of X_0. By assigning $\alpha = 1$ as the starting point, (γ,δ) can be obtained by solving Eqs. (2.19a) and (2.19b).

Tables 2.14 and 2.15 show the comparisons of the different methods in that F1-F5 are the percentiles at dimensionless time points $y_i = i/6$ $(1<i<5)$, respectively. These percentiles were calculated from Eqs. (2.11) or (2.13) depending on how many parameters are involved in the method. SSE_Z is the sum of square error of z_i according to Eq. (2.17) and SSE_p is the sum of square error of p_i according to Eq. (2.16). Obviously, the SSE_p is a good criterion for evaluation of different methods.

Table 2.14 Comparison of Observed and Fitted Duration-Based Dimensionless Mass Curves by Different Methods

Pattern		F1	F2	F3	F4	F5	SSE_Z	SSE_p
1	**observed**	**0.1589220**	**0.3227310**	**0.5053900**	**0.6909720**	**0.8597030**		
	M-3	7.067419E-05	2.739557E-04	7.036851E-04	1.688145E-03	4.851427E-03	52.33640	1.589804
	M-2	0.1079810	0.2393163	0.3788680	0.5364891	0.7325115	0.5980672	6.560345E-02
	LS-1a	0.1529313	0.3350765	0.5105445	0.6839620	0.8592175	2.314079E-03	2.642455E-04
	LS-2a	0.1493081	0.3316787	0.5086638	0.6839314	0.8605611	2.699164E-03	2.335135E-04
	LS-1b	0.1566494	0.3317312	0.5051008	0.6815003	0.8624502	1.535457E-03	1.835123E-04
	LS-2b	0.1546778	0.3300337	0.5049496	0.6833488	0.8655002	1.848085E-03	1.632566E-04
2	**observed**	**0.7865214**	**0.9075214**	**0.9435128**	**0.9666837**	**0.9816025**		
	M-3	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	79.96690	5.876466E-02
	M-2	0.2479784	0.3840427	0.4988455	0.6137442	0.7501863	11.71111	0.9399074
	LS-1a	0.8125654	0.8944928	0.9364548	0.9641149	0.9847665	2.472711E-02	9.144622E-04
	LS-2a	0.7910784	0.8958389	0.9448793	0.9735678	0.9914383	0.1026327	3.032478E-04
	LS-1b	0.7892132	0.9033656	0.9453796	0.9670237	0.9814231	1.012226E-03	2.814941E-05
	LS-2b	0.7870376	0.9046549	0.9465743	0.9673036	0.9805719	1.626376E-03	1.930242E-05
3	**observed**	**0.2724423**	**0.6146058**	**0.8026891**	**0.9036154**	**0.9588782**		
	M-3	1.974693E-05	5.337870E-03	0.1083543	0.5769964	0.9865474	26.18395	1.034968
	M-2	0.1741536	0.2691694	0.3551971	0.4493409	0.5773910	6.851894	0.6811338
	LS-1a	0.3239965	0.5834599	0.7628654	0.8888333	0.9704638	7.597065E-02	5.566555E-03
	LS-2a	0.2893649	0.5931666	0.7978653	0.9239770	0.9869016	0.2579558	1.969192E-03
	LS-1b	0.2715205	0.6184798	0.8031062	0.9013757	0.9594066	3.302416E-04	2.132724E-05
	LS-2b	0.2718428	0.6161817	0.8016383	0.9017442	0.9610731	7.897730E-04	1.226594E-05
4	**observed**	**8.905714E-02**	**0.1248286**	**0.1938857**	**0.3140000**	**0.7046286**		
	M-3*	0.1209881	0.3553683	0.5417147	0.6799692	0.7951576	2.566042	0.3172821
	M-2	2.904795E-02	8.364068E-02	0.1605545	0.2731311	0.4643858	0.7768636	6.579536E-02
	LS-1a	5.733583E-02	0.1454583	0.2540914	0.3946475	0.6003290	0.2302683	2.243896E-02
	LS-2a	1.761542E-02	8.233270E-02	0.1982718	0.3797759	0.6591238	0.6808506	1.332620E-02
	LS-1b	6.538374E-02	0.1335280	0.2314368	0.3827890	0.6368314	0.1151340	1.137458E-02
	LS-2b	3.894272E-02	9.835374E-02	0.1983666	0.3700007	0.6685585	0.2265502	7.669579E-03

Table 2.14 (concluded)

Pattern		F1	F2	F3	F4	F5	SSE_Z	SSE_p
5	**observed**	**5.525000E-02**	**6.583333E-02**	**0.1349167**	**0.7599167**	**0.9330834**		
	M-3	1.368663E-07	3.385330E-06	4.082234E-05	4.570983E-04	7.804650E-03	61.08558	1.458497
	M-2	6.777919E-02	9.468696E-02	0.1197594	0.1491822	0.1949376	8.668016	0.9190750
	LS-1a	1.750361E-02	0.1279803	0.3443987	0.6311932	0.9044441	1.069013	6.655970E-02
	LS-2a	1.011407E-07	2.168072E-03	0.1403544	0.7565243	0.9988175	17.15207	1.146787E-02
	LS-1b	1.005473E-04	3.368941E-02	0.3443987	0.8480632	0.9982314	7.210498	5.997149E-02
	LS-2b	8.432589E-06	4.455411E-03	0.1423098	0.7572291	0.9995108	11.78410	1.129336E-02
6	**observed**	**3.928571E-02**	**6.757143E-02**	**0.9181429**	**0.9842857**	**0.9884286**		
	M-3	1.328572E-06	6.396814E-03	0.1312691	0.5022397	0.8916151	21.61118	0.8661972
	M-2	0.9945288	2.300032E-06	1.903639E-08	1.903639E-08	1.903639E-08	194.5228	3.705851
	LS-1a	3.215922E-02	0.3067996	0.6958591	0.9369914	0.9979799	2.513547	0.1090190
	LS-2a	5.464003E-08	6.754816E-02	0.9181724	0.9999906	1.000000	27.56500	1.923906E-03
	LS-1b	2.707343E-04	0.1154411	0.6958591	0.9868971	0.9999964	8.655032	5.336438E-02
	LS-2b	6.001007E-08	6.753879E-02	0.9181771	0.9999911	1.000000	27.48769	1.923920E-03
7	**observed**	**0.5522500**	**0.6612500**	**0.6865000**	**0.6937500**	**0.7152500**		
	M-3	2.621578E-03	1.002600E-02	2.813572E-02	7.486584E-02	0.2235812	27.34234	1.784384
	M-2	0.3775516	0.4583041	0.5207521	0.5826938	0.6612951	0.7663215	0.1144236
	LS-1a	0.5872116	0.6311018	0.6632354	0.6942194	0.7331084	2.151026E-02	2.991606E-03
	LS-2a	0.5838977	0.6298900	0.6635245	0.6958987	0.7364136	2.171822E-02	2.965415E-03
	LS-1b	0.5685298	0.6378459	0.6765396	0.7029094	0.7227842	7.647642E-03	1.052651E-03
	LS-2b	0.5664715	0.6374129	0.6769818	0.7039192	0.7240878	7.676417E-03	1.042570E-03
8	**observed**	**0.1511429**	**0.1631429**	**0.1765714**	**0.1871429**	**0.2667143**		
	M-3*	0.9999862	1.0000000	1.0000000	1.0000000	1.0000000	188.9185	3.297344
	M-2	0.9142575	0.8363219	0.7536268	0.6526333	0.5017734	14.24036	1.640441
	LS-1a	0.1400837	0.1655081	0.1866080	0.2092973	0.2416555	1.636152E-02	1.347395E-03
	LS-2a	0.1359489	0.1633635	0.1863350	0.2112074	0.2469044	1.711297E-02	1.297763E-03
	LS-1b	0.1423666	0.1624132	0.1826846	0.2080210	0.2492032	1.061679E-02	8.574611E-04
	LS-2b	0.1401350	0.1608606	0.1820813	0.2088588	0.2527730	1.080447E-02	8.226817E-04

***: Fitted parameters out of bounds (Bacon-Shone, 1985)**

Table 2.15 Comparison of Observed and Fitted Event-Based Dimensionless Mass Curves by Different Methods

		F1	F2	F3	F4	F5	SSE_z	SSE_p
1	**observed**	**0.5649250**	**0.8670550**	**0.9132600**	**0.9509200**	**0.9783500**		
	M-3	0.9996665	1.000000	1.000000	1.000000	1.000000	73.76490	0.2170759
	M-2	0.2034837	0.2617125	0.3108431	0.3636013	0.4373535	16.24998	1.497606
	LS-1a	0.6481918	0.8112803	0.8966171	0.9497458	0.9840066	0.1252447	1.0354538E-02
	LS-2a	0.5765975	0.8309220	0.9377584	0.9827696	0.9980069	1.003648	3.4428055E-03
	LS-1b	0.5885988	0.8330464	0.9195151	0.9571854	0.9773445	3.1326428E-02	1.7964263E-03
	LS-2b	0.5717461	0.8413734	0.9304803	0.9655883	0.9816087	5.8736220E-02	1.2283919E-03
2	**observed**	**0.1357792**	**0.3142882**	**0.5217115**	**0.7181115**	**0.8894387**		
	M-3	5.2214921E-02	0.1388108	0.2542078	0.4116257	0.6446325	2.517427	0.2631972
	M-2	0.1262800	0.2615949	0.3990238	0.5504055	0.7364386	0.6745498	6.9453388E-02
	LS-1a	0.1317761	0.3261614	0.5216936	0.7120376	0.8900593	1.7338911E-03	1.9427598E-04
	LS-2a	0.1283264	0.3230919	0.5206363	0.7131366	0.8921120	2.2108043E-03	1.6610170E-04
	LS-1b	0.1336315	0.3242763	0.5186056	0.7106542	0.8915178	1.4996764E-03	1.7395496E-04
	LS-2b	0.1311883	0.3223347	0.5187761	0.7131366	0.8947338	2.0062272E-03	1.4722787E-04
3	**observed**	**0.2876383**	**0.6287833**	**0.8056734**	**0.9031583**	**0.9602917**		
	M-3*	0.9996189	0.9999987	1.000000	1.000000	1.000000	87.43625	0.6934350
	M-2	0.1946028	0.2896057	0.3734757	0.4638309	0.5854027	6.574976	0.6440423
	LS-1a	0.3388027	0.5950288	0.7694151	0.8912761	0.9705927	6.6791877E-02	5.3191213E-03
	LS-2a	0.3055009	0.6045747	0.8025045	0.9244982	0.9864421	0.2325978	2.0544070E-03
	LS-1b	0.2876649	0.6277539	0.8067851	0.9029337	0.9603418	2.5169882E-05	2.3494824E-06
	LS-2b	0.2880281	0.6278328	0.8066850	0.9027805	0.9602096	2.7077258E-05	2.2284264E-06
4	**observed**	**3.7000000E-02**	**5.8800001E-02**	**8.2549997E-02**	**0.1366000**	**0.5076500**		
	M-3*	0.9999995	0.9999972	0.9999928	0.9999863	0.9999777	159.1814	3.642744
	M-2	0.8286012	0.6990954	0.5788293	0.4506570	0.2908670	15.62826	1.428531
	LS-1a	2.2188667E-02	6.3223921E-02	0.1223225	0.2122468	0.3758487	0.3027585	2.4914835E-02
	LS-2a	5.0346524E-04	9.2558898E-03	4.9560431E-02	0.1728296	0.4961525	2.971009	6.3197021E-03
	LS-1b	2.5235035E-02	5.6914199E-02	0.1093429	0.2054266	0.4144396	0.1839481	1.4285115E-02
	LS-2b	3.6436999E-03	1.7605683E-02	5.8702510E-02	0.1737232	0.4963721	1.152534	4.8836409E-03
5	**observed**	**0.9590000**	**0.9672000**	**0.9721000**	**0.9803000**	**0.9902000**		
	M-3	1.000000	1.000000	1.000000	1.000000	1.000000	62.24333	4.0193810E-03
	M-2	0.1414966	8.5412182E-02	5.5531811E-02	3.45854E-02	1.729E-02	65.33846	4.126889
	LS-1a	0.9542153	0.9680323	0.9760238	0.9822649	0.9883476	1.3222400E-02	4.6273643E-05
	LS-2a	0.9572713	0.9685016	0.9752715	0.9807760	0.9864266	1.9071037E-02	2.9206229E-05
	LS-1b	0.9561274	0.9667584	0.9746870	0.9818327	0.9892473	5.1301536E-03	1.8395789E-05
	LS-2b	0.9581161	0.9672843	0.9743191	0.9808576	0.9879456	7.7276174E-03	1.1106136E-05

***: Fitted parameters out of bounds (Bacon-Shone, 1985)**

From these tables, some notable observations can be made:

(1) LS-2b appears to be the best method and M-3 is the worst method.

(2) The least-squares method is better than the moments method.

(3) Basically, M-3 is worse than M-2 because the moments are calculated from samples. The skewness and higher-order moments are not so reliable. According to Johnson and Kitchen (1971), the two moment method is expected to be superior when using sample moments. If the exact moments are known, the four moments method appears to be the best.

(4) The 5-parameter Johnson S_B curves (LS-1b, LS-2b) provide a better fit than the 4-parameter Johnson S_B curves (LS-1a, LS-2a).

(5) Using S_2 as the objective function (LS-2a, LS-2b) leads to a better fit than using S_1 as the objective function (LS-1a, LS-1b).

(6) Using LS-2b, except for pattern 5 of the duration-based storms, every SSE_p, the sum of squared errors of p_i, is less than 10^{-2}. This indicates that Johnson's reduced three-parameter model is good enough to describe the significant storm patterns.

Figures. 2.18 - 2.25 show the comparison of the fitted storm patterns with the original storm patterns. These figures also indicate that LS-2b provides the best fit. The fitted parameters are shown in Tables 2.16 and 2.17. The fitted parameters obtained from the moments and least square method are very different. Using the least square method, there are substantial differences between the parameters fitted by different methods whereas the differences in parameters fitted by the same model with different objective functions are not so great.

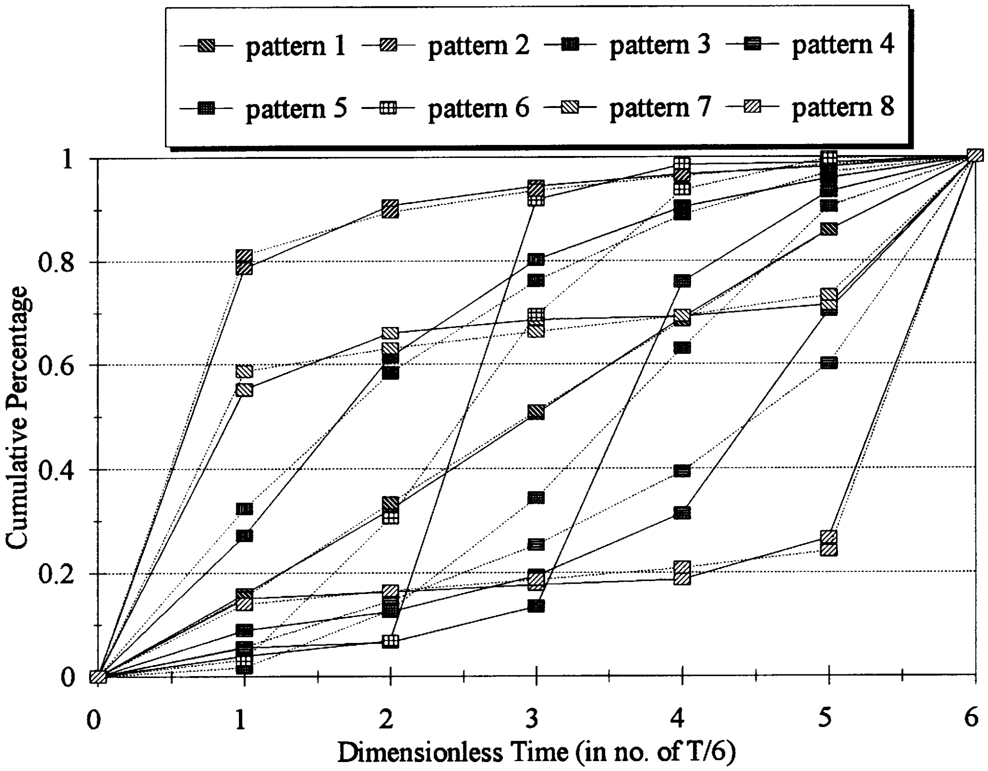


Fig. 2.18 Comparison of 8 Representative Duration-Based Storm Patterns with Those Fitted by the 4-Parameter Johnson SB Curves Using LS-1a (Dash Lines)

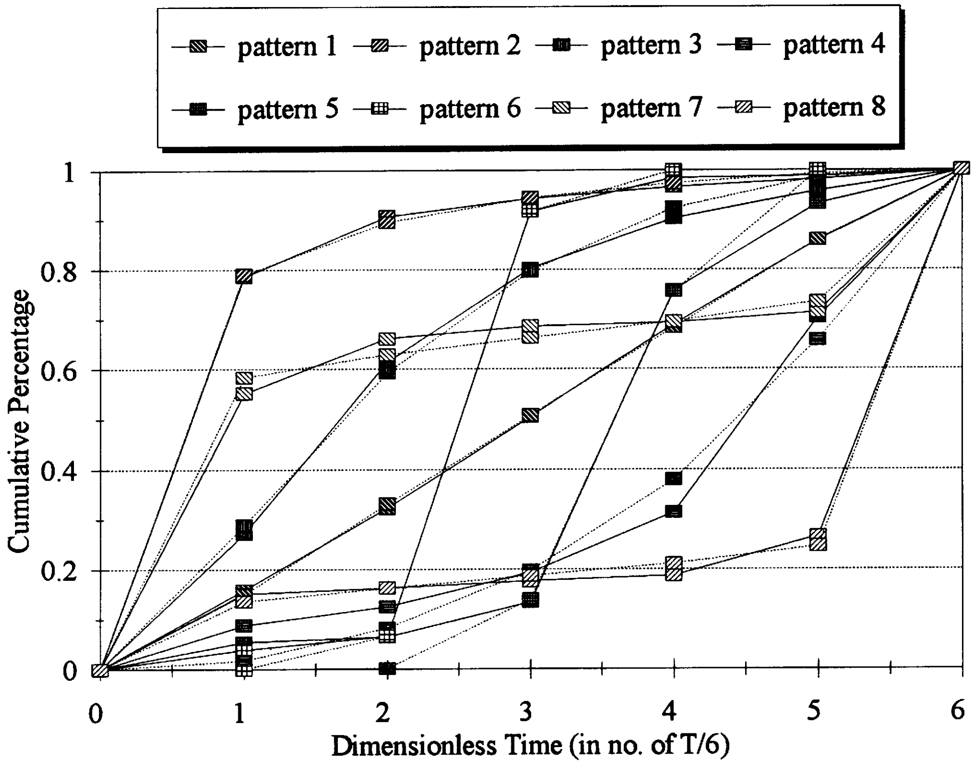


Fig. 2.19 Comparison of 8 Representative Duration-Based Storm Patterns with Those Fitted by the 4-Parameter Johnson SB Curves Using LS-2a (Dash Lines)

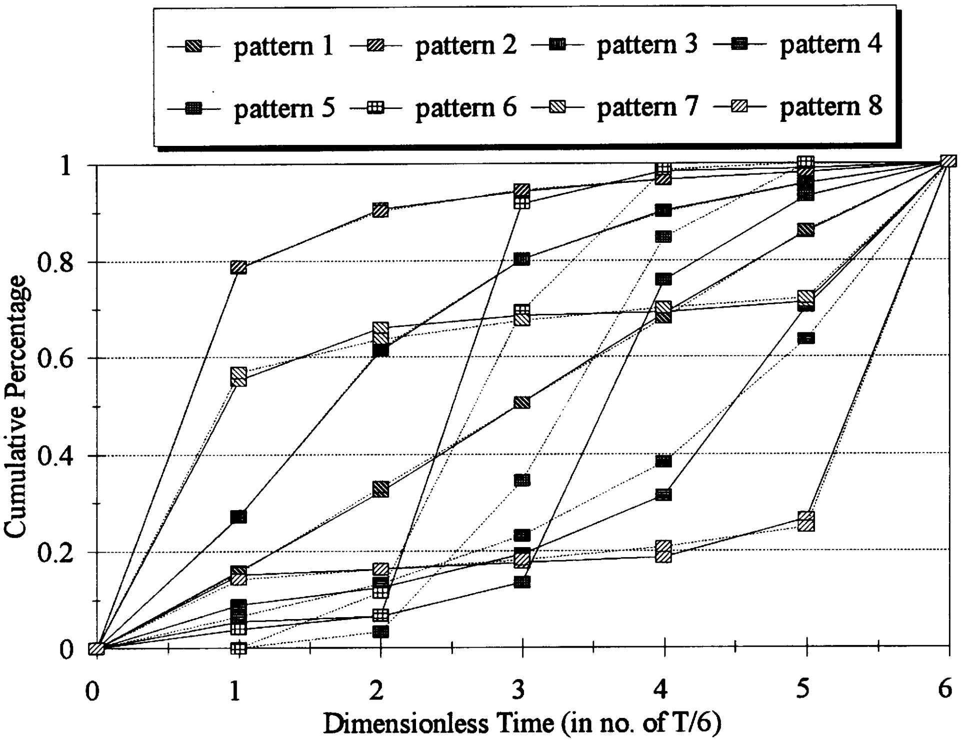


Fig. 2.20 Comparison of 8 Representative Duration-Based Storm Patterns with Those Fitted by the 5-Parameter Johnson SB Curves Using LS-1b (Dash Lines)

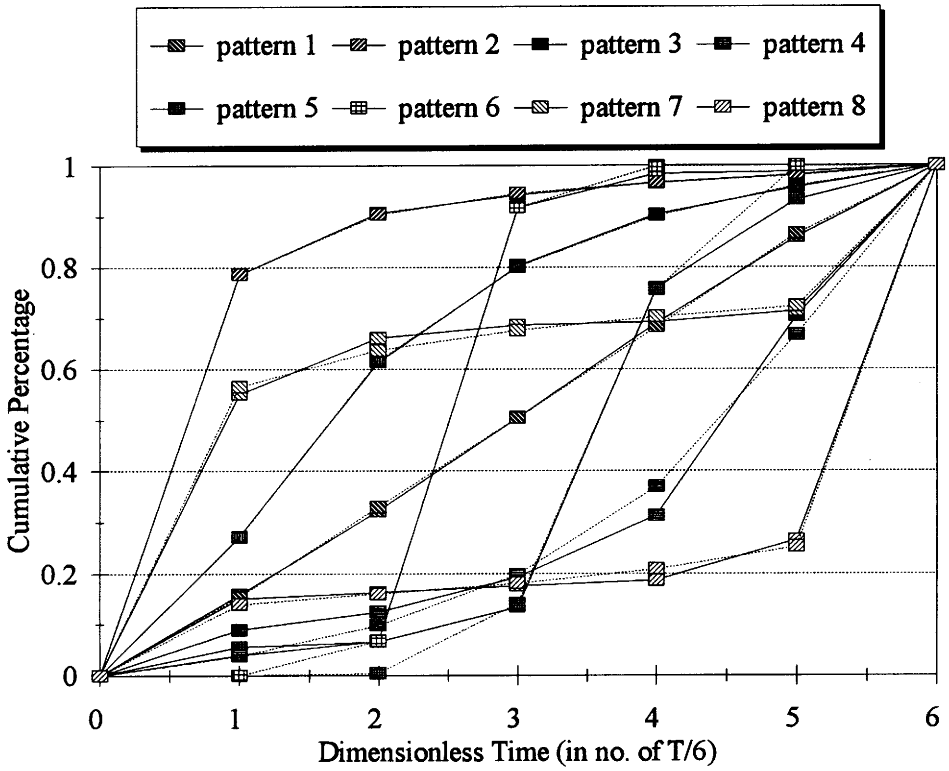


Fig. 2.21 Comparison of 8 Representative Duration-Based Storm Patterns with Those Fitted by the 5-Parameter Johnson SB Curves Using LS-2b (Dash Lines)

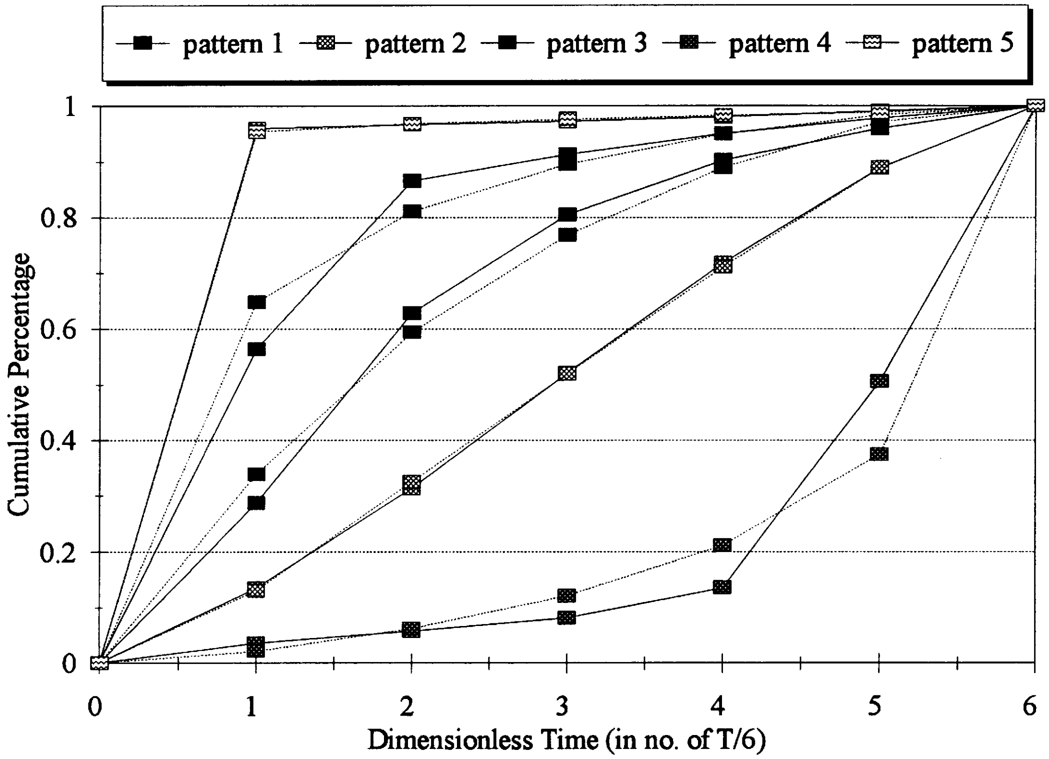


Fig. 2.22 Comparison of 5 Representative Event-Based Storm Patterns with Those Fitted by the 4-Parameter Johnson SB Curves Using LS-1a (Dash Lines)

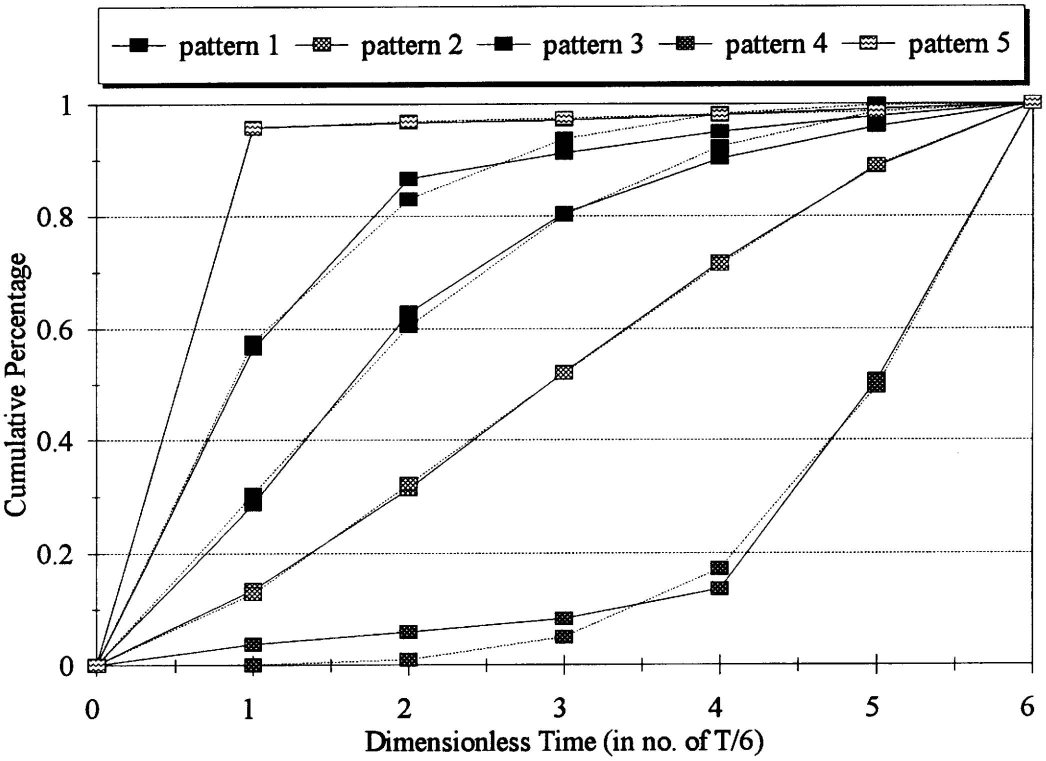


Fig. 2.23 Comparison of 5 Representative Event-Based Storm Patterns with Those Fitted by the 4-Parameter Johnson SB Curves Using LS-2a (Dash Lines)

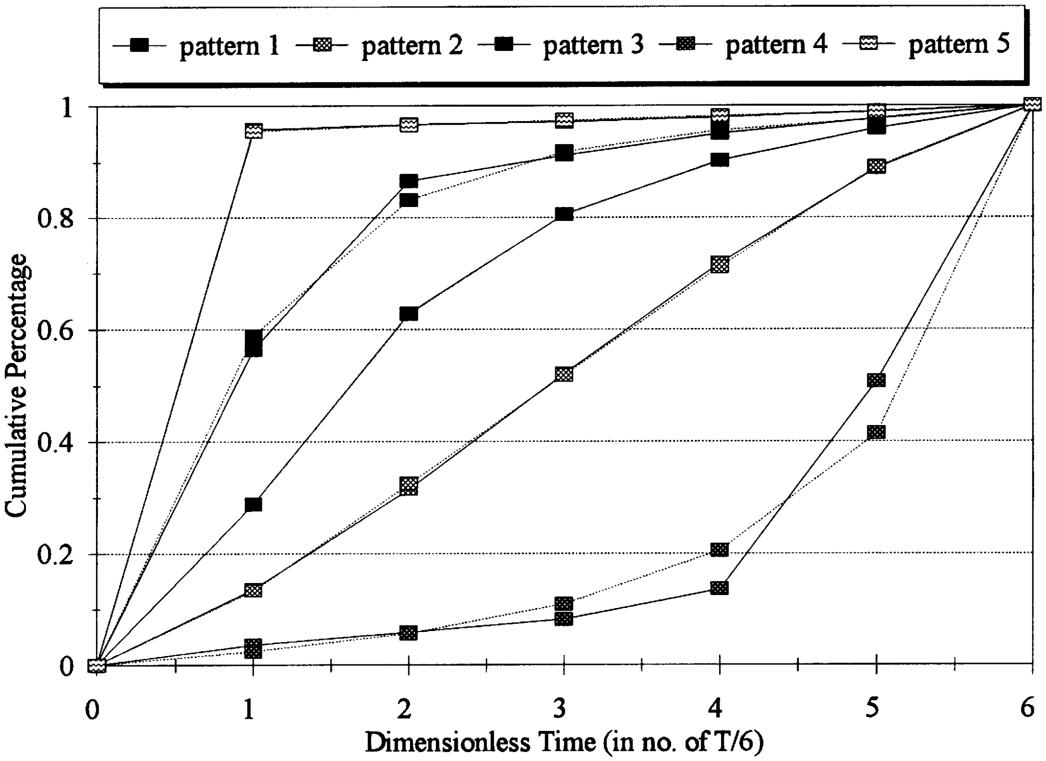


Fig. 2.24 Comparison of 5 Representative Event-Based Storm Patterns with Those Fitted by the 5-Parameter Johnson SB Curves Using LS-1b (Dash Lines)

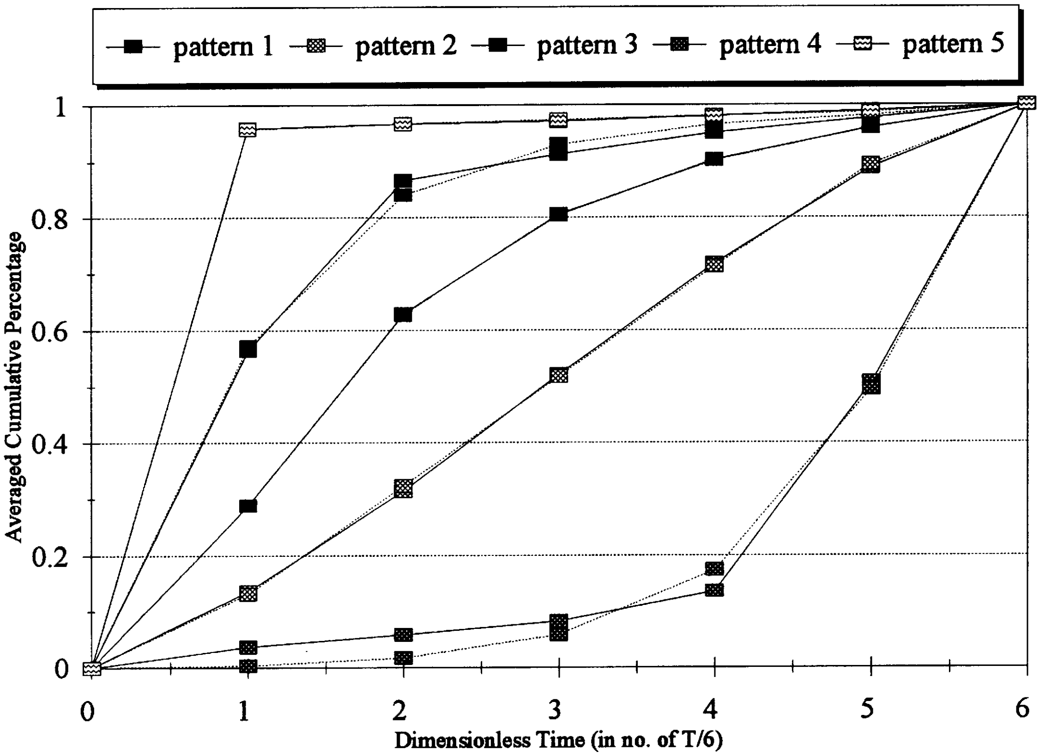


Fig. 2.25 Comparison of 5 Representative Event-Based Storm Patterns with Those Fitted by the 5-Parameter Johnson SB Curves Using LS-2b (Dash Lines)

Table 2.16 Comparison of Fitted Parameters for Duration-Based Storm Patterns by Different Methods

Pattern		GAMMA	DELTA	ALPHA
1	M-3	-3.176377	0.3748699	1.032660
	M-2	-0.3084555	0.5771469	1.000000
	LS-1a	2.6434224E-02	0.6526355	1.000000
	LS-2a	2.1719016E-02	0.6593139	1.000000
	LS-1b	-0.1741563	0.6927671	0.8184373
	LS-2b	-0.1978749	0.7042847	0.8006407
2	M-3	-46.82421	23.76490	3.5421673E-02
	M-2	-2.8944016E-03	0.4212467	1.000000
	LS-1a	1.525677	0.3965901	1.000000
	LS-2a	1.597109	0.4889533	1.000000
	LS-1b	2.081609	0.1894013	3.766300
	LS-2b	2.118697	0.1526248	4.836195
3	M-3	-3.733794	2.378902	0.4327956
	M-2	-0.3713269	0.3520177	1.000000
	LS-1a	0.7155499	0.7282682	1.000000
	LS-2a	0.8340201	0.8631964	1.000000
	LS-1b	1.727899	0.3543890	3.679811
	LS-2b	1.703555	0.3807528	3.388486
4	M-3*	0.8917136	0.2509194	4.586080
	M-2	-0.9921815	0.5609356	1.000000
	LS-1a	-0.6616698	0.5690613	1.000000
	LS-2a	-0.8478103	0.7815669	1.000000
	LS-1b	-4.186930	0.8086880	2.0038918E-02
	LS-2b	-5.791852	0.9594789	8.3160885E-03
5	M-3	-5.609155	1.080621	0.2789566
	M-2	-1.176191	0.1965573	1.000000
	LS-1a	-0.4004879	1.061107	1.000000
	LS-2a	-1.078729	2.559188	1.000000
	LS-1b	-0.4004879	2.061107	1.000000
	LS-2b	-9.341406	3.204761	0.1052760
6	M-3	0.9164577	1.036888	3.023376
	M-2	-9.973822	-7.778093	1.000000
	LS-1a	0.5125276	1.467899	1.000000
	LS-2a	1.392883	4.165331	1.000000
	LS-1b	0.5125276	2.467899	1.000000
	LS-2b	1.237072	4.207398	0.9735287
7	M-3	-3.463716	0.8272812	0.2049949
	M-2	5.2041352E-02	0.2261405	1.000000
	LS-1a	0.4213098	0.1248458	1.000000
	LS-2a	0.4221014	0.1306208	1.000000
	LS-1b	0.6381189	1.3272669E-02	19.57352
	LS-2b	0.6434295	8.8740643E-03	29.93872
8	M-3*	9.571754	5.4989666E-02	54.59815
	M-2	0.6859467	-0.4234407	1.000000
	LS-1a	-0.8904653	0.1177295	1.000000
	LS-2a	-0.8914829	0.1287528	1.000000
	LS-1b	-1.527301	0.1696783	3.6421414E-02
	LS-2b	-1.901894	0.1807582	5.8754357E-03

*:Fitted parameters out of bounds (Bacon-Shone, 1985)

Table 2.17 Comparison of Fitted Parameters for Event-Based Dimensionless Mass Curves by Different Methods

Pattern		GAMMA	DELTA	ALPHA
1	M-3	31.23171	14.20623	4.6731509E-02
	M-2	-0.4934621	0.2086318	1.000000
	LS-1a	1.262509	0.5480583	1.000000
	LS-2a	1.536226	0.8344707	1.000000
	LS-1b	2.142601	0.1581534	6.770798
	LS-2b	2.298673	7.7658139E-02	15.22049
2	M-3	-0.8949274	0.6688797	0.7699425
	M-2	-0.2558750	0.5519188	1.0000000
	LS-1a	5.4404531E-02	0.7284777	1.0000000
	LS-2a	5.1750377E-02	0.7369583	1.0000000
	LS-1b	-5.1960714E-02	0.7505643	0.9083347
	LS-2b	-6.8454258E-02	0.7631938	0.8949277
3	M-3*	6.800911	0.2907065	6.594133
	M-2	-0.3226620	0.3345249	1.000000
	LS-1a	0.7369217	0.7161849	1.000000
	LS-2a	0.8506008	0.8445463	1.000000
	LS-1b	1.710739	0.3642182	3.481001
	LS-2b	1.709685	0.3635737	3.483911
4	M-3*	3.990792	-9.1939420E-03	54.59815
	M-2	0.1988993	-0.4658477	1.000000
	LS-1a	-1.163455	0.5263035	1.000000
	LS-2a	-1.649131	1.018670	1.000000
	LS-1b	-4.561883	0.7566571	1.7545514E-02
	LS-2b	-6.269476	1.160113	2.4806980E-02
5	M-3	-56.94051	40.73217	1.7785141E-02
	M-2	-1.593431	-0.3229766	1.000000
	LS-1a	1.977788	0.1805672	1.000000
	LS-2a	1.964630	0.1520830	1.000000
	LS-1b	0.9116608	0.2566459	2.4578441E-02
	LS-2b	0.8321146	0.2293521	1.1057544E-02

*: Fitted parameters out of bounds (Bacon-Shone, 1985)

2.7 SUMMARY AND CONCLUSION

A temporal pattern of the design storm is often the required input to rainfall-runoff models to produce a flow hydrograph for design and performance evaluation of hydraulic structures. In this study, average storm patterns for complete storms were of interest. 'Significant storms' for each station with depth exceeding that of a 10-year return period at that particular station were extracted. Both duration-based and event-based storm data sets were established. Through the non-dimensionalization, rainfall mass curves and statistical moments of time were used as the attributes in statistical cluster analysis. Eight duration-based storm patterns and five event-based storm patterns were identified in Wyoming. The duration-based storm patterns are useful for constructing design rainfall hyetographs whereas the event-based storm patterns can be used in stochastic generation of typical storms.

Contingency tests were performed to find out whether the occurrence of various storm patterns were affected by climatic region, storm duration, and seasonality. It was found that the event-based storm patterns are independent of climatic region in Wyoming and, therefore, these storm patterns can be used throughout the entire State of Wyoming. On the other hand, the occurrence of duration-based storm patterns are dependent on climatic regions, storm durations, and seasons. One should be cautious when a duration-based storm pattern is to be selected for hydraulic design.

The representative storm patterns were fitted to the Beta and Johnson S_B distributions by various methods. It was found that the Beta distribution is not appropriate for describing the storm patterns under consideration despite its versatility in having different shapes. To fit the Johnson S_B distribution, moments and quantiles of dimensionless time are used. The method that fits the moments does not yield desirable results because of the lack of accurate skewness. The least-squares method using quantiles resulted in a good fit. Therefore, the 3-parameter reduced Johnson S_B distribution is recommended.

CHAPTER 3

GENERATING TEMPORAL DISTRIBUTION OF DESIGN STORMS

3.1 INTRODUCTION

As stated previously, the complete description of a design storm involves the specification of storm depth, duration, and its temporal pattern. The general engineering practice is to pre-select the duration and the return period of the design storm from which the corresponding storm depth is determined by frequency analysis. The temporal variation of the precipitation amount within a storm can be established from the established dimensionless storm patterns such as those presented in Chapter 2.

Note that the representative storm patterns as shown in Figs 2.2 and 2.3 are the averaged values. For each storm type, there exist intrinsic randomness in precipitation amount within each time increment. Therefore, the actual time distribution of precipitation amount within a design storm is subject to uncertainty. To assess the reliability of a hydraulic structure under a specified design storm condition, one should evaluate its performance under several storm distributions that are stochastically possible.

The objective of this chapter is to develop algorithms allowing one to statistically generate the temporal distribution of a design storm satisfying the following two constraints,

$$(Unit\text{-}sum): \qquad P_1 + \cdots + P_D = 1 \tag{3.1a}$$

$$(Non\text{-}negativity): \qquad P_i \geq 0, \quad i = 1, \cdots, D \tag{3.1b}$$

where P_i is the random precipitation percentage in the ith time increment; and D is the total number of time increments within the storm ($D = 6$ in this study).

The random vector $\boldsymbol{P} = (P_1, P_2, \cdots, P_D)$ satisfying the constraint equations (3.1a) and (3.1b) is called compositional data. Aitchison (1986) described many examples of compositional data and presented various problems related to compositional data analysis in different disciplines. To generate compositional data with prescribed statistical properties such as their statistical moments and correlations is not a trivial task due to the presence of the constraints.

Literature about the constrained multivariate Monte Carlo simulation is available. Borgman and Faucette (1993) proposed a practical method to convert the constrained multivariate Gaussian simulation into a conditional multivariate Gaussian simulation (See also Borgman, 1990, for detailed discussions of Gaussian unconditional and conditional simulations). Zhao (1992) applied this method to generate random unit hydrographs. Rubinstein (1981) developed several algorithms for generating random vectors uniformly distributed inside or on the surface of simplex, hypersphere, and hyper-ellipsoid. The basic idea of those algorithms is the acceptance-rejection method as stated in "Algorithm 1" by Rubinstein (1981, p. 205). Unfortunately, the methods mentioned above are not applicable directly to stochastically generate storm events of a selected storm pattern due to the non-normality and non-uniformity.

Since the constrained Monte Carlo simulation involves unconstrained Monte Carlo simulations with non-normal random variables, the procedure to generate unconstrained non-normal random variates is developed first. Based on this procedure, three constrained simulation methods, namely, the acceptance-rejection (AR) method, the cumulative distribution function (CDF) method, and the log-ratio method, are developed for generating storm events of a storm pattern. Using the storm data collected in this study, the performance of the three proposed methods is examined.

3.2 GENERATING UNCONSTRAINED MULTIVARIATE NON-NORMAL RANDOM VARIATES

The common approach to simulate unconstrained multivariate non-normal random variables often involves the following three steps: (1) transformation to the normal space; (2) generation of multivariate normal random variates; (3) inverse transformation to the original space. The transformation to normal space in step (1) is often made through pre-defined marginal distributions (Liu and Kiureghian, 1985; Chang et al., 1994). However, the marginal distributions of the random variables are often unknown in practice. The normal transformation must be found from the statistical properties of the random variable. In this study, the Johnson distribution system was adopted due to its flexibility of covering various distribution types. Three approaches to fit the Johnson system were reviewed to provide a guideline for selecting the distribution type and estimating parameters. To obtain the correlation structure in normal space as required in step (2), the Johnson distribution system was extended to the multivariate setting using the Nataf bivariate model. Formulas were derived which relate the correlation coefficient in the normal space to the correlation coefficient in the original parameter space, the distribution parameters, and the first two moments of the Johnson distributions. Based on these results, a procedure to simulate unconstrained correlated non-normal variables was proposed.

3.2.1 Univariate Johnson System for Normal Transformation

Johnson (1949a) described his system of frequency curves by the method of translation. Like the Pearson system (Johnson and Kotz, 1976), Johnson introduced four parameters in his distribution system,

$$Z = \gamma + \delta f\left(\frac{X-\xi}{\lambda}\right) \tag{3.2}$$

where Z is standard normal random variable; $f(\cdot)$ is a monotonic function of another random variable X; ξ is the location parameter; and λ is the scale parameter. By introducing $Y = (X-\xi)/\lambda$, Eq. (3.2) can be reduced to the standardized form as

$$Z = \gamma + \delta f(Y) \tag{3.3}$$

The first four moments of X and those of Y satisfy the following relationships,

$$\mu_X = \lambda\mu_Y + \xi \tag{3.4a}$$
$$\sigma_X = \lambda\sigma_Y \tag{3.4b}$$
$$\gamma_X = \gamma_Y \tag{3.4c}$$
$$\kappa_X = \kappa_Y \tag{3.4d}$$

The Johnson system consists of three types of frequency curves:

(1) Lognormal system (S_L): $Z = \gamma + \delta \ln(X - \xi), \quad \xi < X;$

(2) Unbounded system (S_U): $Z = \gamma + \delta \sinh^{-1}[(X - \xi)/\lambda];$

(3) Bounded system (S_B): $Z = \gamma + \delta \ln[(X - \xi)/(\xi + \lambda - X)], \quad \xi < X < \xi + \lambda.$

These three curves cover all the feasible area of the moment-ratio diagram (or $\beta_1 - \beta_2$ plane), which suggests every distribution is just a special case of the Johnson system (Johnson, 1949a; Tadikamalla, 1980).

After Johnson introduced his system of frequency curves, numerous efforts have been made to fit data to the Johnson system. Typically, there are three approaches to selecting the Johnson distribution type and estimating its parameters.

Tables to Facilitate Fitting the Johnson System - Based on the work of Leslie (1959), Johnson (1965) constructed a table for fitting S_U curves by their moments. Johnson (1971) provided another table for fitting S_B curves by their moments. These tables involve entries for the skewness $\sqrt{\beta_1}$ and kurtosis β_2 to find the corresponding parameters. However, one has to know the type of Johnson distribution in advance and these tables are not desirable when a high accuracy is required.

Fitting the Johnson System by Moments - Hill et al. (1976) developed an algorithm to fit Johnson system using the first four moments. They include the normal curve and a special case of S_B curves on the $\beta_2 = \beta_1 + 1$ boundary for the sake of completeness.

The S_L curves lie on a line in the $\beta_1 - \beta_2$ plane and the line is described by

$$\beta_1 = (\omega - 1)(\omega + 2)^2 \tag{3.5a}$$
$$\beta_2 = \omega^4 + 2\omega^3 + 3\omega^2 - 3 \tag{3.5b}$$

where $\omega = \exp(\delta^{-2})$. Solving Eq. (3.5a) for ω and evaluating Eq. (3.5b), the distribution type of the Johnson system can be determined. If the actual value of β_2 is greater than the evaluated one, S_U is appropriate; otherwise, S_B is chosen (Johnson, 1949a).

Based on the iterative method proposed by Leslie (1959), the parameters of S_U are fitted. Using Draper's (1952) form of Goodwin's integral, the parameters of S_B are solved iteratively. The tolerance of 0.01 is used by Hill et al. (1976) to select S_L curves. When the difference between the skewness and 0, the kurtosis and 3 are tolerably small, say 0.01, the normal curve is chosen.

Slifker and Shapiro's Method - The procedure of fitting the Johnson system by moments is not always desirable because the sample estimates of higher-order moments tend to have large sampling errors, especially when the sample size is small. Based on the relationship concerning the distances in the tails versus distances in the central portion of the Johnson distributions, Slifker and Shapiro (1980) developed an algorithm to distinguish Johnson distributions and to estimate the parameters.

Suppose x_{3z}, x_z, x_{-z} and x_{-3z} are the values corresponding to $3z$, z, $-z$, and $-3z$ under Johnson's transformation. Let

$$\begin{aligned} m &= x_{3z} - x_z \\ n &= x_{-z} - x_{-3z} \\ p &= x_z - x_{-z} \end{aligned}$$

Slifker and Shapiro (1980) proved the following criterion: (i) $mn/p^2 > 1$ suggests a S_U distribution; (ii) $mn/p^2 < 1$ suggests a S_B distribution; and (iii) $mn/p^2 = 1$ suggests a S_L distribution. They also provided formulas for parameter estimation.

Using the order statistics and certain interpolation of the data, Slifker and Shapiro's method is very easy to implement. However, there is no general rule for selecting the *z* value. Slifker and Shapiro offered an empirical choice of $z = 0.524$. In fact, the choice of *z* value has great influence on the determination of the type of Johnson distributions as well as the parameter estimation. Shayib (1989) investigated this problem by conducting simulations of an exponential distribution. He concluded that "the choice of S_B or S_U from the Johnson family highly depends on the procedure of normalizing used".

Owen (1988) described a method, called STARSHIP, which is able to solve the problem of selecting *z* value. The basic idea of STARSHIP is to choose the optimum normal transformation based on the measure of normality of the transformed data. Owen and Li (1988) gave an example of applying Slifker and Shapiro's method and the idea of the STARSHIP to fit Johnson distributions. Instead of using a predefined single *z* value, different *z* values, $z = 0.05(0.05)5$, were tried. The Shapiro-Wilk test of normality was then applied to select the optimum *z* value for the Johnson transformation.

3.2.2 Bivariate Johnson System

The univariate Johnson system has been shown to be useful in many situations. To extend it to a bivariate case, Johnson (1949b) described a method of using median regression to do such an extension. However, the correlation coefficient in the normal space must be estimated from the data at hand. Schreuder and Hafley (1977) gave an example of using $S_{B,B}$ to describe the stand structure of tree heights and diameters. A practical alternative to extend Johnson system to a bivariate case is to use the Nataf bivariate model (Liu and Kiureghian,

1986) as described below.

Suppose that standard multivariate normal variates $\boldsymbol{Z}=(Z_1,\cdots,Z_n)$ are obtained through a marginal transformation of $X=(X_1,\cdots,X_n)$ and satisfy

$$z_i = g_{X_i}(x_i) = \Phi^{-1}[F_{X_i}(x_i)], \qquad i=1,\cdots,n \tag{3.6}$$

where $\Phi(\cdot)$is the standard normal CDF; and $F_{X_i}(\cdot)$ is the CDF of X_i. The Nataf model for $\boldsymbol{X}$ is obtained by assuming that $\boldsymbol{Z}$ is jointly normal. The joint probability density function (PDF) of $\boldsymbol{X}$ is

$$f_X(x) = f_{X_1}(x_1)f_{X_2}(x_2)\cdots f_{X_n}(x_n)\frac{\phi_n(\boldsymbol{Z},\boldsymbol{R}_Z)}{\phi(z_1)\phi(z_2)\cdots\phi(z_n)} \tag{3.7}$$

where $\phi(\cdot)$ is the standard normal PDF, and $\phi_n(\boldsymbol{Z},\boldsymbol{R}_Z)$ is the n-dimensional joint standard normal PDF with the correlation matrix $\boldsymbol{R}_Z$. The element ρ_{z_i,z_j} of the correlation matrix $\boldsymbol{R}_Z$ is related to the original correlation coefficient ρ_{x_i,x_j} through the following integral equation,

$$\begin{aligned}\rho_{x_i,x_j} &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{x_i-\mu_i}{\sigma_i}\right)\left(\frac{x_j-\mu_j}{\sigma_j}\right)\phi_2(z_i,z_j,\rho_{z_i,z_j})dz_i dz_j \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{g_{X_i}^{-1}(z_i)-\mu_i}{\sigma_i}\right)\left(\frac{g_{X_j}^{-1}(z_j)-\mu_j}{\sigma_j}\right)\phi_2(z_i,z_j,\rho_{z_i,z_j})dz_i dz_j\end{aligned} \tag{3.8}$$

Using the Nataf model, Liu and Kiureghian (1986) derived a series of empirical equations of calculating the correlation coefficient in the standard normal space based on the parameters and the correlation coefficient of the original random variables for 10 commonly used marginal distributions in engineering applications. They also proved several lemmas which are useful for constructing a bivariate distribution using the Nataf model. Some of their results are:

Lemma 1. ρ_X is a strictly increasing function of ρ_Z .

Lemma 2. $\rho_Z = 0$ for $\rho_X = 0$.

Lemma 3. $|\rho_X| \le |\rho_Z|$ where the equality holds when $\rho_X = 0$ or when both marginals are normal.

The Nataf model is valid provided the mappings in Eq. (3.6) are one-to-one and the correlation matrix $\boldsymbol{R}_Z$ is positive definite. For the Johnson distribution system, $g_{X_i}(\cdot)$ are continuous and strictly increasing, which satisfies the monotonic condition. However, due to the sampling error in calculated correlation coefficients, the correlation matrix $\boldsymbol{R}_Z$ obtained from Eq. (3.8) may not necessarily be positive-definite. As a matter of fact, the elements in $\boldsymbol{R}_Z$ may not necessarily exist. According to Lemma 3, there are cases, when the absolute value of the original correlation coefficient is sufficiently large that the correlation coefficient in the normal space may be out of the bounds [-1,1]. The original correlation coefficient must be bounded within a certain range is the limitation of the Nataf model. The correlation coefficient in the normal space will always exist if it is estimated from the data as Johnson (1949b) suggested. Obviously, this method may lead to violation with the above discussion in certain situations.

Table 3.1 lists the formulas of the correlation coefficient ρ_Z in terms of ρ_X and the parameters and the first two moments of Johnson distributions. The derivation is given in Appendix A. As can be seen, the formulas of $S_{B,B}$, $S_{B,U}$, $S_{B,LN}$ are not analytically tractable. The attempt to establish the empirical equations for these cases failed due to the complexities

Table 3.1 Formulas of Correlation Coefficient in Normal Space for Bivariate Johonson Sytems

X_j X_i	N	LN	S_U	S_B
N	ρ_X			
LN	$\dfrac{\rho_X \cdot \omega_i}{\sqrt{\ln(1+\omega_i^2)}}$ where $\omega_i = \sigma_i / \mu_i$	$\dfrac{\ln(1+\rho_X \omega_i \omega_j)}{\sqrt{\ln(1+\omega_i^2) \cdot \ln(1+\omega_j^2)}}$ where $\omega_i = \sigma_i / \mu_i, \ \omega_j = \sigma_j / \mu_j$		
S_U	$\rho_X \sigma_i \delta_i e^{-1/2\delta_i^2} [\cosh(\gamma_i / \delta_i)]^{-1}$	$\delta_i \delta_j \ln(\omega + \sqrt{\omega^2+1}) + \delta_j \gamma_i$ where $\omega = \dfrac{\rho_X \sigma_i \sigma_j + \mu_i \mu_j}{e^{1/2\delta_i^2 + 1/2\delta_j^2 - \gamma_j/\delta_j}}$	$\dfrac{\ln[(B+\sqrt{B^2+AC})/A]}{\Omega_i \Omega_j}$ where $A = \cosh(\omega_i + \omega_j)$ $C = \cosh(\omega_i - \omega_j)$ $B = \rho_X \sigma_i \sigma_j \exp(-\dfrac{\Omega_i^2 + \Omega_j^2}{2})$ $+ \sinh(\omega_i) \sinh(\omega_j)$ $\omega_i = -\gamma_i / \delta_i \ , \ \Omega_i = 1/\delta_i$ $\omega_j = -\gamma_j / \delta_j \ , \ \Omega_j = 1/\delta_j$	
S_B	$\dfrac{\rho_X \sigma_i \delta_i}{\mu_i - \mu_i^2 - \sigma_i^2}$	Not Analytically Tractable	Not Analytically Tractable	Not Analytically Tractable

Note: ρ_X is the correlation coefficient in the original space. μ and σ are the mean and standard deviation of the standardized Johnson variates. γ and δ are parameters for corresponding Johnson system.

of the forms of Johnson distributions. Nevertheless, the value of ρ_{z_i,z_j} on the right-hand side of Eq. (3.8) can be solved by using an appropriate root-finding procedure in conjunction with a proper numerical integration technique. The bracket points are easily determined from Lemmas 1 and 3. If $\rho_X<0$, the search interval is $(-1, \rho_X]$ whereas for $\rho_X>0$, the search can be made in $[\rho_X, 1]$. As stated before, due to the sampling error in the estimate of ρ_X and/or the limitation of the Nataf model, solving Eq. (3.8) for feasible ρ_Z may not always be possible.

3.2.3 Procedure to Generate Unconstrained Multivariate Non-normal Random Variates

Based on the multivariate Johnson system, a procedure to generate unconstrained multivariate non-normal random variates is developed herein. The procedure involves the following steps:

Step [1] - Fit Johnson distributions to each variable and calculate the correlation matrix for the random variables in the original space. If the sample size is large, the procedure by Hill et al. (1976) is recommended. If the sample size is small, Owen's procedure (1988), the STARSHIP, is recommended.

Step [2] - Construct a multivariate distribution model with the Nataf model. Use Table 3.1 to find the correlation matrix $\boldsymbol{R}_Z$. Numerical solutions are necessary to solve Eq. (3.8) to obtain the correlation coefficient in the normal space for $S_{B,LN}$, $S_{B,U}$, and $S_{B,B}$.

Step [3] - Use orthogonal transformation to generate multivariate standard normal random variates $z = (z_1, \cdots, z_n)$ having the correlation matrix $\boldsymbol{R}_Z$. (For detailed discussion of Gaussian unconditional simulation, see Borgman, 1990; Borgman and Faucette, 1993)

Step [4] - Transform the generated multivariate standard normal variates to the original parameter space using the parameters and Johnson distribution types identified in Step [1].

The above procedure is not exactly a full multivariate simulation due to the lack of the information about the marginal PDFs. In fact, this procedure tries to preserve the partial information such as the moments and correlation structure of the given sample.

3.3 GENERATING TEMPORAL STORM PATTERNS

Using the procedure of unconstrained multivariate non-normal simulation, three methods were developed to generate temporal storm patterns as described in Eqs. (3.1a) and (3.1b). They are the acceptance-rejection method, the CDF method, and the log-ratio method. The acceptance-rejection method is intuitively straightforward which deals with the unit-sum constraint explicitly. On the other hand, the CDF method and the log-ratio method are motivated by converting the problem of constrained simulation into a problem of unconstrained simulation.

3.3.1 Acceptance-Rejection (AR) Method

Compositional data can be viewed as random vectors distributed on the surface of a unit simplex. Although the compositional data has its prescribed moments and correlation structure and are not distributed uniformly on the surface of the unit simplex, the idea of the AR method is still applicable. Since the D-dimensional compositional data are in effect (D-1) dimensional, only (D-1) random variables need to be generated.

The steps of the AR method are as follows:

Step [1] - Fit the Johnson distribution system to each random variable representing storm percentage, P_i.

Step [2] - Construct the correlation matrix $\boldsymbol{R}_Z$ from the original correlation matrix $\boldsymbol{R}_P$.

Step [3] - Find the optimum $(D-1) \times (D-1)$ submatrix $\boldsymbol{R}_Z^{\prime}$. The optimum submatrix is defined as the one with the minimal difference between the largest eigenvalue and the smallest eigenvalue.

Step [4] - Generate a set of multivariate standard normal variates $[z_1,\cdots,z_{D-1}]$ based on $\boldsymbol{R}_Z^{\prime}$. Transform them back to obtain the original random storm percentages $[p_1,\cdots,p_{D-1}]$ using the parameters and Johnson distribution types obtained in Step [1].

Step [5] - If the following two conditions are satisfied,

$$
\begin{aligned}
&(i) \quad 0 \leq p_i \leq 1, \qquad \mathit{for} \ 1 \leq i \leq D-1 \\
&(ii) \quad p_1 + \cdots + p_{D-1} \leq 1
\end{aligned}
$$

accept $[p_1,\cdots,p_{D-1}]$ and compute $p_D = 1 - p_1 - \cdots - p_{D-1}$. Otherwise, discard the current simulated vector and repeat Step [4].

3.3.2 CDF Method

Consider P_i as the random percentage precipitation during the time interval $[t_{i-1}, t_i]$ where $t_i = i/D$ as shown in Fig. 2.1(b). Since the dimensionless rainfall hyetograph is directly related to the dimensionless rainfall mass curve, it is reasonable to generate P_i from the dimensionless rainfall mass curve which can be treated as a CDF. Let this CDF describing the dimensionless rainfall mass curve be $F(t|\Theta)$ where $0 \le t \le 1$, Θ is the vector of parameters. It follows that,

$$\begin{aligned} p_1 &= F\left(\frac{1}{D}|\Theta\right) \\ p_2 &= F\left(\frac{i}{D}|\Theta\right) - F\left(\frac{i-1}{D}|\Theta\right), \qquad i = 2, 3, \cdots, D-1 \\ p_D &= 1 - F\left(\frac{D-1}{D}|\Theta\right) \end{aligned} \tag{3.9}$$

The random precipitation percentages generated by Eq. (3.9) will automatically satisfy $0 \le p_i \le 1$ and $p_1 + \cdots + p_D = 1$.

After selecting the form of $F(t|\Theta)$, each observation $[p_1, \cdots, p_D]$ in a specified storm pattern can be fitted to this CDF using an appropriate method. The uncertainty of precipitation percentages $\boldsymbol{P} = (P_1, \cdots, P_D)$ are transferred to the uncertainty of parameters in $F(t|\Theta)$.

The procedure of the CDF method involves the following steps:

Step [1] - Fit each non-dimensionalized rainfall mass curve in a storm pattern to $F(t|\Theta)$.

Step [2] - Calculate the sample moments and correlations of the parameters, Θ.

Step [3] - Fit a Johnson distribution to each of the parameters.

Step [4] - Generate the random parameters Θ using the procedures for simulating unconstrained non-normal random variables.

Step [5] - Use Eq. (3.9) to calculate the components $[p_1,\cdots,p_D]$.

3.3.3 Log-ratio Method

The log-ratio method is developed from the log-ratio transformation proposed by Aitchison (1986). Consider the following transformation,

$$y_i = \log(p_i/p_D), \qquad 1 \le i \le D-1 \tag{3.10}$$

where p_D can be any component of the compositional data. Since $0 \le p_i \le 1$ for $i = 1,2,\cdots,D$, the transformed variable y_i can range from $-\infty$ to ∞. Note that in the above transformation, neither p_i nor p_D can be 0. The inverse transformation of the log-ratio method yields,

$$p_i = \frac{\exp(y_i)}{1+\exp(y_1)+\cdots+\exp(y_{D-1})}, \qquad 1 \le i \le D-1 \tag{3.11a}$$

$$p_D = \frac{1}{1+\exp(y_1)+\cdots+\exp(y_{D-1})} \tag{3.11b}$$

It is obvious that $0<p_i<1$ and $p_1+\cdots+p_D=1$.

After performing the log-ratio transformation, the problem of generating constrained multivariate non-normal variates p_i's is converted to the problem of generating unconstrained multivariate non-normal random variates y_i's. In Aitchison's approach (1986), a multivariate normal distribution is assumed for Y_i's resulting in the logistic-normal distribution. However, the normality condition for the log-ratio, Y_i, is not satisfied in this study. Therefore, the Johnson distribution system was selected to describe random vector ***Y***.

The computational steps of the log-ratio method are as follows:

Step [1] - Choose P_D and carry out the log-ratio transformation (The choice of P_D is arbitrary. A preliminary investigation recommends choosing the largest component as P_D because it leads to small sample moments for the log-ratios).

Step [2] - Calculate the sample moments and correlations of random log-ratio Y_i, $i = 1, 2, \cdots, D\text{-}1$.

Step [3] - Fit Y_i, $i = 1, 2, \cdots, D\text{-}1$, to the Johnson distribution system.

Step [4] - Generate $[y_1, \cdots, y_{D-1}]$ using the procedures for simulating unconstrained non-normal random variables.

Step [5] - Obtain precipitation percentages $[p_1, \cdots, p_D]$ by Eqs. (3.11a) and (3.11b).

3.4 APPLICATIONS

For the purpose of demonstration and performance evaluation, the three proposed procedures were applied to generate storm events. The performance evaluation is focused on the computational efficiency and the ability of the simulated results to preserve the given statistical properties.

Tables 3.2 - 3.4 give the detail simulation results for the duration-based storm patterns 1, 2, and 3, respectively. The first table in each of them contains the sample statistical properties of the given storm pattern with the upper portion showing the first four moments of the six precipitation percentages and the lower portion showing the correlation coefficient among them. The sample sizes for all three storm patterns are sufficiently large so that the procedure of Hill et al. (1976a) were used to fit the data to Johnson distributions in the simulations. To calculate the correlation coefficient in the normal space for $S_{B,LN}$, $S_{B,U}$, and $S_{B,B}$, the value of ρ_{z_i,z_j} in the right-hand side of Eq. (3.8) was solved using a 48-point Gaussian-Legendre integration along with the Van-Wijngaarden-Dekker-Brent for root finding (Press et al., 1989).

By the CDF method, the reduced 2-parameter Johnson S_B distribution was selected as $F(t|\Theta)$

$$F_t = F(t|\gamma,\delta) = \Phi(\gamma + \delta \ln[t/(1-t)]), \qquad 0<t<1 \tag{3.12}$$

where $\Phi(\cdot)$ is the standard normal CDF which can be evaluated using various highly accurate approximation methods (Abramowitz and Stegun, 1972). The parameters γ and δ are fitted by the least square method with the following objective function,

$$S(\gamma,\delta) = \sum_{i=1}^{5} \{F_i - \Phi(\gamma + \delta \ln[t_i/(1-t_i)])\}^2 \tag{3.13}$$

Table 3.2 Results of Simulations of the Duration-Based Storm Pattern 1 (1000 cases)

	P1	P2	P3	P4	P5	P6
Mean	0.15891	0.16380	0.18268	0.18560	0.16873	0.14029
Stdev	0.07720	0.07149	0.09110	0.08548	0.07633	0.07764
Skewness	0.65027	0.32196	1.19350	0.83840	0.41000	0.37661
Kurtosis	3.72483	3.54807	5.92435	4.58673	3.6146	3.24666
P1	1.00000	-0.14052	-0.40608	-0.29759	-0.21022	0.14590
P2	-0.14052	1.00000	-0.06842	-0.29414	-0.24383	-0.13720
P3	-0.40608	-0.06842	1.00000	0.01874	-0.30635	-0.42604
P4	-0.29759	-0.29414	0.01874	1.00000	-0.12957	-0.42887
P5	-0.21022	-0.24383	-0.30635	-0.12957	1.00000	-0.04748
P6	0.14590	-0.13720	-0.42604	-0.42887	-0.04748	1.00000

(a) AR Method

Step [1] - Parameters and types of the Johnson systems

	GAMMA	DELTA	LAMBDA	XI	TYPE
P1	11.31988	4.10057	5.48644	-0.17558	S_B
P2	-1.18047	3.35597	0.21531	0.08298	S_U
P3	-2.88495	2.37821	0.10487	0.00703	S_U
P4	-2.46973	2.80317	0.15635	0.01906	S_U
P5	-1.64385	3.43394	0.22389	0.05259	S_U
P6	3.88304	8.03829	1.00000	-0.48139	S_L

Step [2] - Correlation matrix R_Z

Z1	1.00000	-0.14279	-0.43118	-0.30866	-0.21425	0.14785
Z2	-0.14279	1.00000	-0.07100	-0.30123	-0.24616	-0.13829
Z3	-0.43118	-0.07100	1.00000	0.01968	-0.32028	-0.44620
Z4	-0.30866	-0.30123	0.01968	1.00000	-0.13270	-0.44093
Z5	-0.21425	-0.24616	-0.32028	-0.13270	1.00000	-0.04791
Z6	0.14785	-0.13829	-0.44620	-0.44093	-0.04791	1.00000

Step [3] - The optimum 5 × 5 submatrix is the original one without row and column 3.

Steps [4] & [5] - Simulation (10000 repetitions): C = 10798; Efficiency = 92.6%

	P1'	P2'	P3'	P4'	P5'	P6'
Mean	.156337	.163285	.186824	.184535	.167039	.141980
Stdev	.073799	.069146	.082345	.081636	.072564	.071884
Skewness	.556060	.421261	.093285	.726087	.464494	.437529
Kurtosis	3.287801	3.386889	2.722566	3.874025	3.414015	3.024581
P1'	1.000000	-.148723	-.359941	-.285539	-.237355	.092611
P2'	-.148723	1.000000	-.100999	-.294109	-.228009	-.129357
P3'	-.359941	-.100999	1.000000	-.008505	-.282676	-.383829
P4'	-.285539	-.294109	-.008505	1.000000	-.138797	-.409750
P5'	-.237355	-.228009	-.282676	-.138797	1.000000	-.065018
P6'	.092611	-.129357	-.383829	-.409750	-.065018	1.000000

Table 3.2 (continued)

(b) CDF Method

Step [1] - Fit the each set of data to 2-parameter Johnson S_B

Step [2] - Sample moments and correlations of the parameters for 2-parameter Johnson S_B

	Mean	Stdev	Skewness	Kurtosis	Correlation Matrix	
GAMMA	0.03404	0.24999	1.08434	8.62764	1.00000	0.35721
DELTA	0.69664	0.23655	2.70697	15.89450	0.35721	1.00000

Step [3] - Parameters and types of Johnson systems and correlation matrix R_Z

	Gamma	Delta	Lambda	Xi	Type	Corr. Matrix R_Z	
GAMMA	-0.55145	1.50179	0.27338	-0.09409	S_U	1.00000	0.40361
DELTA	4.48519	1.22089	8.07809	0.42759	S_B	0.40361	1.00000

Steps [4] & [5] - Simulation (10000 repetitions)

	P1'	P2'	P3'	P4'	P5'	P6'
Mean	.151750	.177728	.181676	.174613	.163839	.150395
Stdev	.071149	.041550	.058729	.045873	.033453	.086336
Skewness	.997318	1.411653	2.101315	1.556270	-1.221800	.408924
Kurtosis	8.094628	9.028728	10.066878	6.742286	6.483229	3.176498
P1'	1.00000	.124208	-.567360	-.774161	-.474709	.097353
P2'	.124208	1.00000	.513142	.166481	-.581327	-.795887
P3'	-.567360	.513142	1.00000	.854607	-.320634	-.789485
P4'	-.774161	.166481	.854607	1.00000	.145051	-.611019
P5'	-.474709	-.581327	-.320634	.145051	1.00000	.424533
P6'	.097353	-.795887	-.789485	-.611019	.424533	1.00000

Table 3.2 (concluded)

(c) Log-ratio Method

Step [1] - Selected $P_D = P_4$; Count = 919

Step [2] - Sample moments and correlations of log-ratios.

	Y1	Y2	Y3	Y4	Y5
Mean	-0.18454	-0.10389	-0.03001	-0.07334	-0.28841
stdev	0.88102	0.80977	0.67594	0.74479	0.97044
skewness	-0.28884	-0.29423	0.02207	-0.74249	-0.67676
kurtosis	4.99647	6.06599	5.05152	8.32579	5.46737
Y1	1.000000	0.546312	0.336809	0.434479	0.605829
Y2	0.546312	1.000000	0.545094	0.435590	0.529433
Y3	0.336809	0.545094	1.000000	0.343348	0.383661
Y4	0.434479	0.435590	0.343348	1.000000	0.557576
Y5	0.605829	0.529433	0.383661	0.557576	1.000000

Step [3]- Parameters and types of Johnson system

	GAMMA	DELTA	LAMBDA	XI	TYPE
Y1	0.24722	1.85791	1.39218	0.03022	S_U
Y2	0.17109	1.61976	1.06401	0.03234	S_U
Y3	-0.01737	1.80598	1.03903	-0.04166	S_U
Y4	0.32310	1.44547	0.80663	0.15762	S_U
Y5	0.61851	1.87821	1.47930	0.28312	S_U

Correlation matrix R_Z

Z1	1.00000	0.55423	0.34169	0.44585	0.61384
Z2	0.55423	1.00000	0.55336	0.44857	0.53940
Z3	0.34169	0.55336	1.00000	0.35362	0.39122
Z4	0.44585	0.44857	0.35362	1.00000	0.57094
Z5	0.61384	0.53940	0.39122	0.57094	1.00000

Steps [4] & [5] - Simulation (10000 repetitions)

	P1'	P2'	P3'	P4'	P5'	P6'
Mean	.157393	.164472	.178801	.183208	.170499	.145627
Stdev	.089599	.081753	.088695	.087762	.081676	.081943
Skewness	1.485353	1.576351	1.431104	1.178241	1.272852	1.185645
Kurtosis	7.186072	8.530516	7.529385	5.689450	7.538825	5.780760
P1'	1.000000	-.129061	-.373678	-.293177	-.257292	.010248
P2'	-.129061	1.000000	-.078408	-.291364	-.283792	-.176766
P3'	-.373678	-.078408	1.000000	-.009406	-.266522	-.319854
P4'	-.293177	-.291364	-.009406	1.000000	-.063791	-.385996
P5'	-.257292	-.283792	-.266522	-.063791	1.000000	-.075476
P6'	.010248	-.176766	-.319854	-.385996	-.075476	1.000000

Table 3.3 **Results of Simulations of the Duration-Based Storm Pattern 2 (234 cases)**

	P1	P2	P3	P4	P5	P6
Mean	0.78651	0.12099	0.03599	0.02318	0.01495	0.01837
Stdev	0.16531	0.11856	0.05320	0.04585	0.03353	0.03897
Skewness	-0.31510	0.96983	1.88161	2.72222	3.32915	2.51301
Kurtosis	1.76890	3.26556	6.85497	10.8720	16.4610	9.16871
P1	1.00000	-0.78978	-0.49540	-0.37907	-0.38247	-0.38770
P2	-0.78978	1.00000	0.15683	-0.01416	0.06031	0.05855
P3	-0.49540	0.15683	1.00000	0.15858	0.00892	0.06477
P4	-0.37907	-0.01416	0.15858	1.00000	0.20376	0.08254
P5	-0.38247	0.06031	0.00892	0.20376	1.00000	0.32649
P6	-0.38770	0.05855	0.06477	0.08254	0.32649	1.00000

(a) AR method

Step [1] - Parameters and types of the Johnson systems

	GAMMA	DELTA	LAMBDA	XI	TYPE
P1	-0.26934	0.51637	0.54351	0.47130	S_B
P2	1.07234	0.80823	0.59407	-0.03520	S_B
P3	1.78826	0.75398	0.36565	-0.01439	S_B
P4	1.72482	0.47710	0.28799	-0.00381	S_B
P5	2.12910	0.56005	0.29542	-0.00448	S_B
P6	1.55015	0.42056	0.21052	-0.00412	S_B

Step [2] - Correlation matrix R_Z

Z1	1.00000	-0.82524	-0.56329	-0.48858	-0.50771	-0.49783
Z2	-0.82524	1.00000	0.18002	-0.01899	0.08126	0.07689
Z3	-0.56329	0.18002	1.00000	0.21102	0.01305	0.08979
Z4	-0.48858	-0.01899	0.21102	1.00000	0.29113	0.12571
Z5	-0.50771	0.08126	0.01305	0.29113	1.00000	0.43501
Z6	-0.49783	0.07689	0.08979	0.12571	0.43501	1.00000

Step [3] - The optimum 5 × 5 submatrix is the original one without row and column 1.

Steps [4] & [5] - Simulation (10000 repetitions): C = 51838; Efficiency = 19.3%

	P1'	P2'	P3'	P4'	P5'	P6'
Mean	.663453	.152110	.057458	.047031	.036172	.043776
Stdev	.173254	.118114	.057821	.058625	.045695	.050045
Skewness	-.711008	.851555	1.565885	1.764444	2.121523	1.391674
Kurtosis	3.281164	2.986275	5.349764	5.607735	7.826441	4.022533
P1'	1.000000	-.724028	-.451602	-.403310	-.398644	-.394917
P2'	-.724028	1.000000	.102136	-.034084	.041417	.030497
P3'	-.451602	.102136	1.000000	.122072	-.017368	.039855
P4'	-.403310	-.034084	.122072	1.000000	.139616	.036718
P5'	-.398644	.041417	-.017368	.139616	1.000000	.225777
P6'	-.394917	.030497	.039855	.036718	.225777	1.000000

Table 3.3 (continued)

(b) CDF Method

Step [1] - Fit the each set of data to 2-parameter Johnson S_B

Step [2] - Sample moments and correlations of parameters for 2-parameter Johnson S_B

	Mean	Stdev	Skewness	Kurtosis	Correlation Matrix	
GAMMA	2.85158	1.41037	0.07110	1.78565	1.00000	0.18726
DELTA	0.84144	0.58485	-0.03114	1.58930	0.18726	1.00000

Step [3] - Parameters and types of Johnson systems and correlation matrix R_Z

	Gamma	Delta	Lambda	Xi	Type	Corr. Matrix R_Z	
GAMMA	0.06822	0.62800	5.04390	0.42368	S_B	1.00000	0.19923
DELTA	-0.02558	0.46856	1.80953	-0.07770	S_B	0.19923	1.00000

Steps [4] & [5] - Simulation (10000 repetitions)

	P1'	P2'	P3'	P4'	P5'	P6'
Mean	.785698	.119758	.051132	.020422	.007832	.015157
Stdev	.278602	.158952	.092447	.041848	.017999	.044689
Skewness	-1.287751	1.207792	2.197128	2.699639	2.824335	4.038524
Kurtosis	3.428676	3.137985	7.217025	10.586363	11.378494	20.872132
P1'	1.000000	-.911469	-.941785	-.826079	-.547708	-.049884
P2'	-.911469	1.000000	.795460	.568683	.283061	-.166610
P3'	-.941785	.795460	1.000000	.906336	.524605	-.086693
P4'	-.826079	.568683	.906336	1.000000	.773346	.004450
P5'	-.547708	.283061	.524605	.773346	1.000000	.195565
P6'	-.049884	-.166610	-.086693	.004450	.195565	1.000000

Table 3.3 (continued)

(c) Log-ratio Method

Step [1] - Selected $P_D = P_1$; Count = 29

Step [2] - Sample moments and correlations of log-ratios.

	Y1	Y2	Y3	Y4	Y5
Mean	-1.81122	-2.56081	-2.72805	-2.86728	-2.76335
Stdev	1.42012	0.92034	1.03210	0.96135	1.26081
Skewness	-1.20849	-1.02459	-0.57984	-0.45977	-0.37763
Kurtosis	3.31594	3.53497	2.52211	2.47782	1.78462
Y1	1.000000	0.404898	0.315404	0.170917	0.332036
Y2	0.404898	1.000000	0.190569	0.228076	0.371045
Y3	0.315404	0.190569	1.000000	0.219972	0.081788
Y4	0.170917	0.228076	0.219972	1.000000	0.386972
Y5	0.332036	0.371045	0.081788	0.386972	1.000000

Step [3] - Parameters and types of Johnson system

	GAMMA	DELTA	LAMBDA	XI	TYPE
Y1	-0.90381	0.44834	5.27284	-5.82932	S_B
Y2	-1.22947	0.88894	5.09380	-6.39571	S_B
Y3	-0.72054	0.89468	4.95180	-5.97569	S_B
Y4	-0.66550	1.03769	5.01473	-6.03248	S_B
Y5	-0.31328	0.48770	4.06407	-5.18091	S_B

Correlation matrix R_Z

Z1	1.00000	0.44545	0.34875	0.19042	0.37562
Z2	0.44545	1.00000	0.20198	0.23995	0.39871
Z3	0.34875	0.20198	1.00000	0.22703	0.08726
Z4	0.19042	0.23995	0.22703	1.00000	0.40579
Z5	0.37562	0.39871	0.08726	0.40579	1.00000

Steps [4] & [5] - Simulation (10000 cases)

	P1'	P2'	P3'	P4'	P5'	P6'
Mean	.617640	.157595	.059589	.055804	.046609	.062763
Stdev	.134554	.098604	.034477	.041415	.034654	.051820
Skewness	.517087	-.264261	.223280	.763603	.936254	.552549
Kurtosis	2.334532	1.599887	2.127932	2.803674	3.310642	2.037398
P1'	1.000000	-.773753	-.387428	-.288440	-.249693	-.468989
P2'	-.773753	1.000000	.115930	.057473	-.125037	.066841
P3'	-.387428	.115930	1.000000	-.043683	.009346	.148729
P4'	-.288440	.057473	-.043683	1.000000	.042428	-.158922
P5'	-.249693	-.125037	.009346	.042428	1.000000	.177396
P6'	-.468989	.066841	.148729	-.158922	.177396	1.000000

Table 3.3 (concluded)

Sample moments and correlations recalculated from 29 storm events

	P1"	P2"	P3"	P4"	P5"	P6"
Mean	0.61672	0.15555	0.06048	0.05690	0.04821	0.06214
Stdev	0.13496	0.10410	0.03846	0.04650	0.04180	0.05143
Skewness	1.05532	0.15530	0.54407	1.36175	1.93245	0.51166
Kurtosis	2.98354	1.90722	2.53731	4.55380	7.56498	1.75386
P1"	1.000000	-0.740549	-0.267119	-0.247343	-0.173148	-0.561143
P2"	-0.740549	1.000000	-0.065117	-0.017591	-0.172955	0.124434
P3"	-0.267119	-0.065117	1.000000	-0.066063	-0.064816	0.197290
P4"	-0.247343	-0.017591	-0.066063	1.000000	-0.132156	-0.062640
P5"	-0.173148	-0.172955	-0.064816	-0.132156	1.000000	0.159690
P6"	-0.561143	0.124434	0.197290	-0.062640	0.159690	1.000000

Table 3.4 Results of Simulations of the Duration-Based Storm Pattern 3 (312 cases)

	P1	P2	P3	P4	P5	P6
Mean	0.27244	0.34217	0.18809	0.10093	0.05525	0.04112
Stdev	0.11982	0.18609	0.11601	0.07592	0.05408	0.04755
Skewness	-0.20223	0.92652	0.72358	0.49444	1.05751	1.14574
Kurtosis	2.25020	3.74566	4.20065	2.83959	3.8973	3.64115
P1	1.00000	-0.58518	-0.24033	-0.00591	0.11171	0.23906
P2	-0.58518	1.00000	-0.38447	-0.49986	-0.34521	-0.31032
P3	-0.24033	-0.38447	1.00000	0.10841	-0.22726	-0.24411
P4	-0.00591	-0.49986	0.10841	1.00000	0.14363	-0.05326
P5	0.11171	-0.34521	-0.22726	0.14363	1.00000	0.25728
P6	0.23906	-0.31032	-0.24411	-0.05326	0.25728	1.00000

(a) AR method

Step [1] - Parameters and types of the Johnson systems

	GAMMA	DELTA	LAMBDA	XI	TYPE
P1	-0.30300	1.06550	0.60941	-0.06858	S_B
P2	1.79789	1.34188	1.49585	-0.00337	S_B
P3	-2.63088	3.10887	0.24497	-0.05723	S_B
P4	1.10995	1.48738	0.54768	-0.08343	S_B
P5	1.56498	1.08702	0.36854	-0.02786	S_B
P6	1.13624	0.72062	0.23216	-0.01372	S_B

Step [2] - Correlation matrix R_Z

Z1	1.00000	-0.60159	-0.24499	-0.00602	0.11755	0.25819
Z2	-0.60159	1.00000	-0.40729	-0.52865	-0.38011	-0.35074
Z3	-0.24499	-0.40729	1.00000	0.11071	-0.24248	-0.26756
Z4	-0.00602	-0.52865	0.11071	1.00000	0.15030	-0.05751
Z5	0.11755	-0.38011	-0.24248	0.15030	1.00000	0.27807
Z6	0.25819	-0.35074	-0.26756	-0.05751	0.27807	1.00000

Step [3] - The optimum 5 × 5 submatrix is the original one without row and column 2.

Steps [4] & [5] Simulation (10000 repetitions): C = 16144; Efficiency = 61.9%

	P1'	P2'	P3'	P4'	P5'	P6'
Mean	.278386	.321580	.175999	.107642	.065039	.051354
Stdev	.115671	.160081	.099469	.067924	.050770	.045129
Skewness	-.206410	.199390	.757732	.605964	1.068439	1.088674
Kurtosis	2.233066	2.483245	3.775126	2.892892	3.862558	3.480481
P1'	1.000000	-.598099	-.261945	-.048014	.031570	.172557
P2'	-.598099	1.000000	-.336809	-.428148	-.293030	-.297766
P3'	-.261945	-.336809	1.000000	.054132	-.195940	-.199040
P4'	-.048014	-.428148	.054132	1.000000	.080341	-.073006
P5'	.031570	-.293030	-.195940	.080341	1.000000	.144482
P6'	.172557	-.297766	-.199040	-.073006	.144482	1.000000

Table 3.4 (continued)

(b) CDF Method

Step [1] - Fit the each set of data to 2-parameter Johnson S_B

Step [2] - Sample moments and correlations of the parameters for 2-parameter Johnson S_B

	Mean	Stdev	Skewness	Kurtosis	Correlation Matrix	
GAMMA	1.03229	0.73674	1.55057	4.73242	1.00000	0.89502
DELTA	1.03265	0.56067	0.78745	2.18133	0.89502	1.00000

Step [3] - Parameters and types of Johnson systems and correlation matrix R_Z

	Gamma	Delta	Lambda	Xi	Type	Corr. Matrix R_Z	
GAMMA	1.23511	0.55335	3.43922	0.39205	S_B	1.00000	0.84007
DELTA	0.57304	0.37161	1.72760	0.48777	S_B	0.84007	1.00000

Steps [4] & [5] - Simulation (10000 repetitions)

	P1'	P2'	P3'	P4'	P5'	P6'
Mean	.286894	.319763	.191449	.101512	.061485	.038896
Stdev	.131974	.143732	.089358	.049158	.044355	.042299
Skewness	-.056818	.891705	1.048341	-.524242	-.231013	.610520
Kurtosis	3.285165	2.624081	3.834022	2.883278	1.385447	1.779548
P1'	1.000000	-.504832	-.885109	-.255640	.330526	.415732
P2'	-.504832	1.000000	.296256	-.648906	-.883653	-.768045
P3'	-.885109	.296256	1.000000	.392374	-.333630	-.463815
P4'	-.255640	-.648906	.392374	1.000000	.652252	.327578
P5'	.330526	-.883653	-.333630	.652252	1.000000	.869595
P6'	.415732	-.768045	-.463815	.327578	.869595	1.000000

Table 3.4 (continued)

(c) Log-ratio Method

Step [1] - Selected $P_D = P_2$; Count = 173

Step [2] - Sample moments and correlations of log-ratios.

	Y1	Y2	Y3	Y4	Y5
Mean	-0.01533	-0.55796	-1.02316	-1.53465	-1.74346
Stdev	0.98553	1.07962	1.06322	1.17601	1.18992
Skewness	-1.20555	-0.96115	-0.54264	-0.73777	-0.40043
Kurtosis	7.35613	4.99267	3.79177	3.54360	2.79601
Y1	1.000000	0.466502	0.559266	0.581370	0.659808
Y2	0.466502	1.000000	0.556200	0.291814	0.248605
Y3	0.559266	0.556200	1.000000	0.489552	0.351432
Y4	0.581370	0.291814	0.489552	1.000000	0.600812
Y5	0.659808	0.248605	0.351432	0.600812	1.000000

Step [3] - Parameters and types of Johnson system

	GAMMA	DELTA	LAMBDA	XI	TYPE
Y1	1.02050	1.75247	1.21214	0.86306	S_U
Y2	2.61711	2.64274	1.69658	1.55667	S_U
Y3	2.32167	3.41277	2.77972	1.10661	S_U
Y4	-2.38077	1.88120	12.79249	-11.35107	S_B
Y5	-1.07474	1.68616	9.34767	-7.75818	S_B

Correlation matrix R_Z

Z1	1.00000	0.48077	0.57190	0.59603	0.67652
Z2	0.48077	1.00000	0.56477	0.29995	0.25444
Z3	0.57190	0.56477	1.00000	0.49635	0.35509
Z4	0.59603	0.29995	0.49635	1.00000	0.60774
Z5	0.67652	0.25444	0.35509	0.60774	1.00000

Steps [4] & [5] - Simulation (10000 repetitions)

	P1'	P2'	P3'	P4'	P5'	P6'
Mean	.282089	.283687	.182984	.113828	.074826	.062585
Stdev	.119238	.149237	.111947	.074345	.055419	.050896
Skewness	.128745	1.195503	.775219	1.139609	1.235696	1.436138
Kurtosis	2.825106	4.558992	3.310599	4.600213	4.955305	5.402026
P1'	1.000000	-.560912	-.326912	-.153302	.006666	.237647
P2'	-.560912	1.000000	-.291245	-.322483	-.231198	-.254700
P3'	-.326912	-.291245	1.000000	.036577	-.281562	-.326489
P4'	-.153302	-.322483	.036577	1.000000	-.013742	-.221479
P5'	.006666	-.231198	-.281562	-.013742	1.000000	.212796
P6'	.237647	-.254700	-.326489	-.221479	.212796	1.000000

Table 3.4 (concluded)

Sample moments and correlations recalculated from 173 storm events

	P1"	P2"	P3"	P4"	P5"	P6"
Mean	0.28045	0.28627	0.18284	0.11257	0.07462	0.06326
Stdev	0.10819	0.13769	0.10723	0.06160	0.04969	0.04716
Skewness	-0.25871	1.07890	0.95275	0.43355	0.70585	0.83016
Kurtosis	2.41979	5.35910	5.07843	2.82434	3.10975	3.07150
P1"	1.000000	-0.601846	-0.391708	-0.083044	0.145984	0.308466
P2"	-0.601846	1.000000	-0.271456	-0.364738	-0.214682	-0.219130
P3"	-0.391708	-0.271456	1.000000	0.121808	-0.339822	-0.383813
P4"	-0.083044	-0.364738	0.121808	1.000000	-0.080431	-0.242980
P5"	0.145984	-0.214682	-0.339822	-0.080431	1.000000	0.116094
P6"	0.308466	-0.219130	-0.383813	-0.242980	0.116094	1.000000

where $t_i = i/6, i = 1,\cdots,5$; and $F_i = \sum_{j=1}^{i} p_j$. The estimates of the parameters are obtained by using the multidimensional downhill simplex method (Press et al., 1989) to minimize the above objective function, Eq. (3.13). It should be noted that the objective function $S(\gamma,\delta)$ is exactly the same as the objective function $S_2(\gamma,\delta,1.0)$ in the previous chapter.

As mentioned before, neither p_i nor p_D is allowed to be 0 when the log-ratio transformation is performed. Those storm events with at least one precipitation percentage being 0 were dropped from the computation of log-ratios for the given storm pattern. Sample moments and correlations of log-ratios were calculated from the reduced storm set. The number of cases of the reduced storm set was presented as 'Count' in Tables 3.2 - 3.4.

To generate 10000 storm events of storm pattern 1 using the AR method, it requires 10798 runs of generating normal variates. The corresponding efficiency in this case is 92.6%. As can be seen in Table 3.2(a), the moments and correlations of simulated storm events using the AR method are very close to the observed ones. The maximum difference in the mean and the standard deviation between the simulated storm events and the observed ones is 0.0088. The maximum difference in the correlation coefficient is 0.055. The simulation results using the CDF method are shown in Table 3.2(b). It is observed that the moments of the precipitation percentages are preserved well by the simulated storms. However, some correlation coefficients of the precipitation percentages differ significantly from the observed ones. This problem might be caused by the inefficiency of the downhill simplex method or by the improper choice of the 2-parameter Johnson S_B as the CDF function. After deleting those observations with zero precipitation percentages, the log-ratio transformation was performed on 919 storm cases for storm pattern 1. The moments and the correlation matrix of simulated storms are preserved well although not as well as those obtained from the AR method.

Table 3.3 contains the simulation results for the duration-based storm pattern 2. Unlike the previous case, the efficiency of the AR method (see Table 3.3(a)) is very low, only 19.3%. In this case, the moments of simulated storm events cannot preserve what were specified. However, the correlation matrix of simulated precipitation percentages is reasonable. Using the CDF method (Table 3.3(b)), the moments, especially the means and standard deviations, are almost identical to the given values. Unfortunately, the correlation matrix are not preserved well. By the log-ratio method, only 29 observations were retained after deleting storm events with zero percentage. The performance of this method is not satisfactory. Since the great majority of storm cases were dropped due to the computational restriction of the log-ratio method, the retained storms might not capture the essential features of this storm pattern. In fact, there might be great changes in the storm moments and the correlation structure. This reason may be the cause of the poor performance. However, this does not imply the log-ratio method is not good. The moments and the correlation coefficients of the 29 storms were calculated and are shown at the end of Table 3.3. As can be seen, the moments and the correlation coefficients of the precipitation percentages were preserved very well by the log-ratio method.

Table 3.4 contains the simulation results for the duration-based storm pattern 3. The efficiency of the AR method is 61.9%. The moments and correlation matrix by the simulated storm events are not well preserved. The results are the same as simulations for the duration-based storm pattern 2 using the CDF method. The moments of the precipitation percentages are preserved well while their correlation matrix is not. Using the log-ratio method, almost half of the storm events in pattern 3, 149 cases, were deleted. The correlation structure is preserved well while the simulated moments are not as good as those obtained from the AR method. Compared with the moments and correlation matrix recalculated for the retained 173 storms, the performance of the log-ratio method is satisfactory.

3.5 DISCUSSION

From these three tables, it should be noted that the AR method can outperform the other two methods when its efficiency is high. On the other hand, the performance of the AR method is the worst when its efficiency is low. In this application, the efficiency of the AR method can be expressed as

$$\begin{aligned} \frac{1}{C} &= P\{0 \le P_i \le 1 \ \mathit{for}\ 1 \le i \le D-1 \quad \mathit{and} \quad P_1 + \cdots + P_{D-1} \le 1\} \\ &= P\{P_1 + \cdots + P_{D-1} \le 1 \mid 0 \le P_i \le 1 \ \mathit{for}\ 1 \le i \le D-1\} \cdot P\{0 \le P_i \le 1 \ \mathit{for}\ 1 \le i \le D-1\} \\ &= P\{P_1 + \cdots + P_{D-1} \le 1 \mid 0 \le P_i \le 1 \ \mathit{for}\ 1 \le i \le D-1\} \cdot P\{g_i(0) \le z_i \le g_i(1) \ \mathit{for}\ 1 \le i \le D-1\} \end{aligned} \tag{3.14}$$

where C is the number of trials to be successful; and $g_i(\cdot)$ is the corresponding Johnson transformations of P_i. The conditional probability in Eq. (3.14) can be evaluated approximately by the reliability index in the reliability analysis. Let the performance function be $W(\boldsymbol{P}) = P_1 + \cdots + P_{D-1} - 1 = -P_D$, and the reliability index be $\beta = \mu_W / \sigma_W = -\mu_{P_D} / \sigma_{P_D}$. The conditional probability is simply the CDF of P_D, that is,

$$
\begin{aligned}
& P\{P_1 + \cdots + P_{D-1} \le 1 \mid 0 \le P_i \le 1 \ \mathit{for}\ 1 \le i \le D-1\} \\
&= P\{W(P) \le 0\} \\
&= 1 - F_W(\beta) \\
&= F_{P_D}(\mu_{P_D}/\sigma_{P_D})
\end{aligned}
$$

The second probability in Eq. (3.14) is the joint probability of D-1 multivariate normal random variables bounded between $g_i(0)$ and $g_i(1)$. Unfortunately, this probability is not easy to calculate. The bounds of this probability can be approximated by Ditlevsen's method (Ditlevsen, 1979) or Rackwitz's method (Rackwitz, 1978).

The CDF method can always preserve the moments of the precipitation percentages. However, its ability to preserve the correlation structure of precipitation percentages within a storm is limited. This may be due to the method selected to preserve the statistical properties of the parameters in the adopted CDF for describing the dimensionless rainfall mass curve. Due to the nonlinear relationship between the distributional parameters and the incremental probabilities, the correlation matrix of the precipitation percentages may not be well preserved. Since the selection of the CDF function $F(t|\Theta)$ is arbitrary, different choices will have direct impact on the performance of this method.

The performance of the log-ratio method is satisfactory when only a few or no observations in a given storm pattern contain zero precipitation percentages. The requirement of a non-zero component may limit the performance of this method greatly.

Computationally, the log-ratio method is direct and most efficient. The computation time of the AR method depends on its efficiency which is generally unknown in advance. When the efficiency is too low, this method is not a practical tool. The CDF method requires an extra step of fitting each observation to the CDF which could be time consuming.

Based on the above discussions, a guideline for selecting a method to generate stochastic storm patterns is provided as the following:

(1) If only a small portion of the observations in a given storm pattern contains zero precipitation percentages, the log-ratio method is the best choice;

(2) If a considerable number of observations in a given storm pattern have zero precipitation percentages, the AR method should be tried first; if the efficiency of the AR method is low, one can consider using the CDF method to generate the storm pattern.

3.6 SUMMARY AND CONCLUSIONS

Storm patterns are often required for assessing the reliability of a hydraulic structure under a specified design storm condition. In this study, the storm pattern in its dimensionless form were the same as compositional data. Generating compositional data requires a procedure of generating unconstrained multivariate non-normal random variables which, in turn, can be efficiently done in the multivariate standard normal space after normal transformation is made. In this study, the Johnson distribution system was adopted as the means for normal transformation due to its flexibility of covering a wide variety of distribution types. Different methods of selecting the Johnson distribution type and estimating its parameters were reviewed.

The Nataf model was then studied based on which the Johnson system was extended to a multivariate setting. Using the Johnson distribution system coupled with the use of the bivariate Nataf model, the procedure of generating correlated non-normal random variables without constraint was developed. The developed procedure preserves only the marginal distribution and the correlated structure of the involved random variables.

Three methods, namely, the AR method, the CDF method, and the log-ratio method were developed for generating temporal distribution of storm patterns. The log-ratio method requires the least amount of computer time to run while the AR method is the most accurate provided the efficiency of this method is high. A guideline as to which method to choose for generating stochastic storm patterns under different situations was provided.

CHAPTER 4

ALTERNATIVE NORMAL TRANSFORMATIONS

4.1 INTRODUCTION

A common practice to derive the distribution representation for a given sample is to fit a parametric distribution by matching their first four moments. Chapter 3 illustrates the using of the Johnson system to fit storm patterns. Although the Johnson system has the flexibility of covering various distribution types, it consists of three curves whose parameters are not very easy to estimate. Furthermore, fitting the Johnson system requires skewness and kurtosis whose sample estimators are associated with large standard errors when the sample size is small. The objective of this chapter is to explore appropriate normal transformations with favorable small sample qualities to serve as the alternative of using the Johnson system. In particular, the use of partial moments and L-moments is considered.

In the following sections, the statistical properties of the partial moments and the L-moments are discussed first. Then, two typical normal transformations based on linear transformation and polynomial transformation are described. Using different combinations of moment types and transformations, four procedures, namely, the linear transformation with partial moments (LTPM), the polynomial transformation with complete moments (PTCM), the polynomial transformation with partial moments (PTPM), and the polynomial transformation with L-moments (PTLM), are proposed as the alternatives for the Johnson distribution systems in normal transformation. The performance of the different procedures is examined by conducting a Monte Carlo simulation study under various conditions. The effect of sample size on the selection of different procedures were also studied by comparing the percentage errors of quantiles under nine generalized extreme-value (GEV) distributions. Finally, the extension of the alternative normal transformations to deal with multivariate non-normal variables is described.

4.2 PARTIAL MOMENTS AND L-MOMENTS

In statistics, the *n*th partial moment of a random variable X about the origin is the partial expectation of X^n over $(-\infty, z]$, that is,

$$E_{-\infty}^{z}(X^{n}) = \int_{-\infty}^{z} x^{n} dF(x) \tag{4.1}$$

If the above integral is not finite, the partial moments do not exist. In practice, the 50% quantile is often chosen as the upper bound in Eq. (4.1), that is, $z = x_{0.5}$.

The partial moments are widely used in statistical decision theory. Winkler et al. (1972) listed several applications using partial moments. They also discussed different methods for determining the partial moments. Buck and Askin (1986) discussed the use of partial mean in the economic risk analysis. Since the partial moments also capture some information of a distribution, it is worth trying to use the first two partial moments to describe the behavior of random variable instead of using skewness and kurtosis. Therefore, the impact of large sampling errors associated with the sample skewness and kurtosis can be reduced.

The L-moments are defined as linear combinations of order statistics (Hosking, 1986). They are analogous to the conventional product moments and are estimated by linear combinations of the observed order statistics. The L-moments are a subset of probability weighted moments (PWM) which are defined as

$$M_{p,r,s} = E\left[X^{p}\{F(X)\}^{r}\{1 - F(X)\}^{s}\right] \tag{4.2}$$

where $M_{p,r,s}$ is the *p*th order probability weighted moment of the random variable X with r values less than X and s values greater than X, and $F(X)$ represents the cumulative distribution function. As can be seen, the conventional product moments are a special case of the probability weighted moments, that is, $\mu_p{}' = M_{p,0,0}$. Consider the case of $p=1$, and $s=0$, one has

$$\beta_r = M_{1,r,0} = E\left[X\{F(X)\}^{r}\right] \quad r=0,1,... \tag{4.3}$$

where β_r is also a probability weighted moment.

The *r*th L-moment is defined as

$$\lambda_r = \frac{1}{r}\sum_{k=1}^{n}\left[(-1)^{k}\binom{r-1}{k} EX_{r-k:r}\right] \quad r=1,2,... \tag{4.4}$$

where $E[X_{r-k:r}]$ is the expectation of the $(r-k)$th order statistic out of a sample of r observations. In terms of β_r, the first four L-moments can be computed as,

$$\begin{aligned}\lambda_1 &= \beta_0 \\ \lambda_2 &= 2\beta_1-\beta_0 \\ \lambda_3 &= 6\beta_2-6\beta_1+\beta_0 \\ \lambda_4 &= 20\beta_3-30\beta_2+12\beta_1-\beta_0\end{aligned} \qquad (4.5)$$

The unbiased sample L-moments can be estimated by the following equations,

$$\begin{aligned}\hat{\lambda}_1 &= \frac{1}{n}\sum_{i=1}^{n} X_{(i)} \\ \hat{\lambda}_2 &= \frac{1}{2}\binom{n}{2}^{-1}\sum_{i=2}^{n}\sum_{j=1}^{i-1}(X_{(i)}-X_{(j)}) \\ \hat{\lambda}_3 &= \frac{1}{3}\binom{n}{3}^{-1}\sum_{i=3}^{n}\sum_{j=2}^{i-1}\sum_{k=1}^{j-1}(X_{(i)}-2X_{(j)}+X_{(k)}) \\ \hat{\lambda}_4 &= \frac{1}{4}\binom{n}{4}^{-1}\sum_{i=4}^{n}\sum_{j=3}^{i-1}\sum_{k=2}^{j-1}\sum_{m=1}^{k-1}(X_{(i)}-3X_{(j)}+3X_{(k)}-X_{(m)})\end{aligned}$$

Similar to the product moment ratios, the L-moment ratios such as L-coefficient of variation (τ_2), L-skew coefficient (τ_3), and L-kurtosis coefficient (τ_4) are defined as

$$\begin{aligned}\tau_2 &= \frac{\lambda_2}{\lambda_1} \\ \tau_3 &= \frac{\lambda_3}{\lambda_2} \\ \tau_4 &= \frac{\lambda_4}{\lambda_2}\end{aligned} \qquad (4.6)$$

The L-moment ratios are analogous in interpretation to their product moment ratio equivalents and have favorable small sample qualities. The feasible region for possible distributions is defined by

$$\begin{gathered} -1 \le \tau_3 \le 1 \\ (5\tau_3^2 - 1)/4 \le \tau_4 < 1 \end{gathered} \tag{4.7}$$

Hosking (1986, 1989) presents a unified approach to probability weighted moments and L-moments in statistical estimation. He also demonstrates that the L-moments are competitive with the conventional product moments and maximum likelihood techniques. Theoretically, the L-moments can characterize a wider range of distributions and are more robust to the presence of outliers in the data. The L-moment estimators tend to be less biased, approximate their asymptotic normal distribution more closely in finite samples, and often give more accurate estimates of the parameters of a fitted distribution. The parameter estimates from the L-moments are sometimes more accurate in small samples than are the maximum likelihood estimates. Hosking (1986) also developed a series of FORTRAN programs to calculate the L-moments for different distributions, and to estimate L-moments from samples. These programs were used in this study to assess the ability of the L-moments to describe a distribution.

4.3 LINEAR TRANSFORMATION WITH PARTIAL MOMENTS

Let Y be a standardized random variable with a distribution function $F(y)$, and let Z be a standardized random variable with a symmetric distribution with the distribution function $G(z)$. Shore (1986) suggested a four-parameter linear transformation of Z to approximate Y by

$$Y = \begin{cases} A_1 Z + B_1, & Z<0 \\ A_2 Z + B_2, & Z>0 \end{cases} \tag{4.8}$$

Let the *i*th partial moment of *Z* be denoted by M_i. Since the distribution of *G(z)* is symmetric, $M_i = \int_{1/2}^{1} z^i dG(z)$. Shore (1995) provided a set of equations to determine the coefficients in Eq. (4.8) using the first two complete moments and the first two partial moments,

$$\begin{aligned} A_1^2 &= \{(\sigma^2 + \mu^2) - M_2 Y - 2[\mu - M_1(Y)]^2\} / [(1/2) - 2M_1^2] \\ B_1 &= 2[\mu - M_1(Y) + A_1 M_1] \\ A_2^2 &= \{M_2(Y) - 2[M_1(Y)]^2\} / [(1/2) - 2M_1^2] \\ B_2 &= 2[M_1(Y) - A_2 M_1] \end{aligned} \tag{4.9}$$

where $M_1(Y)$ and $M_2(Y)$ are the first and second partial moment of Y; μ and σ are the mean and the standard deviation of *Y*.

Several candidate distributions may be used for *G(z)*. Shore (1995) investigated two cases: the logistic distribution and the standard normal distribution. The transformation with the logistic variable covers a wider range in the moment-ratio diagram than that with the standard normal variable. However, it is impossible to extend this two-part linear transformation to a bivariate case when the logistic variate is chosen for *Z*. Using the result of Kamat (1953, 1958), the two-part linear transformation with the standard normal variable *Z* can be extended to a bivariate case. However, the formulas are so messy that they are of little practical value and, hence, are not presented here. When the standard normal variable is adopted for *Z*, the first six partial moments of *Z* are

$$M_0 = 1/2, \quad M_1 = 1/\sqrt{2\pi}, \quad M_2 = 1/2, \quad M_3 = \sqrt{(2/\pi)}, \\ M_4 = 3/2, \quad M_5 = \sqrt{(32/\pi)}, \quad M_6 = 15/2. \tag{4.10}$$

4.4 POLYNOMIAL TRANSFORMATION

Based on the notion that most distributions can be adequately characterized by their first four central moments, namely, mean, variance, skewness, and kurtosis, Fleishman (1978) proposed a simple method for simulating non-normal distributions using the following polynomial transformation

$$Y = a + bZ + cZ^2 + dZ^3 \tag{4.11}$$

where Z is a standard normal random variable; Y is the original non-normal random variable; a, b, c, d are the coefficients for polynomial transformation. In this section, three methods using different types of moments for finding the coefficients in Eq. (4.11) are presented.

4.4.1 Polynomial Transformation with Complete Moments

Using the standardized variable $Y_S = (Y - \mu_Y)/\sigma_Y$, Fleishman (1978) provided a set of non-linear equations which can be used to solve the constants in Eq. (4.11)

$$\begin{aligned} 0 &= a_s + c_s \\ 1 &= b_s^2 + 6b_s d_s + 2c - s^2 + 15d_s^2 \\ \gamma_Y &= 2c_s(b_s^2 + 24b_s d_s + 105d_s^2 + 2) \\ \kappa_Y &= 24[b_s d_s + c_s^2(1 + b_s^2 + 28b_s d_s) + d_s^2(12 + 48b_s d_s + 141c_s^2 + 225d_s^2)] + 3 \end{aligned} \tag{4.12}$$

where a_s, b_s, c_s, d_s are the corresponding coefficients to the standardized Y. The polynomial constants in Eq. (4.11) are easy to obtain after solving Eq. (4.12).

$$\begin{aligned} a &= \sigma_Y a_S + \mu_Y \\ b &= \sigma_Y b_S \\ c &= \sigma_Y c_S \\ d &= \sigma_Y d_S \end{aligned} \tag{4.13}$$

The polynomial transformation is easy to implement. However, this method has some potentially severe drawbacks. One is that polynomial transformation is not necessarily monotonic, rendering the calculation of quantiles of certain distributions unreasonable. The necessary condition for the polynomial transformation to be monotonic is that $Y' = b + 2cZ + 3dZ^2$ is always either negative or positive, which requires,

$$c^2 - 3bd < 0 \tag{4.14}$$

Another disadvantage is that this transformation does not cover the entire feasible area in the moment-ratio diagram. The relationship between skewness and kurtosis of the polynomial transformation can be described by the following parabola (Fleishman, 1978)

$$\kappa > 1.58837\gamma^2 + 1.8683 \tag{4.15}$$

Therefore, this method is unable to generate some extreme combinations of skewness and kurtosis, which makes it less attractive in certain conditions (Tadikamalla, 1980).

4.4.2 Polynomial Transformation with Partial Moments

It is also possible to fit a polynomial transformation using the first two complete moments and first two partial moments. Suppose the polynomial transformation is monotonic, i.e., Eq. (4.14) holds, the partial mean can be evaluated by

$$\begin{aligned} M_1(Y) &= \int_{1/2}^{1} y dF(y) \\ &= \int_{0}^{\infty} (a + bz + cz^2 + dz^3)\phi(z)dz \\ &= aM_0 + bM_1 + cM_2 + dM_3 \end{aligned} \tag{4.16}$$

Similarly, the second partial moment can be expressed as

$$\begin{aligned} M_2(Y) &= \int_{1/2}^{1} y^2 dF(y) \\ &= \int_{0}^{\infty} (a + bz + cz^2 + dz^3)^2 \phi(z)dz \\ &= a^2M_0 + 2abM_1 + (b^2 + 2ac)M_2 + (2bc + 2ad)M_3 + (c^2 + 2bd)M_4 + 2cdM_5 + d^2M_6 \end{aligned} \tag{4.17}$$

In Eqs. (4.16) and (4.17), M_i is the ith partial moment of the standard normal variable and its value is given in Eq. (4.10). Together with the following two equations for the complete moments,

$$\mu_Y = a+c \tag{4.18a}$$
$$\sigma_Y = b^2+6bd+2c^2+15d^2 \tag{4.18b}$$

the constant in Eq. (4.11) can be solved.

4.4.3 Polynomial Transformation with L-Moments

Consider the probability weighted moments for the random variable Y in Eq. (4.11). Under the condition of monotonicity, the first four probability weighted moments can be expressed as,

$$\begin{aligned}
\beta_m(Y) &= \int_0^1 yF^m(y)dF(y) \qquad\qquad m=0,1,2,3 \\
&= \int_{-\infty}^{\infty}(a+bz+cz^2+dz^3)\Phi^m(z)\phi(z)dz \\
&= a\cdot\int_{-\infty}^{\infty}\Phi^m(z)\phi(z)dz + b\cdot\int_{-\infty}^{\infty}z\Phi^m(z)\phi(z)dz \\
&\quad + c\cdot\int_{-\infty}^{\infty}z^2\Phi^m(z)\phi(z)dz + d\cdot\int_{-\infty}^{\infty}z^3\Phi^m(z)\phi(z)dz \\
&= aC_{m,0}+bC_{m,1}+cC_{m,2}+dC_{m,3}
\end{aligned} \tag{4.19}$$

where $C_{m,n}$ $(m=0,1,2,3; n=0,1,2,3)$ is a constant and can be evaluated numerically. These constants were evaluated using a 48-point Gaussian-Legendre integration and are presented in Table 4.1.

Since the unbiased estimator of the L-moments can be calculated through Eq. (4.6) and the probability weighted moments can be obtained from the L-moments using the inverse

Table 4.1 Cofficients $C_{m,n}$ for Ploynomial Transformation with L-moments

	n			
m	0	1	2	3
0	1.0	0.0	1.0	0.0
1	0.5	0.28209621	0.5	0.70527822
2	0.33333610	0.28209621	0.42522901	0.70527822
3	0.25000027	0.25734526	0.38783976	0.67514771

form of Eq. (4.5), the coefficients in Eq. (4.11) can be obtained by solving the following system of linear equations

$$\begin{bmatrix} C_{0,0} & C_{0,1} & C_{0,2} & C_{0,3} \\ C_{1,0} & C_{1,1} & C_{1,2} & C_{1,3} \\ C_{2,0} & C_{2,1} & C_{2,2} & C_{2,3} \\ C_{3,0} & C_{3,1} & C_{3,2} & C_{3,3} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} \beta_0(Y) \\ \beta_1(Y) \\ \beta_2(Y) \\ \beta_3(Y) \end{bmatrix} \tag{4.20}$$

4.5 SIMULATIONS AND DISCUSSION

To assess the ability of the partial moments and L-moments to describe a distributional property, four different methods, namely, linear transformation with partial moments (LTPM), polynomial transformation with complete moments (PTCM), polynomial transformation with partial moments (PTPM), and polynomial transformation with L-moments (PTLM), are considered in the simulation study. The performance of the above four methods are examined to select appropriate alternatives for normal transformation. The effects of sample size on the accuracy of those alternatives for normal transformation are also investigated.

4.5.1 The Numerical Experiment

The numerical experiment contains two phases. The objective of Phase-I is to identify the methods which can properly serve as the alternatives for normal transformation. Methods with poor accuracy are discarded from further consideration. Phase-II focuses on the investigation of the effects of sample size and other factors on the accuracy of the qualified methods from the Phase I study.

Phase-I - Three distributions were considered and they are (1) the exponential distribution with the parameter $\beta = 1$; (2) the GEV distribution with parameter $(\xi, \alpha, \kappa) = (0.8, 0.5, 0.3)$; and (3) the GEV distribution with parameters $(\xi, \alpha, \kappa) = (0.3, 0.8, 1.1)$. The true quantiles and those approximated from the four candidate methods mentioned above were compared based on which the performance of these methods was examined. Under the condition of monotonicity, the *p*th order quantile of the random variable *Y* satisfies

$$p = P(Y \leq y_p) = \Phi(z_p) \tag{4.21}$$

and it can be calculated as

$$y_p = f(z_p) \tag{4.22}$$

where $f(\cdot)$ is the transformation function corresponding to the LTPM, PTCM, PTPM, PTLM.

Phase-II - In this phase, the GEV distributions with $\lambda_1 = 1$; $\tau_2 = 0.1, 0.2, 0.3$; and $\tau_3 = 0.0, 0.2, 0.4$ were employed. Sample sizes were chosen at 20, 50, and 100. For each sample size, the GEV random variates were generated from which the sample complete moments and sample L-moments were calculated. Using the methods selected from Phase-I, the coefficients in Eq. (4.11) were solved and the approximated quantiles were calculated. Three criteria were used for performance evaluation and they are:

(1) Relative biasness (e_b)

$$e_b = \int_0^1 \left(\frac{\hat{y}_p - y_p}{y_p} \right) dp \tag{4.23}$$

(2) Relative mean-absolute error (e_{mae})

$$e_{mae} = \int_0^1 \left| \frac{\hat{y}_p - y_p}{y_p} \right| dp \tag{4.24}$$

(3) Relative root-mean-squared error (e_{rmse})

$$e_{rmse} = \sqrt{\int_0^1 \left(\frac{\hat{y}_p - y_p}{y_p} \right)^2 dp} \tag{4.25}$$

where y_p is the true *p*th order quantile and $\hat{y}_p$ is the quantile calculated from Eqs. (4.21) and (4.22). In this study, 29 p-values, 0.02, 0.05(0.05)0.95, 0.96(0.01)0.99, 0.992(0.02)0.998, 0.999, were used to numerically calculate the above three error criteria. To examine the performance of the proposed methods in different parts of the sample space, the following three conditional relative errors were also calculated,

$$ce_b(p_1<p<p_2) = \frac{\int_{p_1}^{p_2}\left(\frac{\hat{y}_p - y_p}{y_p}\right)dp}{p_2 - p_1}$$

$$ce_{ame}(p_1<p<p_2) = \frac{\int_{p_1}^{p_2}\left|\frac{\hat{y}_p - y_p}{y_p}\right|dp}{p_2 - p_1} \qquad (4.28)$$

$$ce_{rmse}(p_1<p<p_2) = \frac{\sqrt{\int_{p_1}^{p_2}\left(\frac{\hat{y}_p - y_p}{y_p}\right)^2 dp}}{p_2 - p_1}$$

In this study, four probability regions in the sampling space, that is, $p = 0 - 0.3$, $0.3 - 0.8$, $0.8 - 0.95$, $0.95 - 1.0$, were selected.

4.5.2 Results and Discussion of the Phase-I Experiment

Table 4.2 shows the coefficient values in the different transformations for the given three distributions. By the LTPM, the coefficients were obtained directly from Eq. (4.9). The Newton-Raphson method (Press et al., 1989) was used to solve the nonlinear system of equations, Eqs. (4.12), to obtain the coefficient values for the PTCM, and to solve Eqs. (4.16) - (4.18) for the PTPM. By the PTLM, the coefficients in the polynomial transformation were obtained by solving the linear system of equations, Eq. (4.20), with $C_{m,n}$ listed in Table 4.1. As can be seen in Table 4.2, the coefficients of the PTCM are very close to those of the PTLM while the coefficients of the PTPM differ greatly from those of the PTCM and the PTLM.

Table 4.2 Coefficients Fitted by Four Different Procedures

(a) Exponential distribution with $\beta = 1$

	A_1	B_1	A_2	B_2
LTPM	1.658897	3.016755	0.3280003	4.5146435E-02
	a	b	c	d
PTCM	0.6862539	0.8263242	0.3137461	2.2707479E-02
PTPM	0.6547011	-0.8612461	0.3452989	-3.7885467E-03
PTLM	0.697746	0.799182	0.302254	0.034814

(b) GEV distribution with $(\xi, \alpha, \kappa) = (0.8, 0.5, 0.3)$

	A_1	B_1	A_2	B_2
LTPM	0.3591964	1.678975	0.4893914	0.1589089
	a	b	c	d
PTCM	0.9770508	0.5149320	-6.1686193E-03	-6.9037438E-03
PTPM	1.107523	-0.6554525	-0.1366406	6.3565090E-02
PTLM	0.975493	0.515552	-0.004611	-0.007118

(c) GEV distribution with $(\xi, \alpha, \kappa) = (0.3, 0.8, 1.1)$

	A_1	B_1	A_2	B_2
LTPM	0.3995391	1.143420	1.411512	-1.418473
	a	b	c	d
PTCM	0.5488653	0.6368845	-0.2826732	3.1954534E-02
PTPM	0.3884259	-0.5197452	-0.1222338	-9.0115815E-02
PTLM	0.539178	0.618190	-0.272986	0.040584

The true quantiles and those approximated from the different methods are presented in Table 4.3. As can be seen, the quantiles obtained from the LTPM and PTPM differ greatly from the true quantiles. On the other hand, the PTCM and the PTLM approximate the true quantiles very well. This suggests that the partial moments cannot capture the full feature of a distribution while both the L-moments and complete moments can. Hence, the PTCM and the PTLM are the appropriate alternatives for normal transformation and are used in the further investigation in Phase-II experiment.

4.5.3 Results and Discussion of Phase-II Experiment

A total of 5000 simulation runs were made for each combination of the GEV distributions and sample sizes. Due to the constraint in Eq. (4.15), the PTCM fails in some cases when (1) the sample product moments lie outside the feasible region; (2) the Newton-Raphson method fails to converge. However, the sample L-moments are always feasible according to Eq. (4.7) and corresponding polynomial coefficients can always be found using the PTLM. The percentages of infeasible cases for the PTCM are shown in Table 4.4. Basically, the percentage of infeasible cases for a given GEV distribution decreases when the sample size increases. When the GEV distribution becomes more skewed, i.e., as the value of τ_3 increase, the percentage of infeasible cases increases. When $\tau_3 = 0.4$, the percentages of feasible cases for using product moments are less than 30% even for the sample size of 100.

Tables 4.5 - 4.13 show the percentage errors of the PTCM and PTLM for the nine GEV distributions. The first two columns are the averaged relative errors of the PTCM,

Table 4.3 Comparison of Exact and Approximated Quantiles Obtained by Four Procedures

(a) Exponential distribution with $\beta = 1$

P-Value	Quantile	Q-LTPM	Q-PTCM	Q-PTPM	Q-PTLM
0.100	0.105361	0.890499	0.094747	2.333829	0.096690
0.200	0.223144	1.620865	0.199557	1.626148	0.218477
0.300	0.356675	2.147490	0.336139	1.201352	0.356752
0.400	0.510826	2.597165	0.496954	0.894691	0.514109
0.500	0.693147	0.045147	0.686254	0.654701	0.697746
0.600	0.916291	0.128109	0.915698	0.458893	0.920182
0.700	1.203973	0.217019	1.208664	0.297673	1.204977
0.800	1.609438	0.321145	1.617247	0.172231	1.605204
0.900	2.302585	0.465554	2.308622	0.110107	2.291630
0.950	2.995732	0.584776	2.996074	0.155526	2.984976
0.980	3.912024	0.718921	3.904421	0.309754	3.915516
0.990	4.605171	0.808333	4.593581	0.472466	4.630998
0.995	5.298318	0.890153	5.285655	0.662896	5.356715
0.999	6.907768	1.058840	6.907025	1.179235	7.081181

(b) GEV distribution with $(\xi, \alpha, \kappa) = (0.8, 0.5, 0.3)$

P-Value	Quantile	Q-LTPM	Q-PTCM	Q-PTPM	Q-PTLM
0.100	0.326175	-1.792794	0.321451	1.589311	0.322196
0.200	0.544235	-1.053588	0.543503	1.524438	0.542571
0.300	0.704554	-0.520590	0.706525	1.404317	0.704896
0.400	0.843143	-0.065472	0.846524	1.263538	0.844700
0.500	0.973541	0.489391	0.977051	1.107523	0.975493
0.600	1.104185	0.529585	1.106788	0.934024	1.105695
0.700	1.243374	0.572660	1.244189	0.735692	1.243555
0.800	1.403934	0.623106	1.401863	0.497111	1.401883
0.900	1.618166	0.693069	1.612383	0.176780	1.613644
0.950	1.782964	0.750830	1.776783	-0.057620	1.779348
0.980	1.949689	0.815820	1.948947	-0.264497	1.953197
0.990	2.047388	0.859138	2.054824	-0.356604	2.060274
0.995	2.126364	0.898778	2.144653	-0.401095	2.151220
0.999	2.256815	0.980503	2.305753	-0.346914	2.314575

(c) GEV distribution with $(\xi, \alpha, \kappa) = (0.3, 0.8, 1.1)$

P-Value	Quantile	Q-LTPM	Q-PTCM	Q-PTPM	Q-PTLM
0.100	-0.792992	-1.066015	-0.799117	1.043543	-0.786830
0.200	-0.200277	-0.562599	-0.206230	0.792912	-0.198661
0.300	0.135251	-0.199615	0.132923	0.640176	0.134076
0.400	0.366680	0.110330	0.369175	0.513525	0.364380
0.500	0.541307	1.411512	0.548865	0.388426	0.539178
0.600	0.679899	1.052733	0.692388	0.247687	0.678933
0.700	0.793283	0.668230	0.809576	0.069550	0.794140
0.800	0.887591	0.217928	0.903668	-0.189155	0.890291
0.900	0.966088	-0.406585	0.968080	-0.668309	0.968496
0.950	0.999555	-0.922176	0.973855	-1.198815	0.998042
0.980	1.017327	-1.502300	0.941336	-1.976146	1.008919
0.990	1.022659	-1.888971	0.902919	-2.617870	1.010881
0.995	1.025126	-2.242810	0.859904	-3.302668	1.013890
0.999	1.026908	-2.972316	0.760519	-5.045460	1.040281

Table 4.4 Percentages of Infeasible Cases by the PTCM in Simulation Study

τ_2	N	τ_3		
		0.0	0.2	0.4
0.1	20 50 100	0.3336 0.0340 0	0.0016 0.1820 0.0470	0.8338 0.8322 0.7442
0.2	20 50 100	0.3340 0.0346 0	0.5478 0.0058 0.0472	0.8338 0.8320 0.7910
0.3	20 50 100	0.3336 0.0350 0	0.5474 0.2312 0.0707	0.8334 0.8322 0.7958

Table 4.5 Relative Errors of Different Methods Using GEV Distribution with $(\tau_2, \tau_3) = (0.1, 0.0)$

(a) Sample size = 20

Prob. Range	Crit.	CM (3332)	LM (3332)	LM (5000)
0.00 - 1.00	BIAS	0.164385E-01	0.178887E-01	0.100246E-01
	MAE	0.454278E-01	0.485666E-01	0.494331E-01
	RMSE	0.549458E-01	0.579561E-01	0.586824E-01
0.00 - 0.30	BIAS	0.271374E-01	0.358193E-01	0.210636E-01
	MAE	0.505079E-01	0.583301E-01	0.576961E-01
	RMSE	0.115022E+00	0.124977E+00	0.124632E+00
0.30 - 0.80	BIAS	0.183176E-01	0.147992E-01	0.864922E-02
	MAE	0.424587E-01	0.438461E-01	0.292190E-01
	RMSE	0.622599E-01	0.659587E-01	0.606046E-01
0.80 - 0.95	BIAS	0.471896E-02	-0.596152E-02	-0.285168E-02
	MAE	0.428168E-01	0.434620E-01	0.394925E-01
	RMSE	0.114048E+00	0.115189E+00	0.105162E+00
0.95 - 1.00	BIAS	-0.313878E-01	0.127505E-01	-0.382608E-02
	MAE	0.524737E-01	0.525077E-01	0.517483E-01
	RMSE	0.264399E+00	0.275759E+00	0.271398E+00

(b) Sample size = 50

Prob. Range	Crit.	CM (4830)	LM (4830)	LM (5000)
0.00 - 1.00	BIAS	-0.205372E-02	-0.163914E-02	-0.157365E-02
	MAE	0.262110E-01	0.265817E-01	0.264456E-01
	RMSE	0.321332E-01	0.328194E-01	0.326766E-01
0.00 - 0.30	BIAS	-0.641137E-02	-0.400530E-02	-0.410901E-02
	MAE	0.344227E-01	0.342108E-01	0.343699E-01
	RMSE	0.724760E-01	0.726778E-01	0.731403E-01
0.30 - 0.80	BIAS	0.164877E-03	-0.867198E-03	-0.707484E-03
	MAE	0.219309E-01	0.223667E-01	0.216062E-01
	RMSE	0.330086E-01	0.336149E-01	0.338799E-01
0.80 - 0.95	BIAS	0.205021E-02	-0.101919E-02	-0.475991E-03
	MAE	0.224437E-01	0.227842E-01	0.232313E-01
	RMSE	0.600218E-01	0.612429E-01	0.623988E-01
0.95 - 1.00	BIAS	-0.104055E-01	0.297871E-02	0.168402E-02
	MAE	0.310445E-01	0.343509E-01	0.351552E-01
	RMSE	0.155845E+00	0.176601E+00	0.181348E+00

(c) Sample size = 100

Prob. Range	Crit.	CM (5000)	LM (5000)	LM (5000)
0.00 - 1.00	BIAS	-0.431173E-02	-0.420618E-02	-0.420618E-02
	MAE	0.172041E-01	0.175631E-01	0.175631E-01
	RMSE	0.211758E-01	0.219523E-01	0.219523E-01
0.00 - 0.30	BIAS	-0.896933E-02	-0.771310E-02	-0.771310E-02
	MAE	0.235851E-01	0.235497E-01	0.235497E-01
	RMSE	0.496736E-01	0.505755E-01	0.505755E-01
0.30 - 0.80	BIAS	-0.252898E-02	-0.347605E-02	-0.347605E-02
	MAE	0.136361E-01	0.138839E-01	0.138839E-01
	RMSE	0.205957E-01	0.210371E-01	0.210371E-01
0.80 - 0.95	BIAS	-0.115145E-02	-0.185973E-02	-0.185973E-02
	MAE	0.148408E-01	0.152762E-01	0.152762E-01
	RMSE	0.395661E-01	0.410161E-01	0.410161E-01
0.95 - 1.00	BIAS	-0.367485E-02	0.249527E-02	0.249527E-02
	MAE	0.216866E-01	0.252985E-01	0.252985E-01
	RMSE	0.105202E+00	0.127128E+00	0.127128E+00

Table 4.6 Relative Errors of Different Methods Using GEV Distribution with $(\tau_2, \tau_3) = (0.1, 0.2)$

(a) Sample size = 20

Prob. Range	Crit.	CM (4992)	LM (4992)	LM (5000)
0.00 - 1.00	BIAS	-0.319799E-01	-0.310531E-01	-0.310072E-01
	MAE	0.391487E-01	0.341868E-01	0.342298E-01
	RMSE	0.517931E-01	0.454438E-01	0.454868E-01
0.00 - 0.30	BIAS	-0.329641E-02	0.240917E-03	0.196546E-03
	MAE	0.270830E-01	0.100930E-01	0.101344E-01
	RMSE	0.646488E-01	0.235687E-01	0.236469E-01
0.30 - 0.80	BIAS	-0.289571E-01	-0.296808E-01	-0.296211E-01
	MAE	0.289994E-01	0.297259E-01	0.296784E-01
	RMSE	0.412287E-01	0.443944E-01	0.444029E-01
0.80 - 0.95	BIAS	-0.598654E-01	-0.675189E-01	-0.673704E-01
	MAE	0.599196E-01	0.675705E-01	0.675842E-01
	RMSE	0.162173E+00	0.177959E+00	0.178000E+00
0.95 - 1.00	BIAS	-0.150665E+00	-0.123165E+00	-0.123023E+00
	MAE	0.150703E+00	0.123217E+00	0.123235E+00
	RMSE	0.717855E+00	0.568810E+00	0.568955E+00

(b) Sample size = 50

Prob. Range	Crit.	CM (4090)	LM (4090)	LM (5000)
0.00 - 1.00	BIAS	0.317675E-02	0.478825E-02	0.399134E-02
	MAE	0.338716E-01	0.311348E-01	0.296368E-01
	RMSE	0.414442E-01	0.360834E-01	0.343111E-01
0.00 - 0.30	BIAS	0.476401E-02	0.915369E-02	0.702673E-02
	MAE	0.306810E-01	0.249785E-01	0.240425E-01
	RMSE	0.714032E-01	0.497992E-01	0.492475E-01
0.30 - 0.80	BIAS	0.136355E-02	0.356353E-02	0.309205E-02
	MAE	0.298880E-01	0.284988E-01	0.233120E-01
	RMSE	0.447943E-01	0.416443E-01	0.444848E-01
0.80 - 0.95	BIAS	0.129745E-01	0.143443E-02	0.292209E-02
	MAE	0.422904E-01	0.412326E-01	0.425458E-01
	RMSE	0.112660E+00	0.109701E+00	0.112755E+00
0.95 - 1.00	BIAS	-0.176080E-01	0.904926E-03	-0.201976E-02
	MAE	0.675919E-01	0.641243E-01	0.595685E-01
	RMSE	0.326637E+00	0.304857E+00	0.286173E+00

(c) Sample size = 100

Prob. Range	Crit.	CM (4765)	LM (4765)	LM (5000)
0.00 - 1.00	BIAS	0.219635E-02	0.276318E-02	0.269845E-02
	MAE	0.260017E-01	0.203450E-01	0.204219E-01
	RMSE	0.336741E-01	0.251398E-01	0.252235E-01
0.00 - 0.30	BIAS	0.813606E-02	0.349613E-02	0.358091E-02
	MAE	0.282987E-01	0.179080E-01	0.181652E-01
	RMSE	0.640731E-01	0.364265E-01	0.370927E-01
0.30 - 0.80	BIAS	-0.486527E-03	0.349125E-02	0.288334E-02
	MAE	0.209346E-01	0.176945E-01	0.168628E-01
	RMSE	0.322475E-01	0.265665E-01	0.265839E-01
0.80 - 0.95	BIAS	0.536144E-02	0.211341E-02	0.260235E-02
	MAE	0.297988E-01	0.250706E-01	0.254758E-01
	RMSE	0.809110E-01	0.682277E-01	0.693781E-01
0.95 - 1.00	BIAS	-0.161084E-01	-0.696606E-02	-0.415697E-02
	MAE	0.515025E-01	0.472972E-01	0.490386E-01
	RMSE	0.259380E+00	0.229119E+00	0.238114E+00

Table 4.7 Relative Errors of Different Methods Using GEV Distribution with $(\tau_2, \tau_3) = (0.1, 0.4)$

(a) Sample size = 20

Prob. Range	Crit.	CM (831)	LM (831)	LM (5000)
0.00 - 1.00	BIAS	-0.643092E-02	-0.550890E-02	0.519373E-02
	MAE	0.518603E-01	0.522168E-01	0.486456E-01
	RMSE	0.810402E-01	0.772411E-01	0.778768E-01
0.00 - 0.30	BIAS	0.562773E-04	0.607571E-02	0.132400E-01
	MAE	0.177217E-01	0.187663E-01	0.289120E-01
	RMSE	0.392829E-01	0.392363E-01	0.599513E-01
0.30 - 0.80	BIAS	0.165464E-01	0.152743E-01	0.428016E-02
	MAE	0.429374E-01	0.445593E-01	0.740576E-02
	RMSE	0.664270E-01	0.704081E-01	0.581991E-01
0.80 - 0.95	BIAS	-0.269722E-01	-0.372006E-01	0.954329E-02
	MAE	0.923807E-01	0.947741E-01	0.810170E-01
	RMSE	0.252945E+00	0.254842E+00	0.226417E+00
0.95 - 1.00	BIAS	-0.213504E+00	-0.187774E+00	-0.469970E-01
	MAE	0.224363E+00	0.201822E+00	0.188922E+00
	RMSE	0.112729E+01	0.996357E+00	0.934861E+00

(b) Sample size = 50

Prob. Range	Crit.	CM (839)	LM (839)	LM (5000)
0.00 - 1.00	BIAS	-0.122247E-01	-0.108377E-01	0.240370E-02
	MAE	0.298211E-01	0.258147E-01	0.257960E-01
	RMSE	0.552836E-01	0.494294E-01	0.498544E-01
0.00 - 0.30	BIAS	0.319870E-02	0.109543E-03	0.864062E-02
	MAE	0.224048E-01	0.109870E-01	0.211575E-01
	RMSE	0.578388E-01	0.224405E-01	0.434485E-01
0.30 - 0.80	BIAS	-0.294631E-02	0.324019E-02	-0.643237E-02
	MAE	0.166316E-01	0.153641E-01	0.257809E-02
	RMSE	0.263625E-01	0.244222E-01	0.355251E-01
0.80 - 0.95	BIAS	-0.227354E-01	-0.303584E-01	0.189491E-01
	MAE	0.417579E-01	0.446937E-01	0.555151E-01
	RMSE	0.125077E+00	0.130640E+00	0.159263E+00
0.95 - 1.00	BIAS	-0.166016E+00	-0.158738E+00	0.370658E-02
	MAE	0.170404E+00	0.162650E+00	0.145750E+00
	RMSE	0.882515E+00	0.829487E+00	0.722232E+00

(c) Sample size = 100

Prob. Range	Crit.	CM (1279)	LM (1279)	LM (5000)
0.00 - 1.00	BIAS	-0.946428E-02	-0.114760E-01	0.252955E-02
	MAE	0.676823E-01	0.181670E-01	0.184260E-01
	RMSE	0.997373E-01	0.374898E-01	0.402524E-01
0.00 - 0.30	BIAS	0.825171E-01	0.112915E-03	0.979842E-02
	MAE	0.985697E-01	0.673682E-02	0.186939E-01
	RMSE	0.239469E+00	0.142129E-01	0.385808E-01
0.30 - 0.80	BIAS	-0.354759E-01	-0.400457E-02	-0.104864E-01
	MAE	0.398879E-01	0.103769E-01	0.265442E-02
	RMSE	0.626433E-01	0.163545E-01	0.273070E-01
0.80 - 0.95	BIAS	-0.580158E-01	-0.224199E-01	0.228196E-01
	MAE	0.690151E-01	0.318918E-01	0.465838E-01
	RMSE	0.185720E+00	0.909607E-01	0.137306E+00
0.95 - 1.00	BIAS	-0.155584E+00	-0.122891E+00	0.282059E-01
	MAE	0.156309E+00	0.123475E+00	0.132794E+00
	RMSE	0.801930E+00	0.658715E+00	0.668842E+00

Table 4.8 Relative Errors of Different Methods Using GEV Distribution with $(\tau_2, \tau_3) = (0.2, 0.0)$

(a) Sample size = 20

Prob. Range	Crit.	CM (3330)	LM (3330)	LM (5000)
0.00 - 1.00	BIAS	0.425403E-01	0.433160E-01	0.261074E-01
	MAE	0.103793E+00	0.110895E+00	0.112118E+00
	RMSE	0.148414E+00	0.152487E+00	0.151934E+00
0.00 - 0.30	BIAS	0.857124E-01	0.953290E-01	0.611732E-01
	MAE	0.157054E+00	0.175327E+00	0.173743E+00
	RMSE	0.408067E+00	0.420712E+00	0.422141E+00
0.30 - 0.80	BIAS	0.361041E-01	0.305329E-01	0.175822E-01
	MAE	0.832047E-01	0.861195E-01	0.573556E-01
	RMSE	0.121923E+00	0.129782E+00	0.119860E+00
0.80 - 0.95	BIAS	0.826329E-02	-0.101117E-01	-0.473763E-02
	MAE	0.728495E-01	0.739683E-01	0.671761E-01
	RMSE	0.194141E+00	0.196188E+00	0.178937E+00
0.95 - 1.00	BIAS	-0.492924E-01	0.193594E-01	-0.649519E-02
	MAE	0.829390E-01	0.828453E-01	0.816672E-01
	RMSE	0.414948E+00	0.431419E+00	0.424574E+00

(b) Sample size = 50

Prob. Range	Crit.	CM (4827)	LM (4827)	LM (5000)
0.00 - 1.00	BIAS	-0.451310E-02	-0.439848E-02	-0.403844E-02
	MAE	0.607728E-01	0.614699E-01	0.611274E-01
	RMSE	0.864890E-01	0.880975E-01	0.875412E-01
0.00 - 0.30	BIAS	-0.139108E-01	-0.116371E-01	-0.111265E-01
	MAE	0.103044E+00	0.102838E+00	0.103436E+00
	RMSE	0.242310E+00	0.244957E+00	0.247213E+00
0.30 - 0.80	BIAS	-0.133790E-03	-0.177259E-02	-0.145867E-02
	MAE	0.433661E-01	0.441881E-01	0.426592E-01
	RMSE	0.655788E-01	0.666981E-01	0.671987E-01
0.80 - 0.95	BIAS	0.356068E-02	-0.163753E-02	-0.641107E-03
	MAE	0.381629E-01	0.387454E-01	0.395290E-01
	RMSE	0.101938E+00	0.104032E+00	0.106060E+00
0.95 - 1.00	BIAS	-0.161413E-01	0.449143E-02	0.250055E-02
	MAE	0.490431E-01	0.542523E-01	0.555125E-01
	RMSE	0.244472E+00	0.276527E+00	0.283911E+00

(c) Sample size = 100

Prob. Range	Crit.	CM (5000)	LM (5000)	LM (5000)
0.00 - 1.00	BIAS	0.547681E-02	0.560581E-02	0.560581E-02
	MAE	0.477811E-01	0.489870E-01	0.489870E-01
	RMSE	0.664957E-01	0.691644E-01	0.691644E-01
0.00 - 0.30	BIAS	0.488073E-02	0.632166E-02	0.632166E-02
	MAE	0.816440E-01	0.839975E-01	0.839975E-01
	RMSE	0.187667E+00	0.195048E+00	0.195048E+00
0.30 - 0.80	BIAS	0.834986E-02	0.757548E-02	0.757548E-02
	MAE	0.352909E-01	0.358697E-01	0.358697E-01
	RMSE	0.530629E-01	0.543847E-01	0.543847E-01
0.80 - 0.95	BIAS	0.229457E-02	-0.308515E-03	-0.308515E-03
	MAE	0.270544E-01	0.277244E-01	0.277244E-01
	RMSE	0.721251E-01	0.741352E-01	0.741352E-01
0.95 - 1.00	BIAS	-0.101308E-01	-0.643137E-03	-0.643137E-03
	MAE	0.316835E-01	0.338808E-01	0.338808E-01
	RMSE	0.156584E+00	0.172404E+00	0.172404E+00

Table 4.9 Relative Errors of Different Methods Using GEV Distribution with $(\tau_2, \tau_3) = (0.2, 0.2)$

(a) Sample size = 20

Prob. Range	Crit.	CM (2261)	LM (2261)	LM (5000)
0.00 - 1.00	BIAS	0.399019E-01	0.429990E-01	0.173421E-01
	MAE	0.102501E+00	0.107664E+00	0.103534E+00
	RMSE	0.122742E+00	0.125355E+00	0.122861E+00
0.00 - 0.30	BIAS	0.406192E-01	0.607895E-01	0.347414E-01
	MAE	0.797718E-01	0.102526E+00	0.992217E-01
	RMSE	0.184235E+00	0.217548E+00	0.214216E+00
0.30 - 0.80	BIAS	0.623064E-01	0.572474E-01	0.178972E-01
	MAE	0.103341E+00	0.103003E+00	0.465782E-01
	RMSE	0.152559E+00	0.154127E+00	0.121887E+00
0.80 - 0.95	BIAS	0.160504E-01	-0.575916E-02	-0.167269E-02
	MAE	0.124718E+00	0.118869E+00	0.100463E+00
	RMSE	0.333249E+00	0.316164E+00	0.269578E+00
0.95 - 1.00	BIAS	-0.116895E+00	-0.599471E-01	-0.355561E-01
	MAE	0.163830E+00	0.151486E+00	0.151852E+00
	RMSE	0.802478E+00	0.717991E+00	0.729347E+00

(b) Sample size = 50

Prob. Range	Crit.	CM (4971)	LM (4971)	LM (5000)
0.00 - 1.00	BIAS	-0.268703E-01	-0.256494E-01	-0.255032E-01
	MAE	0.581946E-01	0.406193E-01	0.406663E-01
	RMSE	0.810565E-01	0.518043E-01	0.518614E-01
0.00 - 0.30	BIAS	-0.475473E-01	-0.474197E-01	-0.470204E-01
	MAE	0.958444E-01	0.565368E-01	0.565641E-01
	RMSE	0.209577E+00	0.109727E+00	0.109889E+00
0.30 - 0.80	BIAS	-0.198006E-01	-0.162790E-01	-0.163504E-01
	MAE	0.401029E-01	0.326888E-01	0.324993E-01
	RMSE	0.674974E-01	0.502572E-01	0.503942E-01
0.80 - 0.95	BIAS	0.110892E-01	-0.359516E-02	-0.347453E-02
	MAE	0.297636E-01	0.181287E-01	0.183818E-01
	RMSE	0.880304E-01	0.631461E-01	0.638171E-01
0.95 - 1.00	BIAS	-0.874052E-01	-0.548784E-01	-0.539982E-01
	MAE	0.984887E-01	0.919501E-01	0.922984E-01
	RMSE	0.519569E+00	0.450114E+00	0.451822E+00

(c) Sample size = 100

Prob. Range	Crit.	CM (4764)	LM (4764)	LM (5000)
0.00 - 1.00	BIAS	0.680098E-02	0.590094E-02	0.568510E-02
	MAE	0.551548E-01	0.417229E-01	0.418571E-01
	RMSE	0.748534E-01	0.501180E-01	0.502943E-01
0.00 - 0.30	BIAS	0.248810E-01	0.831308E-02	0.861449E-02
	MAE	0.759399E-01	0.466357E-01	0.473257E-01
	RMSE	0.182068E+00	0.971342E-01	0.990486E-01
0.30 - 0.80	BIAS	-0.170762E-02	0.681269E-02	0.554468E-02
	MAE	0.421383E-01	0.357914E-01	0.341020E-01
	RMSE	0.647257E-01	0.536770E-01	0.536614E-01
0.80 - 0.95	BIAS	0.933798E-02	0.349230E-02	0.428882E-02
	MAE	0.500695E-01	0.422718E-01	0.429180E-01
	RMSE	0.135281E+00	0.114327E+00	0.116163E+00
0.95 - 1.00	BIAS	-0.242049E-01	-0.104635E-01	-0.629804E-02
	MAE	0.758620E-01	0.699122E-01	0.724467E-01
	RMSE	0.374761E+00	0.333975E+00	0.346742E+00

Table 4.10 Relative Errors of Different Methods Using GEV Distribution with $(\tau_2, \tau_3) = (0.2, 0.4)$

(a) Sample size = 20

Prob. Range	Crit.	CM (831)	LM (831)	LM (5000)
0.00 - 1.00	BIAS	-0.383352E-02	-0.176956E-02	0.135849E-01
	MAE	0.966454E-01	0.979888E-01	0.884117E-01
	RMSE	0.132206E+00	0.129335E+00	0.124672E+00
0.00 - 0.30	BIAS	0.102324E-02	0.135042E-01	0.323306E-01
	MAE	0.443932E-01	0.463819E-01	0.713137E-01
	RMSE	0.100606E+00	0.977480E-01	0.149430E+00
0.30 - 0.80	BIAS	0.342290E-01	0.328156E-01	0.906524E-02
	MAE	0.879817E-01	0.912293E-01	0.151623E-01
	RMSE	0.133805E+00	0.141811E+00	0.118511E+00
0.80 - 0.95	BIAS	-0.431157E-01	-0.612630E-01	0.166294E-01
	MAE	0.158123E+00	0.162372E+00	0.138263E+00
	RMSE	0.428884E+00	0.433204E+00	0.381368E+00
0.95 - 1.00	BIAS	-0.295751E+00	-0.260784E+00	-0.628239E-01
	MAE	0.312359E+00	0.282077E+00	0.264545E+00
	RMSE	0.151935E+01	0.135211E+01	0.127564E+01

(b) Sample size = 50

Prob. Range	Crit.	CM (840)	LM (840)	LM (5000)
0.00 - 1.00	BIAS	-0.167386E-01	-0.144656E-01	0.494515E-02
	MAE	0.567933E-01	0.465809E-01	0.463487E-01
	RMSE	0.914963E-01	0.742514E-01	0.743663E-01
0.00 - 0.30	BIAS	0.113621E-01	0.564167E-03	0.211850E-01
	MAE	0.572209E-01	0.272400E-01	0.525525E-01
	RMSE	0.152389E+00	0.561550E-01	0.108977E+00
0.30 - 0.80	BIAS	-0.651760E-02	0.756693E-02	-0.135901E-01
	MAE	0.346705E-01	0.317517E-01	0.533429E-02
	RMSE	0.545940E-01	0.497013E-01	0.727746E-01
0.80 - 0.95	BIAS	-0.364563E-01	-0.497594E-01	0.324814E-01
	MAE	0.701593E-01	0.751211E-01	0.943084E-01
	RMSE	0.205942E+00	0.215271E+00	0.265995E+00
0.95 - 1.00	BIAS	-0.228399E+00	-0.219088E+00	0.102489E-01
	MAE	0.235356E+00	0.225297E+00	0.205533E+00
	RMSE	0.117342E+01	0.110778E+01	0.992600E+00

(c) Sample size = 100

Prob. Range	Crit.	CM (1045)	LM (1045)	LM (5000)
0.00 - 1.00	BIAS	0.725039E-02	0.125904E-03	0.101070E-01
	MAE	0.979642E-01	0.422267E-01	0.380402E-01
	RMSE	0.152359E+00	0.610731E-01	0.577190E-01
0.00 - 0.30	BIAS	0.124208E+00	0.831565E-02	0.272103E-01
	MAE	0.154958E+00	0.295359E-01	0.458539E-01
	RMSE	0.403135E+00	0.601263E-01	0.938735E-01
0.30 - 0.80	BIAS	-0.342478E-01	0.134885E-01	-0.621285E-02
	MAE	0.629867E-01	0.341415E-01	0.713558E-02
	RMSE	0.975353E-01	0.515221E-01	0.548957E-01
0.80 - 0.95	BIAS	-0.329412E-01	-0.116113E-01	0.324192E-01
	MAE	0.772082E-01	0.555654E-01	0.704707E-01
	RMSE	0.213100E+00	0.156725E+00	0.198487E+00
0.95 - 1.00	BIAS	-0.158937E+00	-0.147427E+00	0.374629E-02
	MAE	0.168044E+00	0.159207E+00	0.154973E+00
	RMSE	0.872634E+00	0.824731E+00	0.776434E+00

Table 4.11 Relative Errors of Different Methods Using GEV Distribution with $(\tau_2, \tau_3) = (0.3, 0.0)$

(a) Sample size = 20

Prob. Range	Crit.	CM (3332)	LM (3332)	LM (5000)
0.00 - 1.00	BIAS	0.228927E-01	0.897979E-01	0.317647E-01
	MAE	0.266918E+00	0.277335E+00	0.278471E+00
	RMSE	0.652394E+00	0.632757E+00	0.626392E+00
0.00 - 0.30	BIAS	-0.105935E-01	0.220942E+00	0.640349E-01
	MAE	0.618259E+00	0.644417E+00	0.641052E+00
	RMSE	0.212266E+01	0.205809E+01	0.208359E+01
0.30 - 0.80	BIAS	0.547608E-01	0.484944E-01	0.276796E-01
	MAE	0.123897E+00	0.128628E+00	0.857176E-01
	RMSE	0.182053E+00	0.194980E+00	0.180671E+00
0.80 - 0.95	BIAS	0.114781E-01	-0.128149E-01	-0.583520E-02
	MAE	0.956305E-01	0.970548E-01	0.880533E-01
	RMSE	0.254992E+00	0.257625E+00	0.234629E+00
0.95 - 1.00	BIAS	-0.606245E-01	0.238177E-01	-0.819926E-02
	MAE	0.102926E+00	0.102762E+00	0.101168E+00
	RMSE	0.512778E+00	0.532484E+00	0.523290E+00

(b) Sample size = 50

Prob. Range	Crit.	CM (4825)	LM (4825)	LM (5000)
0.00 - 1.00	BIAS	-0.267912E-01	-0.715891E-02	-0.969034E-02
	MAE	0.155779E+00	0.158039E+00	0.156984E+00
	RMSE	0.363836E+00	0.372779E+00	0.369626E+00
0.00 - 0.30	BIAS	-0.870272E-01	-0.192823E-01	-0.284522E-01
	MAE	0.376146E+00	0.380207E+00	0.382376E+00
	RMSE	0.118400E+01	0.121192E+01	0.122421E+01
0.30 - 0.80	BIAS	-0.809296E-03	-0.267058E-02	-0.235280E-02
	MAE	0.648509E-01	0.660647E-01	0.637525E-01
	RMSE	0.989976E-01	0.100602E+00	0.101287E+00
0.80 - 0.95	BIAS	0.476261E-02	-0.208436E-02	-0.829595E-03
	MAE	0.497908E-01	0.505432E-01	0.515663E-01
	RMSE	0.132950E+00	0.135648E+00	0.138304E+00
0.95 - 1.00	BIAS	-0.198550E-01	0.547484E-02	0.292508E-02
	MAE	0.608168E-01	0.672639E-01	0.688696E-01
	RMSE	0.301906E+00	0.341119E+00	0.350412E+00

(c) Sample size =100

Prob. Range	Crit.	CM (5000)	LM (5000)	LM (5000)
0.00 - 1.00	BIAS	-0.452619E-03	0.880106E-02	0.880106E-02
	MAE	0.121810E+00	0.124928E+00	0.124928E+00
	RMSE	0.275355E+00	0.285935E+00	0.285935E+00
0.00 - 0.30	BIAS	-0.215071E-01	0.105393E-01	0.105393E-01
	MAE	0.293534E+00	0.301567E+00	0.301567E+00
	RMSE	0.895902E+00	0.929429E+00	0.929429E+00
0.30 - 0.80	BIAS	0.123939E-01	0.115407E-01	0.115407E-01
	MAE	0.529843E-01	0.538730E-01	0.538730E-01
	RMSE	0.806198E-01	0.826482E-01	0.826482E-01
0.80 - 0.95	BIAS	0.294333E-02	-0.465202E-03	-0.465202E-03
	MAE	0.352793E-01	0.361728E-01	0.361728E-01
	RMSE	0.940690E-01	0.967128E-01	0.967128E-01
0.95 - 1.00	BIAS	-0.127798E-01	-0.122587E-02	-0.122587E-02
	MAE	0.393060E-01	0.419071E-01	0.419071E-01
	RMSE	0.193595E+00	0.212233E+00	0.212233E+00

Table 4.12 Relative Errors of Different Methods Using GEV Distribution with $(\tau_2, \tau_3) = (0.3, 0.2)$

(a) Sample size = 20

Prob. Range	Crit.	CM (2263)	LM (2263)	LM (5000)
0.00 - 1.00	BIAS	0.745287E-01	0.731909E-01	0.354278E-01
	MAE	0.170345E+00	0.182349E+00	0.174035E+00
	RMSE	0.221992E+00	0.231647E+00	0.227586E+00
0.00 - 0.30	BIAS	0.103378E+00	0.110766E+00	0.803251E-01
	MAE	0.192983E+00	0.238347E+00	0.233947E+00
	RMSE	0.507581E+00	0.566407E+00	0.568111E+00
0.30 - 0.80	BIAS	0.945340E-01	0.894155E-01	0.277091E-01
	MAE	0.156418E+00	0.156943E+00	0.710322E-01
	RMSE	0.229433E+00	0.234619E+00	0.185829E+00
0.80 - 0.95	BIAS	0.213566E-01	-0.757762E-02	-0.242639E-02
	MAE	0.162946E+00	0.155268E+00	0.130911E+00
	RMSE	0.435175E+00	0.412654E+00	0.350531E+00
0.95 - 1.00	BIAS	-0.139109E+00	-0.722008E-01	-0.432175E-01
	MAE	0.196010E+00	0.181682E+00	0.181239E+00
	RMSE	0.952158E+00	0.856267E+00	0.864273E+00

(b) Sample size = 50

Prob. Range	Crit.	CM (3844)	LM (3844)	LM (5000)
0.00 - 1.00	BIAS	0.741697E-02	-0.963043E-03	-0.306598E-02
	MAE	0.112610E+00	0.903081E-01	0.912292E-01
	RMSE	0.186852E+00	0.124389E+00	0.126136E+00
0.00 - 0.30	BIAS	0.389797E-01	-0.338033E-02	-0.454672E-02
	MAE	0.192989E+00	0.133130E+00	0.136978E+00
	RMSE	0.542551E+00	0.321336E+00	0.325007E+00
0.30 - 0.80	BIAS	-0.744416E-02	0.431305E-02	-0.507095E-02
	MAE	0.746161E-01	0.673211E-01	0.517564E-01
	RMSE	0.114855E+00	0.101164E+00	0.105952E+00
0.80 - 0.95	BIAS	0.133102E-01	-0.506828E-02	0.534970E-02
	MAE	0.754986E-01	0.733449E-01	0.793189E-01
	RMSE	0.204857E+00	0.197805E+00	0.215095E+00
0.95 - 1.00	BIAS	-0.510282E-01	-0.269046E-01	0.621397E-03
	MAE	0.121597E+00	0.114141E+00	0.130539E+00
	RMSE	0.589221E+00	0.541963E+00	0.619266E+00

(c) Sample size = 100

Prob. Range	Crit.	CM (4765)	LM (4765)	LM (5000)
0.00 - 1.00	BIAS	0.259131E-01	0.103641E-01	0.101663E-01
	MAE	0.105984E+00	0.722970E-01	0.725169E-01
	RMSE	0.181665E+00	0.956600E-01	0.958972E-01
0.00 - 0.30	BIAS	0.904993E-01	0.162913E-01	0.174458E-01
	MAE	0.196241E+00	0.107911E+00	0.109772E+00
	RMSE	0.542676E+00	0.247065E+00	0.252993E+00
0.30 - 0.80	BIAS	-0.227655E-02	0.107774E-01	0.885347E-02
	MAE	0.645289E-01	0.549737E-01	0.523899E-01
	RMSE	0.995449E-01	0.827497E-01	0.826720E-01
0.80 - 0.95	BIAS	0.101449E-01	0.474360E-02	0.584780E-02
	MAE	0.674104E-01	0.550397E-01	0.559846E-01
	RMSE	0.181459E+00	0.148311E+00	0.150951E+00
0.95 - 1.00	BIAS	-0.324062E-01	-0.124701E-01	-0.742638E-02
	MAE	0.946985E-01	0.836118E-01	0.867181E-01
	RMSE	0.462305E+00	0.396674E+00	0.412027E+00

Table 4.13 Relative Errors of Different Methods using GEV Distribution with $(\tau_2, \tau_3) = (0.3, 0.4)$

(a) Sample size = 20

Prob. Range	Crit.	CM (833)	LM (833)	LM (5000)
0.00 - 1.00	BIAS	0.356347E-02	0.673392E-02	0.260645E-01
	MAE	0.144463E+00	0.146714E+00	0.129424E+00
	RMSE	0.182748E+00	0.180567E+00	0.167693E+00
0.00 - 0.30	BIAS	0.672773E-02	0.229612E-01	0.634563E-01
	MAE	0.896488E-01	0.916827E-01	0.142121E+00
	RMSE	0.213357E+00	0.197253E+00	0.304789E+00
0.30 - 0.80	BIAS	0.533991E-01	0.533795E-01	0.144374E-01
	MAE	0.136331E+00	0.141320E+00	0.235438E-01
	RMSE	0.204253E+00	0.216645E+00	0.183352E+00
0.80 - 0.95	BIAS	-0.539670E-01	-0.785559E-01	0.222892E-01
	MAE	0.208861E+00	0.214629E+00	0.181762E+00
	RMSE	0.563488E+00	0.570331E+00	0.497757E+00
0.95 - 1.00	BIAS	-0.341188E+00	-0.301214E+00	-0.706838E-01
	MAE	0.361485E+00	0.327107E+00	0.307525E+00
	RMSE	0.173385E+01	0.154881E+01	0.146715E+01

(b) Sample size = 50

Prob. Range	Crit.	CM (840)	LM (840)	LM (5000)
0.00 - 1.00	BIAS	-0.167724E-01	-0.165037E-01	0.825707E-02
	MAE	0.899786E-01	0.683063E-01	0.676984E-01
	RMSE	0.144097E+00	0.968468E-01	0.961605E-01
0.00 - 0.30	BIAS	0.314588E-01	-0.450796E-03	0.405404E-01
	MAE	0.118413E+00	0.534543E-01	0.104905E+00
	RMSE	0.332994E+00	0.112499E+00	0.222209E+00
0.30 - 0.80	BIAS	-0.120399E-01	0.119178E-01	-0.218653E-01
	MAE	0.542969E-01	0.490808E-01	0.824558E-02
	RMSE	0.854584E-01	0.760452E-01	0.112895E+00
0.80 - 0.95	BIAS	-0.467710E-01	-0.644539E-01	0.421232E-01
	MAE	0.914804E-01	0.980332E-01	0.123275E+00
	RMSE	0.264875E+00	0.277227E+00	0.343964E+00
0.95 - 1.00	BIAS	-0.263489E+00	-0.253186E+00	0.141821E-01
	MAE	0.271683E+00	0.260494E+00	0.238831E+00
	RMSE	0.133144E+01	0.126024E+01	0.114126E+01

(c) Sample size = 100

Prob. Range	Crit.	CM (1021)	LM (1021)	LM (5000)
0.00 - 1.00	BIAS	0.380104E-01	0.646443E-02	0.174011E-01
	MAE	0.166702E+00	0.647009E-01	0.570801E-01
	RMSE	0.288847E+00	0.836728E-01	0.771161E-01
0.00 - 0.30	BIAS	0.258191E+00	0.161995E-01	0.521542E-01
	MAE	0.316584E+00	0.591560E-01	0.913727E-01
	RMSE	0.862643E+00	0.121938E+00	0.190567E+00
0.30 - 0.80	BIAS	-0.496745E-01	0.230094E-01	-0.989852E-02
	MAE	0.948131E-01	0.543744E-01	0.111033E-01
	RMSE	0.146428E+00	0.815693E-01	0.858289E-01
0.80 - 0.95	BIAS	-0.373546E-01	-0.108593E-01	0.423907E-01
	MAE	0.983033E-01	0.717080E-01	0.924112E-01
	RMSE	0.271171E+00	0.200650E+00	0.257633E+00
0.95 - 1.00	BIAS	-0.180129E+00	-0.165425E+00	0.690863E-02
	MAE	0.191482E+00	0.180213E+00	0.180210E+00
	RMSE	0.979029E+00	0.918551E+00	0.890712E+00

indicated by CM, and the PTLM, indicated by LM, based on the generated cases which are both feasible to the two methods. The number of feasible cases in 5000 simulation runs is presented in parenthesis. The last column is the averaged relative errors for the PTLM for all 5000 feasible simulated cases. When all the cases from a distribution are feasible using the PTCM, the second and the last columns are identical. Except for the relative biasness, there is no significant difference between the last two columns, which suggests that the performance of the PTLM is quite stable. As can be seen in these tables, the relative errors increase when the GEV distribution becomes more skewed. The relative mean-absolute and root-mean-squared errors decrease when the sample size increase for a given GEV distribution in most cases.

To facilitate the comparison of relative performance of the two methods under the different conditions, Table 4.14 is prepared in which the better method with respect to the various error criteria are identified. In the table, "C" denotes that the PTCM is the better method whereas "L" for the PTLM. When the relative error of one method is significantly less than that of the other, the letter "C" or "L" is followed by an *. From this table, several observations can be made:

(1) The performance of the PTCM and PTLM is practically identical when the sample size is not large.

(2) When the sample size is 20 or 50 and $\tau_3 = 0.2$ or 0.4, the PTLM outperforms the PTCM in the region of p = 0.95 - 1 since every relative error for all cases is less than that of the PTCM. This information is essential in hydrologic frequency analysis because many hydrologic records are short and the focus of the analysis is often

Table 4.14 Identification of Better Method (C - PTCM; L - PTLM)

(a) Sample size = 20

Prob. Region	τ_2	τ_3								
		0.0			0.2			0.4		
		e_b	e_{mae}	e_{rmse}	e_b	e_{mae}	e_{rmse}	e_b	e_{mae}	e_{rmse}
0.0 - 1.0	0.1	C	C	C	L	L	L	L	C	L
	0.2	C	C	C	C	C	C	L*	C	L
	0.3	C	C	L	L	C	C	C*	C	L
0.0 - 0.3	0.1	C	C	C	L*	L*	L*	C*	C	L
	0.2	C	C	C	C	C	C	C*	C	L
	0.3	C*	C	L	C	C	C	C*	C	C
0.3 - 0.8	0.1	L	C	C	C	C	C	L	C	C
	0.2	L	C	C	L	L	C	L	C	C
	0.3	L	C	C	L	C	C	L	C	C
0.8 - 0.95	0.1	C	C	C	C	C	C	C	C	C
	0.2	C	C	C	L*	L	L	C	C	C
	0.3	C	C	C	L*	L	L	C	C	C
0.95 - 1	0.1	L	L	C	L	L	L	L	L	L
	0.2	L	C	C	L*	L	L	L	L	L
	0.3	L*	L	C	L*	L	L	L	L	L

(b) Sample size = 50

Prob. Region	τ_2	τ_3								
		0.0			0.2			0.4		
		e_b	e_{mae}	e_{rmse}	e_b	e_{mae}	e_{rmse}	e_b	e_{mae}	e_{rmse}
0.0 - 1.0	0.1	L	C	C	C	L	L	L	L	L
	0.2	L	C	C	L	L	L	L	L	L
	0.3	L*	C	C	L*	L	L*	L	L	L
0.0 - 0.3	0.1	L	L	C	C*	L	L	L*	L*	L*
	0.2	L	L	C	L	L*	L*	L*	L*	L*
	0.3	L*	C	C	L*	L	L*	L*	L*	L*
0.3 - 0.8	0.1	C	C	C	C*	L	L	C	L	L
	0.2	C*	C	C	L	L	L	C	L	L
	0.3	C*	C	C	L*	L	L	L*	L*	L*
0.8 - 0.95	0.1	L	C	C	L*	L	L	C	C	C
	0.2	L*	C	C	L*	L	L	C	C	C
	0.3	L*	C	C	L*	L	L	L*	L	L
0.95 - 1	0.1	L*	C	C	L*	L	L	L	L	L
	0.2	L*	C	C	L*	L	L	L	L	L
	0.3	L*	C	C	L*	L	L	L	L	L

Table 4.14 (concluded)

(c) Sample size = 100

Prob. Region	τ_2	τ_3								
		0.0			0.2			0.4		
		e_b	e_{mae}	e_{rmse}	e_b	e_{mae}	e_{rmse}	e_b	e_{mae}	e_{rmse}
0.0 - 1.0	0.1	L	C	C	C	L	L	C	L*	L*
	0.2	C	C	C	L	L	L	L*	L*	L*
	0.3	C*	C	C	L*	L	L*	L*	L*	L*
0.0 - 0.3	0.1	L	L	C	L*	L*	L*	L*	L*	L*
	0.2	C	C	C	L*	L*	L	L*	L*	L*
	0.3	L*	C	C	L*	L*	L*	L*	L*	L*
0.3 - 0.8	0.1	C	C	C	L*	L	L	L*	L*	L*
	0.2	L	C	C	C*	L	L	L*	L*	L*
	0.3	C*	C	C	C*	L	L	L*	L*	L*
0.8 - 0.95	0.1	C	C	C	L*	L	L	L*	L*	L*
	0.2	L*	C	C	L*	L	L	L*	L	L
	0.3	L*	C	C	L*	L	L	L*	L	L
0.95 - 1	0.1	L	C	C	L*	L	L	L	L	L
	0.2	L*	C	C	L*	L	L	L	L	L
	0.3	L*	C	C	L*	L	L	L	L	L

placed on the tail part of the distribution.

(3) When the sample size is 100, for $\tau_3 = 0.0$, the GEV distribution is almost symmetric, the relative mean-absolute error and the relative root-mean squared error of the PTCM is always less than those of the PTLM while the relative biasness of the PTLM tends to be less when the region moves to the upper tail of the distribution. For the cases with high skewness, $\tau_3 = 0.4$, almost every relative error of the PTLM is less than that of the PTCM.

(4) When the GEV distribution becomes more dispersed and skewed, i.e., with large (τ_2, τ_3), the PTLM tends to outperform the PTCM.

(5) The relative biasness of the PTLM is often less than or significantly less than that of the PTCM in the upper tail region, with $p = 0.8$ - 1.0.

The PTLM is more capable of describing skewed GEV distributions than the PTCM. Practically, the PTCM may fail due to (1) the violation of the feasibility requirement by the sample skewness and kurtosis; and (2) the divergence of the Newton-Raphson method. This is especially likely when the sample size is not large. However, the coefficients in polynomial transformation are always solvable using the PTLM regardless of the sample size. Furthermore, the PTLM is computationally much easier than the PTCM because it is required to solve a set of linear equations rather than to solve a system of nonlinear equations as required by the PTCM. Hence, the use of L-moments is recommended for the polynomial transformation to normality.

4.6 EXTENSION TO BIVARIATE POLYNOMIAL TRANSFORMATION

As shown in the previous section, the PTCM and PTLM are effective alternatives for normal transformation. It is always desirable to use these procedures to generate multivariate non-normal variates. A general method of extending the polynomial transformation to a bivariate case is discussed in this section.

Vale and Maurelli (1983) extended the polynomial transformation to a bivariate case. Let Y_1 and Y_2 be the two correlated non-normal variables, expressible in polynomial transformation as

$$\begin{aligned} Y_1 &= a_1 + b_1 Z_1 + c_1 Z_1^{\,2} + d_1 Z_1^{\,3} \\ Y_2 &= a_2 + b_2 Z_2 + c_2 Z_2^{\,2} + d_2 Z_2^{\,3} \end{aligned} \tag{4.27}$$

where Z_1 and Z_2 are the two standard normal random variables. Let $\rho_{Y_1 Y_2}$ denote the correlation coefficient of Y_1 and Y_2, then, the correlation coefficient in the normal space, $\rho_{Z_1 Z_2}$, can be obtained by solving the following third-order polynomial equation

$$\rho_{Y_1 Y_2} = \rho_{Z_1 Z_2}(b_1 b_2 + 3b_1 d_2 + 3d_1 b_1 + 9d_1 d_2) + \rho_{Z_1 Z_2}{}^2(2c_1 c_2) + \rho_{Z_1 Z_2}{}^3(6d_1 d_2) \tag{4.28}$$

Based on the bivariate polynomial transformation, an alternative procedure to generate unconstrained multivariate non-normal random variates can be easily developed. This procedure involves the following steps:

Step [1] - Fit the polynomial transformation to each variable and calculate the correlation coefficients for the random variables in the original space.

Step [2] - Calculate correlation coefficients in the normal space using Eq. (4.28).

Step [3] - Use an orthogonal transformation to generate multivariate standard normal random variates $z = (z_1, \cdots, z_n)$ with the prescribed $\rho_{Z_i Z_j}$.

Step [4] - Transform the generated multivariate standard normal variates to the original parameter space using the corresponding polynomial transformation.

4.7 SUMMARY AND CONCLUSIONS

Two simple alternatives for normal transformation, that is, linear transformation and polynomial transformation, were discussed. To avoid using the skewness and kurtosis in parameter estimation, especially when the sample size is small, partial moments and L-moments were considered. Using different types of moments, four procedures, LTPM, PTCM, PTPM, PTLM, were proposed. By comparing the approximated quantiles using different procedures, the LTPM and PTPM were rejected due to their poor accuracy whereas both the PTCM and PTLM are effective alternatives for normal transformation. Simulation were carried out to further investigate the behavior of the PTCM and PTLM. The accuracy of these two methods are almost the same when the sample size is large. The PTLM outperforms the PTCM in the region $p = 0.95$ - 1 when the sample size is not large, say 20 or 50. It is more appropriate to describe a more skewed GEV distribution. The PTCM may fail due to the fact that (1) the sample skewness and kurtosis may not lie in the feasible region for distributions; and (2) the Newton-Raphson method fails to converge. This is especially likely when the sample size is small. Computationally, the PTLM is much easier and the coefficients of polynomial transformation can always be solved. Therefore, the PTLM is the recommended alternative for normal transformation.

CHAPTER 5

SUMMARY AND CONCLUSIONS

5.1 SUMMARY

This study contains two major parts. The first part is concerned with the identification of representative storm patterns in Wyoming. The second part is the development of procedures to stochastically generate temporal distributions for a design storm.

In the first part, average storm patterns for complete storms were of interest. 'Significant storms' for each station with depth exceeding that of a 10-year return period at that particular station were extracted. Both duration-based and event-based storm data sets were established. Through the non-dimensionalization process, dimensionless rainfall mass curves and statistical moments of time were used as the attributes in statistical cluster analysis. Contingency tests were performed to find out whether the occurrence of various storm patterns were affected by climatic region, storm duration, and seasonality. The representative storm patterns were fitted to the Beta distribution and Johnson S_B distribution by various methods.

In the second part, the storm pattern in its dimensionless form were treated as compositional data. Generating compositional data requires a procedure of generating unconstrained multivariate non-normal random variables which, in turn, can be efficiently done in the multivariate standard normal space after a normal transformation is made. The Johnson distribution system was adopted as the means for normal transformation due to its flexibility of covering a wide variety of distribution types. Different methods of selecting the Johnson distribution type and estimating its parameters were reviewed. The Nataf model was then studied based on which the Johnson system was extended to a multivariate setting. Using the Johnson distribution system coupled with the use of the bivariate Nataf model, the procedure of generating correlated non-normal random variables without constraint was developed. Three methods, namely, the AR method, the CDF method, and the log-ratio method were then developed for generating temporal distributions for storm patterns.

Without making a parametric assumption about the distribution form, four procedures with respect to the use of different moments, linear transformation with partial moments (LTPM), polynomial transformation with complete moments (PTCM), polynomial transformation with partial moments (PTPM), and polynomial transformation with L-moments (PTLM), were proposed as the alternative for normal transformation. Their performance was examined by conducting Monte Carlo simulations under various conditions. Simulations were also carried out to investigate closely the behavior of the PTCM and PTLM.

5.2 CONCLUSIONS

In the storm pattern study, eight duration-based storm patterns and five event-based storm patterns were identified in Wyoming. The event-based storm patterns were found to be independent of climatic region but dependent on storm durations and seasons. On the other hand, the occurrence of duration-based storm patterns were dependent on climatic regions, storm durations, and seasons. In the parametric model fitting for storm patterns, the Beta distribution is not appropriate for describing the storm patterns while the 3-parameter reduced Johnson S_B coupled with the least squares method using quantiles provides a good fit.

If only a small portion of the observations in a given storm pattern contains zero precipitation percentages, the log-ratio method is recommended. If a considerable number of observations in a given storm pattern have zero precipitation percentages, the AR method should be tried first. If the efficiency of the AR method is low, one can consider using the CDF method to generate the storm pattern.

By comparing the performance of several alternative normal transformation procedures, the LTPM and PTPM were rejected due to their poor accuracy. Both the PTCM and PTLM can serve as viable alternatives for normal transformation. The PTLM outperforms the PTCM in the probability region $p = 0.95 - 0.1$ when the sample size is not large. Furthermore, the PTLM is more appropriate to describe more skewed distributions. The PTCM may fail in two possible causes: (1) the sample skewness and kurtosis may define an infeasible distribution; and (2) the Newton-Raphson method may not converge. This is especially likely when the sample size is not large. The PTLM is computationally easier and the coefficients of polynomial transformation can always be solved. Hence, the PTLM is the recommended alternative for normal transformation under all conditions.

5.3 RECOMMENDATIONS FOR FUTURE RESEARCH

Due to the small number of expected observations in some cells, the chi-square test for contingency table is not appropriate for testing the dependency of storm patterns and the climatic region, storm duration, and season. The network algorithm developed by Mehta and Patel (1983) should be implemented to perform the exact Fisher's test.

One disadvantage of the AR method for generating storm patterns is that its efficiency is generally unknown in advance. As stated in Chapter 3, this efficiency can be expressed as the product of two probabilities as shown in Eq. (3.14). It would be useful to know or to estimate the efficiency in advance because one may want to discard the AR method even before using it if the efficiency is low. Further effort should be made to estimate this efficiency.

The log-ratio transformation requires non-zero components in the sample which is often violated in reality as shown in storm patterns analysis in Chapter 3. Computationally, we are only interested in the first four central moments of the log-ratios. One possible way to circumvent the problem associated with the zero-valued components is to use some type of approximation procedure to indirectly calculate the moments of log-ratios from the sample moments of the components rather than calculate directly from sample log-ratios. However, finding such a procedure is not a trivial task since the skewness and the kurtosis are also needed to be approximated. Another related issue is how good are the indirect procedures.

Chapter 4 suggests that the polynomial transformation with L-moments can be used as the alternative for normal transformation. As pointed out in Chapter 3, the STARSHIP is the recommended procedure for the Johnson system to deal with small sample size. Simulation should be performed to examine the performance of the PTLM and STARSHIP when the sample size is small. In addition, the PTLM can also be incorporate in the reliability analysis to deal with problems involving multivariate non-normal random variables.

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APPENDIX A.

The formulas of the bivariate Johnson system are developed using the standardized Johnson variates with $\lambda = 1$ and $\xi = 0$. It is easy to prove that $\rho_{y_i y_j} = \rho_{x_i x_j}$, $\mu_y = (\mu_x - \xi)/\lambda$ and $\sigma_y = \sigma_x/\lambda$ with μ and σ being the mean, standard deviation of the standardized Johnson variates, respectively.

The formulas for $S_{LN,N}$ and $S_{LN,LN}$ can be found in Liu and Kiureghian (1986). Johnson (1982,1987) gave the formula for $S_{U,U}$. Let **Y** denote a random vector $\boldsymbol{Y} = [Y_1,\cdots,Y_n] = [\sinh(Z_1'),\cdots,\sinh(Z_n')]$ where

$$Y_i = \sinh(Z_i') = \sinh\left(\frac{Z_i - \gamma_i}{\delta_i}\right), \qquad \textit{for } 1 \le i \le n$$

It is easy to show that $\boldsymbol{Z}'$ is multivariate normal with $E(Z_i') = -\gamma_i/\delta_i$ and $Var(Z_i') = -1/\delta_i^2$. Applying the formula Johnson provided, the formula for $S_{U,U}$ is obtained as shown in Table 3.1.

It is impossible to obtain analytical expressions for $S_{B,L,N}$, $S_{B,U}$ and $S_{B,B}$. The derivations of formulas for $S_{U,N}$, $S_{U,LN}$ and $S_{B,N}$ are presented below.

(1) For $S_{U,N}$

$$\rho_X \sigma_i \sigma_j + \mu_i \mu_j = E(Y_i Y_j) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sinh\left(\frac{z_i - \gamma_i}{\delta_i}\right) z_j \phi(z_i, z_j; \rho_Z)\, dz_i dz_j$$

Note that $\mu_j = 0$ and $\sigma_j = 1$. Consider the transformation

$$\begin{aligned} u &= (1-\rho^2)^{-\frac{1}{2}}(z_i - \rho z_j) \\ v &= z_j \end{aligned}$$

and omit the subscript for ρ_Z, the above integral equation can be written as

$$\begin{aligned} \rho_X\sigma_i &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sinh\left(\frac{\sqrt{1-\rho^2}u+\rho v-\gamma_i}{\delta_i}\right) v\cdot\frac{1}{2\pi}e^{-\frac{u^2}{2}}e^{-\frac{v^2}{2}}dudv \\ &= \int_{-\infty}^{\infty} E\left[\sinh\left(\frac{\sqrt{1-\rho^2}u+\rho v-\gamma_i}{\delta_i}\right)\right]\cdot v\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{v^2}{2}}dv \\ &= \int_{-\infty}^{\infty} e^{\frac{1-\rho^2}{2\delta_i^2}} \sinh\left(\frac{\rho v-\gamma_i}{\delta_i^2}\right)\cdot v\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{v^2}{2}}dv \\ &= \frac{1}{2}\cdot e^{\frac{1-\rho^2}{2\delta_i^2}}\cdot\left(\int_{-\infty}^{\infty} e^{\frac{\rho v-\gamma_i}{\delta_i}}\cdot v\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{v^2}{2}}dv - \int_{-\infty}^{\infty} e^{-\frac{\rho v-\gamma_i}{\delta_i}}\cdot v\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{v^2}{2}}dv\right) \\ &= \frac{1}{2}\cdot e^{\frac{1-\rho^2}{2\delta_i^2}}\cdot\frac{\rho}{\delta_i}\cdot\left(e^{\frac{\rho^2-2\gamma_i\delta_i}{2\delta_i^2}}+e^{\frac{\rho^2+2\gamma_i\delta_i}{2\delta_i^2}}\right) \\ &= e^{\frac{1}{2\delta_i^2}}\cdot\frac{\rho}{\delta_i}\cdot\cosh\left(\frac{\gamma_i}{\delta_i}\right) \end{aligned}$$

The third step follows the fact that $E[\sinh(U)] = \exp(\sigma^2/2)\cdot\sinh(\mu)$ where $U \sim N(\mu,\sigma^2)$.

(2) For $S_{U,LN}$

Use the same transformation, the integral equation can be evaluated as

$$
\begin{aligned}
\rho_X\sigma_i\sigma_j+\mu_i\mu_j &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sinh\left(\frac{z_i-\gamma_i}{\delta_i}\right)\cdot e^{\frac{z_j-\gamma_j}{\delta_j}}\cdot\phi(z_i,z_j;\rho_Z)\,dz_i dz_j \\
&= e^{\frac{1-\rho^2}{2\delta_i^2}}\cdot\int_{-\infty}^{\infty}\sinh\left(\frac{\rho\nu-\gamma_i}{\delta_i}\right)\cdot e^{\frac{\nu-\gamma_j}{\delta_j}}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{\nu^2}{2}}\,d\nu \\
&= e^{\frac{1-\rho^2}{2\delta_i^2}}\cdot\frac{1}{2}\cdot\left(e^{\frac{1}{2}(\frac{\rho}{\delta_i}+\frac{1}{\delta_j})^2-\frac{\gamma_i}{\delta_i}-\frac{\gamma_j}{\delta_j}}-e^{\frac{1}{2}(\frac{\rho}{\delta_i}-\frac{1}{\delta_j})^2+\frac{\gamma_i}{\delta_i}-\frac{\gamma_j}{\delta_j}}\right) \\
&= e^{\frac{1}{2\delta_i^2}+\frac{1}{2\delta_j^2}-\frac{\gamma_j}{\delta_j}}\cdot\sinh\left(\frac{\rho}{\delta_i\delta_j}-\frac{\gamma_i}{\delta_i}\right)
\end{aligned}
$$

By letting $\omega = \dfrac{\rho_X\sigma_i\sigma_j+\mu_i\mu_j}{e^{1/2\delta_i^2+1/2\delta_j^2-\gamma_j/\delta_j}}$, then $\sinh\left(\dfrac{\rho}{\delta_i\delta_j}-\dfrac{\gamma_i}{\delta_i}\right)=\omega$, the above equation is solved.

(3) For $S_{B,N}$

$$
\rho_X\sigma_i\sigma_j+\mu_i\mu_j = E(Y_iY_j) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{1+e^{-(z_i-\gamma_i)/\delta_i}}z_j\phi(z_i,z_j;\rho_Z)\,dz_i dz_j
$$

Note that $\mu_j=0$ and $\sigma_j=1$. Consider the transformation

$$
\begin{aligned}
u &= (1-\rho^2)^{-\frac{1}{2}}(z_j-\rho z_i) \\
v &= z_i
\end{aligned}
$$

and omit the subscript for ρ_Z, the above integral equation can be written as

$$
\begin{aligned}
\rho_X \sigma_i &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{1+e^{-(v-\gamma_i)/\delta_i}} (\sqrt{1-\rho^2}u + \rho v)\phi(u)\phi(v)\,du\,dv \\
&= \rho \cdot \int_{-\infty}^{\infty} \frac{1}{1+e^{-(v-\gamma_i)/\delta_i}} \cdot v \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{v^2}{2}} dv \\
&= -\rho \cdot \int_{-\infty}^{\infty} \frac{1}{1+e^{-(v-\gamma_i)/\delta_i}} \cdot \frac{1}{\sqrt{2\pi}} \cdot de^{-\frac{v^2}{2}} \\
&= 0 + \rho \cdot \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{v^2}{2}} d\left(\frac{1}{1+e^{-(v-\gamma_i)/\delta_i}} \right) \\
&= \frac{\rho}{\delta_i} \cdot e^{\frac{1}{2\delta_i^2}+\frac{\gamma_i}{\delta_i}} \cdot \int_{-\infty}^{\infty} \left(\frac{1}{1+e^{-(v-\gamma_i)/\delta_i}} \right)^2 \frac{1}{\sqrt{2\pi}} e^{-\frac{(v+1/\delta_i)^2}{2}} dv \\
&= \frac{\rho}{\delta_i} \cdot e^{\frac{1}{2\delta_i^2}+\frac{\gamma_i}{\delta_i}} \cdot \mu_2'(\gamma_i + \frac{1}{\delta_i}, \delta_i) \\
&= \frac{\rho}{\delta_i} \cdot [\mu_1'(\gamma_i, \delta_i) - \mu_2'(\gamma_i, \delta_i)]
\end{aligned}
$$

The last step is obtained by using the following recursive equation developed by Johnson (1949a),

$$
\mu'_r(\gamma + \delta^{-1}, \delta) = e^{-(0.5\delta^{-2}+\gamma/\delta)}[\mu'_{r-1}(\gamma, \delta) - \mu'_r(\gamma, \delta)]
$$