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Improved Dynamic Reliability Model for Hydraulic Design

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ABSTRACT

An improved dynamic reliability model is developed for hydraulic design. In the model the mean rate of occurrence of flood events is compatible with the lower bound of integration of the loading function. In addition, the model is consistent with the assumption that flood events above the threshold defined by the design flood follows a Poisson process. If actual design floods are the same, regardless of changes between the safety factor and the design flood, the same reliability results can be obtained from this model. Compared with other reliability models, the results obtained by the current model can be explained on a more rigorous basis physically and theoretically. The improved model considers hydrologic and hydraulic uncertainties and can be used both to design a new hydraulic structure and to evaluate the reliability level of existing structures.

INTRODUCTION

Hydrological data are usually used to analyze the occurrence of certain magnitude of flood for designing a hydraulic structure, such as a levee. A reliability model can integrate the hydrological information with the feature of the hydraulic structure to be designed, and provide relationships between the size and the reliable level of the structure during its intended service period. Therefore, the reliability theory offers a consistent and conceptually complete framework for designing hydraulic structures. In hydraulic design, the project reliability is defined as the probability that it does not fail to perform its intended purpose during the service period. There are several methods for the reliability analysis of a structure, such as direct integration, Monte Carlo simulation, reliability index, and first-order second-moment analysis. Various failure mechanisms of levee were presented by *Vrijling* [1987]. Here only the overtopping is considered in the reliability model among those failure modes.

In the traditional design of a hydraulic structure, only the hydrologic uncertainties are taken into consideration explicitly. Other uncertainties are included by introducing a safety factor to make sure that the structure can withstand the random external loading. A safety factor

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is defined as

$$S = Y_m / X_T \tag{1}$$

where Y_m is the mean values of the resistance Y and X_T is the design flood with return period T of the loading X. Safety factor has been widely used in various engineering fields as a protection against the engineer's ignorance, uncertainties, and randomness of natural phenomena. Therefore, it is important to include safety factor in the reliability model properly.

Tang [1980] presented a procedure for incorporating the uncertainties into a risk analysis, and a direct integration method was employed to calculate the hydrologic risk. *Tung and Mays* [1980a] evaluated risks for a levee by estimating the uncertainties of some parameters through a first-order approximate formula and assumed distribution functions of both loading and resistance. The major contribution of *Tung and Mays* [1980a] is to introduce this kind of model, which includes both hydrologic and hydraulic uncertainties in the design of hydraulic structures. However, it is questionable whether the model satisfies the independent assumption of external loading. Meanwhile, the influence of safety factor on the mean rate of occurrence of flood events was not properly considered, which would result in incorrect reliability results. Noticing the limitations of the reliability model by *Tung and Mays* [1980a], *Lee and Mays* [1983] proposed a revised reliability model based on conditional probability distributions for the loading and resistance. The model satisfies the independent assumption of external loading. Nevertheless, the conditional loading distribution was used improperly and the influence of safety factor on the mean rate of occurrence of flood events was not taken into consideration either, which also may result in unreasonable reliability results. The objective of this study is to develop a dynamic reliability model based on critically reviewing existing reliability models. The improved model is compared with others.

DYNAMIC RELIABILITY MODELS

The term "dynamic" is referred to repeated occurrences of inherent hydrologic events during the service period of a hydraulic structure. The reliability of a hydraulic structure decreases with time due to the increasing number of external loadings that have been encountered, which increases the possibility of at least one loading exceeding the resistance. The reliability in time interval [0, t], $R(t)$, can be mathematically expressed as

$$R(t) = \sum_{i=0}^{\infty} \pi_i(t) R_i \tag{2}$$

where t is the service period of a structure; $\pi_i(t)$ is the probability of i cycles occurring in the time interval [0, t]; and R_i is the probability of all

i successes (resistances > loadings). It is assumed that the number of cycles occurring in a given time interval follows a stationary Poisson process, i.e.

$$\pi_i(t) = P[N_t = i] = \frac{e^{-\alpha t}(\alpha t)^i}{i!} \tag{3}$$

where $\alpha = 1/T$ is the mean rate of occurrences of those events $\geq x_T$ per unit time; x_T is the design flood with a return period T years.

Based on (2), two different dynamic reliability models were developed by *Tung and Mays* [1980a, 1981] and *Lee and Mays* [1983], which are reviewed as follows.

***Tung and Mays'* dynamic reliability model**

For the random-independent loading and random-fixed resistance, the time-dependent reliability was expressed as [*Tung and Mays*, 1980a, 1981]

$$\begin{aligned} R(t) &= \sum_{i=0}^{\infty} \frac{e^{-\alpha t}(\alpha t)^i}{i!} \int_0^{\infty} f_Y(y) \left[\int_0^y f_X(x) dx \right]^i dy \qquad y>0 \\ &= \int_0^{\infty} f_Y(y) e^{-\alpha t[1-F_X(y)]} dy \end{aligned} \tag{4}$$

where $f_X(x)$ is the PDF of random independent loading; $f_Y(y)$ is the PDF of random-fixed resistance; and $F_X(x)$ is the cumulative distribution function of loading. *Tung and Mays* [1980a] used this model to evaluate the reliability (or risk) of a structure, given knowledge of loading and resistance distributions, and the mean rate of occurrence of the loading. In this model its resistance density function was in the form of

$$f_Y(y) = \frac{1}{D_1 \sqrt{2\pi}\, y\, \sigma_{lnY}} \exp[-\frac{1}{2\sigma_{lnY}^2}(lny - \mu_{lnY})^2], \quad y \geq y_0 \tag{5}$$

$$D_1 = \int_{y_0}^{\infty} \frac{1}{y\, \sigma_{lnY} \sqrt{2\pi}} \exp[-\frac{1}{2\sigma_{lnY}^2}(lny - \mu_{lnY})^2] dy \tag{6}$$

where μ_{lnY} and σ_{lnY} are, respectively, the mean and standard deviation of the transformed levee capacity; and D_1 is a normalizing factor [*Kapur and Lamberson*, 1977]. In the model (4), y_0 was set to 0, therefore, $D_1 = 1$.

It is reasonable that an untruncated loading distribution was used in the model. However, the integration of loading distribution from 0 to y (variate of resistance) is questionable if the dynamic model satisfies the independent assumption of external loadings, an assumption used in the procedure of developing the dynamic model. Another problem is that α is assumed to be equal to $1/T$, this is correct only if S is equal to 1. Basically, the influence of the safety factor on α was not considered in the model. The resistance PDF was assumed to be an untruncated lognormal distribution, which may not reflect the actual situation. When floods (loadings) are smaller than some threshold value there will not exist any risk for the levee to conduct the flood safely. This means that the PDF values should be equal to zero for such floods. Therefore, it would be reasonable that the resistance PDF was truncated at some threshold value.

Lee and Mays' dynamic reliability model

Modifying *Tung and Mays* model [1980a], *Lee and Mays* [1983] proposed a revised reliability model. *Lee and Mays*' model satisfies the Poisson process assumption and is theoretically sound. By introducing both a truncated PDF as resistance distribution and x_T as lower bound of integration, the computed reliability results is smaller than that of *Tung and Mays*' model. The mean occurrence of flood events α and the lower bound of integration are consistent with each other when safety factor is equal to 1. Nevertheless, due to using a conditional loading distribution in the model, when the lower bound of integrations (x_T) changes, the loading distribution varies accordingly. This results in following problem: the frequency value for a certain flood magnitude will change when x_T varies. This notation is not consistent with the intention of using some flood models (unconditional or conditional) to represent approximately the relationship between flood magnitude and its frequency (or return period) on the basis of recorded hydrologic data. The relationship between flood magnitude and its frequency should not be changed at any time during the process of reliability calculations after it has been set up. The second problem is that α is always assumed to be equal to $1/T$, which is incorrect if S is not equal to 1.

Improved reliability model

We defined an actual design flood as

$$x_a = S x_T \tag{7}$$

where S is the safety factor and x_T is the design flood with return period T. Notice that T_a (the return period for the actual design flood x_a) is **not** equal to T when S is greater than 1 and α should be equal to $1/T_a$. The following guidelines are considered for the development of the reliability model. First, given the same PDF of resistance, the dynamic reliability is a function of x_a and t, i.e., $R = f(x_a, t)$; if t is specified the reliability is a unique function of x_a, i.e., $R = f(x_a)$. Physically, if $x_{a1} = x_{a2}$, the reliability results should satisfy that $R_1 = f(x_{a1}) = R_2 = f(x_{a2})$. Now the reliability analysis considers the influence of both the safety factor and the design flood, instead of the design flood alone. Secondly, the mean rate of flood recurrence α and the lower bound of integrations should be compatible to ensure that the Poisson process assumption is satisfied. Thirdly, the loading distribution should remain unchanged during the process of reliability computation. Based on these considerations, an improved dynamic reliability model is proposed as follows

$$
\begin{aligned}
R(t) &= \sum_{i=0}^{\infty} \frac{e^{-\alpha t}(\alpha t)^i}{i!} \left\{ \int_{x_a}^{\infty} f'_Y(y) \left[\int_{x_a}^{y} f_X(x)dx \right]^i dy \right\} \\
&= \int_{x_a}^{\infty} f'_Y(y) \exp[-\alpha t(1 - \int_{x_a}^{y} f_X(x)dx)]dy
\end{aligned}
\tag{8}
$$

where x_a is the actual design flood with return period T_a, $f_X(x)$ is the loading PDF, $f'_Y(y)$ is the resistance PDF truncated at x_a.

The reason for setting the lower bound of integration for the loading distribution as x_a is that x_a and α must be compatible with each other according to the assumption that the number of occurrence in a given time interval follows the stationary Poisson process (2). The resistance PDF is truncated at $y=x_a$ because a hydraulic structure should surely be able to withstand flood less than or equal to x_a. The relationship between reliability and actual design flood is shown in Fig. 1 for different service periods of a levee. From Fig. 1 we observe that reliability increases as the actual design flood increases for a specified t, and that reliability deceases as service period increases for a specified actual design flood.

It should be pointed out that the three models assume that the number of occurrence in a given time interval should follow a stationary Poisson process, however, only the models of *Lee and Mays* [1983] and of this paper satisfy the assumption. The mathematical differences among the three models are summarized in Table 1. The reliability values calculated by using three different models are plotted in Fig. 2 for t=100 years. It is clear that only the improved model gives a single curve of R versus x_a. *Tung and Mays*' reliability results are always larger than the results of the improved model, while *Lee and Mays*' results are larger (when S is small) or smaller (when S is larger) than the results of the improved model. The results show that these models do not follow the one to one rule, which is mainly due to incorrect introduction of S in the reliability models.

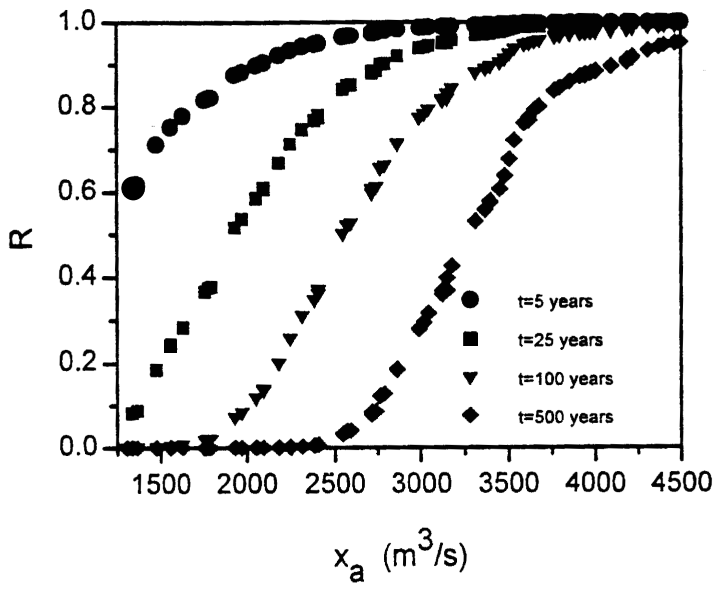

Figure 1. Relationship between actual design flood and reliability of the improved model.

Table 1. The Mathematical Differences among the Reliability Models

Reliability Model	*Tung and Mays*	*Lee and Mays*	*Improved Model*
Lower bound of integrations	0	x_T	x_a
α	$1/x_T$	$1/x_T$	$1/x_a$
Resistance pdf $f_Y(y)$ (5) and (6)	$y_0 = 0$	$y_0 = x_T$	$y_0 = x_a$
Loading pdf $f_X(x)$	untruncated	truncated at x_T	untruncated

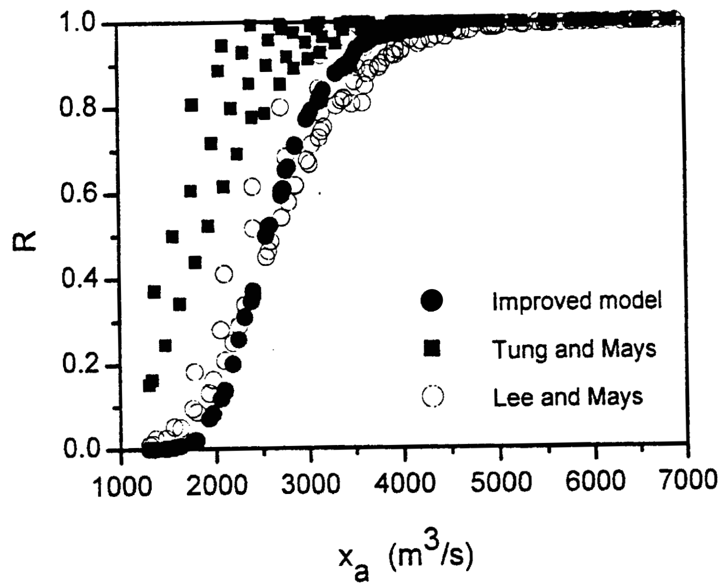

Figure 2. Comparison of reliability results among the three models (t=100 years).

APPLICATIONS OF RELIABILITY MODELS

Annual maximum flood data (from 1903 to 1977) [*Tung and Mays*, 1980b] on the Guadalupe River near Victoria in Texas were used for determining the loading distribution. The extreme value type I (*EV*1) and log-normal distribution were chosen as the loading and resistance PDFs, respectively, for all the three reliability models. The parameters in *EV*1 were estimated using the biased probability weighted methods (*PWM*) [*Cunnane*, 1992]. The detailed information for the resistance PDF was given by *Tung and Mays* [1980b].

For the dynamic reliability model, it is necessary to consider only those floods whose magnitudes (x) exceed the actual design flood x_a=Sx_T during the service period of a structure. Before looking at dynamic reliability results in which both hydrologic and hydraulic uncertainties are been taken into account, let us look at inherent hydrologic uncertainty and its reliability exclusively.

The inherent hydrologic uncertainty is associated only with the inherent randomness of natural processes such as flood events. We may wonder "What is the probability that x_T is exceeded during the expected service period t?" Even though practically a structure is rarely designed for a fixed t, it is instructive to study the relationship between t and T. The relationship may be characterized by a Poisson distribution [*Haan*, 1977; *Cunnane*, 1992]. Suppose x_T is exceeded on average once in T years, the rate of occurrence per unit time is $1/T$ and the mean rate of occurrences in t years is λ = t/T. Hence the probability of no occurrences (i.e., reliability) of x_T in t years of service period of the hydraulic structure is

$$R(t) = P(x < x_T) = e^{-\lambda} = e^{-t/T} \tag{9}$$

By using the multiplicative law of probability, the probability of exceeding or equaling x_T in t years is [*Chow and Takase*, 1977]

$$P(X \geq x_T)_t = 1 - [1 - P(X \geq x_T)]^t = 1 - (1 - 1/T)^t \tag{10}$$

By expanding $(1-t/T)^t$ into a series, the equation is very close to (9) if t/T^2 is much less than 1.

When $T = t$ and $S = 1.0$, the calculated reliability values from the three dynamic models are presented in Table 2. The reliability results obtained by the improved model are near the same as those obtained by (9), while the results obtained by other two models are much larger than the theoretical results. The levee heights are also listed in Table 2. In order to attain the same reliability level the levee height H increases when service period t increases. As pointed out by *Lee and Mays* [1983], the reliability calculated by the dynamic model for T-year flood and t-year service period of the structure should be similar to $e^{(-t/T)}$ as S=1. Fig. 3 presents the reliability results calculated by using *Tung and Mays* [1981], *Lee and Mays* [1983] and the current models, respectively. Only the improved model provided comparable results with the theoretical results, (9), as suggested by *Lee and Mays* [1983]. The improved model fit the curve of (9), which indicates that the improved model is a special case that the hydraulic uncertainties are not included.

Table 2. Reliability Results of Different Dynamic Reliability Models (S=1.0, T=t, and a loading distribution *EV1PWM*, standing for extreme value type I distribution and its parameters being estimated by biased probability weighted methods).

Model	Service Period t (years)			
	50	100	500	1000
Tung and Mays	0.9726	0.9839	0.9962	0.9980
Lee and Mays	0.6254	0.6169	0.6554	0.6743
Improved Model	0.3715	0.3698	0.3684	0.3681
Equation (9)	0.3679	0.3679	0.3679	0.3679
Levee Height , H (m)	0.7315	0.9449	1.4021	1.5679

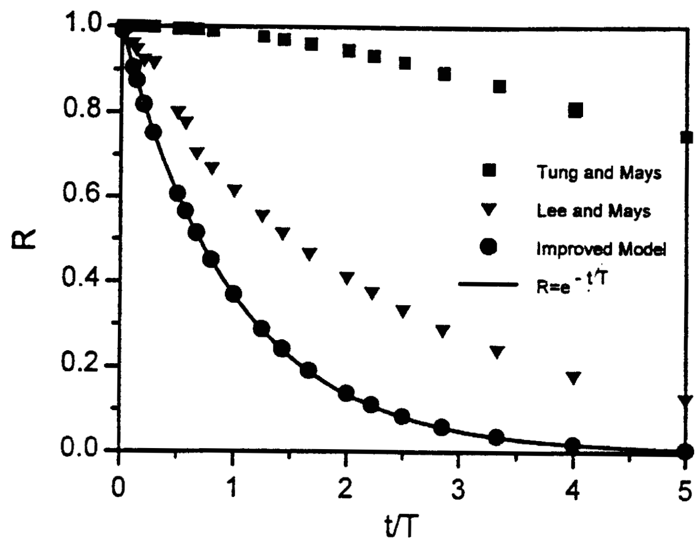

Figure 3. Relationship between t/T and reliability (t=100 years and S=1.0).

The improved model can be used to quantify the relationship between actual design flood and reliability level (see Fig. 1). This relationship is crucial to the design of hydraulic structures, such as the determination of a levee height. In the design of a new hydraulic structure, the reliability value may be obtained from Fig. 1 and levee height can be calculated from Fig. 4 provided that the return period T, service period t and safety factor S are specified. For example, taking T=50 years and t=100 years, the actual design floods (x_a) are 2,092 and 4,184 m^3/s for S=1 and 2, respectively. The corresponding R's are 0.14 and 0.98 from Fig. 1, and the levee heights are, respectively, 0.73 and 1.94 m from Fig. 4. To repeat the process, we may find the magnitude of the actual design flood x_a and levee height H for a specified R.

If the levee heights are the same, i.e., the actual design floods should remain unchanged, therefore the reliability results ought to be the same. From Table 3, it can be noted that the fact is satisfied only by using the improved model, regardless changes between safety factor and design flood. If R=0.9 is expected, by using the improved model and repeating the process, the levee height should be 1.55 m which corresponds to y_c=3,380 m^3/s.

To evaluate the reliability level of an existing structure such as a levee, we can use Fig. 4 to calculate the corresponding capacity for the river channel to conduct flood. At this time x_a is taken to be the corresponding capacity. Then, the reliability value can be obtained from

Fig. 1 for a specific service period t of the levee. This result is used to judge whether or not we need to consolidate the levee. If R is lower than the required value the levee should be heightened. For example, suppose the levee height H is 1.4 m, then the x_a is 3,110 m^3/s from Fig. 4, and R is 0.82 from Fig. 1 given that t=100 years. Under these conditions the reliability results by using three different models are compared in Table 4.

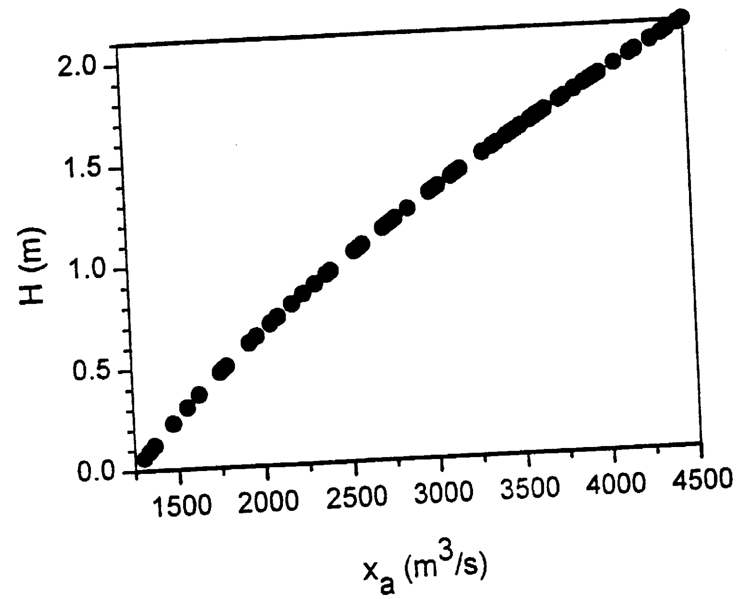

Figure 4. Relationship between actual design flood and levee height.

Table 3. Reliability Comparison for Different S (t = 100 years, x_a=3128 m^3/s).

Model	S=1.76	S=1.0
Tung and Mays	0.9845	0.9992
Lee and Mays	0.7305	0.9222
Improved Model	0.8189	0.8189
Poisson Distribution	0.8187	0.8187

Table 4. Reliability Results of Evaluating Existing Levee by Using Different Models (H=1.4 m, t=100 years).

Model	S=1	S=2
Tung and Mays	0.995	0.975
Lee and Mays	0.920	0.720
Improved Model	0.820	0.820

CONCLUSIONS

An reliability model is developed to improve the existing models. The safety factor is correctly introduced into the model, and the mean rate of flood events are compatible with the lower bound of integrations of the loading function. The independent assumption of external loading is satisfied. Therefore, the improved model provides more accurate and meaningful reliability results. The actual design flood was introduced to consider the influence of both design flood and safety factor on the reliability analysis. The reliability was uniquely determined by the actual design flood. In this way, the reliability becomes an exclusive standard for hydraulic design. The model can be used not only to design a new hydraulic structure with a specific reliability value but also to evaluate the reliability level of existing hydraulic structures.

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