PROBABILISTIC HYDRAULIC DESIGN: A NEXT STEP TO EXPERIMENTAL HYDRAULICS

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Probabilistic hydraulic design: A next step to experimental hydraulics

Dimensionnement probabiliste en hydraulique: Une étape complementaire aux essais hydrauliques

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SUMMARY
Hydraulic engineers conduct experiments based on which empirical laws are developed for the physical processes involved. However, during the design and analysis stage, the step often falls short of recognizing the inherent uncertainty associated with the developed empirical laws. This paper discusses the presence of various uncertainties in the empirical equations derived from hydraulic experiments and illustrates how to incorporate such uncertainties in probabilistic design of hydraulic structures.

RÉSUMÉ
Les ingénieurs hydrauliciens conduisent des études expérimentales sur lesquelles se fondent des lois empiriques décrivant les processus physiques concernés. Cependant, durant la phase de projet et d'analyse, il arrive souvent que le projeteur se heurte à la question des incertitudes inhérentes aux lois empiriques qui ont été élaborées. Le présent article discute de la présence d'incertitudes diverses dans les relations tirées des essais hydrauliques et illustre comment prendre en compte ces incertitudes dans un dimensionnement probabiliste des structures hydrauliques.

1 Introduction

To properly perform a hydraulic design and/or analysis, it is essential to have a good understanding about the hydraulic and hydrologic phenomena and processes involved. During the course of seeking a better understanding of the involved hydraulic and hydrological processes, experiments (including both in the field and in the laboratory) are often conducted to collect relevant data based on which empirical laws are established to compactly characterize the relationship among the observed data. In hydraulics and hydrology, these empirical laws are often presented in the forms of equations, tables, or charts. When these empirical laws are applied to hydraulic design or analysis, they are generally treated as if they are closely correlated to the unknown laws of nature which govern the mechanisms of processes. This is not true in many cases. It is also a well known fact that any empirical law is data dependent; it is subject to possible modification when new observations or evidences become available.

Yen et al. [4] indicated that in hydraulic and hydrologic design and analysis uncertainty could arise from various sources including, but not limited to, natural uncertainties, data uncertainties,

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model uncertainties, parameter uncertainties, and operational uncertainties. Model uncertainties refer to the fact that the empirical laws do not describe the complete feature of the true process. Parameter uncertainties refer to some parameters in the model that cannot be quantified with absolute certainty. Data uncertainties are caused by inconsistency, non-homogeneity, and data measurement and recording errors. Operational uncertainties include those associated with construction, manufacturing, deterioration, maintenance, and other human factors that are not accounted for in the modeling or design procedure.

In hydraulic engineering, many empirical equations are used in the design and analysis. Frequently, these empirical equations are applied in a deterministic fashion to hydraulic design problems. It is a common practice in the hydraulics and hydrology profession to present the final empirical equation or charts without explicitly stating or elaborating the associated uncertainty. The most commonly seen examples are applications of regression analysis or some types of curve fitting techniques to establish the empirical relations. The results presented to the readers are the values of regression coefficients or best-fitted parameter values in the model and some goodness-of-fit indicators such as the coefficient of determination. There are two major disadvantages associated with this common practice. The first one could be potentially more serious; namely, a designer could loose sight of intrinsic uncertainty associated with the empirical equation. This could occur especially when the implementation of design and analysis is on a digital computer by which the original scatterness of the data are not displayed. The second disadvantage is that this common practice does not fully utilize the information contained in the data. Consequently, much of the information in the data is unused resulting in unnecessary waste of information and perhaps flawed design decisions.

The presence of the above mentioned uncertainties contribute to the overall uncertainty of hydraulic design and analysis. Consequently, the performance potential of a hydraulic design can not be assessed with certainty. When this is the case, probabilistic approach becomes a necessary alternative for hydraulic design problems. Yevjivich [6] pointed out that a deterministic solution is only one special case of many the probabilistic solution which encompasses the full spectrum of possible solutions to a design problem.

In the following, an example is used to demonstrate the utilization of information provided by statistical regression analysis about the uncertainty features in an empirical hydraulic model for designing riprap protection. Furthermore, uncertainty features of the empirical equation are incorporated to evaluate the reliability of various riprap designs and optimum risk-based riprap design.

2 An example

To illustrate how to utilize the results from an experiment to perform probabilistic hydraulic design, data from a study for channel bed riprap protection under a pipe outlet by Shafai-Bajestan and Albertson [1] is used herein. Detail descriptions of the hydraulic theories, experimental setup and procedures, and other related subjects are presented in their paper.

One of the main objectives in the experimental study of Shafai-Bajestan and Albertson [1] is to establish a good relationship describing the overall conditions for the incipient motion of bed material under a pipe outlet. Based on their extensive discussions, Shafai-Bajestan and Albertson [1] indicated that the incipient motion of bed material under a pipe outlet can be described by

$$\mathrm{SN}_{\mathrm{D_s}} = h\left(\frac{PI}{D}\right) \tag{1}$$

in which SN_{D_s} is a stability number which is defined as

$$SN_{D_s} = \frac{V}{\sqrt{g(S_s - 1)D_s}}, \tag{2}$$

with V = exit pipe flow velocity; g = gravitational acceleration; S_s = specific gravity of riprap material; D_s = characteristic size of riprap material; D = the pipe diameter; and Pl = the penetration depth of the jet from pipe outlet which can be calculated as.

$$Pl = \frac{TW}{\sin\theta} \tag{3}$$

where

$$\sin\theta = 1 - \left(\frac{V}{V_p}\right)^2 \tag{4}$$

and

$$V_p = \sqrt{V^2 + 2gH'} \tag{5}$$

with TW = tailwater depth; V_p = penetration velocity of the jet at tailwater surface; and H' = vertical distance from the center of the pipe to tailwater surface. The definition sketch for the pipe-riprap system is shown in Fig. 1.

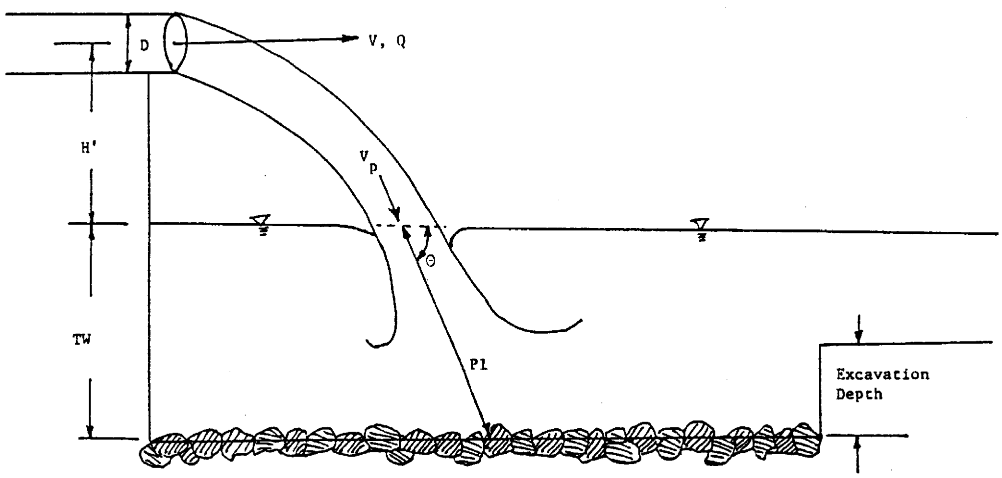

Fig. 1. Definition sketch of pipe outlet-riprap system.

Based on the experimental results, Shafai-Bajestan and Albertson [1] proposed a unified relationship for various riprap gradations by using D_{30} for the incipient motion and D_{90} for the incipient failure as the characteristic riprap size. Fig. 2 is the experimental results for the incipient motion which will be used in this paper for demonstration purpose.

As can be seen from Fig. 2, the data reveal a linear relationship between SN_{30} and Pl/D. Although the regression equation for Fig. 2 has been presented by Shafai-Bajestan and Albertson [1], it contains only the values of regression coefficients and correlation coefficient. Other relevant

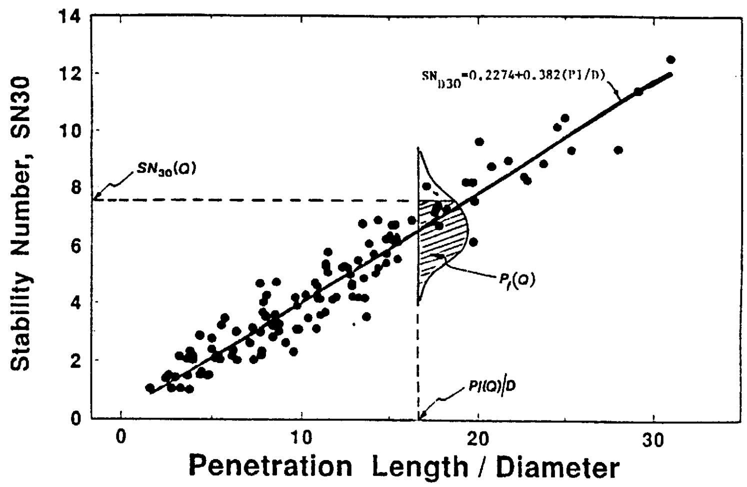

Fig. 2. Variation of Pl/D versus SN_{30} for all data (Shafai-Bajestan and Albertson, 1993).

statistical information with regard to the regression coefficients and the error associated with the regression equation are not presented. For this reason, data in Fig. 2 were re-analyzed using the following regression model

$$SN_{D_{30}} = a + b\left(\frac{Pl}{D}\right) + e \tag{6}$$

in which a and b are regression coefficients and e is the model error term. From the regression analysis, the following information is generally provided by most statistical packages

$$\mu_a = 0.2274; \quad \mu_b = 0.3820;$$
$$\sigma_a = 0.1338; \quad \sigma_b = 0.0102; \quad \sigma_e = 0.6958;$$
$$\varrho_{a,b} = -0.8821$$

in which μ_a and μ_b are the mean values of regression coefficients; σ_a and σ_b are the standard deviations associated with the regression coefficients; σ_e is model standard error representing the amount of uncertainty associated the regression model; and $\varrho_{a,b}$ is the correlation coefficient between the two regression coefficients. The value of coefficient of determination, R^2, is 0.92 indicating 92% of data variation is explained by the regression model, equation (6). As will be illustrated in the following sections, information such as this is essential for probabilistic design and analysis of riprap protection.

3 Reliability analysis of riprap failure due to incipient motion

With regard to this pipe outlet riprap design example, the primary concern is the stability of the riprap on the channel bed downstream of the pipe outlet. According to the physical process involved and the mathematical relationship between the stability number (SN) and dimensionless penetration length (Pl/D) as given in equation (6), one could define the state of the riprap as

$$\text{Unstable: } a + b\left(\frac{Pl}{D}\right) + e < \mathrm{SN}_{D_{30}}$$
$$\text{Stable: } a + b\left(\frac{Pl}{D}\right) + e > \mathrm{SN}_{D_{30}} \tag{7}$$

Due to the presence of uncertainties in regression coefficients and model, there would be a certain degree of risk that the designed riprap would not be stable. The riprap failure probability for being in the unstable state, then, can be expressed as

$$p_f = P\left[a + b\left(\frac{Pl}{D}\right) + e < \mathrm{SN}_{D_{30}}\right]$$
$$= P\left[a + b\left(\frac{\mathrm{TW}_0 + E_d}{D \cdot \sin\theta(Q)}\right) + e < \frac{Q/A}{\sqrt{g(S_s - 1)D_{30}}}\right] \tag{8}$$

in which TW_0 is the initial tailwater depth, E_d is the excavation depth, and A is the pipe cross-sectional area.

Under a specified design discharge, q_{dsgn}, the left-hand-side (LHS) in $P[\,]$ of equation (8) involves stochastic parameters (a, b, and e) and, therefore, is a random variable. On the other hand, the stability number ($\mathrm{SN}_{D_{30}}$) on the right-hand-side (RHS) is deterministic. To evaluate riprap failure probability under a specified design pipe discharge, the statistical properties of the random LHS are obtained as

$$\mu_{LHS}(q_{dsgn}) = \mu_a + \left(\frac{\mathrm{TW}_0 + E_d}{D \cdot \sin\theta(q_{dsgn})}\right)\mu_b \tag{9}$$

$$\sigma^2_{LHS}(q_{dsgn}) = \sigma^2_a + \left(\frac{\mathrm{TW}_0 + E_d}{D \cdot \sin\theta(q_{dsgn})}\right)^2 \sigma^2_b + \sigma^2_e + 2\left(\frac{\mathrm{TW}_0 + E_d}{D \cdot \sin\theta(q_{dsgn})}\right)\sigma_a\sigma_b\varrho_{a,b} \tag{10}$$

Based on the normality condition of the ordinary regression analysis, the LHS of equation (8) is a normal random variable with the mean and variance given in equations (9) and (10), respectively. Hence, the riprap failure probability under a designed pipe discharge can be calculated as

$$p_f(q_{dsgn}) = P\left[Z < \frac{\mathrm{SN}_{D_{30}}(q_{dsgn}) - \mu_{LHS}(q_{dsgn})}{\sigma_{LHS}(q_{dsgn})}\right] = \Phi\left[\frac{\mathrm{SN}_{D_{30}}(q_{dsgn}) - \mu_{LHS}(q_{dsgn})}{\sigma_{LHS}(q_{dsgn})}\right] \tag{11}$$

where Z is the standard normal random variable having mean 0 and unit variance, $\Phi()$ is the cumulative distribution function (CDF) of Z, and

$$\mathrm{SN}_{D_s}(q_{dsgn}) = \frac{q_{dsgn}/A}{\sqrt{g(S_s - 1)D_s}} \tag{12}$$

Equation (11) should be interpreted as the riprap failure probability conditioned on the specified design discharge. Under the condition that $q_{dsgn} = 0.396$ cms; $D = 0.457$ m; $H' = 2.286$ m; $\mathrm{TW}_0 = 0.762$ m (from [1]), Fig. 3 shows the variation of riprap failure probability due to incipient motion with respect to characteristic riprap size (D_{30}) and excavation depth (E_d). As can be expected, the failure probability decreases as the riprap size gets larger or the excavation depth increases. Using the deterministic approach without considering uncertainties in the regression equation, the resulting design would correspond to a 50 percent failure probability.

In the real-life situation, the riprap will be placed on the channel bed for a certain period of time during which the discharge through the pipe outlet would vary. In such circumstances, the

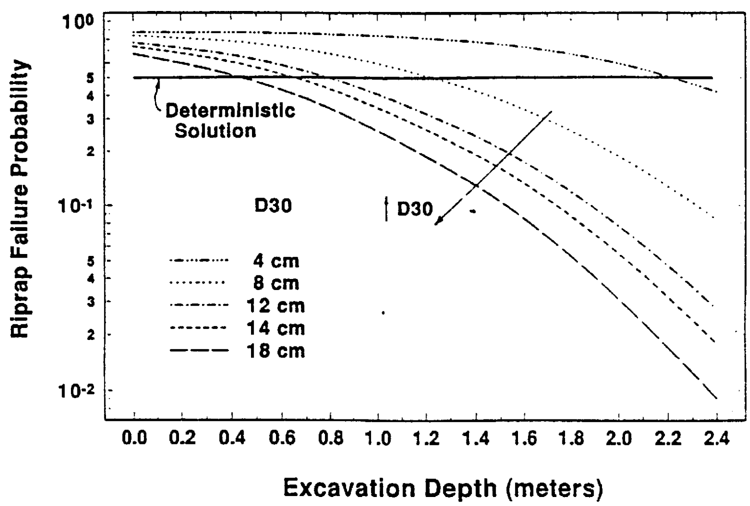

Fig. 3. Conditional riprap failure probability curves considering uncertainties in regression model.

random nature of the pipe outlet discharge should be considered in the evaluation of reliability of riprap protection. Suppose that one is interested in the annual failure probability associated with the riprap protection, then, the distribution of annual maximum discharge through the pipe outlet can be integrated in the above reliability analysis framework. The statistical properties of annual maximum discharge through the pipe outlet could be obtained from considering the interaction between the hydraulic structure and the distribution of annual maximum streamflow. However, for simplicity, we assume in this paper that the distributional properties of annual maximum pipe discharge are derived.

To consider the random nature of pipe outlet discharge, riprap failure probability can be evaluated by equation (8) in which Q is treated as a random variable, along with a, b, and e in the regression equation. To place all random variables on the LHS in the probability statement, equation (8) can be rewritten as

$$p_f = P\left[a + b\left(\frac{TW_0 + E_d}{D \cdot \sin\theta(Q)}\right) + e - \frac{Q/A}{\sqrt{g(S_s - 1)D_{30}}} < 0\right] \tag{13}$$

Due to nonlinearity of the LHS terms in equation (13), analytical derivation of the probability density function (PDF) of the LHS is difficult, if not impossible. Several methods such as first-order second-moment techniques or Monte-Carlo simulation can be applied to evaluate the riprap failure probability. The descriptions of features of various reliability analysis techniques can be found in Yen and Tung [5].

Notice that $p_f(q)$ given in equation (11) is the riprap failure probability conditioned on a specified pipe outlet discharge, q. Referring to Fig. 4, the annual riprap failure probability can alternatively be calculated by integrating the conditional failure probability, $p_f(q)$, over the likelihood of all possible pipe outlet discharge. That is,

$$p_f = \int_0^\infty p_f(q) f(q)\, dq = E_Q[p_f(Q)] \tag{14}$$

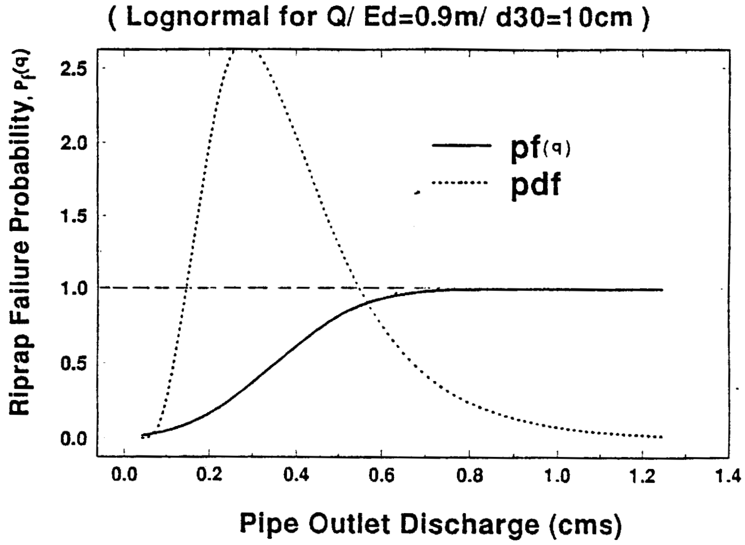

Fig. 4. Variation of conditional riprap failure probability verus discharge.

where $f(q)$ is the PDF of annual maximum pipe outlet discharge and $E[\,]$ is the statistical expectation operator with respect to the random annual maximum pipe outlet discharge, Q. In other words, p_f obtained by equation (14) is the expectation of conditional riprap failure probability.

For illustration, the annual maximum pipe outlet discharge is assumed to have a log-normal distribution with a mean of 0.396 cms and standard deviation of 0.2 cms. The pipe outlet discharge is considered uncorrelated to any of the regression coefficients. Under the identical system setting as in the previous example, a series of annual riprap failure probability curves due to incipient motion can be constructed for various combinations of excavation depth and characteristic riprap size, D_{30}. As shown in Fig. 5, similar behaviors of conditional failure probability càn be observed. The deterministic design procedure would result in a slightly less than 50% annual failure probability.

4 Risk-based design of riprap at pipe outlet

In the riprap design for channel bed protection, the design variables are characteristic riprap size and excavation depth. The design can be made by explicitly considering the trade-off between the required excavation depth, riprap size, and the associated failure probability based on the information generated from the reliability analysis. Accordingly, the riprap design problem is, in essence, a multi-criteria or multi-objective decision-making problem.

To simplify the design, we assume that rock material for riprap protection from a certain source is to be used. Hence, the characteristic riprap size and its associated cost are fixed in advance, leaving excavation depth as the sole design variable. The design problem can be cast into the determination of excavation depth associated with the minimum total costs or total losses. Referring to Fig. 6, the total losses consist of losses due to over-design and losses due to under-

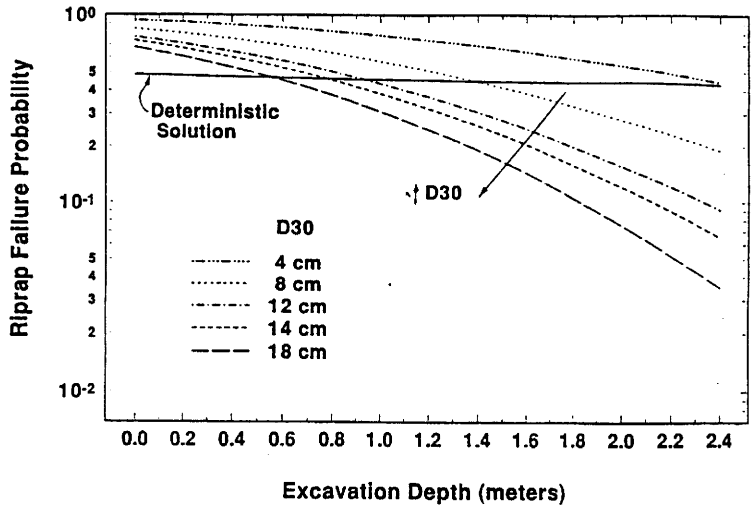

Fig. 5. Annual riprap failure probability curves considering both uncertainties in regression model and discharge.

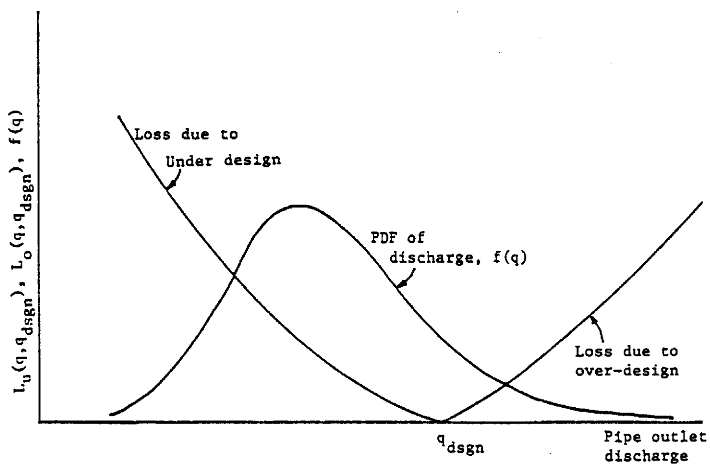

Fig. 6. Schematic sketch of losses due over-design and under-design.

design. Because pipe outlet discharge is random, the annual total losses cannot be computed with absolute certainty. Therefore, the annual expected total losses is frequently used as the criterion for determining the economic merit of an engineering design.

Without considering the uncertainty in the regression equation, the relationship between the pipe outlet discharge and the required excavation depth for stable riprap condition is unique. The annual expected total losses associated with a specified design discharge can be calculated as

$$E[L_T(q_{dsgn})] = E[L_O(q_{dsgn})] + E[L_U(q_{dsgn})] = \int_0^{q_{dsgn}} g_O[E_d(q_{dsgn}) - E_d(q)]f(q)\,dq + \int_{q_{dsgn}}^{\infty} g_U[E_d(q) - E_d(q_{dsgn})]f(q)\,dq \quad (15)$$

in which $E[L_T]$, $E[L_O]$, and $E[L_U]$ are the annual expected total losses, annual expected losses due to over-design, and under-design, respectively; $L_O() = g_O[]$ and $L_U() = g_U[]$ are the loss functions due to over-design and under-design, respectively.

When uncertainties in the regression equation are considered, there exists no such one-to-one relationship between the pipe outlet discharge and required excavation depth. However, referring to equation (11), if the allowable riprap failure probability is specified in advance, the required excavation depth corresponding to a pipe outlet discharge can be uniquely determined. This can be achieved by inversely solving equation (11). Considering uncertainties in the regression equation, the expression in equation (15) can be modified as

$$E\{L_T[q_{dsgn}, p_f(q_{dsgn})]\} = E\{L_O[q_{dsgn}, p_f(q_{dsgn})]\} + E\{L_U[q_{dsgn}, p_f(q_{dsgn})]\}$$
$$= \int_0^{q_{dsgn}} g_O\{E_d[q_{dsgn}, p_f(q_{dsgn})] - E_d(q)\}f(q)\,dq + \int_{q_{dsgn}}^{\infty} g_U\{E_d(q) - E_d[q_{dgn}, p_f(q_{dsgn})]\}f(q)\,dq \quad (16)$$

Referring to Fig. 7, there exists a trade-off between the annual expected losses due to over-design and under-design. One observes that, as the selected design discharge increases, the annual expected losses due to over-design increases whereas the annual expected losses due to under-design decreases. Hence, the objective of optimal risk-based design is to identify the design discharge with a certain return period such that the annual total expected losses is minimum. Hence, in the optimal risk-based design procedure, equation (15) or (16) is used as an objective function in an appropriate optimization algorithm for seeking the optimal design discharge or its corresponding excavation depth.

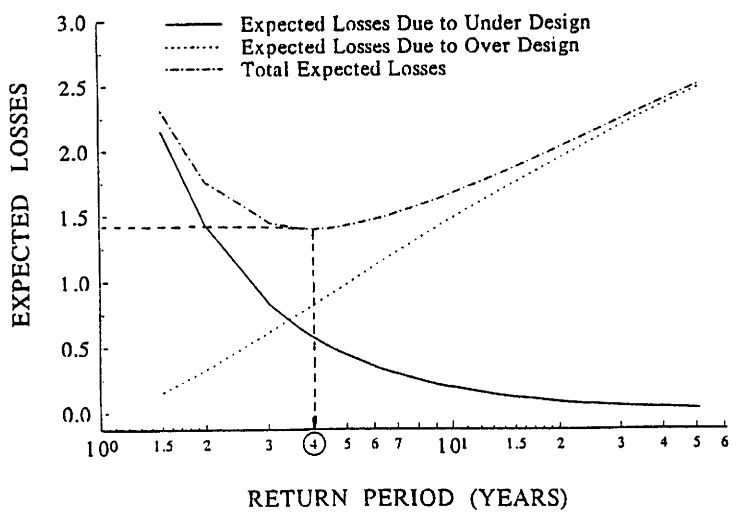

Fig. 7. Variations of annual expected losses with respect to design return period.

In this particular example, the characteristic riprap size is set at $D_{30} = 6$ cm. The adopted loss functions are simple linear functions as

$$\begin{aligned} g_O[E_d(q_{dsgn})] &= [E_d(q_{dsgn}) - E_d(q)] \\ g_U[E_d(q_{dsgn})] &= 3[E_d(q) - E_d(q_{dsgn})] \end{aligned} \tag{17}$$

As indicated by equation (17), losses due to under-design are considered more severe than losses due to over-design.

Since there is only one design variable, namely, the excavation depth, a simple one-dimension optimization algorithm, called Fibonacci search [2], is applied to obtain the optimum design discharge and its corresponding excavation depth. Both equations (15) and (16) are used to examine the effect of incorporating the uncertainties in regression equation on the optimal design frequency and the corresponding required excavation depth. Fig. 7 shows the result of optimal risk-based design of riprap protection at the pipe outlet considering the uncertainties in regression equation and the allowed failure probability for all discharge is set at 5%. The optimum design frequency was found to be 4.0 years which is practically identical to results obtained from using various allowed failure probability and the one using equation (15) without considering the uncertainties in regression equation. Although the optimum design frequency is quite invariant to the type of uncertainty and allowed riprap failure probability considered, the corresponding optimum excavations are quite different. In fact, as shown in Fig. 8, the optimum excavation depth increases as the allowed riprap failure probability decreases. The deterministic application of the regression equation without considering its uncertainties results in the least amount of excavation. However, the deterministic solution corresponds to a conditional riprap failure probability of 39% for all possible pipe outlet discharges.

The current state-of-the-art practice of risk-based hydraulic designs consider the expected losses or expected cost as the sole basis for evaluating the relative merit of competitive designs or alternatives. It, however, should be pointed out that the expected value is only one aspect of

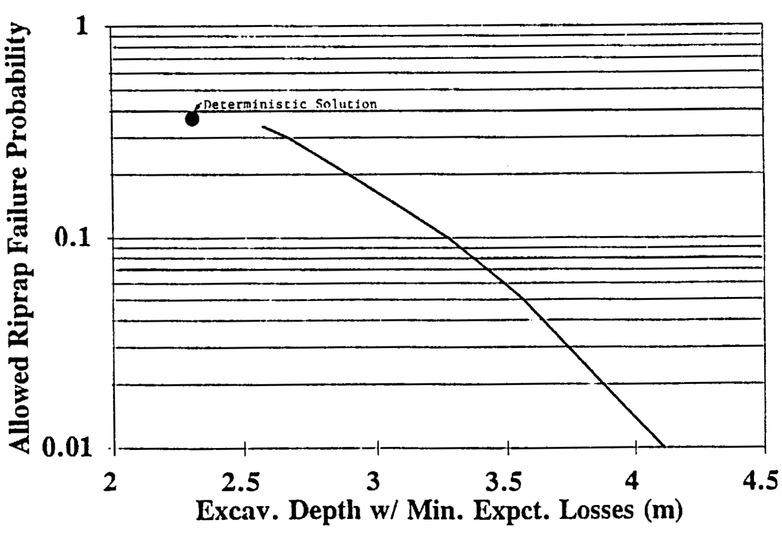

Fig. 8. Allowed riprap failure probability versus excavation depth with minimum annual expected loss.

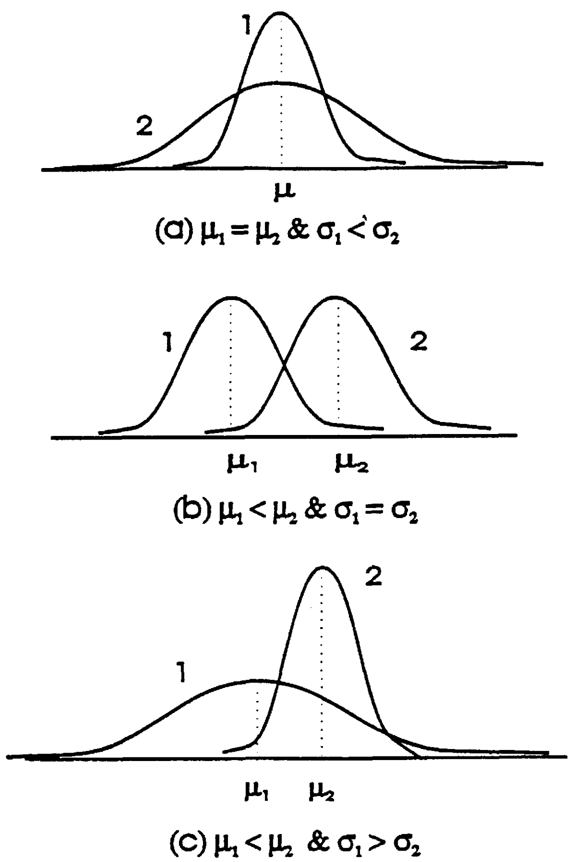

Fig. 9. Designs with different distributions of annual total losses.

uncertainty feature of the random losses or costs. Other aspects of random losses, such as the standard deviation, could be an important parameter influencing the outcome of decision-making process. For example, a design with a smaller annual expected loss is not necessarily a desirable one if its associated standard deviation is large. Decision for cases shown in Figs. 9a and 9b is generally a trivial one. One would select the design with smaller standard deviation in losses if both have the same expected loss whereas one would prefer the design with lower expected loss if both have the identical standard deviation. However, the decision would not be as trivial for case shown in Fig. 9c. In this case, decision-maker would has to resort to his/her attitude toward the risk [3].

In the context of riprap design problem considered herein, the uncertainty of annual total losses associated with different design frequencies (or discharges) can be computed as

$$\sigma_{L_T}[q_{dsgn}, p_f(q_{dsgn})] = \sqrt{E\{L_T^2[q_{dsgn}, p_f(q_{dsgn})]\} - E^2\{L_T[q_{dsgn}, p_f(q_{dsgn})]\}} \tag{18}$$

where

$$E\{L_T^2[q_{dsgn}, p_f(q_{dsgn})]\} = E\{L_O^2[q_{dsgn}, p_f(q_{dsgn})]\} + E\{L_U^2[q_{gsdn}, p_f(q_{gsdn})]\}$$
$$= \int_0^{q_{dsgn}} g_O^2\{E_d[q_{dsgn}, p_f(q_{dsgn})] - E_d(q)\} f(q)\,dq \qquad (19)$$
$$+ \int_{q_{dsgn}}^{\infty} g_U^2\{E_d(q) - E_d[q_{dsgn}, p_f(q_{dsgn})]\} f(q)\,dq$$

with $E\{L_T^2[q_{dsgn}, p_f(q_{dsgn})]\}$ being the second-order moment about the origin for the random annual total losses. Fig. 10 shows the curves of annual total expected losses and the associated standard deviation using equation (18) with $p_f = 5\%$ for all discharges. As can be observed, for design frequencies shorter than 4 years, the standard deviation of annual total losses increases along with the expected value. This indicates that, in that domain, the level of desirability decreases as the design frequency becomes smaller. This is also true for the domain of exceeding about 13 years. Between design frequencies of 4–13 years, the uncertainty associated with the annual total loss decreases monotonically as the corresponding expected value increases. Considering the tradeoff between the expected loss and the associated standard deviation and referring to Fig. 10, a more plausible design frequency could be somewhat larger than 4 years because rate of increase in expected loss appears to be less than the rate of decrease in standard deviation between 4–6 years.

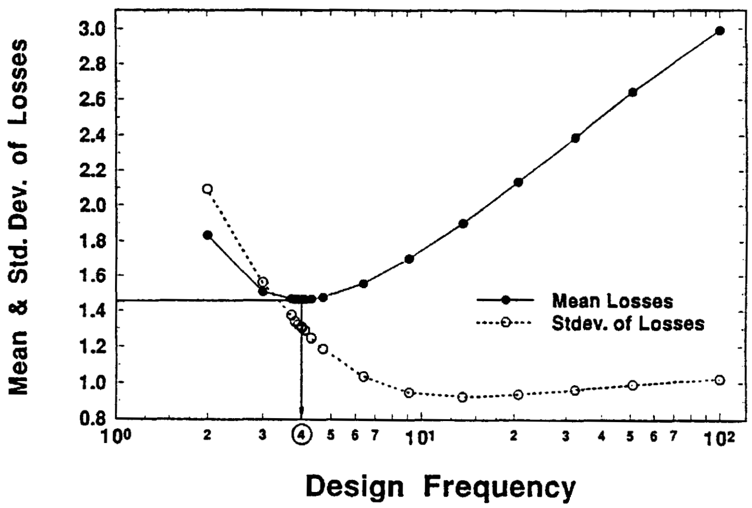

Fig. 10. Mean and standard deviation of annual losses versus design frequency.

5 Summary and conclusions

In hydraulic engineering designs, the common practice is to adopt some relevant empirical equations derived from hydraulic experiments. More often than not, the use of empirical hydraulic equations is in a deterministic manner without explicitly considering their intrinsic uncertainties. This occurs not only in all engineering design practices, but also in our teaching of fluid mechanics, hydraulics and hydrology in universities. In design manuals or textbooks, the scatterness of the original data from experiments is often reduced to a series of simple curves or equations. Using empirical equations as if they are deterministic without uncertainties does not allow an engineer to assess the reliability of the design. Such practice might result in inferior hydraulic design that is either unsafe or uneconomical. After all, it is the goal of our profession to design a hydraulic and water resource project that could achieve project objectives in the most economical manner consistent with acceptable levels of public safety.

Moreover, without considering uncertainties in an empirical equation for hydraulic design, we do not utilize the full information in the experimental data which are so painstakingly and costly to collect. I would like to take this opportunity to make a plea to my colleagues in experimental hydraulics: when experimental data are presented in form of charts or equations, go one step further to include other statistical information which indicates the uncertainty features of the charts or equations. Of course, the feasibility of taking this extra step depends on the type of data and the techniques used for data analysis. Sometimes, it is not an easy task to take this extra step. However, there are other situations, such as when using regression analysis, where the information is already there for our use.

Notations

a, b	regression coefficients
A	pipe cross-sectional area
D	pipe diameter
D_{30}	riprap characteristic size with 30% finer
e	error term of regression model
E_d	excavation depth
$E_X()$	expectation operator with respect to random variable X
$f(q)$	probability density function of pipe outlet discharge
g	gravitational acceleration
$h()$	general expression for function
H'	vertical distance from the pipe center to tailwater surface
$L_O()$	loss function due to over-design
$L_T()$	total loss function
$L_U()$	loss function due to under-design
p_f	riprap failure probability
$p_f(q)$	riprap failure probability conditioned on pipe outlet discharge, q
Pl	penetration depth of the jet from pipe outlet
$P[]$	probability expression
q	pipe outlet discharge
q_{dsgn}	design pipe outlet discharge
SN_{D_s}	stability number of riprap material based on characteristic size D_s
S_s	specific gravity of riprap material
TW	tailwater depth
TW_0	initial tailwater depth
V	exit pipe flow velocity
V_p	penetration velocity of the jet at tailwater surface
μ_x	mean of random variable X
σ_x	standard deviation of random variable X
$\varrho_{x,y}$	correlation coefficient between random variables X and Y
$\Phi(z)$	standard normal cumulative distribution function

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