MULTIPLE-OBJECTIVE STOCHASTIC WASTE-LOAD ALLOCATION

Yeou-Koung Tung

1992

Journal Article WWRC-92-05

In

Water Resources Management

Volume 6

Submitted by

Yeou-Koung Tung
Department of Statistics
and
Wyoming Water Resources Center
University of Wyoming
Laramie, Wyoming

Water Resources Management **6**: 117–133, 1992

Multiple-Objective Stochastic Waste-Load Allocation

YEOU-KOUNG TUNG*
Department of Civil Engineering, National Chiao-Tung University, Hsinchu, Taiwan 30049, Republic of China

(Received: 4 March 1991; accepted in final form: 7 January 1992)

Abstract. The practice of waste-load allocation in water quality management involves several noncommensurate and conflicting objectives. In particular, the objectives considered in this multiobjective stochastic waste-load allocation study are (1) maximization of total waste discharge, (2) maximization of instream dissolved oxygen concentration, (3) minimization of difference in equity measures, and (4) maximization of reliability of water quality compliance. To demonstrate the analysis, the model was applied to an example involving six waste dischargers.

Keywords. Waste-load allocation, water-quality management, multiple-objective analysis, optimization, uncertainty analysis.

1. Introduction

Issues involved in many environmental problems facing water-quality professionals today are becoming more complex. The necessity for improved environmental protection has not precluded the problem of waste-load allocation (WLA) from increasing governmental and societal demands on water-quality assurance. As society progresses with time, the demand placed on water quality will continue to grow. In fact, the decision-making process in most environmental problems is cultivated by the desire to achieve several objectives simultaneously. The WLA problem is without exception to these aspirations. Therefore, in searching for effective and efficient management decision for protecting and preserving water quality in the WLA process, several management objectives or goals should be considered simultaneously. Tung and Hathhorn (1989) presented a deterministic multiple-objective WLA model. Due to the presence of various uncertainties in natural stream environment, this paper further presents a multiobjective analysis for WLA problem in a stochastic stream environment in that uncertainties in water quality parameters are explicitly considered. Because of the rising demands placed on water quality assurance by government and society, the utilization of multi-objective procedures can only lead to improved water quality protection and control.

* On sabbatical leave from the Wyoming Water Research Center, University of Wyoming, Laramie, WY 82071, U.S.A.

2. Multiple-Objective Modeling

In a multi-objective problem, several scalar objective functions are involved. The problem is sometimes referred to as the vector optimization. The general framework of a multi-objective model can be expressed as

$$\text{Max } \mathbf{Z}(\mathrm{X}) = [Z_1(\mathrm{X}), Z_2(\mathrm{X}), \ldots, Z_K(\mathrm{X})], \tag{1}$$

subject to

$$\mathrm{g}(\mathrm{X}) \leqslant 0 \tag{2}$$

in which $\mathbf{Z}$ (X) is a K-dimensional vector of the objective functions, $\mathbf{X}$ is an n-dimensional vector containing the decision variables, and $\mathbf{g}$ ($\mathbf{X}$) is a m-dimensional vector of constraints.

In the context of multi-objective modeling, the concept of 'optimality' in the single-objective problems is no longer appropriate because there normally exists several noncommensurable and conflicting objectives. Without a prior knowledge of the preference among different objective, the solution to a multi-objective problem would be a set of points defining the tradeoff among objectives. Consequently, 'noninferior solution' in the multi-objective analysis replaces 'optimum solution' in the single-objective framework.

The noninferior solution set, in general, is defined by a unique continuous curve or surface depicting the tradeoffs between the various objectives. It is not until the decision-maker provides the information about the preference among objectives, that a best compromising solution can be identified. The best compromising solution to the multi-objective problems is an alternative which possess the property of maximum combined utility. Such an alternative only exists at the point where the indifference curve is tangent to the noninferior solution set (Cohon, 1978).

3. Single-Objective Stochastic WLA Model

In all fields of science and engineering, the outcomes of a system on which decisions are based depend on several parameters and variables. More often than not, one or more of these parameters cannot be assessed with certainty. This is particularly true in decision-making for environmental management problems. The environment in which decisions are to be made concerning instream water quality management are inherently subject to many uncertainties (Ward and Loftis, 1983). The stream system itself, through nature, is an animate environment abundant with ever-changing processes, both physically and biologically.

In this study, the natural inherent uncertainties of water quality parameters in a stochastic stream system were incorporated in the WLA model through the chance-constrained framework (Charnes and Cooper, 1963; Kolbin, 1977). There have been several articles recently utilizing chance-constrained model for water quality management (Lohani and Thanh, 1979; Yaron, 1979; Burn and McBean, 1985; Fujiwara *et al.*, 1986; Ellis, 1987; Tung and Hathhorn, 1990). The single-objective stochastic WLA model, which serves as the basic model for the multi-objective formulation in this study, is expressed in the following.

3.1. OBJECTIVE FUNCTION

The objective function adopted was

$$\text{Maximize} \sum_{j=1}^{N} (B_j + D_j), \tag{3}$$

in which B_j are the biochemical oxygen demand (BOD) concentration (mg/l) and dissolved oxygen (DO) deficit concentration (mg/l) in the effluent at discharge location j, respectively, and N is the total number of waste dischargers.

This objective function was chosen for its simplicity and its economical equivalence to the minimization of treatment cost. Both effluent waste discharge and DO deficit were chosen in attempting to replicate actual design condition because they were controllable.

3.2. CONSTRAINTS

The constraints in a stochastic WLA model basically involve the following types.

Constraints on Water Quality. The most common requirement of a WLA problem has been the assurance of minimum concentrations of DO throughout the river system in an attempt to maintain a desirable environment for aquatic biota. In general, the constraint relating the response of DO to the addition of effluent waste can be defined by the Streeter–Phelps equation (Streeter and Phelps, 1925) or its variations (Dobbins, 1964; Krenkel and Novotny, 1980). In this study, the original Streeter–Phelps equation was employed for deriving the water quality constraints for demonstrating the proposed methodologies without over-complicating the algebraic manipulations.

To ensure the compliance of water quality standard, several control points within each reach of the river system were selected. Constraint equations in the WLA model were established for each control location at which water quality condition was checked. A typical water quality constraint without considering uncertainties in water quality parameters could be expressed as the following:

$$a_{oi} + \sum_{j=1}^{n_i} \Theta_{ij} B_j + \sum_{j=1}^{n_i} \Omega_{ij} D_j \leqslant \mathrm{DO}_i^{\mathrm{sat}} - \mathrm{DO}_i^{\mathrm{std}}, \quad \text{for } i = 1, 2, \ldots, M, \tag{4}$$

in which Θ_{ij} and Ω_{ij} are the transfer coefficients indicating the relative impact on DO concentration at downstream location, i, resulting from a unit waste input at the upstream location, j. The technological transfer coefficients are functions of water-quality parameters such as reaeration and deoxygenation rates, flow velocity, etc. Also in Equation (4), n_i is the number of the waste dischargers upstreams of the control point i; DO_i^{std} and DO_i^{sat} represent the required DO standard and saturated DO concentration at control point i, respectively; a_{0i} is the transfer coefficient relating the DO deficit concentration at control point i as affected by the initial waste load at the upstream end of the entire stream system; M is the total number of control points. Expressions for Θ_{ij} and Ω_{ij} based on the Streeter–Phelps equation can be found elsewhere (Hathhorn, 1986).

In reality, water quality parameters such as reaeration and deoxygenation coefficients, flow velocity, initial DO and BOD concentrations are random (Kothandaraman and Ewing, 1969; Esen and Rathbun, 1976; Hornberger, 1980; Chadderton *et al.*, 1982; Ward and Loftis, 1983). Due to the existence of uncertainty within the stream environment, the compliance of water-quality standard in the stream system cannot be assessed with certainty. Therefore, the water-quality constraints given by Equation (4) should be expressed probabilistically as

$$\Pr\left[a_{oi} + \sum_{j=1}^{n_i} \Theta_{ij} B_j + \sum_{j=1}^{n_i} \Omega_{ij} D_j \leqslant DO_i^{sat} - DO_i^{std}\right] \tag{5}$$

in which Pr{ } is the probability operator and α_i is the specified water quality compliance reliability at control point i.

However, the probabilistic statement given by Equation (5) is not mathematically operational. It has to be transformed into its deterministic equivalent. The corresponding deterministic equivalent of Equation (5) can be derived as

$$\sum_{j=1}^{n_i} E[\Theta_{ij}]B_j + \sum_{j=1}^{n_i} E[\Omega_{ij}]\, D_j + Z_i\,(\alpha_i)\sqrt{(\mathbf{B}^t, \mathbf{D}^t)\,\mathbf{C}(\Theta_i, \Omega_i)\,(\mathbf{B}, \mathbf{D})} \leqslant R_i' \tag{6}$$

in which $R' = DO_i^{sat} - DO_i^{std} - E[a_{oi}]$, $(\mathbf{B}^t, \mathbf{D}^t)$ is the row vector of BOD and DO deficit concentrations in waste effluent, $\mathbf{C}\ (\Theta_i, \Omega_i)$ is the covariance matrix associated with the technological coefficients in the ith water quality constraint, including a_{0i}; $z_i\ (\alpha_i)$ is the α_ith order quantile associated with the standardized random variable z_i

$$z_i = \frac{R_i' - \left[\sum_{j=1}^{n_i} E[\Theta_{ij}]B_j + \sum_{j=1}^{n_i} E[\Omega_{ij}]D_j\right]}{\sqrt{(\mathbf{B}^t, \mathbf{D}^t)\,\mathbf{C}(\Theta_i, \Omega_i)\,(\mathbf{B}, \mathbf{D})}}\,. \tag{7}$$

The above deterministic equivalent of water-quality chance-constraints is nonlinear involving the squared root of a quadratic function of waste-load decision variables. Also, note that to solve the stochastic WLA model with chance constraints such as Equation (6), the knowledge of covariance matrix of technological coefficients in water-quality constraints must be known or estimated.

Because of the nonlinearity of water-quality model, the use of analytical techniques to determine the statistical properties of the random technological coefficients is an extremely formidable task, if not impossible. The level of complexity increases rapidly as the control points move toward downstream. Furthermore, the existence of spatial correlation of water-quality parameters and cross-correlation among the parameters makes such task even more difficult. Even if one ignores the spatial correlation of water-quality parameters, the technological coefficients in the water-quality constraints would not be uncorrelated because they are functions of the same water-quality parameters. As a practical alternative, simulation procedures were used to estimate the mean and covariance structure of the random technological coefficients in a given water-quality constraint. In particular, unconditional simulation developed in geostatistics was applied in this research to generate the random but spatially correlated water-quality parameters. Detailed descriptions of the use of unconditional simulation for estimating statistical properties of the technological transfer coefficients in stochastic water quality constraints were given by Tung and Hathhorn (1990).

Constraints on Treatment Equity. In addition to the constraints for complying water-quality standard, constraints were also employed to define equity between the various dischargers along the river system. Without including equity considerations in the WLA model, any attempts to maximize waste discharge (or to minimize treatment cost) could result in the allocation of large quantities of waste to the upstream users; whereas the downstream dischargers could be required to treat their effluent at levels of maximum possible efficiency. This is especially true for fast moving streams. Several articles have discussed the importance of equity considerations in the WLA problem (Gross, 1965; Loucks *et al.*, 1967; Miller and Gill, 1976; Brill *et al.*, 1976).

Recognizing the importance of equity consideration in the WLA process, the choice must then be made about the type of equity to be used. Based on the conclusion drawn by Chadderton *et al.* (1981), the type of equity measure considered in this study was the equal percent removal which can be expressed mathematically as

$$\left| \left(\frac{B_j}{I_j}\right) - \left(\frac{B_{j'}}{I_{j'}}\right) \right| \leqslant E_A, \quad \text{for } j \neq j' \tag{8}$$

in which I_j is the influent raw waste concentration (mg/l BOD) at discharge location j, E_A is the specified allowable difference in equity measure between any two waste dischargers.

Additionally, note that, for any given stream system, one or more of the waste dischargers considered might be influent tributaries. The waste discharge from a tributary should be excluded from the consideration of equity to prevent an undue restriction being placed on the required treatment levels assigned to other dischargers.

Constraints on Treatment Efficiency. This set of constraints defined the acceptable range of the treatment efficiency. A range of 35 to 90% removal of incoming raw waste was used in this study for illustration. The minimum requirement of 35% removal was to prevent floating solids from being discharged to the stream environment. On the other hand, the upper limit of 90% removal represents the maximum efficiency (assumed) attainable by treatment technology.

The treatment efficiency constraints for each discharge location can be expressed as

$$0.35 \leqslant \frac{B_j}{I_j} \leqslant 0.90, \quad \text{for } j = 1, 2, \ldots, N. \tag{9}$$

Certainly, one might argue that the limits set on treatment efficiency are antiquated. By changing these limits, only the size of the feasible region in which the optimum solution is sought will be affected, not the utility of the model.

Finally, nonnegativity constraints on decision variables should be included in the model.

4. Multiple-Objective Stochastic WLA Model

In this paper model presentation and discussion are based on a four-objective stochastic WLA problem formulation. The objective functions considered are discussed as the following.

As stated previously that it is incomplete in the WLA model without incorporating the idea of fairness into the model formulation. As the requirement of equity measure is raised, the total waste load to the stream system would generally be reduced. Furthermore, from the preserving stream water-quality viewpoint, setting a higher the water-quality standard is more desirable. However, it is intuitively understandable that the waste treatment cost would be increased as the instream water-quality standard is raised. Therefore, the objectives of preserving water-quality and of enhancing economic efficiency are conflicting each other. Lastly, as the requirement of reliability for the complying water-quality standard is raised, the total waste load that can be discharged would expectedly have to be reduced.

All the above intuitive arguments of tradeoff among objectives can be easily made for most of multi-objective problems. However, the exact tradeoff behavior cannot be made without going through the formalism of solving the problem by appropriate techniques.

The four objective functions considered for the stochastic WLA problem in this study are: (1) to maximize the total waste load, (2) to minimize the maximum difference in equity measure between various dischargers, (3) to maximize the lowest allowable DO concentration level in the stream, and (4) to maximize the lowest water-quality compliance reliability.

The first objective function considered is formultated as Equation (3) stated previously

$$\text{Maximize } Z_1 = \sum_{j=1}^{N} (B_j + D_j). \tag{10}$$

For a stream system involving multiple dischargers, the differences in equity measure would generally be varying. To collapse different values of equity measure into a single representative indicator, the worst case associated with the largest difference was adopted in the study. Hence, the second objective can be expressed as

$$\text{Minimize } Z_2 = \delta_{\max} = \max \left| \left(\frac{B_j}{I_j}\right) - \left(\frac{B_{j'}}{I_{j'}}\right) \right| , \quad \text{for all } j \neq j' \tag{11}$$

in which $\delta_{\max}$ is a new decision variable representing the largest difference in equity measure between different dischargers.

The third objective considered is the maximization of the lowest allowable DO concentration level that should be maintained in the stream environment. In the study, this third objective is expressed as

$$\text{Maximize } Z_3 = \text{DO}^{\text{std}}_{\min} , \tag{12}$$

where the new decision variable $\text{DO}^{\text{std}}_{\min}$ is the minimum required DO standard in the stream.

Similar to the difference in equity measure, the water quality compliance reliability at different control points will not be uniform. To utilize a single representative measure of compliance reliability for the entire system, a conservative view of looking at the lowest reliability was adopted. The objective is to maximize this lowest compliance reliability, i.e.

$$\text{Maximize } \alpha_{\min} = \min\,[\alpha_1, \alpha_2, \ldots, \alpha_M] . \tag{13}$$

By the definition of $\alpha_{\min}$, the chance constraints for water-quality compliance, Equation (5), would satisfy the following relation

$$\Pr\left[a_{oi} + \sum_{j=1}^{n_i} \Theta_{ij} B_j + \sum_{j=1}^{n_i} \Omega_{ij} D_j + \text{DO}^{\text{std}}_{\min} \leqslant \text{DO}^{\text{sat}}_i\right] \geqslant \alpha_{\min} \tag{14}$$

The corresponding deterministic equivalent of Equation (14) can be expressed as

$$\sum_{j=1}^{n_i} E[\Theta_{ij}]B_j + \sum_{j=1}^{n_i} E[\Omega_{ij}]\text{D}^{\text{std}}_{\min} + Z(\alpha_{\min}) \sqrt{(\mathbf{B}^t, \mathbf{D}^t)\, \mathbf{C}(\Theta_i, \Omega_i)\, (\mathbf{B}, \mathbf{D})} \leqslant R''_i \tag{15}$$

in which $R''_i = \text{DO}_i - E[a_{0i}]$.

Note that the original objective function in Equation (13) was to maximize $\alpha_{\min}$. However, under the assumption that the standardized left-hand sides of the water-quality constraints, i.e., z_i's, are continuous and unimodal random variables, the decision variable $\alpha_{\min}$ would have a strictly increasing relation with $z(\alpha_{\min})$. Therefore, maximization of $\alpha_{\min}$ is then equivalent to maximizing $z(\alpha_{\min})$. In the actual model solving, it is more convenient to replace Equation (13) by

$$\text{Maximize } Z_4 = z(\alpha_{\min}). \tag{16}$$

Note that the substituted decision variable $z(\alpha_{\min})$ is unrestricted-in-sign. The objective function of maximizing the lowest compliance reliability is equivalent to minimizing the largest water-quality violation risk.

5. Solving Multiple-Objective Stochastic WLA Model

Various methods were developed for solving multiobjective problems (Gohon, 1978; Geocoichea *et al.* 1980; Haimes, 1977). In general, the solution techniques can be categorized into one of the two types: (i) generating techniques and (ii) techniques incorporating preference information (Cohon, 1978). In this study, one of the generating techniques called the constraint method was employed.

The constraint method was first cited by Marglin in the book by Maass *et al.* (1962) and again by Marglin (1967). This approach enables analysts to generate the noninferior solution set in entirety without regards to convexity. The computational simplicity is probably the most distinguished advantage of the constraint method. When using the constraint method, the multiobjective problem is solved by adopting only one objective in the objective function. The remaining objectives are simply transformed into constraints in the problem formulation.

Once the multiobjective problem has been formulated, the constraint method provides a relatively effortless computational methodology for generating the noninferior solution set. Moreover, if the multiobjective formulation follows an LP format, the constraint method can be solved by a parametric LP approach. For a detailed analysis of the attributes of the constraint method readers should consult Cohon (1978).

In summary, the multiobjective stochastic WLA problem described above can be cast into the following format to be solved by the constraint method.

$$\text{Maximize } z(\alpha_{\min}) \,. \tag{17}$$

Subject to

$$\sum_{j=1}^{n_i} E[\Theta_{ij}]B_j + \sum_{j=1}^{n_i} E[\Omega_{ij}]D_j + \mathrm{DO}_{\min}^{\mathrm{std}} + Z(\alpha_{\min}) \sqrt{(\mathbf{B}^t, \mathbf{D}^t)\, \mathbf{C}(\Theta_i, \Omega_i)\, (\mathbf{B}, \mathbf{D})} \leqslant R_i'' \tag{18}$$

$$0.35 \leqslant \frac{B_j}{I_j} \leqslant 0.90, \quad \text{for } j = 1,2,\ldots N. \tag{19}$$

$$\left| \left(\frac{B_j}{I_j}\right) - \left(\frac{B_{j'}}{I_{j'}}\right) \right| - \delta E_{\max} \leqslant 0 \quad \text{for all } j \neq j' \tag{20}$$

$$\sum_{j=1}^{N} (B_j + D_j) \geqslant Z_1^0, \tag{21}$$

$$\mathrm{DO}_{\min}^{\mathrm{std}} \geqslant Z_3^0, \tag{22}$$

$$\delta E_{\max} \leqslant Z_3^0, \tag{23}$$

and nonnegativity constraints for the decision variables except for $z(\alpha_{\min})$. In the above formulation, the right-hand sides Z_1^0, Z_2^0, and Z_3^0 are the values of objective functions 1, 2, and 3, respectively, which are to be varied parametrically.

6. Model Solution Technique

The deterministic equivalent transformation of chance-constrained water-quality constraints resulted in the presence of nonlinearity as shown in Equation (15). The problem became one of the linear optimization which could be solved by various nonlinear programming techniques such as the generalized reduced gradient technique (Lasdon and Warren, 1979) and others.

Alternatively, this study adopted a procedure to linearize the nonlinear terms of the water-quality constraints in the stochastic WLA model and solved the linearized model by an LP technique iteratively.

Tung (1986) used the first-order Taylor's expansion to linearize a nonlinear terms involving the squared root of the variance which is a quadratic function of waste load decision variables. The linearization procedure required an initial guess of the solution to the optimization problem which was not known. As a result, the linearized problem had to be solved iteratively until the solution converges, since the linearization process utilized by Tung (1986) was a cumbersome exercise in this case and the resulting linearized model still had to be solved iteratively. In this study, the assumed solutions to the stochastic WLA model were used to calculate the value of the squared root terms and were treated as a constant associated with the decision variable $z(\alpha_{\min})$. The resulting linearized water-quality constraints in the stochastic WLA model could then be written as

$$\sum_{j=1}^{n_i} E[\Theta_{ij}]B_j + \sum_{j=1}^{n_{iE}} [\Omega_{ij}]D_j + \mathrm{DO}_{\min}^{\mathrm{std}} + z(\alpha_{\min}) \sqrt{(\hat{\mathbf{B}}^t, \hat{\mathbf{D}}^t)\, \mathbf{C}(\Theta_i, \Omega_i)\, (\hat{\mathbf{B}}, \hat{\mathbf{D}})} \leqslant R_i' \tag{24}$$

in which $\hat{\mathbf{B}}$ and $\hat{\mathbf{D}}$ are the assumed solution vectors to the stochastic WLA model.

Consequently, the linearized stochastic WLA model can then be solved by the LP technique iteratively, each time comparing the values of the current solutions with those obtained in the previous iteration. Then, the current solutions were used to compute the covariance of the left-hand sides (LHS_i) in each of the stochastic water quality constraints

$$\mathrm{LHS}_i = a_{oi} + \sum_{j=1}^{n_i} \Theta_{ij} B_j + \sum_{j=1}^{n_i} \Omega_{ij} D_j \tag{25}$$

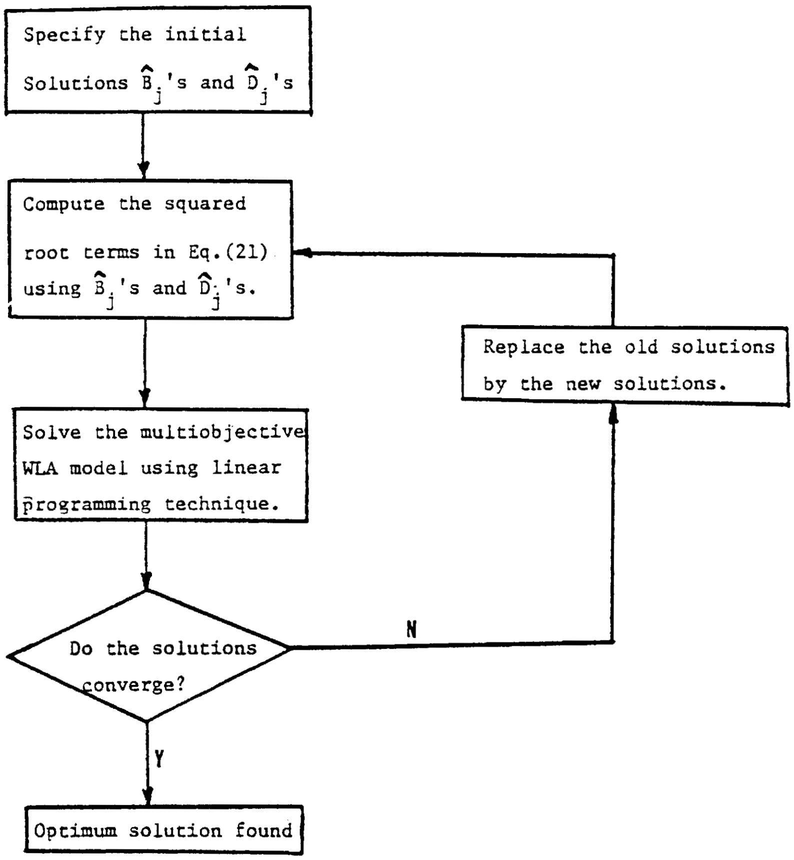

Fig. 1. Flow chart for solving the linear multi-objective stochastic waste-load allocation model.

until convergence criteria were met between any two successive iterations. A flow chart depicting the procedures is shown in Figure 1. Of course, alternative stopping rules could be incorporated in the algorithm to prevent excessive iteration during the computation.

To solve the multiobjective stochastic WLA model as formulated above requires no knowledge about the distribution of random LHS_i's. However, in order to assess the minimum compliance reliability of water-quality constraints, the probability distribution for the LHS_i must be known or assumed. Once such distributional assumption is made, the minimum compliance reliability can be determined when the solution technique converges at which time the means and variances of LHS_i can be evaluated.

It should be noted that the decision variable $z(\alpha_{\min})$ is not without upper bound. The highest value possible for $z_i(\alpha_i)$, as can be observed form Equation (15), could be achieved only when there is no waste discharged into the stream system, i.e. $\mathbf{B} = \mathbf{0}$ and $\mathbf{D} = \mathbf{0}$,

$$z^*_i = \frac{R''_i}{\sqrt{\mathrm{Var}\,(a_{oi})}} \tag{26}$$

in which Var (a) is a variance operator. Therefore, the upper bound of $z(\alpha_{\min})$ is equal to

$$z^*_{\min} = \min \{z^*_1, z^*_2, \ldots, z^*_M\}.$$

As the solution iteration proceeds, the upper bound for $z(\alpha_{\min})$ needs to be updated accordingly.

Under the normality assumption for the LHS_i's in Equation (26), the highest minimum compliance reliability can be easily computed by utilizing the standard normal distribution. However, when lognormal distribution was assumed, the same value for z_i's in different water-quality constraints does not necessarily indicate the same compliance reliability because the higher moments may not be the same. In this case, the procedure is, first, to identify the binding water-quality constraints and, then, calculate the associated compliance reliability. The smallest reliability from the binding constraints will be the largest minimum compliance reliability achievable by the stream system.

Due to the nonlinear nature of the stochastic WLA model, it should also be pointed out that, in general, the global optimum solution cannot be assured. Thus, it was suggested that a few runs of the solution procedure with different initial solutions should be carried out to ensure that model solution converges to the overall optimum. Other suggestions such as how to select proper initial solutions for the iterative procedure, particularly for the optimal WLA problems, can be found elsewhere (Tung and Hathhorn, 1990).

7. Example Application

The means and standard deviations for the stream water-quality parameters are shown in Tables I and II. An illustration of the six-reach example is given in Figure 2. To assess the statistical properties (i.e., the mean and covariance matrix) of the technological transfer coefficients in the water-quality constraints for this example, 200 sets of technological coefficients were generated by the unconditional simulation approach under the condition that all stream water-quality parameters are normally distributed. It was found numerically by Tung and Hathhorn (1990) that the statistical properties of Θ_{ij} and Ω_{ij} reached a very stable values based on 200 sets of simulated parameters. The mean an covariance matrix of the technological coefficients computed from the simulated results were used in this four-objective stochastic WLA model. However, for purpose of illustration, spatial independence of water-quality parameters was considered in estimating the means and covariance matrices of the technological coefficients in the water-quality constraints.

Based on the study by Tung and Hathhorn (1988), a two-parameter lognormal distribution was found to be the best parametric distribution for describing the Do deficit concentration computed by the Streeter–Phelps equation regardless of the probability distribution of water-quality parameters and the correlation between reaeration coefficient and average flow velocity. Therefore, adoption of a lognormal

Table I. The mean values of physical stream parameters used in the example of WLA model

(a) Mean stream characteristics for each reach

Reach i	Deoxygen coeff. K_d	Reaera. coeff. K_a	Avg. stream velocity U	Raw waste concen. I	Effluent flow rate q
1	0.6	1.84	26.4	1370	0.0042
2	0.6	2.13	26.4	6	1.2460
3	0.6	1.98	26.4	665	0.1308
4	0.6	1.64	26.4	910	1.0141
5	0.6	1.64	26.4	1500	0.0906
6	0.6	1.48	26.4	410	0.0221
Units	1/day	1/day	km/day	mg/l	m^3/sec

(b) Background characteristics

Upstream waste concen. L_0	Downstream flow rate Q_0	Upstream DO deficit D_0
5.0	3.2568	1.0
mg/l BOD	m^3/sec	mg/l

Table II. Standard deviations used for the physical stream characteristics

(a) For each reach

Reach i	Deoxygen coeff. K_d	Reaera. coeff. K_a	Avg. stream velocity U
1–6	0.2	0.4	6.4
Units	1/day	1/day	km/day

(b) Background characteristics

Upstream waste concen. L_0	Downstream flow rate Q_0	Upstream DO deficit D_0
1.0	0.561	0.3
mg/l BOD	m^3/sec	mg/l

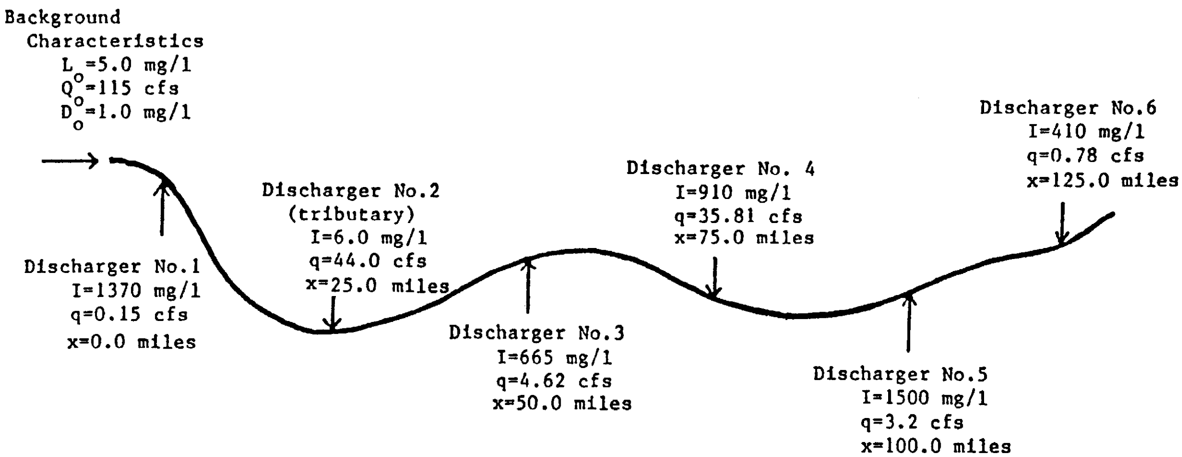

Fig. 2. Schematic sketch of the example system in WLA problem.

distribution for the random left-hand side, LHS_i, given in Equation (25) were made to compute the minimum water-quality compliance reliability once the model is solved.

The tradeoff curves among the various objectives considered for a given minimum DO standard concentration are shown in Figures 3–5. As can be seen that, for a specified minimum DO standard and total waste loading, the largest water-quality violation risk decreases as the largest difference in equity measure increases. Increase in equity measure implies a larger tolerance for the 'unfairness' among waste dischargers. As the level of minimum required DO standard is raised, the set of tradeoff curves move upward. To show the tradeoff for different minimum DO

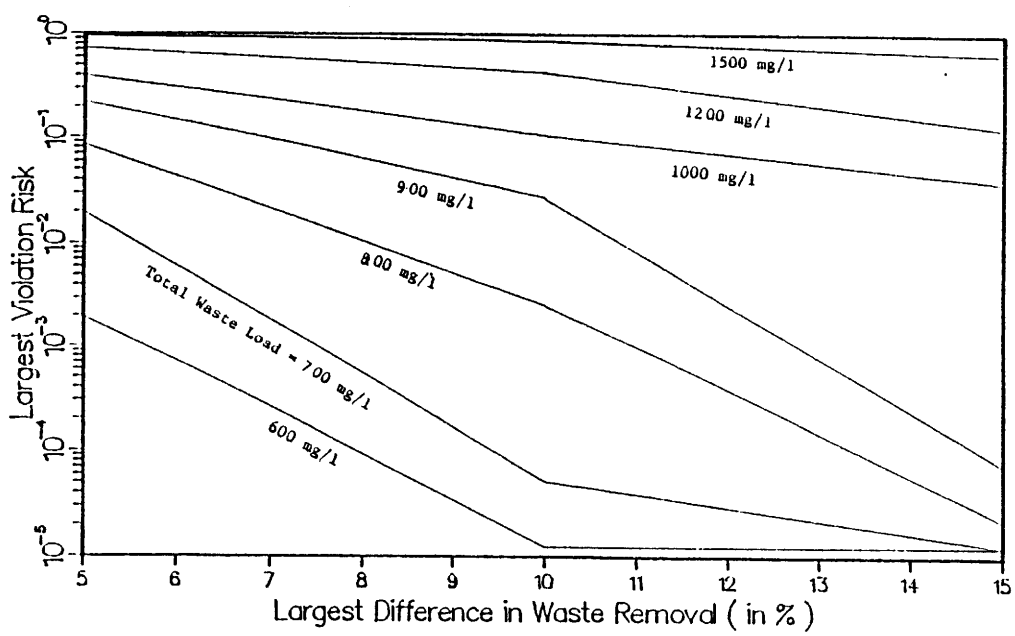

Fig. 3. Tradeoff curves of the various objectives in stochastic WLA problem with 4 mg/l minimum DO standard.

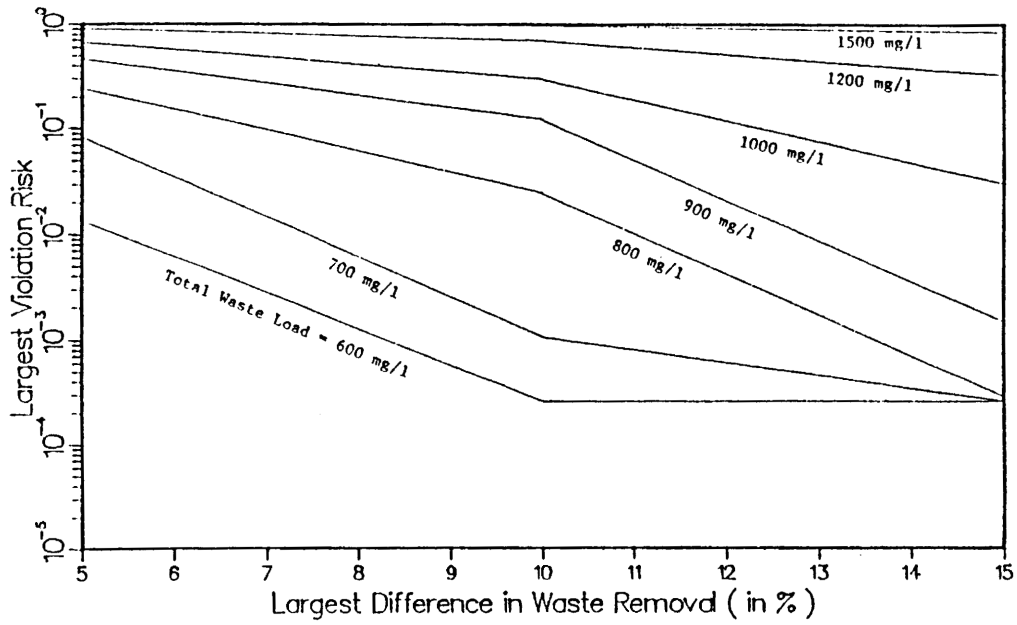

Fig. 4. Tradeoff curves of the various objectives in stochastic WLA problem with 5 mg/l minimum DO standard.

Standard, Figures 6 and 7 were plotted for the risk of water-quality violation, equity measurement, and water-quality standard while the total waste load were fixed at specified levels.

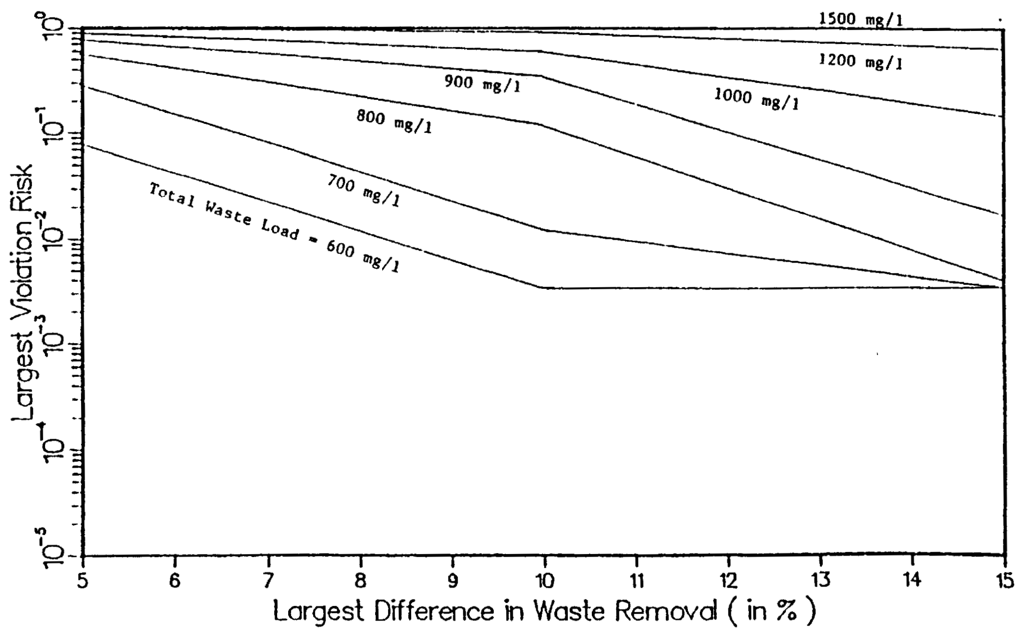

Fig. 5. Tradeoff curves of the various objectives in stochastic WLA problem with 6 mg/l munimum DO standard.

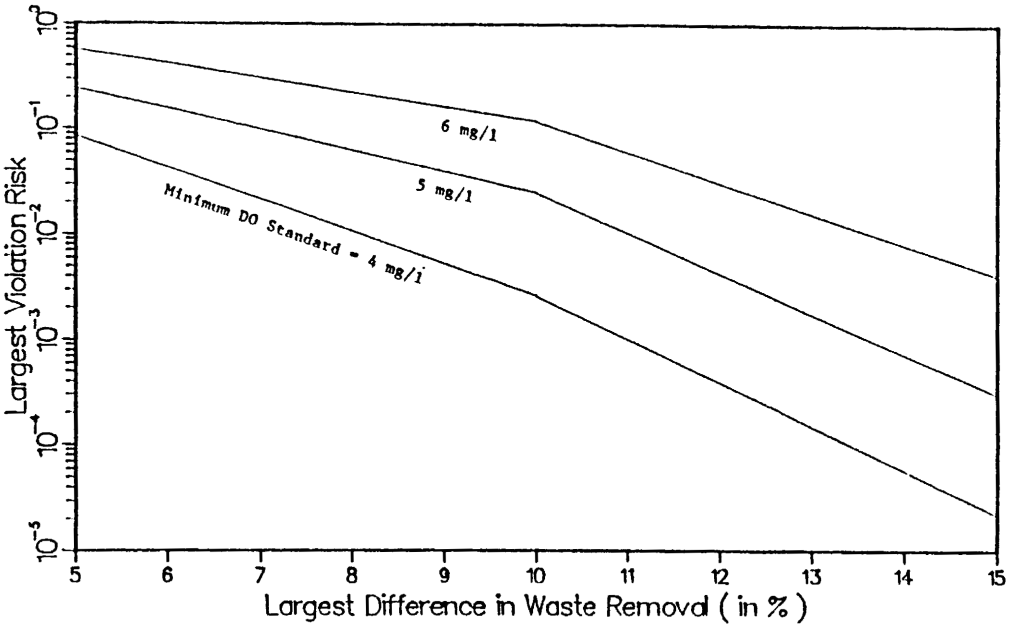

Fig. 6. Tradeoff curves of the various objectives with total waste load fixed at 800 mg/l.

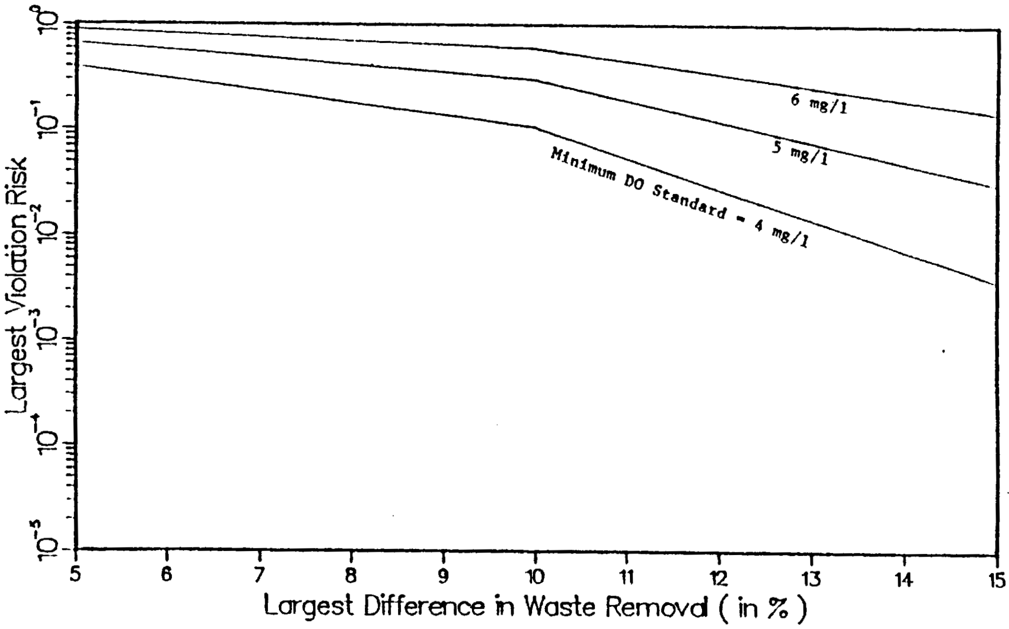

Fig. 7. Tradeoff curves of the various objectives in stochastic WLA problem with the total waste load fixed at 1000 mg/l.

8. Summary and Conclusions

Most environmental management problems, including waste-load allocation, are multiobjective by nature and should be treated accordingly. In an attempt to improve river-water quality management-practice, this paper presented a methodology to analyze a four-objective stochastic WLA problem using the constraint method. The model developed considered explicitly the uncertainties in water-quality parameters. The multi-objective model presented here was applied to a multiple-discharger river system in which the objectives of maximization of total waste discharge, minimization of the largest differences in equity measure among waste dischargers, maximization of minimum DO standard, and maximization of lowest water quality compliance reliability were considered. The relevance of this multi-objective approach to the problem is that a more realistic solution to the problem of WLA could be identified by specifying the tradeoffs (given by the noninferior solution set) among the four objectives. This information can then be passed on to the decision-making entity where the ultimate responsibility of management policy lies. The information provided by this approach will likely enhance the decision-maker's ability to select a 'best-compromising' solution given the set of alternatives to the problem of optimal river water-quality management.

Acknowledgements

The research in part was funded under the USGS Federal Water Research Program. The author is also grateful to the Wyoming Water Research Center for providing additional funding on the study.

References

Brill, E., Liebman, J., and ReVelle, C., 1976, Equity measures for exploring water-quality management alternatives, *Water Resour. Res.* **12**, 845–851.

Burn, D. H. and McBean, E. A., 1985, Optimization modeling of water-quality in an uncertain environment, *Water Resour. Res.* **21**, 934–940.

Chadderton, R. A., Miller, A. D., and McDonnell, A. J., 1981, Analysis of waste load allocation procedure, *Water Resour. Bull.* **17**(5), 760–766.

Chadderton, R. A., Miller, A. C., and McDonnell, A. J., 1982, Uncertainty analysis, of dissolved oxygen model, *J. Environ. Engr.* **108**(5), 1003–1012.

Charnes, A. and Cooper, W. W., 1963, Deterministic equivalents for optimizing and satisficing under chance constraints, *Operations Res.* **11**(1), 18–39.

Cohon, J. L., 1978, *Multiobjective Programming and Planning*, Academic Press, New York.

Dobbins, W. E., 1964, BOD and oxygen relationships in stream, *J. of Sani. Engr. Div.* **90**(4), 53–78.

Ellis, J. H., 1987, Stochatic water-quality optimization using imbedded chance constraints, *Water Resour. Res.* **23**(12), 2227–2238.

Esen, I. I. and Rathbun, R. E., 1976, A stochastic model for predicting the probability distribution of the dissolved oxygen deficit in streams, USGS, *Professional Paper* 913.

Fujiwara, O., Gnanendran, S. K., and Ohgaki, S., 1986, River quality management under stochastic streamflow, *J. Environ. Engr.* **112** [EE2], 185–198.

Goicoechea, A., Hansen, D. R., and Duckstein, L., 1982, *Multi-Objective Decision Analysis with Engineering and Business Applications*, Wiley, New York.

Gross, W. M., 1965, A lawyer looks at stream pollution, *Civil Engineering*; 44–45.

Haimes, Y. Y., 1977, *Hierarchical Analyses of Water Resources Systems*, McGrawHill, New York.

Hathhorn, W. E., 1986, Optimal waste load allocation in a stream environment under uncertainty, MS Thesis, Department of Civil Engineering, University of Wyoming, Laramie, Wyoming.

Hornberger, G. M., 1980, Uncertainty in dissolved oxygen prediction due to variability in algal photosyntheses, *Water Res.* **14**, 355–361.

Kolbin, V. V., 1977, *Stochastic Programming*, D. Reidel, Dordrecht.

Kothandaraman, V. and Ewing, B. B., 1969, A probabilistic analysis of dissolved oxygen-biochemical oxygen demand relationship in stream. *J. Water Pollut. Control Fed.* **41**(2), R73–R90.

Krenkel, P. A. and Novotony, V., 1980, *Water Quality Management*, Academic Press, New York.

Lasdon, L. S. and Warren, A. D., 1979, Generalized reduced gradient software for linear and nonlinear constrained problems, in H. Greenferg (ed.), *Design and Implementation for Optimization Sopftware*, Sijthoff and Noordhoff.

Lohani, B. N. and Thanh, W. R., 1978, Stochastic programming model for water-quality management in a river, *J. Water Pollut. Control Fed.* **50**, 2175–2182.

Loucks, D. P., Revelle, C. S., and Lynn, W. R., 1967, Linear programming models for water pollution control, *Management Sci.*, B166–B181.

Mass, A., *et al.*, 1962, *Design of Water Resource Systems*, Harvard University Press, Cambridge, MA.

Marglin, S. A., 1967, *Public Investment Criteria*, MIT Press, Cambridge, Mass.

Miller, W. L. and Gill, J. H., 1976, Equity considerations on controlling nonpoint pollution from agricultural sources, *Wat. Resour. Bull.* **12**(2), 253–261.

Streeter, H. W. and Phelps, E. B., 1925, A study of the pollution and natural purification of the Ohio River, *Public Health Bulletin* 146, U.S. Public Health Service, Washington, D.C.

Tung, Y. K., 1986, Groundwater management by chance-constrained model, *J. Water Resour. Plan. Mgmt.* **122**(1), 1–19.

Tung, Y., K. and Hathhorn, W. E., 1988, Assessment of probability distribution of dissolved oxygen deficit, *J. Environ. Engr,.* **114**(6), 1421–1435.

Tung, Y. K. and Hathhorn, W. E., 1989, Multiple-objective waste load allocation, *Intl. J. Water Resour. Mgmt.* 3; 129–140.

Tung, Y. K., and Hathhorn, W. E., 1990, Stochastic waste load allocation, *J. Ecol. Model.* **51**, 29–46.

Ward, R. C. and Loftis, J. C., Incorporating the stochastic nature of water-quality into management, *J. Water Pollut Control Fed.* **55**(4), 408–414.

Yaron, D., 1979, A method for the analysis of seasonal aspects of water-quality control, *J. Environ. Econ. Mgmt.* **6**.