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COMPARISON OF CORRECTION FACTOR FOR DISPERSION COEFFICIENT BASED ON ENTROPY AND LOGARITHMIC VELOCITY PROFILES

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Abstract Recently, new velocity distribution equations have been developed by Chiu (1987, 1989) through the use of maximum entropy principle. A comparison will be made of the correction factor (β) corresponding to entropy and logarithmic velocity profiles when applied to the diffusion coefficient in computing the suspended solid concentration in sediment-laden flow.

Introduction

Suspended solid (SS) concentration in streams in one of the major concern in non-point source pollution control and management. In the modeling of SS concentration, the diffusion coefficient and velocity distribution are the two essential factors which govern the mass transport in rivers. Because the diffusion coefficient is a function of velocity gradient, the common thread in the assessment of diffusion coefficient and SS concentration is the velocity distribution.

When apply the diffusion equation to sediment-laden flow, the turbulent diffusion coefficient for clear water is replaced by the diffusion coefficient of solid which would depend on particle size and solid concentration. A common approach to account for the effect of suspended solid is to multiply a correction factor to the turbulent diffusion coefficient.

Recently, new velocity distribution equations have been developed by Chiu (1987, 1989) through the use of maximum entropy principle. Chiu and Karaffa (1989) compared the depth-averaged diffusion coefficient using velocity distri-

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butions based on power law, logarithmic law, and entropy principle. The paper investigate the relations between the correction factors based on logarithmic and entropy velocity distributions.

Velocity Distribution Equations

Two types of velocity distribution equations in open channel hydraulics are commonly used. The first type is based on the power law

$$u = m\ y^{1/n} \tag{1}$$

in which y is the vertical distance from the channel bed, and m and n are constants. The second type of velocity distribution uses logarithmic law as

$$u = \frac{u^*}{\kappa} \ln(y/a) + u_a\ ,\quad a<y\le D \tag{2}$$

in which u_* is the shear velocity ($\sqrt{\tau_o/\rho}$) with ρ being the density of water and τ_o being the shear stress on channel bed; κ is von Karman's constant, 0.4; a is the equivalent distance from channel bed at which the velocity is zero; and D is depth of water.

A simplified version of the new velocity distribution developed by Chiu (1987) can be written as

$$u = \frac{1}{\lambda_1} \ln(1+\theta\delta D) \tag{3}$$

where λ_1 is the constant, parameter θ is a function of maximum velocity (u_{max}) as $\theta=\exp(\lambda_1\ u_{max})-1$, and $\delta=y/D$.

The velocity profiles described by eqs. 1-3 have the maximum velocity occurred on the free surface. A more complete velocity distribution equation based on entropy principle has been derived by Chiu (1989). It allows one to model vertical velocity profile having maximum velocity occurring beneath the free surface which is frequently observed during flow measurements. For purpose of simplifying the analysis, this paper adopts eq. 3 in the following investigation.

Diffusion Coefficient

The SS concentration in open channel under non-uniform and steady state condition can be modeled as

$$\epsilon_s(y) \frac{dC}{dy} + v_{ss} C = 0 \tag{4}$$

in which C is SS concentration, v_{ss} is the settling velocity of particle, and ϵ_s is the diffusion coefficient in the sediment-laden flow. In eq. 4, both C and ϵ_s are function of flow depth y. Utilizing eq. 4 to compute SS concentration a commonly used approach is to replace ϵ_s by $\beta\epsilon$ in which ϵ is the turbulent diffusion coefficient in clear water and β is the correction factor to account for the presence of SS.

Based on turbulent diffusion theory, the diffusion coefficient for momentum transport (ϵ) along the vertical direction (y-direction) in a wide channel is related to the shear stress as

$$\tau(y) = \rho\ \epsilon_s(y) \frac{du}{dy} \tag{5}$$

in which $\tau(y)$ is the shear stress at distance y above channel bed. From eq. 5, the diffusion coefficient can be obtained as

$$\epsilon_s(y) = u_*^2 \left(\frac{du}{dy}\right)^{-1} \left(1 - \frac{y}{D}\right) \tag{6}$$

Substituting eqs. 2-3, respectively, in to eq. 6 the expressions for diffusion coefficient $\epsilon(y)$ can be derived, for logarithmic law as

$$\epsilon_L(y) = u_*\ \kappa\ D\ \delta(1-\delta) \tag{7}$$

and for entropy principle as

$$\epsilon_E(y) = \frac{u_*^2\ \lambda_1\ D}{\theta} (1-\delta)\ (1+\theta\delta) \tag{8}$$

From eq. 7, the diffusion coefficient based on logarithmic velocity distribution reaches it maximum at y=D/2 and decreases symmetrically toward the channel bed and water surface. On the other hand, the diffusion coefficient based on entropy velocity profile has the maximum value at the channel bed and it decreases to zero as y approaches to D.

In a recent study about the diffusion coefficient, Chiu and Karaffa (1989) utilized the depth-averaged diffusion

coefficient to study the SS concentration profile. The depth-averaged diffusion coefficient based on logarithmic law ($\bar{\epsilon}_L$) and entropy principle ($\bar{\epsilon}_E$) can be derived using eqs. 7 and 8 as (Chiu and Karaffa, 1989)

$$\bar{\epsilon}_L = \frac{u_*\ \kappa\ D}{6} \left(\frac{1 - 3a'^2 - 2a'^3}{1 - a'}\right) \tag{9}$$

with a'=a/D and

$$\bar{\epsilon}_E = \frac{u_*^2\ \lambda_1\ D}{6} \left(1 + \frac{1}{\theta}\right) \tag{10}$$

SS Concentration Profile

Equations 7 and 8 can be substituted back to eq. 4 and the expressions for SS concentration profile can be derived as

$$\ln\left(\frac{C}{C_a}\right)_L = \frac{z}{\beta_L} \ln\left(\frac{1-\delta}{\delta}\ \frac{a'}{1-a'}\right) \quad ,\ a' < \delta \le 1 \tag{11}$$

and

$$\ln\left(\frac{C}{C_a}\right)_E = \frac{w}{\beta_E} \ln\left(\frac{1-\delta}{1+\theta\delta}\ \frac{1+\theta a'}{1-a'}\right),\ 0 \le a' < \delta \le 1 \tag{12}$$

in which $\ln(C/C_a)_L$ and $\ln(C/C_a)_E$ are the natural logarithm of ratios of SS concentration at vertical distance y to that of at a reference distance a from channel bed, $z=v_{ss}/(u_*\kappa)$, and $w=(v_{ss}\theta)/(u_*^2\lambda)$. Equation 11 is the well-known Rouse concentration equation.

Using the depth-averaged diffusion coefficient ($\bar{\epsilon}_L$ and $\bar{\epsilon}_E$) the expressions for SS concentration profile (Chiu and Karaffa, 1989) are greatly simplified as

$$\ln\left(\frac{C}{C_a}\right)_L = - \frac{v_{ss}}{\bar{\beta}_L \bar{\epsilon}_L}\ \ln[(\delta - a')D] \tag{13}$$

and

Table 1. Relevant data used in numerical examples (from Karim and Kennedy, 1986)

Run #	D(cm)	u_{max} (cm/s)	a (cm)	u_* (cm/s)	v_{ss} (cm/s)
1	8.7	49.2	0.0574	4.329	0.719
2	7.4	73.5	0.0448	3.119	0.712
3	7.8	83.1	0.0490	3.220	0.728
4	7.7	81.9	0.0724	3.735	1.652

Table 2. Ratios of correction factors based on logarithmic velocity distribution to entropy velocity distribution.

	β_L/β_E			
δ	Run 1	Run 2	Run 3	Run 4
0.10	0.1057E+03	0.1238E+03	0.1360E+03	0.1352E+03
0.20	0.6529E+02	0.7435E+02	0.8150E+02	0.8266E+02
0.30	0.4872E+02	0.5410E+02	0.5915E+02	0.6070E+02
0.40	0.3926E+02	0.4259E+02	0.4644E+02	0.4807E+02
0.50	0.3292E+02	0.3492E+02	0.3798E+02	0.3960E+02
0.60	0.2821E+02	0.2927E+02	0.3174E+02	0.3333E+02
0.70	0.2441E+02	0.2477E+02	0.2677E+02	0.2830E+02
0.80	0.2107E+02	0.2087E+02	0.2249E+02	0.2394E+02
0.90	0.1775E+02	0.1709E+02	0.1834E+02	0.1970E+02
0.99	0.1321E+02	0.1213E+02	0.1294E+02	0.1414E+02

$$\ln\left(\frac{C}{C_a}\right)_E = -\frac{v_{ss}}{\bar{\beta}_E\bar{\epsilon}_E}\ln[(\delta-a')D] \tag{14}$$

Comparison of Correction Factor (β)

In the modeling of SS concentration, it is common that the correction factor (β) is treated as a constant for a specified velocity distribution equation. In fact, the correction factor may vary with respect to y. The relationship between the correction factor based on different velocity distributions that yields the same SS concentration profile can be developed utilizing the SS concentration equations. Under the condition that the SS concentration profiles are equal, the ratio of correction factor using logarithmic velocity distribution to that using entropy velocity distribution can be derived from eqs. 11 and 12 as

$$\frac{\beta_L}{\beta_E} = \frac{u_*\,\lambda_1}{\kappa\;\theta}\;\frac{\ln\left(\dfrac{1-\delta}{\delta}\;\dfrac{a'}{1-a'}\right)}{\ln\left(\dfrac{1-\delta}{1+\theta\delta}\;\dfrac{1+\theta}{1-a'}\right)} \tag{15}$$

Table 2 shows the numerical values of ratio of correction factors based on logarithmic velocity distribution and entropy velocity distribution. The data use in the numerical computations are extracted from Karim and Kennedy (1986) which are given in Table 1.

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