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STOCHASTIC WASTE LOAD ALLOCATION

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ABSTRACT

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This paper considers solving a stochastic waste load allocation model in a chance-constrained format. The model explicitly takes into account the uncertainty of the water quality parameters including their spatial and cross-correlations. Random characteristics of the coefficients in the water quality constraints are obtained by using unconditional simulation. A hypothetical example is used to demonstrate the methodologies and investigate the effects of water quality compliance reliability and correlation structures of the water quality parameters on the optimal solution of the waste load allocation problem.

INTRODUCTION

Water quality management is the practice of protecting the physical, chemical and biological characteristics of various water resources. Historically, such efforts have been guided toward the goal of assessing and controlling the impacts of human activities on the quality of water. To implement water quality management measures in a conscious manner, one must acknowledge both the activities of the society and the inherently random nature of the stream environment itself (Ward and Loftis, 1983).

To date, much of the research in developing predictive water quality models has been based on a deterministic evaluation of the stream environment. Only in recent years has the random nature of the stream environment been recognized in the waste load allocation (WLA) process. Some notable pilot works in the development of stochastic WLA models include Lohani and Thanh (1979) and Yaron (1979). Nevertheless, their models are not complete in that only the streamflow or background pollution is considered a random variable. The limitations of such models was pointed out by Brill et al. (1979). Invariably, the main reason for having such idealizations is the ability to solve the stochastic WLA problem by well-known linear programming (LP) techniques.

More recently, Burn and McBean (1985) and Fujiwara et al. (1986) have developed stochastic WLA models using a chance-constrained formulation in an attempt to incorporate some of the uncertainty of the system into the optimization framework. Nevertheless, stream flow remained the only random variable considered. Later, Fujiwara et al. (1987) expand their previous model to include randomness of tributary flow and storm runoff. One of the earliest works to broaden the realm of uncertainty considered was that presented by Ellis (1987). Ellis (1987) used the management model of Fujiwara et al. (1986) to develop an imbedded chance-constrained formulation considering stochastic water quality parameters.

In this context, it should be noted that the left-hand-side (LHS) coefficients of the water quality constraints in a WLA model are functions of various random water quality parameters. As a result, these LHS coefficients are random variables as well. Furthermore, correlation exists among these LHS coefficients because: (1) they are functions of the same water quality parameters; and (2) some water quality parameters are correlated with each other. Moreover, the water quality parameters along a stream are spatially correlated. Therefore, to reflect the reality of a stream system, a stochastic WLA model should account for the randomness of the water quality parameters, including spatial and cross-correlations of each parameter.

The main objective of this paper is to present methodologies to solve a stochastic WLA problem in a chance-constrained frame-work. The randomness of the water quality parameters and their spatial and cross-correlations are also taken into account. A six-reach example is utilized to demonstrate these methodologies. Factors affecting the model solution to be examined are: (1) the distribution of the LHS coefficients in water quality constraints; and (2) the spatial correlation of water quality parameters.

GENERALIZED CHANCE-CONSTRAINED FORMULATION

In all fields of science and engineering, the decision-making process is dependent on several variables. More often than not at least one of these variables cannot be assessed with certainty. In particular, the environment in which decisions are to be made concerning instream water quality management are inherently subject to many uncertainties. The stream system itself, through nature, is an environment abundant with ever-changing and complex processes, both physically and biologically.

Attempts to manage such an environment deterministically implies that the compliance of water quality standards at all control points in the stream system could be assured with absolute certainty. This, of course, is unrealistic. The existence of the uncertainties associated with stream environments should not be ignored. Thus, it is more appropriate in such an environment to examine the performance of the constraints of a mathematical programming model in a probabilistic context.

Consider a generalized LP model formulation. By imposing a reliability restriction, α, on the system constraints, the LP model can be transformed into the following chance-constrained formulation:

$$\text{Maximize } \boldsymbol{c}^{\mathrm{t}}\boldsymbol{x} \tag{1}$$

subject to

$$\Pr\{\boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{b}\} \geqslant \alpha \tag{2}$$

$$\boldsymbol{x} \geq 0$$

where $\boldsymbol{\alpha}$ represents an m-dimensional column vector of desired performance reliability for each constraint, $0 \leq \boldsymbol{\alpha} \leq 1$; $\Pr\{\ \}$ is the probability operator; $\boldsymbol{x}$ and $\boldsymbol{c}$ are n-dimensional column vectors of decision variables and the associated objective function coefficients, respectively; $\boldsymbol{b}$ is an m-dimensional vector of the right-hand-side (RHS); and $\boldsymbol{A}$ is an m by n matrix of the technological coefficients (Taha, 1982); the subscript 't' represents the transpose of a vector. For a detailed analysis of chance-constrained problems, readers are referred to Charnes and Cooper (1963), Vajda (1972) and Kolbin (1977).

In chance-constrained models, the elements in $\boldsymbol{A}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ can be random variables. When the objective function coefficient c_j's are random variables, it is common to replace them by their expected values. Hence, three cases remain: (1) elements of the technological coefficient matrix (a_{ij}'s) are random variables; (2) elements of the RHS vector b_i's are random variables; and (3) elements a_{ij} and b_i are simultaneously random variables.

It should be noted that a probabilistic statement of the constraints, like the one in equation (2), is not mathematically operational. It is necessary to develop a deterministic equivalent for equation (2) if the model is to be solved. In doing so, the statistical characteristics of a random variable can be described by its probability distribution and statistical moments.

Consider the ith constraint whose technological coefficients a_{ij}'s are random. The deterministic equivalent of the chance constraint:

$$\Pr\left[\sum_{j=1}^{n} a_{ij}x_j \leq b_i\right] \geq \alpha_i \tag{3}$$

can be derived as (Vajda, 1972; Kolbin, 1977):

$$\sum_{j=1}^{n} E\left[a_{ij}\right] x_j + F_{Z_i}^{-1}(\alpha)\sqrt{\boldsymbol{x}'\boldsymbol{C}\boldsymbol{x}} \le b_i \tag{4}$$

where E[ ] is an expectation operator, $F_Z^{-1}(\alpha_i)$ the appropriate quantile for the α_i percentage given by the CDF of Z_i, and $\boldsymbol{C}$ an n by n covariance matrix of n technological coefficients $(a_{i1}, a_{i2}, \ldots, a_{in})$ in the ith constraint. If all a_{ij}'s are independent random variables, i.e. $\rho(a_{ij}, a_{ij'}) = 0$ for $j \neq j'$, $\boldsymbol{C}$ then is a diagonal matrix:

$$\boldsymbol{C} = \mathrm{diag}\left(\sigma_{i1}^2, \sigma_{i2}^2, \ldots, \sigma_{in}^2\right)$$

where $\rho(\ )$ is a correlation coefficient, and σ_{ij}^2 the variance of coefficient a_{ij}. In the case that correlation between a_{ij}'s exists, off-diagonal elements of $\boldsymbol{C}$ are non-zero. The resulting deterministic equivalents of the chance constraints, when a_{ij}'s are random, are no longer linear functions of the decision variables. The treatment of these nonlinearities is addressed in detail in a later section of this paper.

DETERMINISTIC WASTE LOAD ALLOCATION MODEL

Any number of pollutants may be considered in the overall quality management of a river system. However, the use of BOD–DO interactions in the WLA have been implemented in various optimization framework (Loucks et al., 1967; Lohani and Thanh, 1978; Burn and McBean, 1985). In this paper, BOD–DO water quality model is also adopted.

In LP format, the deterministic WLA model considered herein can be written as:

$$\text{Maximize} \sum_{j=1}^{N} \left(B_j + D_j\right) \tag{5}$$

subject to

- constraints on water quality:

$$a_{0i} + \sum_{j=1}^{n_i} \Theta_{ij} B_j + \sum_{j=1}^{n_i} \Omega_{ij} D_j \le \mathrm{DO}_i^{\mathrm{sat}} - \mathrm{DO}_i^{\mathrm{std}} \qquad \text{for} \quad i = 1, 2, \ldots, M \tag{6}$$

- constraints on treatment equity:

$$\left|\left(B_j/I_j\right) - \left(B_{j'}/I_{j'}\right)\right| \le E_{\mathrm{A}} \qquad \text{for} \quad j \neq j' \tag{7}$$

- constraints on treatment efficiency:

$$\underline{e}_j \le 1 - B_j/I_j \le \bar{e}_j \qquad \text{for} \quad j = 1, 2, \ldots, N \tag{8}$$

where B_j, D_j, and I_j are the effluent waste concentration (mg/L BOD), effluent dissolved oxygen (DO) deficit concentration (mg/L), and raw waste influent concentration (mg/L BOD) at discharge location j, respectively; N is the total number of waste dischargers. The LHS coefficients a_{0i}, Θ_{ij} and Ω_{ij} in equation (6) are the technological transfer coefficients relating impact on DO concentrations at downstream locations, i, resulting from the background waste and waste input at an upstream location, j. These technological transfer coefficients are functions of water quality parameters such as reaeriation and deoxygenation rates, flow velocity, etc. $\mathrm{DO}_i^{\mathrm{std}}$ and $\mathrm{DO}_i^{\mathrm{sat}}$ represent the required DO standard and saturated DO concentration at control point i, respectively. Finally, E_{A} is the allowable difference (i.e., equity) in treatment efficiency between two waste dischargers; and $\underline{e}_j$ and $\bar{e}_j$ are the lower and upper bounds of waste removal efficiency for the jth discharger, respectively. The importance of incorporating the treatment equity in the WLA problems is discussed by many researchers (Gross, 1965; Loucks et al., 1967; Brill et al., 1976; Miller and Gill, 1976; Chadderton et al., 1981).

Water quality constraint relating the response of DO to the effluent waste can be defined by water quality models such as the Streeter–Phelps equation (Streeter and Phelps, 1925) or its variations (Dobbins, 1964; Krenkel and Novotny, 1980). To demonstrate the proposed methodologies the original Streeter-Phelps equation is used to derive the water quality constraints. Expressions for Θ_{ij}, Ω_{ij} based on the Streeter–Phelps equation can be found elsewhere (Hathhorn and Tung, 1987).

CHANCE-CONSTRAINED WASTE LOAD ALLOCATION MODEL

The deterministic WLA model presented above, equations (5)–(8), serves as the basic model for deriving the stochastic WLA model. In considering the existence of uncertainty within the stream environment (Ellis, 1987), the water quality constraints given by equation (6) can be expressed probabilistically as:

$$\Pr\left[a_{01} + \sum_{j=1}^{n_i} \Theta_{ij} B_j + \sum_{j=1}^{n_i} \Omega_{ij} D_j \le \mathrm{DO}_i^{\mathrm{sat}} - \mathrm{DO}_i^{\mathrm{std}}\right] \ge \alpha_i \tag{9}$$

Based on equation (4) the deterministic equivalent of (9) can be derived as:

$$\sum_{j=1}^{n_i} E\left[\Theta_{ij}\right] B_j + \sum_{j=1}^{n_i} E\left[\Omega_{ij}\right] D_j + F_Z^{-1}(\alpha_i)\sqrt{(\boldsymbol{B}, \boldsymbol{D})^{\mathrm{t}}\, C(\boldsymbol{\Theta}_i, \boldsymbol{\Omega}_i)\,(\boldsymbol{B}, \boldsymbol{D})} \le R_i' \tag{10}$$

in which $R_i' = \mathrm{DO}_i^{\mathrm{sat}} - \mathrm{DO}_i^{\mathrm{std}} - \mathrm{E}[a_{ij}]$, $(\boldsymbol{B}, \boldsymbol{D})$ is the column vector of BOD and DO deficit concentrations in waste effluents, and $\boldsymbol{C}(\boldsymbol{\Theta}_i, \boldsymbol{\Omega}_i)$ the covariance matrix associated with the technological transfer coefficients in the ith water quality constraint, including a_{0i}. The stochastic WLA model to be solved consists of equations (5), (10), (7) and (8).

ASSESSMENTS OF THE STATISTICAL PROPERTIES OF RANDOM TECHNOLOGICAL COEFFICIENTS IN STOCHASTIC WLA MODEL

To solve the stochastic WLA model, it is necessary to assess the statistical properties of the random LHS in the chance-constraint equation (10). As mentioned previously, the technological transfer coefficients Θ_{ij} and Ω_{ij} are nonlinear functions of the stochastic water quality parameters which are cross-correlated amongst themselves within each stream reach and spatially correlated between stream reaches. Furthermore, the complexity of functional relationships between these transfer coefficients and the water quality parameters increases rapidly as the control point moves downstream. Hence, the analytical derivation of the statistical properties of Θ_{ij} and Ω_{ij} becomes a formidable task given even a small number of reaches. As a practical alternative, simulation procedures may be used to estimate the mean and covariance structure of the random technological coefficients within a given water quality constraint. In particular, the method of unconditional simulation developed in geostatistics is applied in this research to generate the random, spatially correlated water quality parameters (Journal and Huijbregts, 1978).

The assumptions made in the unconditional simulation to generate water quality parameters in all reaches of the stream system are as follows:

(1) The representative values for the reaeriation coefficient, deoxygenation coefficient, and average flow velocity in each reach are second-order stationary. That is, the spatial covariance functions of water quality parameters are dependent only on the 'space lag' or separation distance.

(2) Correlation between the reaeriation coefficient and average flow velocity exists only within the same stream reach.

(3) Background DO and BOD concentrations at the upstream end of the entire stream system are independent of each other and of all other water quality parameters.

(4) All water quality parameters follow a normal distribution.

Some investigators have considered a positive correlation between the reaeriaton and deoxygenation coefficients (Esen and Rathbun, 1976; Padgett, 1978). Although statistical analysis of a given data set might reveal a correlation between these parameters, it does not necessarily imply that such correlation has any meaningful physical representation of the system behavior. The reaeriation coefficient is a function of the physical characteristic of the stream whereas deoxygenation coefficient is characterized by the biological composition of the waste discharge and stream environment. Thus, the correlation between reaeriation and deoxygenation coefficients appears to be spurious and, hence, is not considered in this study.

In the unconditional simulation, variance-covariance matrices representing the spatial correlation of water quality parameter can be derived from the variogram models (Journal and Huijbregts, 1978). The three variogram models used in this study are:

– transitive variogram model:

$$C(|\boldsymbol{h}|) = \sigma^2\left[1 - \frac{|\boldsymbol{h}|}{h_0}\right] \qquad |\boldsymbol{h}| < h_0 \tag{11}$$

– spherical variogram model:

$$C(|\boldsymbol{h}|) = \sigma^2\left[1 - \frac{3}{2}\left(\frac{|\boldsymbol{h}|}{h_0}\right) + \frac{1}{2}\left(\frac{|\boldsymbol{h}|}{h_0}\right)^3\right] \qquad |\boldsymbol{h}| < h_0 \tag{12}$$

– Gaussian variogram model:

$$C(|\boldsymbol{h}|) = \sigma^2\left[\exp\left(\frac{-|\boldsymbol{h}|^2}{2h_0^2}\right)\right] \qquad |\boldsymbol{h}| < \sqrt{3}\,h_0 \tag{13}$$

in which $C(|\boldsymbol{h}|)$ represents the value of covariance between two measurements of the same water quality parameter separated by a distance $|\boldsymbol{h}|$ apart, h_0 is the length of zone of influence, and σ^2 the variance of water quality parameter within a given reach. When the distance between the centers of reaches exceeds h_0, the value of the covariance function goes to zero. Graphically, these three variograms are shown in Fig. 1.

To illustrate the concept, consider the water quality parameters reaeriation coefficient, K_a, and average flow velocity, U. From the variogram models, the variance–covariance matrix for the two parameters can be constructed as follows:

$$\boldsymbol{C}(\boldsymbol{K}_a, \boldsymbol{U}) = \begin{bmatrix} \boldsymbol{C}_{\boldsymbol{K}_a,\boldsymbol{K}_a} & \boldsymbol{C}_{\boldsymbol{K}_a,\boldsymbol{U}} \\ \boldsymbol{C}_{\boldsymbol{U},\boldsymbol{K}_a} & \boldsymbol{C}_{\boldsymbol{U},\boldsymbol{U}} \end{bmatrix} \tag{14}$$

in which $\boldsymbol{K}_a = (K_{a,1}, K_{a,2}, \ldots, K_{a,N})$ and $\boldsymbol{U} = (U, U_2, \ldots, U_N)$ are vectors of the reaeriation coefficient and average velocity in each stream reach, respectively (see Table 1). In equation (14), $\boldsymbol{C}_{\boldsymbol{K}_a,\boldsymbol{K}_a}$, $\boldsymbol{C}_{\boldsymbol{K}_a,\boldsymbol{U}}$, $\boldsymbol{C}_{\boldsymbol{U},\boldsymbol{U}}$ are N by N square, symmetric covariance matrices with N being the number of stream reaches in the WLA model. Submatrices $\boldsymbol{C}_{\boldsymbol{K}_a,\boldsymbol{K}_a}$ and $\boldsymbol{C}_{\boldsymbol{U},\boldsymbol{U}}$ define the spatial correlation of K_a and U between the reaches, while submatrix $\boldsymbol{C}_{\boldsymbol{K}_a,\boldsymbol{U}}$ defines the cross-correlation between K_a and U within the same reach. Under the assumption (2) mentioned above, the submatrix $\boldsymbol{C}_{\boldsymbol{K}_a,\boldsymbol{U}}$ is a diagonal matrix.

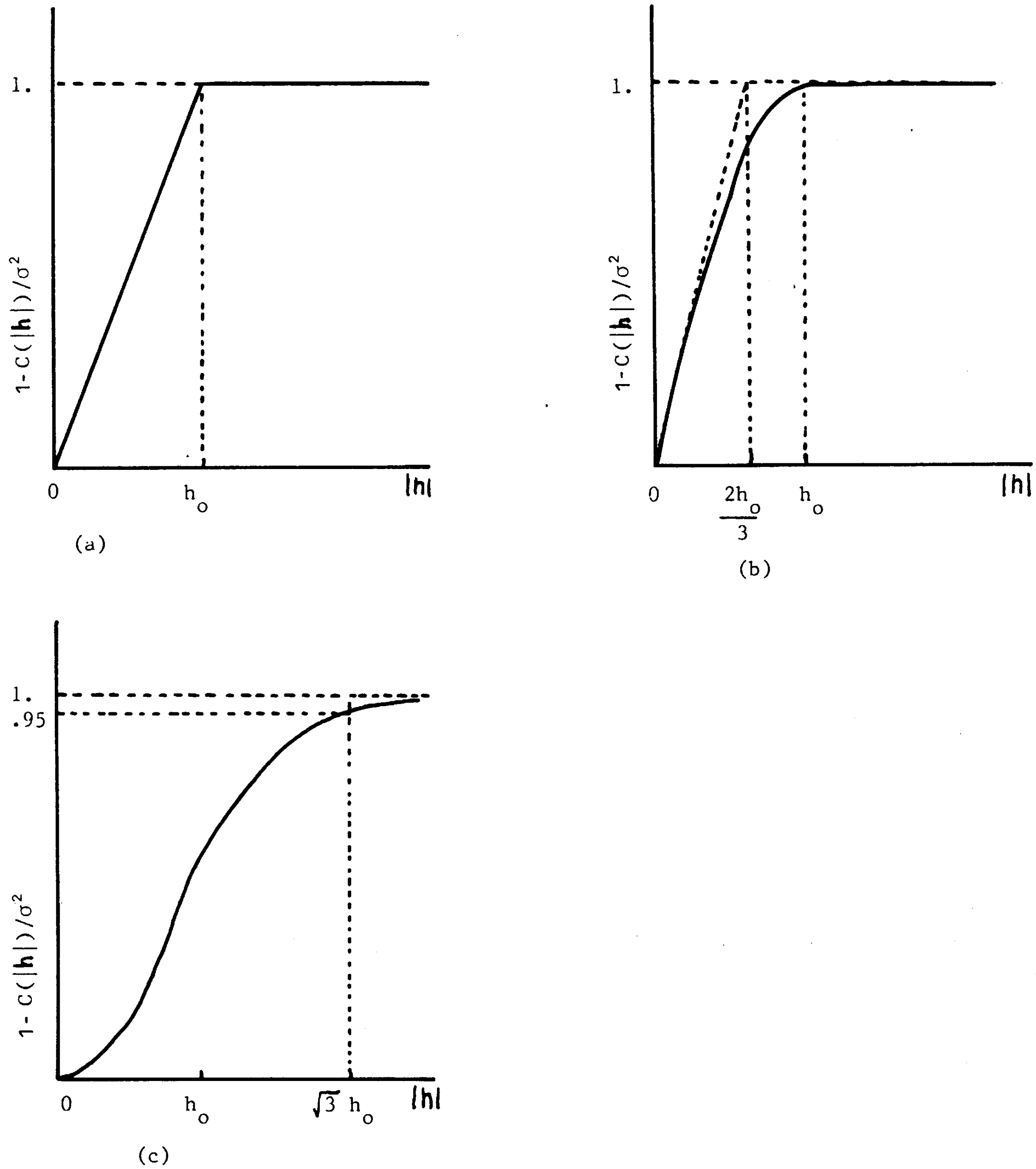

Fig. 1. Graphs of the three variogram models used: (a) transitive model; b) spherical model; (c) Gaussian model.

For water quality parameters which are not cross-correlated with other parameters, but are spatially correlated, the associated variance-covariance matrix has the form similar to $C_{U,U}$. For parameters that are spatially independent, their covariance matrices are diagonal.

To present the procedures of unconditional simulation, the arguments in the covariance matrix $C(K_a, U)$ are dropped. Note that the covariance matrix constructed using variogram model is positive semidefinite. Consider

TABLE 1
Structure of covariance matrix $C(K_a, U)$ for N-reach stream system

$$\left[\begin{array}{cccc|cccc}
C(K_{a_1},K_{a_1}) & C(K_{a_1},K_{a_2}) & \cdots & C(K_{a_1},K_{a_N}) & C(K_{a_1},U_1) & & & \emptyset \\
C(K_{a_2},K_{a_1}) & C(K_{a_2}) & \cdots & C(K_{a_2},K_{a_N}) & & C(K_{a_2},U_2) & & \\
\vdots & \vdots & \cdots & \vdots & & & \ddots & \\
C(K_{a_N},K_{a_1}) & C(K_{a_N},K_{a_2}) & \cdots & C(K_{a_N},K_{a_N}) & \emptyset & & & C(K_{a_N},U_N) \\
\hline
C(U_1,K_{a_1}) & & & \emptyset & C(U_1,U_1) & C(U_1,U_2) & \cdots & C(U_1,U_N) \\
 & C(U_2,K_{a_2}) & & & C(U_2,U_1) & C(U_2,U_2) & \cdots & C(U_2,U_N) \\
 & & \ddots & & \vdots & \vdots & \cdots & \vdots \\
\emptyset & & & C(U_N,K_{a_N}) & C(U_N,U_1) & C(U_N,U_2) & \cdots & C(U_N,U_N)
\end{array}\right]$$

the case that matrix $\boldsymbol{C}$ is strictly positive definite. Matrix $\boldsymbol{C}$ can be decomposed into:

$$\boldsymbol{C} = \boldsymbol{V}\boldsymbol{L}\boldsymbol{V}^{t} \tag{15}$$

in which matrix $\boldsymbol{L} = \text{diag}(\delta_1, \delta_2, \ldots, \delta_N)$ with all its eigenvalues $\delta_j > 0$, and $\boldsymbol{V}$ is an N by N matrix composed of the corresponding eigenvectors (Quimby, 1986). Based on the eigenvalues and eigenvectors matrices, a random water quality parameter for N stream reaches with a covariance matrix C can be generated as:

$$\boldsymbol{P} = \boldsymbol{\mu}_p + \boldsymbol{V}\boldsymbol{L}^{1/2}\boldsymbol{z} \tag{16}$$

in which $\boldsymbol{p}$ is an N by 1 column vector for a given water quality parameter, $\boldsymbol{\mu}_p$ is the vector of the mean values, and $\boldsymbol{z}$ is an N by 1 column vector of standardized independent random variables with zero mean and unit variance. If the random water quality parameter is normally distributed, the elements of vector $\boldsymbol{z}$ in equation (16) are the independent standard normal variates.

In summary, the unconditional simulation for generating spatial and cross-correlated water quality parameters can be outlined as follows:

(1) Identify the variogram model for a given water quality parameter and construct the corresponding covariance matrix $\boldsymbol{C}$.
(2) Apply eigenvalue-eigenvector decomposition to the covariance matrix $\boldsymbol{X}$ by using equation (15).
(3) Generate vector of standard random variates $\boldsymbol{z}$.
(4) Apply equation (16) to obtain the values of the water quality parameter for each reach in the WLA model.
(5) Repeat steps (1)–(4) for all water quality parameters.

For each set of the water quality parameters generated by steps (1)–(4), the value of each technological coefficient is computed and incorporated into the water quality constraints. In simulation, parameter sets are produced repeatedly and their corresponding technological coefficients computed. Based on the simulated values, the mean and covariance matrices of the random technological coefficients for each water quality constraint is calculated and used in solving the stochastic WLA problem.

TECHNIQUE FOR SOLVING OPTIMAL STOCHASTIC WLA MODEL

The deterministic WLA model presented previously follows an LP format which can be solved using the simplex algorithm. However, the deterministic equivalent of the chance-constrained wáter quality constraints are nonlinear. Thus, the problem is one of nonlinear optimization which can be solved by various nonlinear programming techniques such as the generalized reduced gradient technique (Lasdon and Warren, 1979). Moreover, if the covariance matrix $\boldsymbol{C}(\boldsymbol{\Theta}_{ij}, \boldsymbol{\Omega}_{ij})$ in equation (10) is diagonal, the model can be transformed into a separable programming model (Taha, 1982).

Alternatively, this study adopts a linearization procedure for the water quality constraints of the stochastic WLA model and solves the linearized model using the standard LP techniques.

Tung (1986) proposed an approach using first-order Taylor's expansion to linearize the terms involving the square root of the variance. The linearization procedure requires and a-priori estimation of the solution to the optimization problem. As a result, the linearized problem is solved iteratively until the solution converges. Nevertheless, the linearization process utilized by Tung (1986) is not a practical approach for the WLA problem because the functional relationships involving the technological transfer coefficients Θ_{ij} and Ω_{ij} are highly nonlinear. The first-order Taylor expansion of such functions is both cumbersome and insufficient in representing the functions' highly nonlinear nature. By contrast, a simpler procedure is employed wherein the assumed solution to the stochastic WLA model is used to calculate the value of the nonlinear terms. The nonlinear terms become constants and are then moved to the RHS of the constraints. The resulting linearized water quality constraints can be written as:

$$\sum_{j=1}^{n_i} E\left[\Theta_{ij}\right] B_j + \sum_{j=1}^{n_i} E\left[\Omega_{ij}\right] D_j \leq R_i' - F_Z^{-1}(\alpha_i)\sqrt{(\hat{\boldsymbol{B}}, \hat{\boldsymbol{D}})^t \boldsymbol{C}(\boldsymbol{\Theta}_i, \boldsymbol{\Omega}_i)(\hat{\boldsymbol{B}}, \hat{\boldsymbol{D}})} \tag{17}$$

in which $\hat{\boldsymbol{B}}$ and $\hat{\boldsymbol{D}}$ are assumed solution vectors to the stochastic WLA model.

The linearized stochastic WLA model, replacing equation (10) by equation (17), can be solved using the LP techniques repeatedly, each time

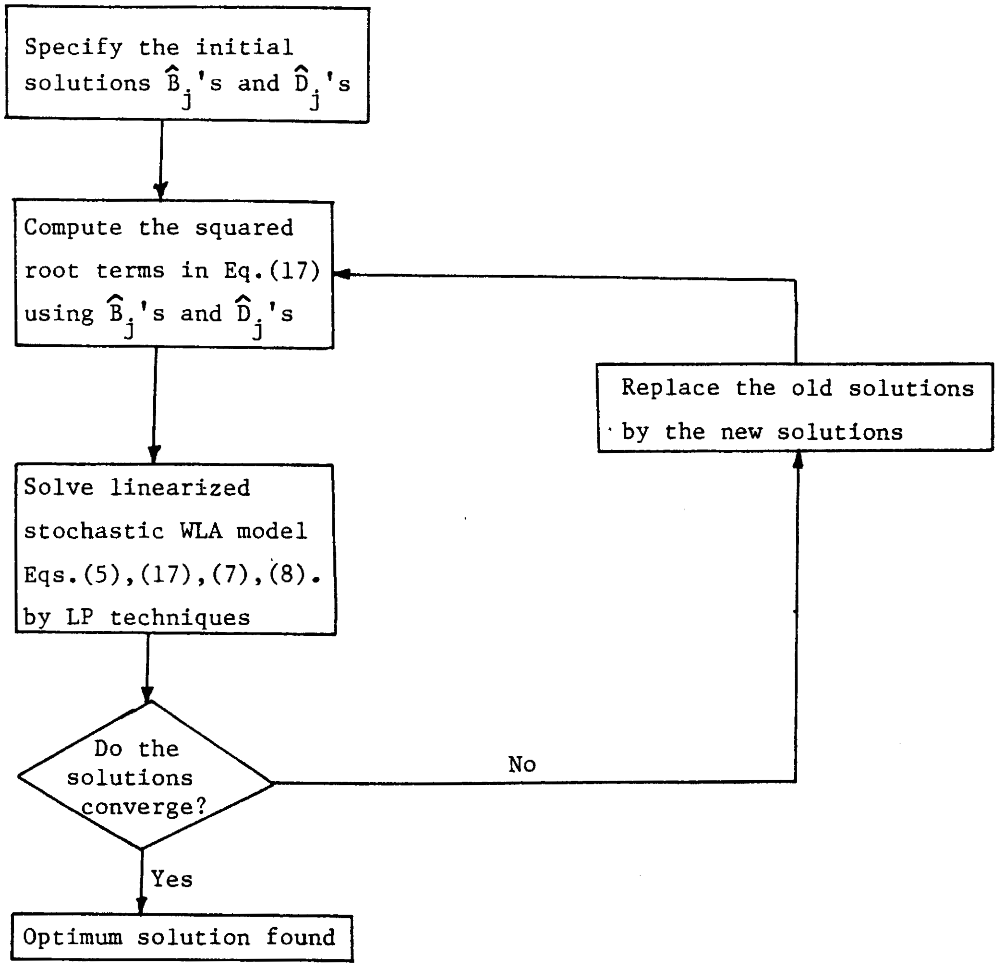

Fig. 2. Flow chart for solving the linearized stochastic WLA model.

updating the previous solution values with those obtained from the current iteration, resulting in new values for the RHS. These steps are repeated until convergence criteria are met between any two successive iterations. A flow chart depicting the procedures is shown in Fig. 2. Of course, alternative stopping rules could be incorporated in the algorithm to prevent excessive iteration during the computation. Prior to the application of these solution procedures, an assumption for the distribution of the random LHS must be made to determine the appropriate value for the term F_Z^{-1} (α_i) in equation (17).

Due to the nonlinear nature of the stochastic WLA model, the global optimum solution, in general, cannot be guaranteed. It is suggested that a few runs of the solution procedure with different initial solutions be carried out to ensure that model solution converges to the overall optimum. A reasonable initial solution is to select the waste effluent concentration for each discharger associated with the upper bounds of their respective treatment levels. By doing so, the initial solution corresponds to the waste discharge at their respective lower limits. If the stochastic WLA solution is infeasible during the first iteration, it is likely that the feasible solution to the stochastic WLA problem does not exist. Hence, time and computational effort could be saved in searching for an optimal solution which might not exist.

NUMERICAL EXAMPLE AND DISCUSSION OF MODEL PERFORMANCE

The model is applied to a six-reach example which is shown in Fig. 3. The means and standard deviations for the water quality parameters in each reach are given in Tables 2 and 3. The waste influent and mean water quality parameters in each reach are obtained from Chadderton et al. (1981). Standard deviations of water quality parameters are artificially imposed such that the range of coefficient of variation (ratio of standard deviation to mean) of the water quality parameters are within those reported in the literature (Churchill et al., 1962; Chadderton et al., 1982; Zielinski, 1988).

To assess the statistical properties (i.e. the mean and covariance matrix) of the random technological coefficients in the water quality constraints, the unconditional simulation procedure described earlier is implemented to generate multivariate normal water quality parameters. Different numbers of simulation sets are generated to examine the stability of resulting means and covariance matrix of technological coefficients. It was found that the statistical properties of Θ_{ij} and Ω_{ij} become stable using 200 sets of simulated parameters. In the example, a positive correlation coefficient of 0.8 between the reaeriation coefficient and average flow velocity is used. Both

Discharger	x (miles)	I (mg/L)	q (ft^3/s)
#1	0.0	1370	0.5
#2[a]	10.0	6.0	44.0
#3	20.0	665	4.62
#4	30.0	910	35.81
#5	40.0	1500	3.2
#6	50.0	410	0.78

[a] Tributary. Background characteristics: $L_0 = 5.0$ mg/L; $Q_0 = 115$ ft^3/s; $D_0 = 1.0$ mg/L

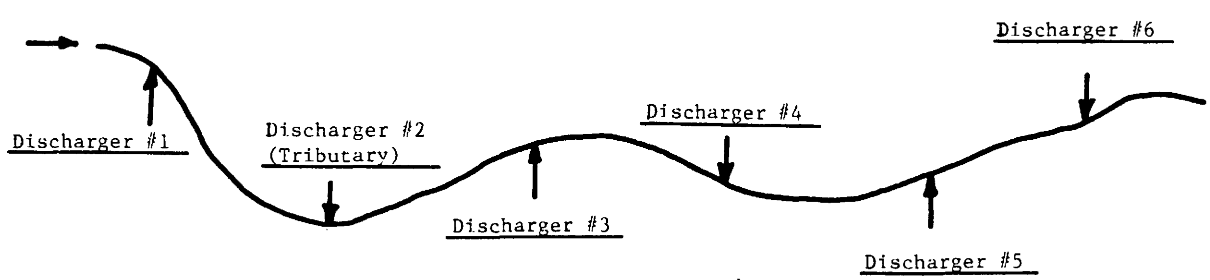

Fig. 3. Schematic sketch of the example system in WLA problem.

TABLE 2

Mean values of physical stream parameters used in the example of WLA model

(a) Mean stream characteristics for each reach

Reach i	Deoxygenation coefficient K_d	Reaeration coefficient K_a	Average stream velocity U	Raw waste concentration I	Effluent flow rate q
1	0.6	1.84	16.4	1370	0.15
2	0.6	2.13	16.4	6	44.00
3	0.6	1.98	16.4	665	4.62
4	0.6	1.64	16.4	910	35.81
5	0.6	1.64	16.4	1500	3.20
6	0.6	1.48	16.4	410	0.78
Units	day^{-1}	day^{-1}	miles/day	mg/L BOD	ft^3/s

(b) Background characteristics

Upstream waste concentration L_0	Upstream flow rate Q_0	Upstream DO deficit D_0
5.0	115.0	1.0
mg/L BOD	ft^3/s	mg/L

mile ≈ 1.609 km; mile/day ≈ 67 m/h. $ft^3 \approx 28.32$ L.

TABLE 3

Standard deviations selected for the physical stream characteristics

(a) For each reach

Reach	Deoxygenation coefficient K_d	Reaeration coefficient K_a	Average stream velocity U
1–6	0.2	0.4	4.0
Units	day^{-1}	day^{-1}	ft^3/s

(b) Background characteristics

Upstream Waste concentration L_0	Upstream flow rate Q_0	Upstream DO deficit D_0
1.0	20.0	0.3
mg/L BOD	ft^3/s	mg/L

normal and lognormal distributions are assumed for the random LHS of the water quality constraints:

$$a_{0i} + \sum_{j=1}^{n_i} \Theta_{ij} B_j + \sum_{j=1}^{n_i} \Omega_{ij} D_j$$

in equation (9). Various reliability levels, α_i, ranging from 0.85 to 0.99, for the water quality constraints are considered.

In addition, the stochastic WLA model is solved under the conditions that the water quality parameters have different spatial correlation structures. The spatial correlation of representative water quality parameter values between two reaches is computed based on a separation distance between the centers of the two reaches. Two zones of influence ($h_0 = 15$ miles and $h_0 = 30$ miles) along with the three variogram models, equations (11)–(13), are used for examining the effect of spatial correlation structure on the optimal waste load allocation. In Fig. 3 the length of each reach in the system is 10 miles. A value of $h_0 = 15$ miles implies that the water quality parameters in a given reach are spatially correlated only with the two immediate adjacent reaches. For $h_0 = 30$ miles, the spatial correlation extends two reaches upstream and downstream of the reach under consideration. To create spatial independence of water quality parameters, the zone of influence, h_0, can be set shorter than 10 miles. The optimal solutions to the stochastic WLA problem under these various conditions are presented in Tables 4 and 5.

In examining the results in Tables 4 and 5, the maximum total BOD discharge, under a given spatial correlation structure, reduces as the reliability of water quality constraints increases. This behavior is expected since an increase in water quality compliance reliability is equivalent to imposing stricter standards on water quality assurance. To meet this increased water quality compliance reliability, the amount of waste discharge must be

TABLE 4

Maximum total BOD load that can be discharged for different reliability levels and spatial correlation structures under normal distribution

α	I [a]	$h_0 = 15$ miles			$h_0 = 30$ miles		
		T	S	G	T	S	G
0.85	671 [b]	734	737	679	659	664	694
0.90	633	693	695	639	624	625	656
0.95	588	644	646	593	580	578	610
0.99	521	570	572	524	516	511	541

[a] I, independence; T, transitive model; S, spherical model; G, Gaussian model
[b] Total BOD Load Concentration in mg/L.

TABLE 5

Maximum total BOD load that can be discharged for different reliability levels and spatial correlation structures under lognormal distribution

α	I [a]	$h_0 = 15$ miles			$h_0 = 30$ miles		
		T	S	G	T	S	G
0.85	691 [b]	753	755	699	676	686	712
0.90	633	692	694	640	623	626	655
0.95	560	614	616	565	554	661	582
0.99	424	496	498	425	420	388	471

[a] I, independence; T, transitive model; S, spherical model; G, Gaussian model.
[b] Total BOD Load concentration in mg/L.

reduced to lower the risk of water quality violation at the various control points. Continuing to increase the required reliability for the water quality constraints, at some point these restrictions could becomes too stringent and feasible solution to the problem no longer obtainable.

From Tables 4 and 5, using a lognormal distribution for the LHS of water quality constraints yields a higher total BOD discharge than that under a normal distribution when the performance reliability requirement is 0.85. However, the results reverse themselves when reliability requirements are greater than or equal to 0.90. This indicates that the optimal solution to the stochastic WLA model depends on the distribution used for the LHS of the water quality constraints. From the previous empirical investigation (Tung and Hathhorn, 1988), lognormal distribution was found to best describe the DO deficit concentration in a single-reach case. In other words, each term of the LHS in water quality constraints could be considered as a lognormal random variable. Therefore, the LHS is the sum of correlated lognormal random variables. For the first two or three reaches from the upstream end of the system, the distribution of the LHS may close to be lognormal because the number of terms in the LHS is few. However, when consider the control point for much downstream reaches, the number of terms in the LHS increase and the resulting distribution may approach to normal from the argument of central limit theorem. Since the true distribution for the LHS of water quality constraints is not known, it is suggested that different distributions are used for model solutions and, from a conservative viewpoint, adopt the least amount of total BOD load for implementation.

From the tables of results, the impacts of the extent of the spatial correlation of the water quality parameters (represented by the length of h_0) and the structure (represented by the form of the variogram) on the results of stochastic WLA model can also be observed. When $h_0 = 15$ miles, where the spatial correlation of the water quality parameters extends only one reach, the maximum allowable total BOD load, for all three variogram models, is higher than that of spatially independent case. When the spatial correlation extends over two reaches (i.e., $h_0 = 30$ miles), the use of transitive and spherical variogram models results in lower maximum total BOD loads than that of the spatial independence case, while the use of Gaussian variogram yields a higher total BOD load. The model results using a transitive variogram are very similar to that of a spherical model.

As a final comment on the computational aspects of the proposed technique for solving the stochastic nonlinear WLA model formulated in this study, it was observed that the iterative technique proposed takes three to five iterations to converge for all the cases investigated. Therefore, the proposed solution procedure is quite efficient in solving the stochastic WLA model.

SUMMARY AND CONCLUSIONS

A practical approach for solving a chance-constrained stochastic WLA model is presented. The method consists of a simple linearization procedure for solving a nonlinear stochastic WLA model in which the statistical properties (i.e. the mean and covariance matrix) of the left-hand-side coefficients in water quality constraints are estimated using the unconditional simulation. The stochastic WLA model presented here considers uncertainty in all the water quality parameters in the Streeter–Phelps equation. In addition, spatial and cross-correlation of the water quality parameters are also modeled by means of unconditional simulation. The results observed from the hypothetical example used clearly demonstrated the existence of the tradeoff between the requirement on water quality performance reliability and the maximum total allowable BOD discharge. An increase in water quality compliance reliability results in a reduction of the total allowable BOD to be discharged into the stream system. This can only be achieved with an increase in treatment cost. This tradeoff implies that persons in charge of managing the stream environment must be cognizant of both the need to ensure water quality protection and the desire to meet this goal at a reduced cost.

Furthermore, the results from the example application revealed the significant effect the spatial correlation has on the solution of stochastic WLA problems. The maximum total BOD that can be discharged into the stream system is dependent on the extent and structure of the spatial correlation. It should be emphasized that, even if the water quality parameters are assumed to be spatially independent, the resulting technological coefficients in the water quality constraints are not independent of each other. A full accounting of such correlation for the technological coefficients in the water quality constraints is essential for solving the stochastic WLA problems correctly.

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REFERENCES

Brill, E., Liebman, J. and ReVelle, C., 1976. Equity measures for exploring water quality management alternatives. Water Resour. Res., 12: 845–851.

Brill, E.D., Eheart, J.W. and Liebman, J.C., 179. Discussion: stochastic programming model for water quality management in a river. J. Water Pollut. Control. Fed., 12: 2958.

Burn, D.H. and McBean, E.A., 1985. Optimization modeling of water quality in an uncertain environment. Water Resour. Res., 21: 934–940.

Chadderton, R.A., Miller, A.C. and McDonnell, A.J., 1981. Analysis of Waste load allocation procedure. Water Resour. Bull. AWRA, 17: 760–766.

Chadderton, R.A., Miller, A.C. and McDonnell, A.J., 1982. Uncertainty analysis of dissolved oxygen model. J. Environ. Eng. ASCE, 108: 1003–1012.

Charnes, A. and Cooper, W.W., 1963. Deterministic equivalents for optimizing and satisficing under chance constraints. Oper. Res., 11: 18–39.

Churchill, M.A., Elmore, H.L. and Buckingham, R.A., 1964. The prediction of stream reaeriation rates. J. Sanit. Eng. Div. ASCE, 88: 1–46.

Dobbins, W.E., 1964. BOD and oxygen relationships in stream. J. Sanit. Eng. Div. ASCE, 90: 53–78.

Ellis, J.H., 1987. Stochastic water quality optimization using imbedded chance constraints. Water Resour. Res., 23: 2227–2238.

Esen, I.I. and Rathbun, R.E., 1976. A stochastic model for predicting the probability distribution of the dissolved oxygen deficit in streams. USGS Prof. Pap. 913, 47 pp.

Fujiwara, O., Gnanendran, S.K. and Ohgaki, S., 1986. River quality management under stochastic streamflow. J. Environ. Eng. ASCE, 112: 185–198.

Fujiwara, O., Gnanendran, S.K. and Ohgaki, S., 1987. Chance constrained model for river water quality management. J. Environ. Eng. ASCE, 113: 1018–1031.

Gross, W.M.., 1965. A lawyer looks at stream pollution. Civil Eng. ASCE, 44–45.

Hathhorn, W.E. and Tung, Y.K., 1987. Waste load allocation in stochastic stream environments. Technical report, Wyoming Water Research Center, University of Wyoming, laramie, WY, 392 pp.

Journel, A.G. and Huijbregts, C.J., 1978. Mining Geostatistics. Academic Press, New York, 600 pp.

Kolbin, V.V., 1977. Stochastic Programming. Reidel, Boston, MA, 195 pp.

Krenkel, P.A. and Novotny, V., 1980. Water Quality Management, Academic Press, New York, 671 pp.

Lasdon, L.S. and Warren, A.D., 1979. Generalized reduced gradient software for linear and nonlinear constrained problems. In: H. Greenberg (Editor), Design and Implementation for Optimization Software, Sijthoff/Noordhoff, The Netherlands, pp. 363–397.

Lohani, B.N. and Thanh, W.R., 1978. Stochastic programming model for water quality management in a river. J. Water Pollut. Control Fed., 50: 2175–2182.

Loucks, D.P., Revelle, C.S. and Lynn, W.R., 1967. Linear programming models for water pollution control. Manage. Sci., 14: 166–181.

Miller, W.L. and Gill, J.H., 1976. Equity considerations on controlling nonpoint pollution from agricultural sources. Water Resour. Bull., 12: 253–261.

Padgett, W.J., 1978. A stream pollution model with random deoxygenation and reaeriation coefficients. Math. Biosci., 42: 137–148.

Quimby, W.F., 1986. Selected topics in spatial statistical analysis: Nonstationary vector kriging, large scale conditional stimulation of three dimensional Gaussian random fields, and hypothesis testing in a correlated random fields. Ph.D. thesis, Department of Statistics, University of Wyoming, Laramie, WY, 141 pp.

Streeter, H.W. and Phelps, E.B., 1925. A study of the pollution and natural purification of the Ohio River. Publ. Health Bull. 146, U.S. Public Health Service, Washington, DC, pp. 127–147.

Taha, H.A., 1982. Operations Research: An Introduction. McMillan New York, 648 pp.

Tung, Y.K., 1986. Groundwater management by chance-constrained model. J. Water Resour. Plann. Manage. ASCE, 112: 1–19.

Tung, Y.K. and Hathhorn, W.E., 1988. Assessment of probability distribution of instream dissolved oxygen deficit. J. Environ. Eng. ASCE, 114: 1421–1435.

Vajda, S., 1972. Probabilistic Programming. Academic Press, New York, 127 pp.

Ward, R.C. and Loftis, J.C., 1983. Incorporating the stochastic nature of water quality into management. J. Water Pollut. Control Fed., 55: 408–414.

Yaron, D., 1979. A method for the analysis of seasonal aspects of water quality control. J. Environ. Econ. Manage., 6.

Zielinski, P.A., 1988. Stochastic dissolved oxygen model. J. Environ. Eng. ASCE, 114: p. 74–90.