RELIABILITY ANALYSIS USING LOAD-RESISTANCE ANALYSIS

Y.K. Tung
L.W. Mays

1989
Book Chapter WWRC-89-20

In

Reliability Analysis of
Water Distribution Systems

Chapter 8

Y.K. Tung
Wyoming Water Research Center
University of Wyoming
Laramie, Wyoming

L.W. Mays
Center for Research in Water Resources
University of Texas
Austin, Texas

CHAPTER 8

RELIABILITY ANALYSIS USING LOAD-RESISTANCE ANALYSIS

by

Y. K. Tung and Larry W. Mays

The resistance or strength of a component is defined as the ability of the component to accomplish its required mission satisfactorily without a failure when subjected to an external stress. Stress is the loading of the component, which may be a mechanical load, an environmental exposure, a flow rate, temperature fluctuation, etc. The stress or loading tends to cause failure of the component. When the strength of the component is less than the stress imposed on it, the failure occurs. This type of analysis can be applied to the reliability analysis of components of water distribution systems.

8.1 STATIC RELIABILITY ANALYSIS

The reliability of a hydraulic system is defined as the probability of the resistance Y to exceed the loading X, i.e., the probability of survival. The terms "stress" and "strength" are more meaningful to structural engineers, whereas the terms "loading" and resistance" are more descriptive to water resources engineers. The risk of a hydraulic component, subsystem, or system is defined as the probability of the loading exceeding the resistance, i.e., the probability of failure. The mathematical representation of the reliability R can be expressed as

$$R = P(Y > X) = P(Y - X > 0) \tag{8.1.1}$$

where P( )refers to probability, Y is the resistance, and X is the load. The relationship between reliability and risk $\overline{R}$ is

$$R = 1 - \overline{R} \tag{8.1.2}$$

The resistance of a hydraulic system is essentially the flow carrying capacity of the system, and the loading is essentially the magnitude of flows through or pressure imposed on the system by demands. Since the loading and resistance are random variables due to the various hydraulic and demand uncertainties, a knowledge of the probability distributions of Y and X is required to develop reliability models. The computation of risk and reliability can be referred to as "loading-resistance interference." Probability distributions for load and resistance are illustrated in Fig. 8.1.1. The reliability is the probability that the resistance is greater than the loading for all possible values of the loading.

The word "static," from the reliability computation point of view, represents the worst single stress, or load, applied. Actually, the loading applied to many hydraulic systems is a random variable. Also, the number of times a loading is imposed is random.

8.1.1 Reliability Computation By Direct Integration

Following the reliability definition given in equation (8.1.1), the reliability and risk of a hydraulic structure can be expressed as

$$R = \int_{o}^{\infty} f_Y(y) \left[\int_{o}^{y} f_X(x)dx \right] dy = \int_{o}^{\infty} f_Y(y)\, F_X(y)dy \tag{8.1.3}$$

in which $f_Y(\,)$ and $f_X(\,)$ represent the probability density functions of resistance and loading, respectively. The reliability computations for a hydraulic structure require the knowledge of the probability distributions of loading and resistance. A schematic diagram of the reliability computation by equation (8.1.3) is shown in Fig. 8.1.2.

To illustrate the computation procedure involved, we consider that the loading X and the resistance Y are exponentially distributed, i.e.,

$$f_X(x) = \lambda_x e^{-\lambda_x x}, \;\; x \geq 0 \tag{8.1.4}$$

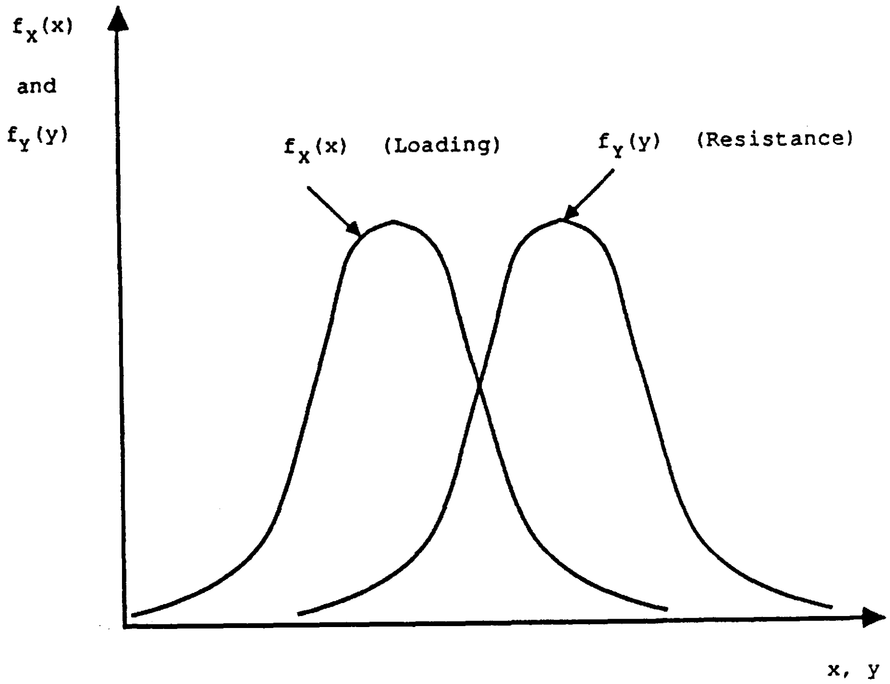

Figure 8.1.1 Load-Resistance Interference Reliability Analysis

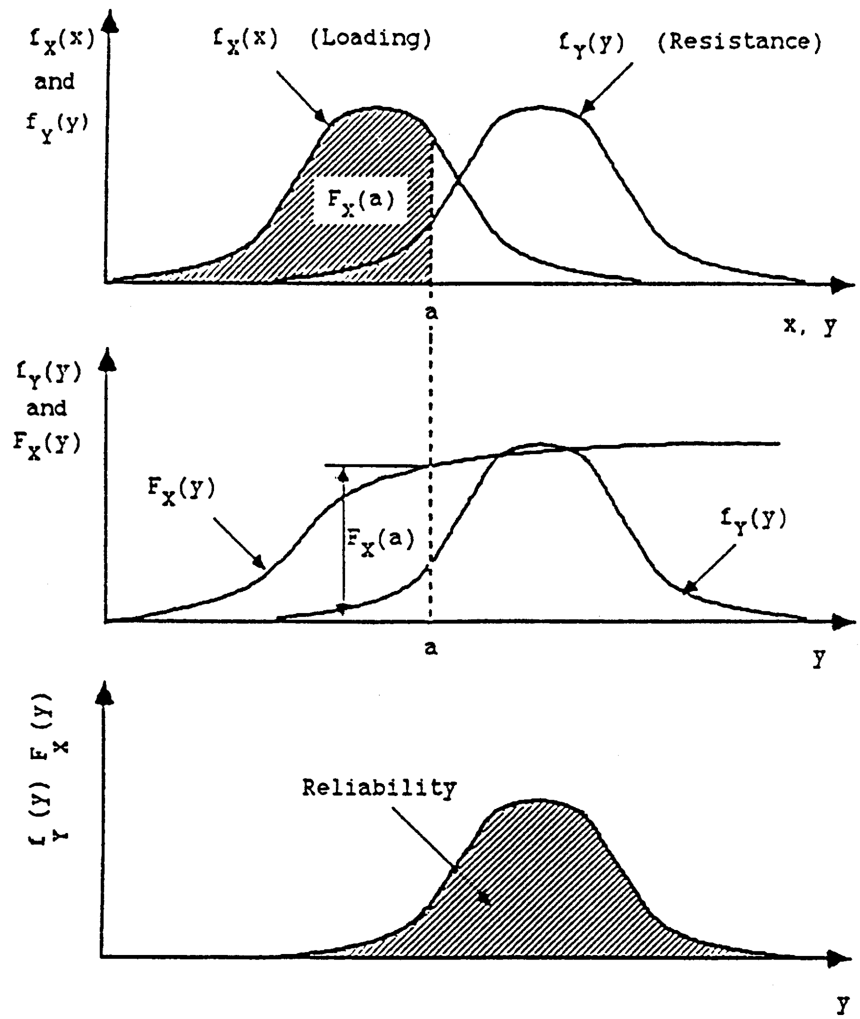

Figure 8.1.2 Graphical Illustration of the Steps Involved in Reliability Computation by Equation (8.1.3)

$$f_Y(y) = \lambda_y e^{-\lambda_y y}, \quad y \geq 0 \tag{8.1.5}$$

Then the static reliability can be derived by applying equation (8.1.3) in a straight forward manner as

$$R = \int_o^\infty \lambda_y e^{-\lambda_y y} \left[\int_o^y \lambda_x e^{-\lambda_x x} \, dx \right] dy$$

$$= \int_o^\infty \lambda_y e^{-\lambda_y y} \left[1 - e^{-\lambda_x y} \right] dy$$

$$= \frac{\lambda_x}{\lambda_x + \lambda_y} \tag{8.1.6}$$

For some special combinations of load and resistance distributions, the static reliability can be derived analytically in the closed-form. In cases in which both the loading X and resistance Y are log-normally distributed, the reliability can be computed as (Kapur and Lamberson, 1977)

$$R = \int_{-z}^{\infty} \phi(z) dz = \Phi(z) \tag{8.1.7}$$

where $\phi(z)$ and $\Phi(z)$ are the probability density function and the cumulative distribution function, respectively, for the standard normal deviate z given as

$$z = \frac{\mu_{y'} - \mu_{x'}}{\sqrt{\sigma_{y'}^2 + \sigma_{x'}^2}} \tag{8.1.8}$$

where x' = lnx and y' = lny. The table of values of the cumulative distribution function $\Phi(z)$ for the standard normal deviate is available in any standard statistics textbook.

In cases in which the loading X is exponentially distributed and the resistance is normally distributed, the reliability can be expressed as (Kapur and Lamberson, 1977)

$$R = 1 - \Phi\left(\frac{\mu_y}{\sigma_y}\right) - \exp\left[-\frac{1}{2}\left(2\mu_y\lambda_x - \lambda_x^2\sigma_y^2\right)\right] \left[1 - \Phi\left(-\frac{\mu_y - \lambda_x\sigma_y^2}{\sigma_y}\right)\right] \tag{8.1.9}$$

Example

Consider a water distribution system (see Fig. 8.1.3) consisting of a storage tank serving as the source, and a 2-ft diameter cast-iron pipe 1 mile long, leading to a user. The head elevation at the source is maintained at a constant height of 100 feet above the user. It is also known that, at the user end, the required pressure head is fixed at 20 psi with variable demand on flow rate. Assume that the demand in flow rate is random, having a log-normal distribution with the mean 1 cfs and standard deviation 0.3 cfs. Because of the uncertainty in pipe roughness, the supply to the user is not certain. We know that the pipe has been installed for about 3 years. Therefore, our estimation of the pipe roughness in the Hazen-Williams equation is about 130, with some errors ± 20. Again, we further assume that the Hazen-William's C coefficient has a lognormal distribution with a mean of 130 and a standard deviation of 20. It is required to estimate the reliability that the water demand by the user will be satisfied.

In this example, the resistance of the system is the water supply from the source, while the load is the water demand by the user. Both supply and demand are random variables. By Hazen-William's equation, the supply is calculated as

$$Y = Q_s = \frac{C}{149.2}\left(\frac{\Delta h}{L}\right)^{0.54} D^{2.63}$$

where Δh is the head difference (in ft) between the source and the user, D is the pipe diameter in feet, and L is the pipe length in feet. Because roughness coefficient C is a random variable, so is the supply. Due to the multiplicative form of the Hazen-Williams equation, the logarithmic transformation leads to a linear relation among variables, i.e.

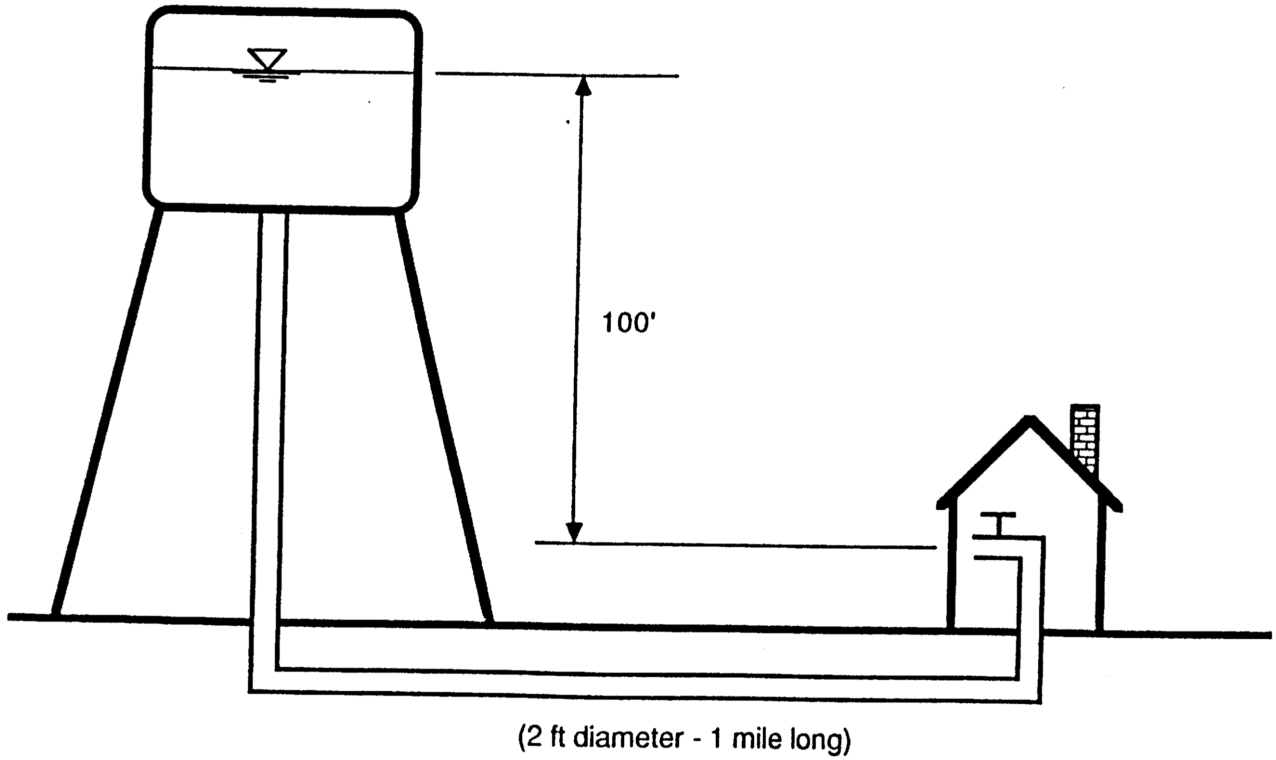

Figure 8.1.3 Example System

$$\ln Y = \ln C - \ln (149.2) + 0.54 \ln \left[\frac{100 - \frac{(20)(144)}{62.4}}{5280} \right] + 2.63 \ln (2)$$

$$= \ln C - 5.659$$

Assume that the roughness coefficient C is lognormally distributed, then lnC is normally distributed, as is the log-transformed water supply (resistance). From the moment relations given in Table 6.1.2 for lognormal distribution, the mean and the standard deviation of ln C are 4.856 and 0.153, respectively. From these, the mean and standard deviation of ln Y are 0.337 and 0.153, respectively.

Because the water demand (loading) has a lognormal distribution, the mean and standard deviation of its log-transformed scale can be calculated, in the same manner as for roughness coefficient C, as -0.0431 and 0.294, respectively.

Knowing the distributions and statistical properties of the load (water demand) and resistance (water supply), both log-normal in this example, the reliability of the system can be calculated by equation (8.1.7) as

$$R = \Phi \left[z = \frac{0.337 - (-0.0431)}{\sqrt{0.153^2 + 0.294^2}} \right]$$

$$= \Phi [z = 1.147]$$

$$= 0.873$$

This means that the water demanded by the user will be met 87.3% of the time.

8.1.2 Reliability Computation Using Safety Margin/Safety Factor

Safety Margin. The safety margin is defined as the difference between the project capacity (resistance) and the value calculated for the design loading SM = Y - X. The reliability is equal to the probability that Y > X, or equivalently,

$$R = P(Y - X > 0) = P(SM > 0) \tag{8.1.10}$$

If Y and X are independent random variables, then the mean value of SM is given by $\mu_{SM} = \mu_Y - \mu_X$ and its variance by $\sigma^2_{SM} = \sigma^2_Y + \sigma^2_X$. If the safety margin is normally distributed, then $z = (SM - \mu_{SM})/\sigma_{SM}$ is a standard normal variate z. By subtracting μ_{SM} from both sides of the inequality in equation (8.1.10) and dividing both sides by σ_{SM}, it can be seen that

$$R = P\left(z > \frac{\mu_{SM}}{\sigma_{SM}}\right) = \Phi\left(\frac{\mu_{SM}}{\sigma_{SM}}\right) \tag{8.1.11}$$

The key assumption of this analysis is that it considers that the safety margin is normally distributed but does not specify what the distributions of loading and capacity must be. Ang (1973) indicates that provided R > 0.001, R is not greatly influenced by the choice of distribution for Y and X and the assumption of a normal distribution for SM is satisfactory. For lower risk than this (e.g., R = 0.00001), the shape of the tails of the distributions for Y and X becomes critical in which case accurate assessment of the distribution of SM or direct integration procedure should be used to evaluate the risk or probability of failure.

Applying the safety-margin approach to evaluate the reliability of the simple water distribution system described in the previous subsection, we can calculate the mean and standard deviation of the resistance (i.e., water supply) as

$$\mu_Y = \exp\left(\mu_{\ln x} + \frac{1}{2}\sigma^2_{\ln x}\right)$$

$$= \exp\left[0.337 + \frac{1}{2}(0.153)^2\right] = 1.417 \text{ cfs}$$

and

$$\sigma_Y = \sqrt{\mu^2_Y\left[\exp\left(\sigma^2_{\ln x}\right) - 1\right]} = 0.218 \text{ cfs}$$

From the problem statement, we know that the mean and standard deviation of the load (water demand) are $\mu_X = 1$ cfs and $\sigma_X = 0.3$ cfs, respectively. Therefore, the mean and variance of the safety margin can be calculated as

$$\mu_{SM} = \mu_y - \mu_x = 1.417 - 1.0 = 0.417 \text{ and}$$

$$\sigma_{SM}^2 = \sigma_y^2 + \sigma_x^2 = (0.218)^2 + (0.3)^2 = 0.138$$

Now, the reliability of the system can be assessed by the safety margin approach, as

$$R = \Phi\left[\frac{0.417}{\sqrt{0.138}}\right] = \Phi\,[1.124] = 0.869$$

The reliability computed by the safety-margin method is not identical to that of direct integration; the difference is practically negligible. It should, however, be pointed out that the distribution of the safety-margin in this example is not exactly normal as assumed. Thus, the reliability obtained should be regarded as an approximation to the true reliability.

Safety Factor. The safety factor SF is given by the ratio of Y/X and the reliability can be specified by P(SF > 1). Several safety factor measures and their usefulness in hydraulic engineering are discussed by Yen (1978). By taking logarithms of both sides of this inequality

$$R = P(SF > 1) = P[1n(SF) > 0] = P\,[\ln(Y/X) > 0] \tag{8.1.12}$$

If the resistance and loading are independent and log-normally distributed, then the risk can be expressed as

$$\overline{R} = \Phi\left\{\frac{\ln\left[\dfrac{\mu_y}{\mu_x}\sqrt{\dfrac{1+CV_x^2}{1+CV_y^2}}\right]}{\ln\left[\left(1+CV_y^2\right)\left(1+CV_x^2\right)\right]^{1/2}}\right\} \tag{8.1.13}$$

where CV are the coefficients of variations defined in Table 6.1.1. Applying the safety-factor approach to the simple water distribution system would yield the same reliability as that of direct integration because the exact distribution of SF, in this example, is lognormal.

8.2 DYNAMIC RELIABILITY ANALYSES

Dynamic or time-dependent reliability analyses consider repeated application of loading and, in addition, can consider the change of the resistance with time. The practical motivation behind considering time-dependent risk and reliability models is that, for hydraulic structures, there is uncertainty about the random loading and resistance variables with respect to time and loading cycles.

Repeated loadings on a hydraulic structure are characterized by the frequency or time each load is applied and the distribution of time intervals between the loadings. For reliability analysis purposes, the uncertainty about the loading and resistance variables may be classified into three categories: deterministic, random-fixed, and random-independent (Kapur and Lamberson, 1977). For the deterministic case, the variables assume values that are exactly known a priori. For the random-fixed case, the randomness varies in time in a known manner. For the random-independent case, the variables are not only random but the successive values of the variables are statistically independent.

The objective of the reliability computations for the dynamic models is to determine the reliability over n cycles or occurrences of loading R_n, i.e., the probability of not having a failure during any of the n cycles or loadings.

Reliability computations for dynamic (time-dependent) analysis can be made for deterministic and random cycle times. The loading on water distribution systems can be deterministic under normal loading conditions and random under emergency loading conditions. The model for deterministic cycles will be developed which naturally leads to the model for random cycle times. For deterministically known cycle times, the reliability of the system after n cycles or occurrences of loading R_n can be expressed as

$$\begin{aligned} R_n &= P\left[\left(X_1 < Y\right) \cap \left(X_2 < Y\right) \cap \ldots \cap \left(X_n < Y\right)\right] \\ &= P\left[\left(\max\left(X_1, X_2 \ldots X_n\right) < Y\right)\right] \end{aligned} \tag{8.2.1}$$

By letting the maximum loading, $X_{max} = \max(X_1, X_2, \ldots X_n)$, the distribution function of X_{max}, $F_n(x)$, is

$$F_n(x) = \left[F_X(x)\right]^n \tag{8.2.2}$$

provided that the loadings are independent and identically distributed, in which $F_X(x)$ is the cumulative distribution of the loadings or hydrologic events.

For the time-dependent reliability model with deterministic cycles, the reliability is expressed as

$$R_n = \int_0^\infty f_Y(y)\left[F_X(y)\right]^n dy \tag{8.2.3}$$

Since the number of occurrences of loading is, in general, random, the reliability of the system under random loading cycles in the time interval [0,t] can be expressed as

$$R(t) = \sum_{n=0}^{\infty} \pi_n(t) R_n \tag{8.2.4}$$

where $\pi_n(t)$ = the probability of n loadings occurring in the time interval [0,t]. It is now evident that the case of deterministic cycle times is a special case of the preceding reliability equation for random cycle times.

A Poisson distribution can be used to describe the probability of the number of events occurring in a given time interval, given as

$$\pi_n(t) = \frac{e^{-\alpha t}(\alpha t)^n}{n!} \tag{8.2.5}$$

in which α = the mean rate of occurrence of the loading which may be estimated from historical data. For example, if annual data are being used, $\alpha = 1/T_r$ in which T_r is the return period. Other distributions may also be applicable but they lead to more complicated analysis.

For the random independent loading and random fixed resistance, the time-dependent reliability can be expressed as

$$R(t) = \sum_{n=0}^{\infty} \frac{e^{-\alpha t}(\alpha t)^n}{n!} \int_0^{\infty} f_Y(y)\left[F_X(y)\right]^n dy = \int_0^{\infty} f_Y(y) e^{-t[1-F_X(y)]} dy \quad (8.2.6)$$

For random-independent loading and random-fixed resistance, R for one loading cycle is expressed by equation (8.1.3) and R(t) is expressed by equation (8.2.4). Thus, using the Poisson distribution, equation (8.2.5) the reliability is expressed as

$$R(t) = \sum_{n=0}^{\infty} \frac{e^{-\alpha t}(\alpha t)^n}{n!} R^n = e^{-\alpha t(1-R)} \quad (8.2.7)$$

A computer program for computing risk-SF curves for the dynamic case has been developed by Tung and Mays (1980) which can consider various distributions such as normal, log-normal, extremal type I, Pearson type III, log-Pearson type III, and Weibull distribution loading.

REFERENCES

Ang, A. H.-S., "Structural Risk Analysis and Reliability-Based Design," *Journal of the Structural Engineering Division*, American Society of Civil Engineers, Vol. 99, No. ST9, pp. 1891-1910, 1973.

Kapur, K. C. and L.R. Lamberson, Reliability in Engineering Design, John Wiley and Sons, New York. 1977.

Tung, Y.-K. and L.W. Mays, "Risk Analysis for Hydraulic Design," *Journal of the Hydraulics Division*, American Society of Civil Engineers, Vol. 106, No. HY5, pp. 893-913, 1980.

Yen, B. C., "Safety Factor in Hydrologic and Hydraulic Engineering Design," Proceedings, International Symposium on Risk Reliability in Water Resources, University of Waterloo, Waterloo, Ontario, Canada, June 1978.