METHODS FOR COMPONENT RELIABILITY ANALYSIS

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CHAPTER 6

METHODS FOR COMPONENT RELIABILITY ANALYSIS

by

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6.1 PROBABILITY CONCEPTS

6.1.1 Random Variable

A random variable, X, is a variable described by a probability distribution. The distribution specifies the chance that an observation, x, of the variable will fall in a specified range of X.

A set of observations, $x_1, x_2, \ldots, x_n$, of the random variable, X, is called a sample. It is assumed that samples are drawn from a population (generally unknown) possessing constant statistical properties while the properties of a sample may vary from one sample to another. The possible range of variation of all of the samples which could be drawn from the population is called the sample space, and an event is a subset of the sample space.

The probability of an event, P(A), is the chance that it will occur when an observation of the random variable is made. Probabilities of events can be estimated. If a sample of n observations has n_A values in the range of event A, then the relative frequency of the occurrence of A is n_A/n and, as the sample size is increased, the relative frequency becomes a progressively better estimate of the probability of the event, i.e.,

$$P(A) = \lim_{n \to \infty} \frac{n_A}{n} \tag{6.1.1}$$

Such probabilities are called objective or posterior probabilities because they depend completely on observations of random variable. People are accustomed to estimating the chance that a future event will occur based on their judgement and experience. Such estimates are called subjective or prior probabilities.

In case that there exists more than one interrelated event, joint probability and conditional probability may be used. Consider, for example, that a process involves two interdependent events, A and B. The probability of joint occurrence of both events is denoted, P(A,B), and the probability of occurrence of event A conditioned on the occurrence of event B is denoted, P(A/B). The relation between joint probability and conditional probability is

$$P(A,B) = P(A/B) \bullet P(B) \tag{6.1.2}$$

Random variables can, in general, be discrete or continuous. The sample space of a discrete random variable can be finite or countably infinite. For a discrete random variable, the probability occurs discretely only at all elements in the sample space. While, for a continuous random variable, the probability occurs continuously over the sample space. A listing of probability values or a mathematical relation that describe how probability is distributed over the different values of the random variable is called the probability density function (pdf). For a continuous random variable X, the probability distribution function value F(x) is the cumulative probability of occurrence of x, $P(X \leq x)$, and it is given by the integral of the pdf, f(x), over the range $X \leq x$, i.e.,

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(h)\, dh \tag{6.1.3}$$

where h is a dummy variable of integration.

Similar to that of joint probability and conditional probability, the terms joint pdf and conditional pdf are employed for cases when more than one random variable is involved. If there are two continuous random variables X and Y, the relation between joint pdf and conditional pdf of X and Y is

$$f\left(\frac{x}{y}\right) = \frac{f(x,y)}{f(y)} \tag{6.1.4}$$

where f(x/y) is the conditional pdf of X on Y, f(x,y) is the joint pdf of X and Y, and f(y) is the marginal pdf of Y obtained by integrating f(x,y) over the entire sample space of X.

6.1.2 Statistical Properties of a Random Variable

A common way to characterize the statistical properties of a random variable is by examining its statistical moments. The r-th moment of a continuous random variable X about any point $X = x_o$ is defined as

$$E\left[\left(X - x_o\right)^r\right] = \int_{-\infty}^{\infty} \left(x - x_o\right)^r f(x)\,dx \qquad (6.1.5)$$

In practice, the first three moments are used to describe the central tendency, variability, and asymmetry of the distribution of a random variable. The common descriptors for statistical properties of a random variable are the following.

For measuring the central tendency, the expectation of a random variable X is defined as

$$E(X) = \mu = \int_{-\infty}^{\infty} x f(x)\,dx \qquad (6.1.6)$$

is frequently used. This expectation is known as the mean of a random variable. Other descriptors for central tendency of a random variable are shown in Table 6.1.1.

For measuring the variability, the variance of a random variable

$$\mathrm{Var}(X) = \sigma^2 = E\left[\left(X - \mu\right)^2\right] = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)\,dx \qquad (6.1.7)$$

is frequently used. As can be seen, the variance is the second moment about the mean. The positive square root of variance is called the standard deviation which is often used as the measure of the degree of uncertainty associated with a random variable.

Table 6.1.1 Commonly Used Statistical Properties of a Random Variable with Their Sample Estimators

Statistical Properties	Sample Statistic
1. Central Tendency	
Arithmetic Mean	
$\mu = E(X) = \int_{-\infty}^{\infty} x\ f(x)\ dx$	$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$
Median	
x such that F(x) = 0.5	50th percentile value of data
Geometric mean	
Antilog E[log(X)]	$[\prod_{i=1}^{n} x_i]^{1/n}$
2. Variability	
Variance	
$\sigma^2 = E(X-\mu)^2$	$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$
Standard Deviation	
$\sigma = [E(X-\mu)^2]^{1/2}$	$s = [\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2]^{1/2}$
Coefficient of Variation	
$C_v = \sigma / \mu$	$C_v = s / \bar{x}$
3. Symmetry	
Coefficient of Skewness	
$\gamma = \frac{E(X-\mu)^3}{\sigma^3}$	$C_s = \frac{n \sum_{i=1}^{n} (x_i - \bar{x})^3}{(n-1)(n-2)s^3}$

The standard deviation has the same units as the random variable. To compare the degree of uncertainty of two random variables of different units and values, a nondimensionalized measure σ/μ, called the coefficient of variation, is useful.

To measure the asymmetry of the distribution function of a random variable, skew coefficient, γ, defined as

$$\gamma = E\left[(X-\mu)^3\right]/\sigma^3 \tag{6.1.8}$$

is used. The skew coefficient is dimensionless and is related to the third central moment. The sign of the skew coefficient indicates the extent of symmetry of the probability distribution about its mean. If $\gamma = 0$, the distribution is symmetric about its mean; $\gamma > 0$, the distribution has a long tail to the right; $\gamma < 0$, the distribution has a long tail to the left.

In practice, statistical moments higher than three are rarely used because their accuracy decreases rapidly when estimated from the limited sample. Equations used to compute the sample estimates of the above statistical properties are shown in Table 6.1.1.

6.1.3 Fitting a Probability Distribution

A probability distribution is a function representing the frequency of occurrence of the value of a random variable. By fitting a distribution to a set of data, a great deal of the probabilistic information in the sample can be compactly summarized in the function and its associated parameters. Fitting distributions can be accomplished by the method of moments or the method of maximum likelihood.

Between the two methods, the method of moments is more widely used primarily for its computational simplicity. The method relates the parameters in a probability distribution model to the statistical moments to which the parameter-moment relationships for commonly used distributions in reliability analysis are immediately available (see Table 6.1.2). In practice, the true mechanism that generates the observed random process is not entirely known. Therefore, to estimate the parameter values in a probability distribution model by the method of moments, sample moments are used.

6.2 RELIABILITY CONCEPTS

The analysis of reliability and availability requires an understanding of some basic terms, which are defined in this section. The concepts represented by these terms will be used in later sections to quantify reliability and availability.

6.2.1 Failure Density Functions

The common thread in the analysis of reliability and availability is the selection of an appropriate failure density function. Failure density functions are used to model a variety of reliability-associated events including time to failure and time to repair. Some of the more common failure density functions used in reliability analysis and their associated unreliability, failure rate, and mean time to failure functions are presented in Table 6.1.2.

6.2.2 Reliability

The reliability R(t) of a component is defined as the probability that the component experiences no failures during the time interval (o,t) from time zero to time t, given that it is new or repaired at time zero. In other words, the reliability is the probability that the time to failure T exceeds t, or

$$R(t) = \int_t^\infty f(t)\, dt \qquad (6.2.1)$$

where f(t) is the probability density function of the time to failure. Values for R(t) range between 0 and 1. The probability density function f(t) may be developed from equipment failure data, using various statistical methods. In many cases, a simple exponential distribution is found appropriate. Using the exponential distribution as an example, the reliability of a component in interval (o,t) is the area under the failure density curve to the right of point t (see Fig. 6.2.1).

6.2.3 Unreliability

The unreliability F(t) of a component is defined as the probability that the component will fail by time t. Unreliability can be defined mathematically as

Table 6.1.2 Probability Distributions Commonly Used In Reliability Analysis

Distribution	Probability Density Function	Range	Parameter-Moment Relations
Normal	$f(x) = \frac{1}{\beta\sqrt{2\pi}} e^{-\frac{(x-\alpha)^2}{2\beta^2}}$	$-\infty < x < \infty$	$\alpha = \mu_x, \beta = \sigma_x$
Lognormal	$f(x) = \frac{1}{x\beta\sqrt{2\pi}} e^{-\frac{(y-\alpha)^2}{2\beta^2}}$ where $y = \log x$	$x > 0$	$\alpha = \ln \mu_x - \frac{\beta^2}{2}$ $\beta^2 = \ln(1+C_v^2)$ $C_v = \sigma_x / \mu_x$
Exponential	$f(x) = \lambda e^{-\lambda x}$	$x \geq 0$	$\lambda = \frac{1}{\mu_x}$
Gamma	$f(x) = \frac{\lambda^\beta x^{\beta-1} e^{-\lambda x}}{\Gamma(\beta)}$ where Γ = Gamma function	$x \geq 0$	$\lambda = \frac{\mu_x}{\sigma_x^2}$, $\beta = \frac{\mu_x^2}{\sigma_x^2} = \frac{1}{C_v^2}$
Weibull	$f(x) = \frac{\beta}{\theta}\left[\frac{x-\gamma}{\theta}\right]^{\beta-1} e^{-\left[\frac{x-\gamma}{\theta}\right]^\beta}$	$x \geq 0$	$\mu_x = \gamma + \theta\Gamma(1+\frac{1}{\beta})$ $\sigma_x^2 = \theta^2\left[\Gamma(1+\frac{2}{\beta}) - \Gamma^2(1+\frac{1}{\beta})\right]$
Extreme Value Type I	$f(x) = \frac{1}{\alpha} e^{-(x-\beta)/\alpha - e^{-(x-\beta)/\alpha}}$	$-\infty < x < \infty$	$\alpha = \sqrt{6}\,\sigma_x/\pi$ $\beta = \mu_x - 0.5772\,\alpha$

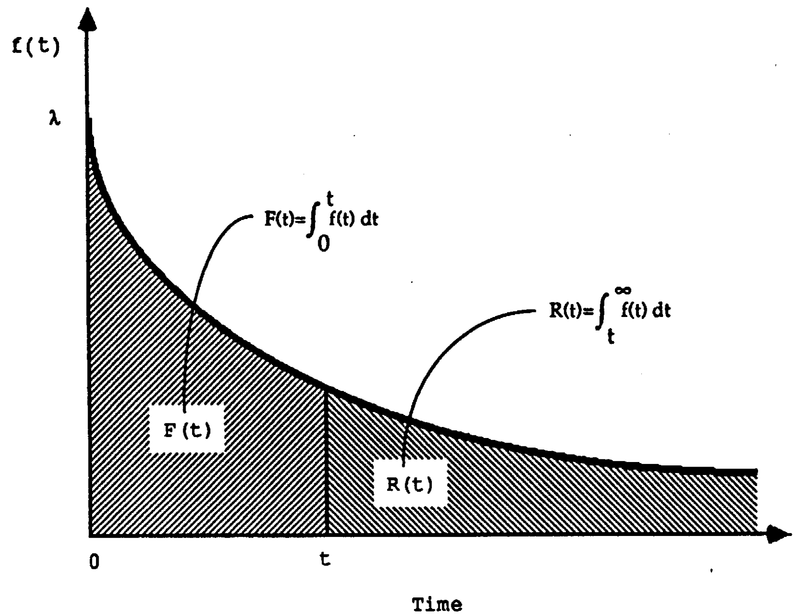

Figure 6.2.1 Exponential reliability density functions with areas showing F(t) and R(t)

$$F(t) = \int_0^t f(t)dt = 1 - R(t) \qquad (6.2.2)$$

Graphically, the unreliability function F(t) is the area under the failure density function to the left of point t (see Fig. 6.2.1)

6.2.4 Failure Rate

The failure rate m(t) is the probability that a component experiences a failure per unit of time t given that the component was operating at time zero and has survived to time t. Note that the failure rate m(t) is a conditional probability. The relationship of m(t) to f(t) and F(t) is given as

$$m(t) = \frac{f(t)}{R(t)} \qquad (6.2.3)$$

Sometimes, the failure rate is called hazard function. The quantity m(t)dt is the probability that a component fails during time (t, t + dt). Values for m(t)dt range from 0 to 1. Given the failure rate, the failure density function and the component reliability can be obtained as equations (6.2.4) and (6.2.5), respectively (Kapur and Lamberson, 1977).

$$f(t) = m(t) \exp\left[-\int_o^t m(h)dh\right] \qquad (6.2.4)$$

$$R(t) = \exp\left[-\int_o^t m(h)\,dh\right] \qquad (6.2.5)$$

6.3 TIME TO FAILURE ANALYSIS

Since the time to failure of a component is not certain, it is always desirable to have some idea of the expected life of the component under investigation. Furthermore, for a repairable component, the time required to repair the failed component might also be uncertain. This section briefly describes and defines some of the useful terminology in the field of reliability theory that is relevant in the reliability assessment of water distribution systems.

6.3.1 Mean Time to Failure

The mean time to failure (MTTF) is the expected value of the time to failure, stated mathematically as

$$MTTF = \int_0^{\infty} t f(t) dt \qquad (6.3.1)$$

which is expressed in hours.

6.3.2 Repair Density Function and Probability of Repair

Similar to the failure density function, the repair density function, g(t), describes the random characteristics of the time required to repair a failed component when failure occurs at time zero. The probability of repair, G(t), is the probability that the component repair is completed before time t, given that the component failed at time zero. Note that the repair process starts with a failure at time zero and ends at the completion of the repair at time t.

6.3.3 Repair Rate

Similar to the failure rate, the repair rate r(t) is the probability that the component is repaired per unit time t given that the component failed at time zero and is still not repaired at time t. The quantity r(t)dt is the probability that a component is repaired during time (t, t + dt) given that the components failure occurred at time t. The relation between repair rate, repair density and repair probability function is

$$r(t) = \frac{g(t)}{G(t)} \qquad (6.3.2)$$

Given a repair rate function r(t), the repair density function and the repair probability are, respectively,

$$g(t) = r(t) \exp\left[- \int_0^t r(h) dh \right] \qquad (6.3.3)$$

$$G(t) = 1 - \exp\left[-\int_0^t r(h)\,dh\right] \qquad (6.3.4)$$

6.3.4 Mean Time to Repair

The mean time to repair (MTTR) is the expected value of the time to repair a failed component. The MTTR is defined mathematically as

$$MTTR = \int_0^{\infty} t\,g(t)\,dt \qquad (6.3.5)$$

where g(t) is the probability density function for the repair time. The MTTR is expressed in hours.

6.3.5 Mean Time Between Failures

The mean time between failures (MTBF) is the expected value of the time between two consecutive failures. For a repairable component, the MTBF is defined mathematically as

$$MTBF = MTTF + MTTR \qquad (6.3.6)$$

6.3.6 Mean Time Between Repairs

The mean time between repairs (MTBR) is the expected value of the time between two consecutive repairs and equals the MTBF.

6.4 AVAILABILITY AND UNAVAILABILITY CONCEPTS

The reliability of a component is a measure of the probability that the component would be continuously functional without interruption through the entire period (o,t). This measure is appropriate if a component is nonrepairable and has to be discarded when the component fails. However, many of the components in a water distribution system are generally repairable and can be put back in service again. In that situation, a measure that has a broader meaning than that of the reliability is needed.

6.4.1 Availability

The availability A(t) of a component is the probability that the component is in operating condition at time t, given that the component was as good as new at time zero. The reliability generally differs from the availability because reliability requires the continuation of the operational state over the whole interval (0,t). Subcomponents contribute to the availability A(t) but not to the reliability R(t) if the subcomponent that failed before time t is repaired and is then operational at time t. As a result, the availability A(t) is always larger than or equal to the reliability R(t), i.e., $A(t) \geq R(t)$. For a nonrepairable component, it is operational at time t, if and only if, it has been operational to time t, i.e., $A(t) = R(t)$. As shown in Fig. 6.4.1, the availability of a nonrepairable component decreases to zero as t becomes larger, whereas the availability of a repairable component converges to a nonzero positive number.

6.4.2 Unavailability

The unavailability U(t) at time t is the probability that a component is in the failed state at time t, given that it started in the operational state at time zero. In general, the U(t) is less than or equal to the unreliability F(t), and for nonrepairable components they are equal. Because a component is either in the operational state or in the failed state at time t; therefore,

$$A(t) + U(t) = 1 \tag{6.4.1}$$

6.4.3 Conditional Failure Intensity

Conditional failure intensity, $\lambda(t)$, is the probability that a component fails per unit time at time t, given that it is in the operational state at time zero and is operational at time t. The quantity $\lambda(t)dt$ is the probability that a component fails during a small time interval (t, t + dt) given that the component was as good as new at time zero and operational at time t. The quantity m(t)dt is the probability that a component fails during the time interval given that the component was repaired at time zero and has been operational to time t. The quantities $\lambda(t)dt$ and m(t)dt differ because m(t)dt assumes the continuation of the operational state to time t or that no failure occurred in the interval (0,t), whereas $\lambda(t)dt$ only assumes that the component is operational at time t, i.e., intermediate failures between time zero and time t are not important to the calculation.

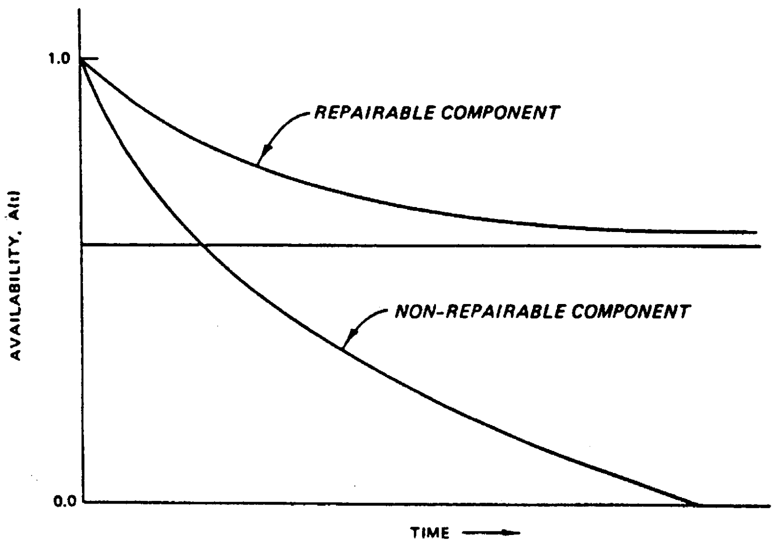

Figure 6.4.1 Availability for Repairable and Nonrepairable Components

$$\begin{aligned} &\lambda(t) \neq m(t) && \text{general case} \\ &\lambda(t) = m(t) && \text{nonrepairable component} \\ &\lambda(t) = m && \text{constant failure rate r} \end{aligned} \tag{6.4.2}$$

6.4.4 Unconditional Failure Intensity

The unconditional failure intensity, w(t), is the probability that a component fails per unit time at time t, given that it started in the operational state at time zero. The unconditional failure intensity is obtained from the analysis of equipment failure data (Henley and Kumamoto, 1981).

6.4.5 Expected Number of Failures

The expected number of failures W(t, t + dt), given that the component started in the operational state at time zero, is defined as

$$W(t,t + dt) = \int_{t}^{t+dt} w(h)dh \tag{6.4.3}$$

For a nonrepairable component, W(0,t) = F(t) and approaches unity as t gets larger. For a repairable component, W(0,t) diverges to infinity as t becomes larger. Typical curves of W(0,t) are shown in Fig. 6.4.2.

6.4.6 Conditional Repair Intensity

The conditional repair intensity, u(t), is the probability that a component is repaired per unit time at time t, given that it started in the operational state at time zero and failed at time t. The repair rate, r(t), and u(t) differ in a manner similar to the relation between $\lambda(t)$ and m(t).

$$\begin{aligned} &u(t) = r(t) = 0 && \text{nonrepairable component} \\ &u(t) = r && \text{constant repair rate r} \end{aligned} \tag{6.4.4}$$

6.4.7 Unconditional Repair Intensity

An unconditional repair intensity, v(t), is the probability that a component is repaired per unit time t, given that it started in the operational state at time zero.

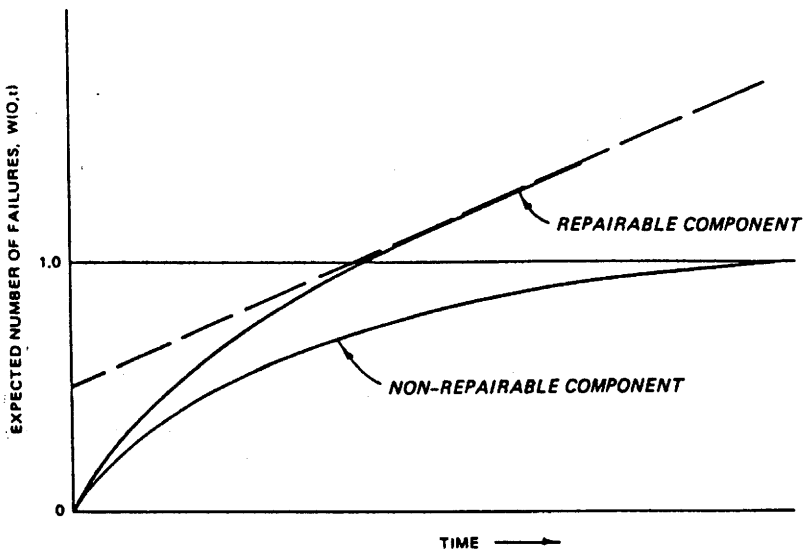

Figure 6.4.2 Expected Number of Failures for Repairable and Nonrepairable Components

6.4.8 Expected Number of Repairs

The expected number of repairs during (t, t + dt) given that the component started in the operational state at time zero is

$$V(t,t+dt) = \int_{t}^{t+dt} v(t)dt \tag{6.4.5}$$

For a nonrepairable component V(0,t) = 0 and for a repairable component, V(0,t) $\rightarrow \infty$ as t gets larger. Henley and Kumamoto (1981) developed a number of relationships between the various reliability (availability) parameters. The more significant of these are summarized in Table 6.4.1.

6.5 EXAMPLE CALCULATIONS OF AVAILABILITY AND UNAVAILABILITY

Because of its relative simplicity for performing reliability computations, the exponential distribution is probably the most widely used failure density function. Suppose the time to failure of a pump in a water distribution system is assumed to follow an exponential distribution with the parameter λ = 0.0008/hr (7.0/yr). The failure density function of the pump can be expressed as

$$f(t) = \lambda e^{-\lambda t} \qquad t \geq 0, \lambda = 0.0008/hr \tag{6.5.1}$$

The parameter λ in equation (6.5.1) is the failure rate m [refer to equation (6.2.3)]. The reliability of the pump at any time t > 0 is calculated, according to equation (6.2.1), as

$$R(t) = \int_{0}^{\infty} \lambda e^{-\lambda t} dt = e^{-0.0008t} \tag{6.5.2}$$

The reliability of the pump in the period of (0, 100 hours) is R(t = 100) = exp (-0.08) = 0.9231, and the associated unreliability is F(t = 100 hrs) = 1 - R(t = 100 hrs) = 0.0769.

Table 6.4.1 Relations Among Probabilistic Parameters*

	Repairable	Non-repairable
Fundamental Relations	(1) $A(t) + U(t) = 1$	$A(t) + U(t) = 1$
	(2) $A(t) > R(t)$	$A(t) = R(t)$
	(3) $U(t) < F(t)$	$U(t) = F(t)$
	(4) $w(t) = f(t) + \int_0^t f(t-u)\, v(u)\, du$	$w(t) = f(t)$
	(5) $v(t) = \int_0^t g(t-u)\, w(u)\, du$	$v(t) = 0$
	(6) $W(t, t + dt) = w(t)\, dt$	$W(t, t + dt) = w(t)\, dt$
	(7) $V(t, t + dt) = v(t)\, dt$	$V(t, t + dt) = 0$
	(8) $W(t_1, t_2) = \int_{t_1}^{t_2} w(u)\, du$	$W(t_1, t_2) = \int_{t_1}^{t_2} w(u)\, du$ $= F(t_1) - F(t_2)$
	(9) $V(t_1, t_2) = \int_{t_1}^{t_2} v(u)\, du$	$V(t_1, t_2) = 0$
	(10) $u(t) = W(0, t) - V(0, t)$	$u(t) = W(0, t) = F(t)$
	(11) $\lambda(t) = \frac{w(t)}{1 - u(t)}$	$\lambda(t) = \frac{w(t)}{1 - u(t)}$
	(12) $\eta(t) = \frac{v(t)}{u(t)}$	$\eta(t) = 0$
Stationary Value	(13) MTBF = MTBR = MTTF + MTTR	MTBF = MTBR = ∞
	(14) $0 < A(\infty) < 1,\ 0 < u(\infty) < 1$	$A(\infty) = 0,\ u(\infty) = 1$
	(15) $0 < w(\infty) < \infty,\ 0 < v(\infty) < \infty$	$w(\infty) = 0,\ v(\infty) = 0$
	(16) $w(\infty) = v(\infty)$	$w(\infty) = v(\infty) = 0$
	(17) $W(0, \infty) = \infty,\ V(0, \infty) = \infty$	$W(0, \infty) = 1,\ V(0, \infty) = 0$
Remark	(18) $w(t) \neq \lambda(t),\ v(t) \neq \eta(t)$	$w(t) \neq \lambda(t),\ v(t) = \eta(t) = 0$
	(19) $\lambda(t) \neq r(t),\ \eta(t) \neq m(t)$	$\lambda(t) = r(t),\ \eta(t) = m(t) = 0$
	(20) $w(t) \neq f(t),\ v(t) \neq g(t)$	$w(t) = f(t),\ v(t) = g(t) = 0$

* Adapted from Reliability Engineering and Risk Assessment, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1981, p. 187.

The MTTF of the pump , by equation (6.3.1), is

$$\text{MTTF} = \int_0^{\infty} t\lambda e^{-\lambda t} dt = \frac{1}{0.0008/\text{hr}} = 1250 \text{ hrs} \qquad (6.5.3)$$

From equation (6.5.3), the MTTF of a component having an exponential failure distribution of the form as shown by equation (6.5.1) is simply the inverse of the parameter λ.

Similarly, the mean time to repair of the pump can also be calculated assuming an exponential repair density function with the parameter $\eta = 0.02/\text{hr}$ as

$$g(t) = \eta e^{-\eta t}, t > 0, \eta = 0.02/\text{hr}. \qquad (6.5.4)$$

so that

$$\text{MTTR} = \int_0^{\infty} t \ \eta e^{-\eta t} dt = \frac{1}{\eta} = \frac{1}{0.02/\text{hr}} = 50 \text{ hrs}. \qquad (6.5.5)$$

The MTTR can be estimated using an arithmetical mean of the time to repair data for various types of components.

Using exponential failure and repair density functions, the resulting failure rate and repair rate, according to the definitions given previously, are constants equal to their respective parameters. For a constant failure rate and a constant repair rate the analysis of the whole process can be simplified to analytical solutions. Henley and Kumamoto (1981) use Laplace transforms to derive the unavailability as

$$U(t) = \frac{\lambda}{\lambda + \eta}\left[1 - e^{-(\lambda+\eta)t}\right] \qquad (6.5.6)$$

and the availability

$$A(t) = 1 - U(t) = \frac{\eta}{\lambda + \eta} + \frac{\lambda}{\lambda + \eta} e^{-(\lambda+\eta)t} \qquad (6.5.7)$$

Substituting $\lambda = 0.0008$ and $\eta = 0.02$ into equations (6.5.6) and (6.5.7), the corresponding unavailability and availability of the pump at time t = 100 hrs are 0.0336 and 0.9667, respectively.

The steady state or stationary unavailability $U(\infty)$ and the stationary availability $A(\infty)$ for t approaches ∞ are, respectively,

$$U(\infty) = \frac{\lambda}{\lambda + \eta} = \frac{MTTR}{MTTF + MTTR} \tag{6.5.8}$$

and

$$A(\infty) = \frac{\eta}{\lambda + \eta} = \frac{MTTF}{MTTF + MTTR} \tag{6.5.9}$$

As time gets larger, the steady state (or stationary) unavailability and availability for the pump can be calculated, using MTTF = 1250 hrs and MTTR = 50 hrs, as $U(\infty) = 0.03846$ and $A(\infty) = 0.96154$, respectively. The following relation is also true

$$\frac{U(t)}{U(\infty)} = 1 - e^{-(\lambda+\eta)t} \tag{6.5.10}$$

6.6 TIME TO FAILURE ANALYSIS FOR PUMPS

Time to failure analysis can be applied to the evaluation of pumping systems. For the sake of simplicity, the exponential distribution is used to illustrate a procedure for the time to failure analysis of a pump in which the pump failure data are lumped, i.e., failure data for the pump's individual subsystems are lumped into one parameter. A more detailed analysis in which the reliability and availability of the individual subsystem is presented in the following section.

Damelin, Shamir and Arad (1972) presented data for a pump indicating an MTTF of 1,200 hr and a MTTR of 50 hr for a 100-m^3/hr-capacity pump. Using the exponential distribution for time to failure and time to repair, the values for failure rate λ and repair rate η of 0.00083/hr (7.3/yr) and 0.02/hr (175.2/yr), respectively. Table 6.6.1 presents reliability, unreliability, and availability values for these values of λ and η.

The reliability of a system is the probability that the system experiences no failures during the time interval (0,t). The reliability curve for λ

Table 6.6.1

Reliability, Unreliability, Availability, and Unavailability
Data for $\lambda = 0.000833/\text{hr}$ and $\eta = 0.02/\text{hr}$

Time (hr)	Reliability	Unreliability	Availability	Unavailability
0	1.	0.	1.	0.
10	0.9917	0.0083	0.9925	0.0075
20	0.9834	0.0166	0.9637	0.0363
30	0.9753	0.0247	0.9814	0.0186
40	0.9672	0.0328	0.9774	0.0226
50	0.9592	0.0408	0.9741	0.0259
100	0.9200	0.0800	0.9650	0.0350
200	0.8465	0.1535	0.9606	0.0394
300	0.7788	0.2212	0.9601	0.0399
400	0.7165	0.2835	0.9600*	0.0400*
500	0.6592	0.3408	0.9600	0.0400
1000	0.4346	0.5654	0.9600	0.0400
2000	0.1889	0.8111	0.9600	0.0400
3000	0.0821	0.9179	0.9600	0.0400
4000	0.0357	0.9643	0.9600	0.0400
5000	0.0155	0.9845	0.9600	0.0400
10000	0.0002	0.9998	0.9600	0.0400

*Point of stationary availability and unavailability.

= 0.00083 is shown in Fig. 6.6.1. The availability of a system is the probability that the system is operational at time t. For repairable systems, availability is a more appropriate measure of the probability that a system will be operational. Availability is affected by both the MTTF and MTTR. Figure 6.6.2 presents availability curves for λ = 0.00083/hr and η = 0.02, 0.01, and 0.005/hr.

For repairable systems, the availability is always greater than or equal to the reliability. This concept is illustrated graphically in Fig. 6.6.3 for λ = 0.0008/hr and η = 0.01/hr. For repairable systems, as t approaches infinity, the availability approaches a constant value greater than 0 (stationary availability). A comparison is of the effect of both η and λ on the stationary availability and unavailability (Fig. 6.6.4).

6.7 TIME TO FAILURE ANALYSIS FOR WATER DISTRIBUTION PIPING

Regression equations can be developed for the break rates of water mains using data from specific water distribution systems. As an example, Walski and Pelliccia (1982) developed break rate regression equations (Fig. 6.7.1) for the Binghamton, New York system. These equations are

Pit Cast Iron: $$N(t) = 0.02577\ e^{0.0270t} \tag{6.7.1a}$$

Sandspun Cast Iron: $$N(t) = 0.0627\ e^{0.0137t} \tag{6.7.1b}$$

where N(t) = break rate in breaks/mile/year and t = age of pipe years.

Walski and Pelliccia (1982) also developed a regression equation for the time required to repair pipe breaks,

$$t_r = 65\,d^{0.285} \tag{6.7.2}$$

where t_r= time to repair, hr, and d = pipe diameter, in.

Techniques for evaluating the reliability and availability of water mains can best be illustrated through use of a simple example. Consider a 5-mile water main of sandspun cast iron pipe. The break rate per year (failure rate) can be calculated as follows

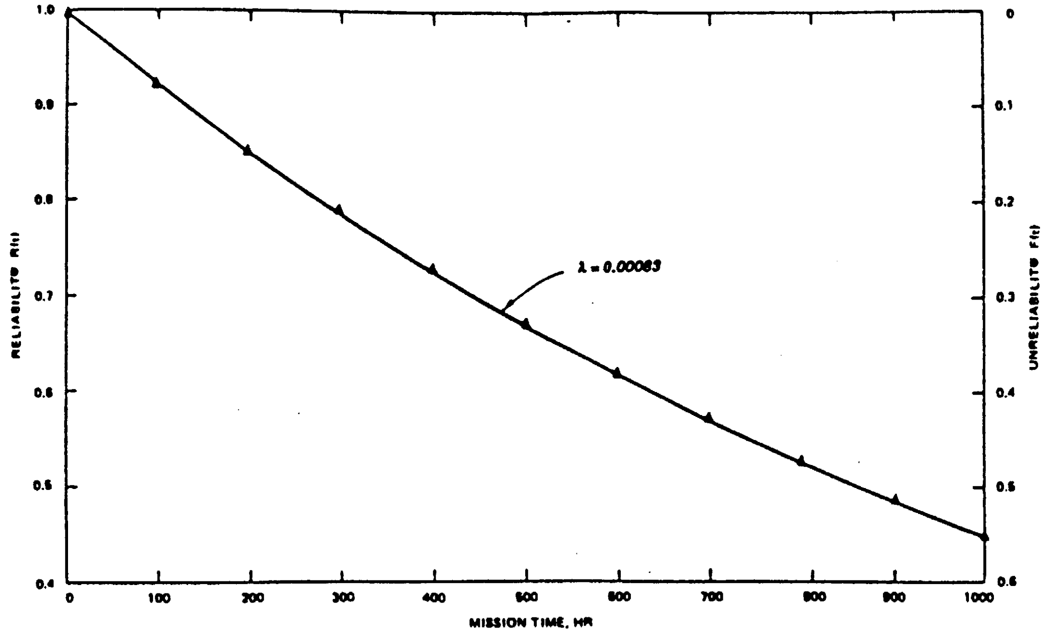

Figure 6.6.1 Reliability Curve for a Pump System With
$\lambda = 0.00083/\text{hr}$

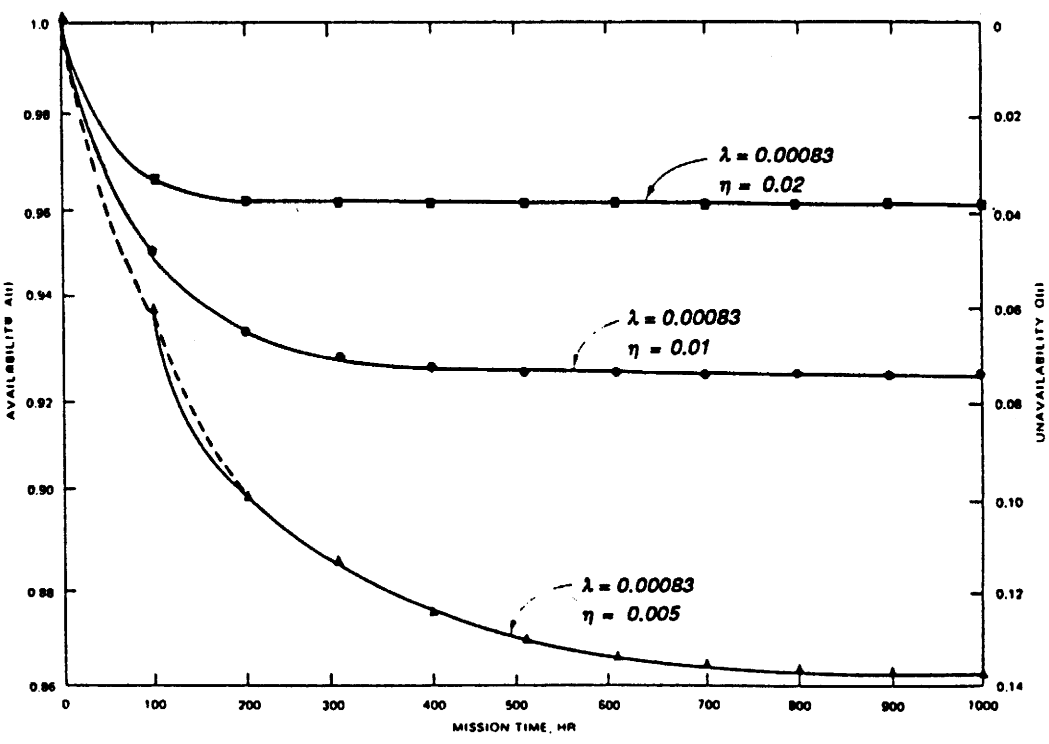

Figure 6.6.2 Availability Curves for a Pump System With
$\lambda = 0.00083/\text{hr}$ and $\eta = 0.02, 0.01,$ and $0.005/\text{hr}$

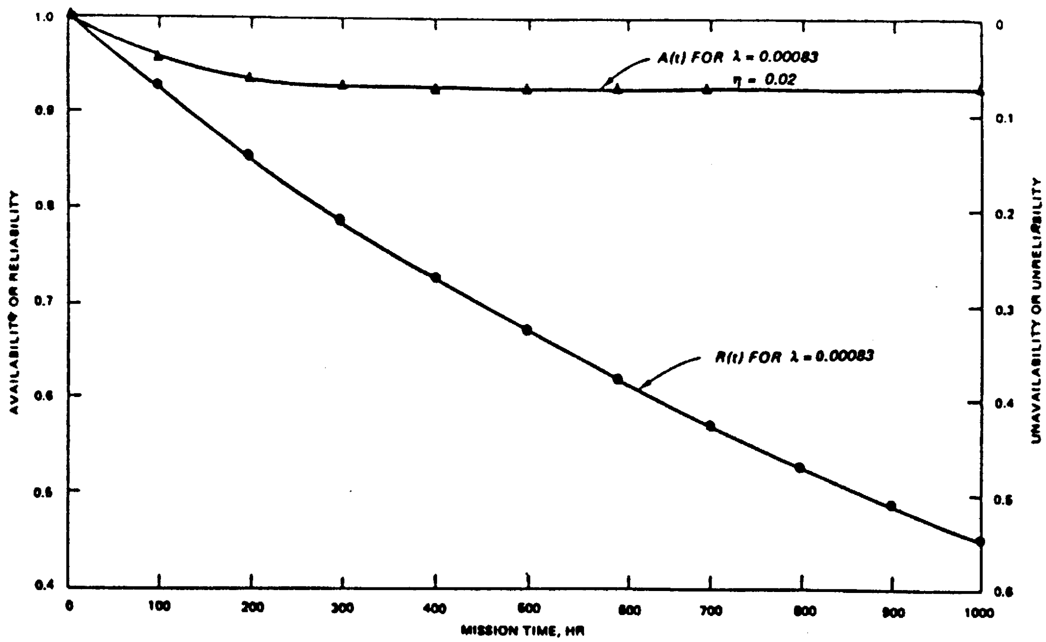

Figure 6.6.3 Comparison of Reliability and Availability for $\lambda = 0.00083/\text{hr}$ and $\eta = 0.02/\text{hr}$

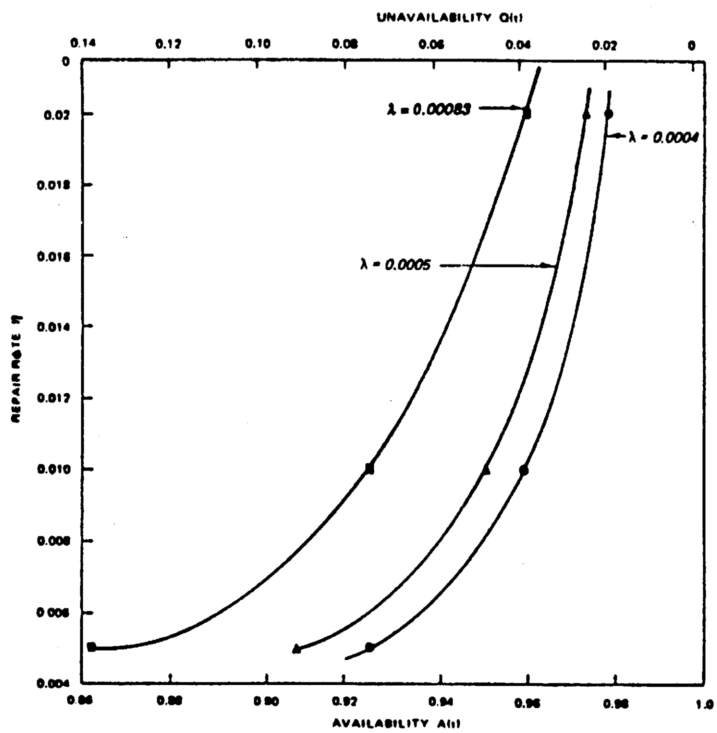

Figure 6.6.4 Effect of λ and η on Stationary Availability and Unavailability

Pipe Diameter in.	MTTR hr.
6	8.717
8	9.079
10	11.746
12	16.460
16 & larger	24.360

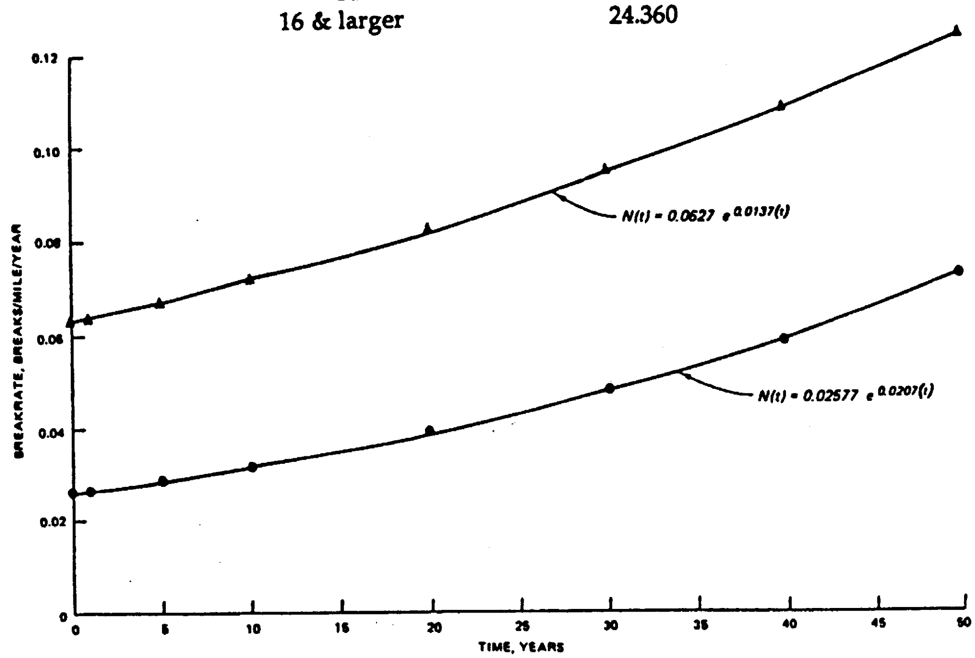

Figure 6.7.1 Break Rate Curves for Pit Cast Iron and Sandspun Cast Iron Pipes

$$r(t) = 5 \text{ miles} \times N(t) = 5 \times 0.0627\, e^{0.0137t} = 0.3185\, e^{0.0137t} \qquad (6.7.3)$$

The reliability for the newly installed 5-mile water main can be computed using equations (6.7.3) and (6.2.5) as follows

$$R(t) = \exp\left[- \int_0^t \left(0.3185\, e^{0.0137h} \right) dh \right] \qquad (6.7.4)$$

$$R(t) = \exp\left[23.25 \left(1 - e^{0.0137t} \right) \right] \qquad (6.7.5)$$

The failure density f(t) can be calculated, using equation (6.2.4), as

$$f(t) = 0.3185\, e^{0.0137t} \exp\left[23.25 \left(1 - e^{0.0137t} \right) \right] \qquad (6.7.6)$$

In a similar manner, the reliability based on the failure rate per mile can be calculated to be

$$R(t) = \exp\left[4.577 \left(1 - e^{0.0317t} \right) \right] \qquad (6.7.7)$$

Reliability curves for various mission times for equations (6.7.5) and (6.7.7) are plotted in Fig. 6.7.2.

Determining the availability of a water main is substantially more difficult because the failure rate increases as pipe age increases. Numerical integration or Laplace transform methods may be used to compute availability. However, a simplified procedure can be used to evaluate water main availability if a constant failure rate is assumed. For example, the average failure rate for the above 5-mile pipe link can be estimated from Fig. 6.7.3 to be 0.48. Assuming an MTTR of 16.460 hr (0.69 days or 0.0019 yrs), the availability can be calculated as follows

$$A(\infty) = \frac{\text{MTTF}}{\text{MTTF + MTTR}} = \frac{2.08}{2.08 + 0.0019} = 0.999 \qquad (6.7.8)$$

An availability of 0.999 indicates that, on average, the main will be out of service approximately 9 hrs. per year.

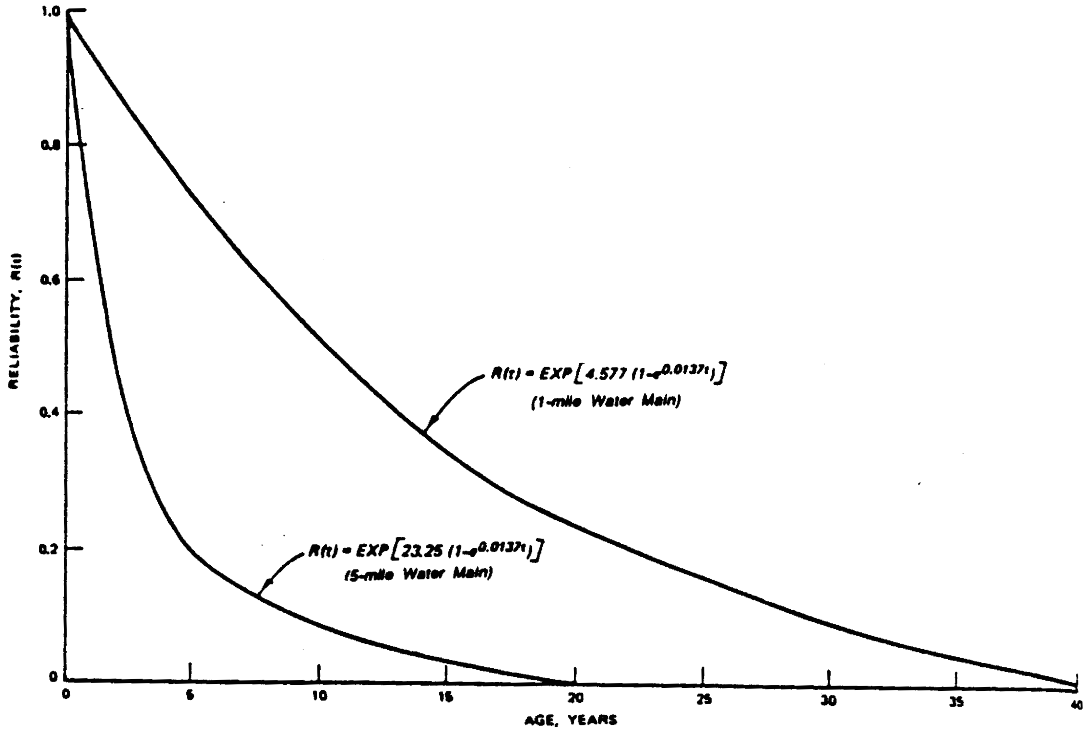

Figure 6.7.2 Reliability Curves for Pipe Evaluation Example

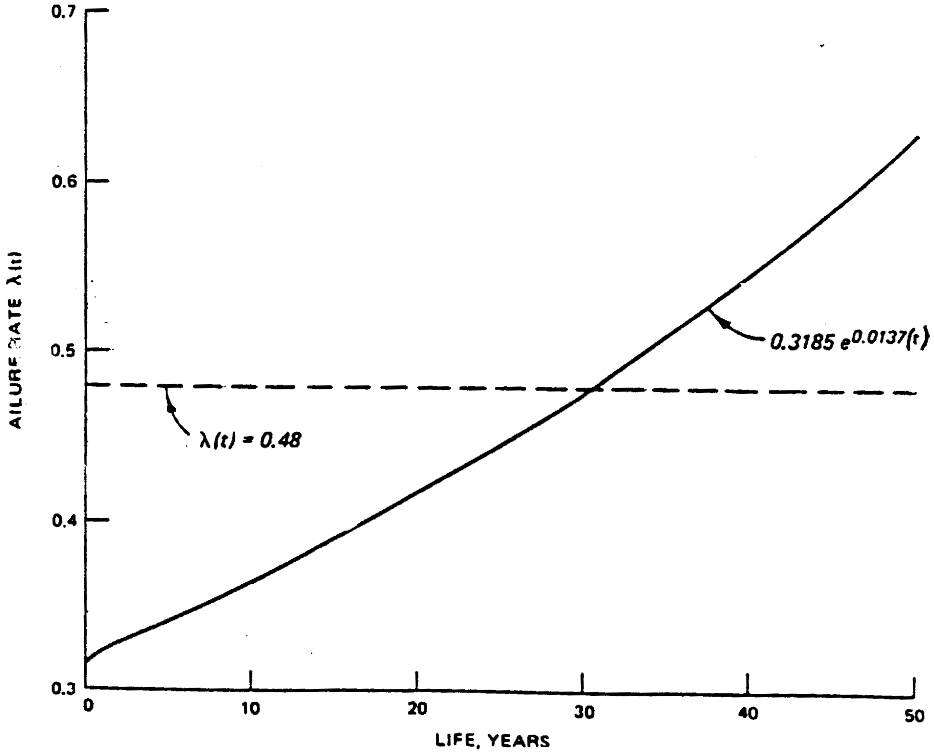

Figure 6.7.3 Failure Rate Curve for Sandspun Cast Iron Pipe

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