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Multiple-Objective Waste Load Allocation

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Abstract. In an attempt to improve managing river water quality, this paper presents a model to a three-objective WLA problem using the constraint method in conjunction with the parametric linear programming technique. The three objectives considered are: (1) maximization of total waste load discharge, (2) minimization of the largest difference in equity between dischargers, and (3) maximization of the lowest allowable water quality standard.

Key words. Waste load allocation, water quality management, multi-objective analysis, linear programming.

1. Introduction

The solution to a growing number of environmental problems facing water quality professionals today are becoming more complex. The necessity for improved environmental protection has not precluded the problem of waste load allocation (WLA) from increasing governmental and societal demands on water quality assurance. As society progresses with time, the demand placed on water quality will continue to grow, resulting in the continued need for improved water quality prediction and protection techniques. Consequently, the solution to such problems will become ever increasingly complex.

Past research attempts to solve the optimal WLA problems have been centered around a single goal or objective to be attained in the problem formulation, i.e., the minimization of treatment cost or the maximization of waste discharge. From a decision-making viewpoint, an optimum solution to such a problem can only be obtained by including the entirety of possible physical, legal, and economic considerations in the problem formulation. In reality, most environmental problems, including WLA, are multiobjective by nature. It is unlikely that a 'true' optimum solution to such problems are obtained by considering a single objective in the decision process. The decision-making process in most environmental problems is cultivated by the desire to achieve several goals simultaneously. The problem of optimal WLA is without exception to these aspirations. The identification of a single objective to obtain an optimum solution to the WLA problem is obviously unrealistic.

The importance of considering a multiobjective approach in the area of water resources has been cited in a number of previous works (Monarchi *et al.*, 1973; Cohon and Marks, 1973; Taylor *et al.*, 1976). By incorporating multiobjective procedures in the decisionmaking process, three major improvements are accomplished: (1) the role of the analyst and decision-maker are more clearly defined, (2) the results from the multiobjective approach provide a greater number of alternatives to the decision-making process, and (3) models utilizing such techniques are generally more realistic. The use of multiobjective procedures possess the distinct advantage of allowing a variety of problems to be solved, while simultaneously considering several noncommensurable and conflicting objectives (Cohon, 1978).

It is the intention of this paper to present a methodology for formulating and solving the WLA problem within a multiobjective framework using the procedure known as the 'constraint method' in conjunction with the parametric linear programming (LP) technique. Given the rising demands placed on water quality assurance by government and society, the utilization of multiobjective procedures can only lead to improved water quality prediction and control.

2. General Framework of Multi-Objective Model

2.1. VECTOR OPTIMIZATION MODEL

Within the multiobjective framework the problem consists of more than one scalar objective function. The problem is one of the 'vector optimization' which can be expressed as

$$\text{Max } \mathbf{Z}(\mathbf{X}) = [Z_1(\mathbf{X}), Z_2(\mathbf{X}), \ldots, Z_K(\mathbf{X})], \tag{1}$$

subject to

$$\mathbf{g}(\mathbf{X}) \leq \mathbf{0}, \tag{2}$$

where $\mathbf{Z}(\mathbf{X})$ is a K-dimensional vector of the objective functions, $\mathbf{X}$ is an n-dimensional vector containing the decision variables, and $\mathbf{g}(\mathbf{X})$ is an m-dimensional vector of constraints.

2.1. NONINFERIOR SOLUTION SET

In contrast to the single-objective problem, the ideological theme of 'optimality' is no longer appropriate in the context of the multiobjective framework because there are normally several objectives which are noncommensurable and conflicting with each other. It is also important to realize that without prior knowledge of the preference between the objectives, the mathematical programming solution to the multiobjective problem results in a set of points defining the tradeoff between each objective. Here, the goal of 'optimality' (in the single-objective framework) is replaced by the concept of 'noninferiority' in the multiobjective analysis. Cohon

(1978) defined the noninferiority in the following passage:

> A feasibility solution to a multiobjective programming problem is noninferior if there exists no other feasible solution that will yield an improvement in one objective without causing a degradation in at least one other objective.

The noninferior solution set, in general, is defined by a unique continuous curve or surface depicting the tradeoffs between the various alternatives. From this, it is obvious that, in theory, an infinite number of solutions exist to the multiobjective problem. It is not until the decision-maker provides the characterization of preference between each objective that a best compromising solution is identified. The 'best-compromising' solution to the multiobjective problems is a unique set of alternatives which possess the property of maximum combined utility and are elements in both the noninferior solutions set and indifference curve. Such an alternative only exists at the point where the indifference curve and noninferior solution set are tangent (Cohon, 1978).

3. Generalized Constraint Method Approach

The constraint method was first cited by Marglin in the book by Maass *et al.* (1962) and again in Marglin (1967). This approach enables the analyst to generate the noninferior solution set in entirety without regards to convexity. The computational simplicity is probably the most distinguished advantage of the constraint method, although, in general, such procedures are usually confined to multiobjective formulations containing fewer than four objectives. When using the constraint method, the multiobjective problem is solved by adopting only one objective in the objective function. The remaining objectives are simply transformed into constraints in the problem formulation.

Once the multiobjective problem has been formulated, the constraint method provides a relatively effortless computational methodology for generating the noninferior solution set. Moreover, if the multiobjective formulation follows an LP format, the constraint method can be solved by a parametric LP approach. For a detailed analysis of the attributes of the constraint method, the readers should consult Cohon and Marks (1975) and Cohon (1978).

4. The Multi-Objective Waste Load Allocation

In this paper, model presentation and discussion are based on a three-objective LP problem formulation. The objective functions and the model constraints are discussed in the following subsections.

4.1. OBJECTIVE FUNCTIONS

The three objective functions considered for the WLA problem in this study are: (1) the maximization of wase discharge, where both biochemical oxygen demand

(BOD) and dissolved oxygen (DO) deficit from each discharger are the decision variables, (2) the minimization of the maximum difference in equity between the various users of the stream environment, and (3) the maximization of the lowest allowable DO concentration level in the stream.

In review of treatment plant operations, a tradeoff exists between the allowable waste discharge and the DO deficit in each plant effluent. By reducing the DO deficit in the effluent through an induced reaeration process, greater quantities of waste can be discharged without violating the minimum DO requirements within the stream environment, hence, waste removal costs are reduced. Of course, a price must be paid to provide this reaeration. Here an analogy can be drawn between the maximization of waste discharge and minimization of treatment cost, in fact, both goals are economically quite similar. By maximizing waste output, the associated overall treatment costs are generally reduced. Though not exactly identical, the economic parallelism between these objectives is evident.

Therefore, the first objective function is formulated as

$$\text{Maximize } Z_1 = \sum_{j=1}^{N} (B_j + D_j), \tag{3}$$

where B_j and D_j are the waste concentration (mg/1 BOD) and DO deficit (mg/1) in the effluent at each discharge location j, and N is the total number of dischargers.

The second objective considered here is that a minimizing the maximum difference in equity between the various dischargers to the stream environment. It seems unreasonable to consider the WLA model complete without incorporating the idea of 'fairness' into the model formulation. For instance, without considering equity among waste dischargers, the attempt to maximize waste discharge would result in an allocation of large quantities of waste to the upstream users, while the downstream dischargers would be required to treat their influents at levels of maximum possible efficiency. (This is especially true for fast-moving streams.) Therefore, as the requirement of fairness measure is raised, the total waste load to the stream system would generally be decreased. Equity between the various users can be measured in a number of ways. In this study, the equity consideration of equal percent removals among the various dischargers was utilized. For a system involving multiple dischargers, the difference in equity measure would be varying. In this study, the worst case associated with the largest difference was adopted. Hence, the second objective can be expressed as

$$\text{Minimize } Z_2 = \delta E_{\max} = \max [\, |E_j - E_{j'}|\,], \quad \forall\, j \neq j', \tag{4}$$

where $\delta E_{\max}$ is a new decision variable representing the largest difference in equity measure between the various dischargers and E_j is the equity measure for the waste discharger i.

The third objective considered is the lowest allowable DO concentration level that should be maintained in the stream environment. From the perspective of preserving stream water quality, the higher the water quality standard is set, the more desirable the water quality would be maintained. However, it is intuitively understandable that the waste treatment cost would be increased as the instream water quality standard is raised. Therefore, the perspectives of preserving water quality and of enhancing economic efficiency are conflicting each other. In the study, this third objective is expressed as

$$\text{Maximize } Z_3 = \text{DO}_{\text{min}}^{\text{std}}, \tag{5}$$

where $\text{DO}_{\text{min}}^{\text{std}}$ is the minimum required DO standard in the stream.

4.2. CONSTRAINTS

The constraints set in a mathematical programming model defines the physical, biological, legal, and economical limitations of the system itself. With the above-mentioned objective functions in the WLA model, unrestricted waste discharge to a stream environment will pose detrimental effects to the aquatic biota, eventually producing an anaerobic environment in which all forms of desired life cease to exist. Hence, the inclusion of constraints which properly define and protect the use of limited resources within a stream environment are essential in the WLA problem formulation.

Constraints on Water Quality

The most common requirement of the WLA problem has been the assurance of minimum concentrations of DO throughout the river system in an attempt to maintain a desirable living environment for aquatic biota. Specifically, the constraint relating the response of DO to the addition of in-stream waste, is generally defined by the Streeter-Phelps equation (Streeter and Phelps, 1925) or a variation of this equation (ReVelle *et al.*, 1968; Bathala *et al.*, 1979). By utilizing the Streeter-Phelps equation, each control point and discharge location becomes a constraint in a mathematical programming model providing a check on water quality at that location. In a general framework, a typical water quality constraint would be as follows:

$$\sum_{j=1}^{n_i} \Theta_{ij} \cdot B_j + \sum_{j=1}^{n_i} \Omega_{ij} \cdot D_j + \text{DO}_{\text{min}}^{\text{std}} \leqslant \text{DO}_i^{\text{sat}} - D_{0i}, \quad i=1,2,\ldots,M, \tag{6}$$

where Θ_{ij} and Ω_{ij} are the technological transfer coefficients indicating the relative impact on DO concentrations at downstream locations, i, resulting from a waste input at an upstream location, j; n_i is the number of the dischargers upstream of the control point i; $\text{DO}_{\text{min}}^{\text{std}}$ represents the minimum DO standard desired for the stream environment; DO_i^{sat} is the saturated DO concentration in reach i; D_{0i} is the initial DO concentration at the upstream end of the i^{th} reach; and M is the total number of control points.

Constraints on Treatment Equity

In addition to the constraints satisfying water quality, constraints satisfying water quality, constraints are also required for defining equity between the various dischargers along the river system. As stated previously, without the inclusion of equity considerations in the WLA model, any attempts to maximize waste discharge would result in the allocation of large quantities of waste to the upstream users, while the downstream dischargers would be required to treat their effluent at levels of maxiumum possible efficiency. There have been several articles citing the importance of equity considerations in the WLA problem (Gross, 1965; Loucks *et al.*, 1967; Miller and Gill, 1976; Brill *et al.*, 1976).

From a decision-making viewpoint, the objective of the WLA problem is to obtain an optimum solution from a model formulation which has incorporated as many factors as possible concerning actual system behavior. In doing so, the excecution of such a model will result in an optimum solution attaining the highest degree of consciousness. Hence, any attempts by a legislative body to mandate the compliance of a WLA policy where large equitable differences existed between the various dischargers, would unquestionably be tried in both social and legal arenas. The implementation or regulatory enforcement of an optimum policy derived from the solution of any WLA model, in which equity is not considered, is probably neither acceptable nor justifiable.

Recognizing the importance of equity consideration in the WLA process, the choice must then be made as to the type of equity to be used. Based on the conclusion drawn by Chadderton *et al.* (1981), the type of equity considered in this study is the equal percent removal. In mathematical form, constraints for equity can be generally expressed as

$$| E_j - E_{j'} | - \delta E_{\max} \leqslant 0 , \quad \text{for } j \neq j' \tag{7}$$

where E_j represents the equity measure considered for discharger j, $\delta E_{\max}$ (a decision variable) represents the largest difference in equity between the two dischargers j and j'. In order to incorporate this constraint into an LP model, it must be expressed as linear functions of the decision variables (i.e., effluent waste concentration at each discharge location, B_j). In following this approach, the constraints for equity considering equal percent removal between the dischargers can be written as

$$| (B_j / I_j) - (B_{j'} / I_{j'}) | - \delta E_{\max} \leqslant 0 , \quad \text{for } j \neq j' \tag{8}$$

where I_j is the influent raw waste concentration (mg/1 BOD) at discharge location j.

Additionally, it should be noted that for any stream system considered, one or more of the discharges considered may be an influent tributary. The discharge from a tributary should be excluded from the consideration of equity in order to prevent undue restrictions being placed on the required treatment levels assigned to other dischargers. Therefore, provisions should be included to account for tributary flows and their waste inputs in order to identify the entirety of potential waste sources.

Constraints on Treatment Efficiency

The final set of constraints to consider are those defining the acceptable range of the treatment efficiency. A range between 35 and 90% removal of raw waste at each discharge location is considered in this study. The minimum requirement of 35% removal is to prevent floating solids from being discharged to the stream environment. The discharge of solids of this type is both socially and environmentally objectionable. On the other hand, the upper limit of 90% removal represents the maximum efficiency (assumed) attainable by practical treatment technology (Loucks *et al.*, 1967).

The treatment efficiency constraints for each discharge location can be expressed as

$$0.35 \leqslant 1 - \frac{B_j}{I_j} \leqslant 0.90\,, \quad \text{for } j = 1,2,\ldots,N. \tag{9}$$

Certainly, readers might argue that the limits set on treatment efficiency are antiquated. Nonetheless, these limits were selected solely as a means to illustrate the use of the methods presented here. By changing these limits, only the size of the feasible region in which the optimum solution is sought is affected, not the utility of the methods themselves. The numerical exactness of the values selected for the examples in this article are not the point of the discussion. Rather, it is the intention of the paper to focus on the use of a multiobjective framework in the WLA problem.

In summary, the three-objective WLA model can be expressed, in entirety, as follows:

$$\text{(Objective 1) Maximize } Z_1 = \sum_{j=1}^{N} (B_j + D_j), \tag{3}$$

$$\text{(Objective 2) Minimize } Z_2 = \delta E_{\max}, \tag{4}$$

$$\text{(Objective 3) Maximize } Z_3 = \mathrm{DO}^{\mathrm{std}}_{\min} \tag{5}$$

subject to

$$\sum_{j=1}^{n_i} \Theta_{ij} \cdot B_j + \sum_{j=1}^{n_i} \Omega_{ij} \cdot D_j + \mathrm{DO}^{\mathrm{std}}_{\min} \leqslant \mathrm{DO}^{\mathrm{sat}}_i - D_{0i}, \quad i=1,2,\ldots,M, \tag{6}$$

$$| E_j - E_{j'} | - \delta E_{\max} \leqslant 0 , \quad \text{for } j \neq j', \tag{7}$$

$$0.35 \leqslant 1 - \frac{B_j}{I_j} \leqslant 0.90 , \quad \text{for } j = 1,2,\ldots,N. \tag{9}$$

and

$$B_j \geqslant 0, \qquad D_j \geqslant 0, \qquad \delta E_{\max} \geqslant 0, \qquad \mathrm{DO}_{\min}^{\mathrm{std}} \geqslant 0, \quad \text{for } j=1,2,\ldots,N.$$

5. Multi-Objective WLA Using the Constraint Method

5.1. FORMULATION OF MULTI-OBJECTIVE WLA MODEL USING CONSTRAINT METHOD

Following the general procedures of the constraint method, the three-objective WLA model must first be transformed into a single-objective model formulation. In doing so, the goal to maximize the minimum DO standard was selected to be the objective function. The remaining objectives were transformed into the constraints in the WLA model. Hence, the original three-objective formulation was cast into a single-objective formulation as

$$\text{Maximize } Z_3 = \mathrm{DO}_{\min}^{\mathrm{std}}, \tag{5}$$

subject to

$$\sum_{j=1}^{n_i} \Theta_{ij} \cdot B_j + \sum_{j=1}^{n_i} \Omega_{ij} \cdot D_j + \mathrm{DO}_{\min}^{\mathrm{std}} \leqslant \mathrm{DO}_i^{\mathrm{sat}} - D_{0i}, \quad i=1,2,\ldots,M, \tag{6}$$

$$| E_j - E_{j'} | - \delta E_{\max} \leqslant 0 , \quad \text{for } j \neq j', \tag{7}$$

$$0.35 \leqslant 1 - \frac{B_j}{I_j} \leqslant 0.90 , \quad \text{for } j = 1,2,\ldots,N. \tag{9}$$

$$\delta E_{\max} \geqslant \delta E^0, \tag{10}$$

$$\sum_{j=1}^{N} (B_j + D_j) \geqslant L_{\mathrm{tot}} \tag{11}$$

and

$$\delta E_{\max} \geqslant 0, \qquad \mathrm{DO}_{\min}^{\mathrm{std}} \geqslant 0, \qquad B_j \geqslant 0, \qquad L_j \geqslant 0, \quad \text{for } j=1,2,\ldots,N,$$

in which δE^0 and L_{tot} are the specified and difference in equity measure total waste load amount, respectively.

6. Application

A hypothetical example of a six-reach stream system was utilized in this paper. Data describing the physical parameters of the stream environment and a sketch of the example system are shown in Table I and Figure 1, respectively. Once the three-objective WLA model using the constraint method was formulated, it simply became a matter of solving the model repeatedly using different values of δE^0 and L_{tot} to generate the noninferior solution set. Of course, this can be done via parametric LP approach. The model was solved successively until the solution set become infeasible.

In order to enhance the computational efficiency with reduced constraints, the moving control point approach for controlling water quality in the model constraints was applied. Briefly speaking, the essence of the moving control point approach is to utilize a single moving control point associated with the critical location (i.e., the point of minimum dissolved oxygen) within each reach of the river system. The problem was solved in an interative manner. During each iteration, the water quality constraints were defined at a single control point within each reach, which coincided with the critical location found by

$$X_i^c = \left(\frac{U_i}{K_i^a - K_i^d}\right) \ln \left\{ \frac{K_i^a}{K_i^d} \left[1 - \frac{K_i^a - K_i^d}{K_i^d} \frac{D_j}{B_j}\right]\right\} \tag{12}$$

using the current solutions for B_j and D_j, where $K^a{}_i$ and $K^d{}_i$ are aeration and deoxygenation coefficients for the i^{th} stream segment, respectively. The iterations

Table I. Data of physical characteristics used in the example of WLA models

(a) Stream characteristics for each reach

Reach	Deoxygenation coefficient (1/days)	Reaeration coefficient (1/days)	Average stream velocity (km/day)	Raw waste concen. (mg/l BOD)	Effluent flow rate (m^3/sec)
1	0.6	1.84	26.4	1370	0.0042
2	0.6	2.13	26.4	6.0	1.2460
3	0.6	1.98	26.4	665	0.1308
4	0.6	1.64	26.4	910	1.0141
5	0.6	1.64	26.4	1500	0.0906
6	0.6	1.48	26.4	410	0.0221

(b) Background characteristics

Upstream waste concentration (mg/l BOD)	Upstream flow rate (m^3/sec)	Upstream DO deficit (mg/l)
5.0	3.2568	1.0

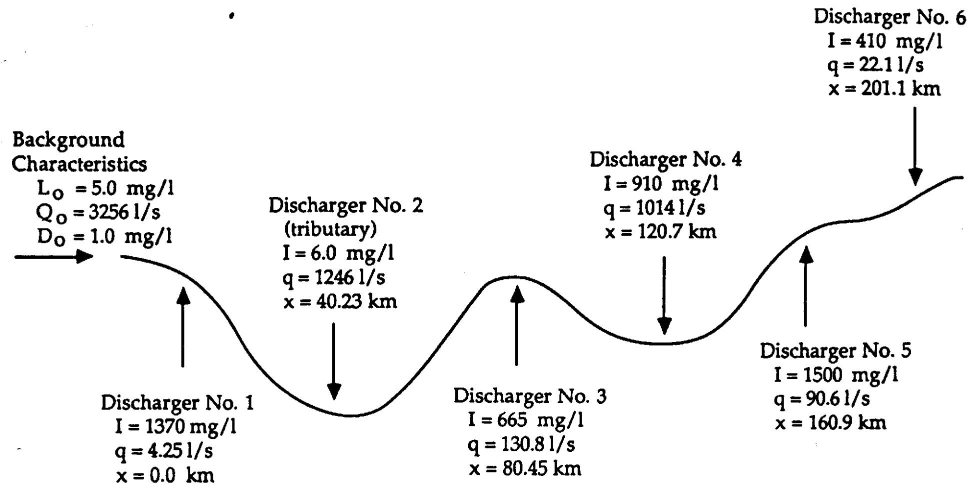

Fig. 1. Schematic sketch of example river system in WLA problem.

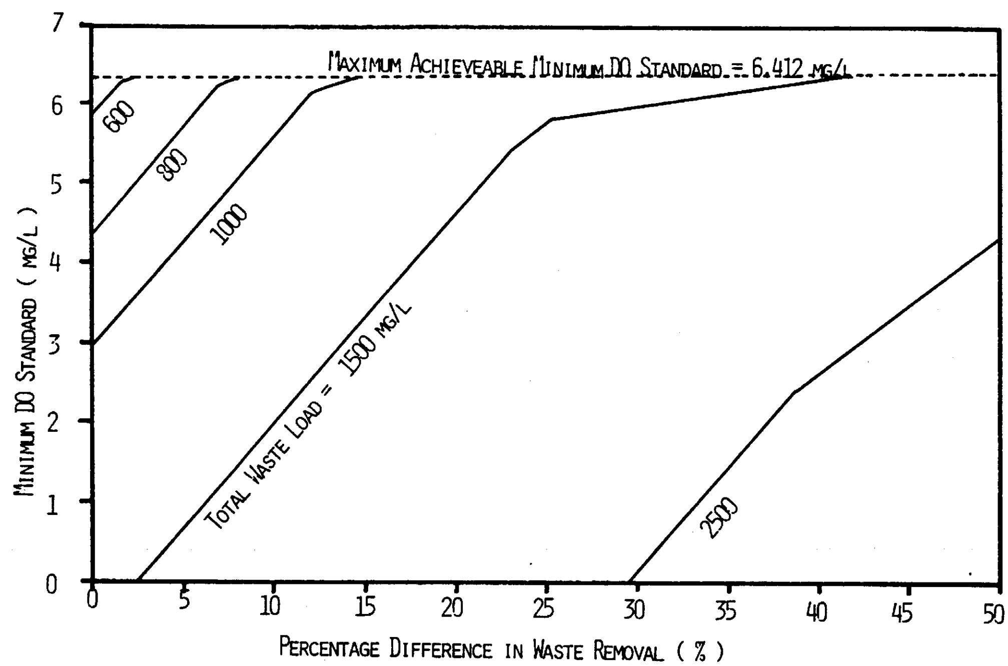

Fig. 2. Indifference solution sets for different levels of total waste load.

cease once the difference between critical locations and/or optimal solutions of successive iterations are less than some specified convergence critaria. The practical importance of such provisions was to take advantage of the savings in computer storage and to improve model performance. For a detailed description of the moving control point method, readers are referred to Hathhorn (1986).

The solution to the noninferior set for the type of equity measure considered is displayed graphically in Figure 2. Clearly, for a given level of minimum DO standard, the amount of total waste that could be discharged can only be increased at the expense of accepting a wider difference in removal percentage among dischargers. The highest achievable DO standard for the example system considered was 6.412 mg/l. As the minimum DO standard was raised, under a fixed level of difference in waste removal percentage, the associated total waste load that can be discharged in the stream system was decreased.

7. Summary and Conclusions

To date, the continued reliance upon a single-objective optimization framework to manage a variety of environmental systems seems unreasonable. Most environmental problems are multiobjective by nature and should be treated accordingly. The problem of WLA is not withstanding from this realization.

In an attempt to improve river water quality management, this paper presented a methodology to solve a three-objective WLA model using the constraint method in conjunction with parametric LP technique. The multiobjective model presented here was applied to a multiple-reach river system in which the goals of maximization of total waste discharge, minimization of the largest differences in equity measure among waste dischargers, and maximization of minimum DO standard were considered. The relevance of this multiobjective approach to the problem is that a more realistic solution to the problem of WLA could be identified by specifying the tradeoffs (given by the noninferior solution set) that exist among the three objectives. This information can then be passed on to the decision-making agency, where the ultimate responsibility of management policy lies. The information provided by this approach will likely enhance the decision-maker's ability to select a 'best-compromising' solution given the set of alternatives to the problem of optimal river water quality management.

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