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Yeou-Koung Tung
Wade E. Hathhorn

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Yeou-Koung Tung
Wyoming Water Research Center

Wade E. Hathhorn
Department of Civil Engineering

University of Wyoming
Laramie, Wyoming

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# DETERMINATION OF THE CRITICAL LOCATIONS IN A STOCHASTIC STREAM ENVIRONMENT

YEOU-KOUNG TUNG

*Wyoming Water Research Center and Statistics Department, University of Wyoming, Laramie, WY 82071 (U.S.A.)*

and WADE E. HATHHORN

*Department of Civil Engineering, University of Texas, Austin, TX 78712 (U.S.A.)*

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## ABSTRACT

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Due to the existence of many uncertainties in a stream environment, the determination of the critical location is not a trivial task. The problem is further complicated by the lack of universally acceptable definition for the critical location in a stochastic stream environment. This paper discusses four potentially usable criteria for defining the critical location in a stochastic stream environment. Furthermore, techniques are developed to find these critical locations. A numerical experimentation is performed to investigate their differences.

## INTRODUCTION

In a deterministic stream system with dissolved oxygen (DO) problems, the critical point represents a unique location at which the DO concentration is at its minimum. From a regulatory viewpoint, it is this critical location which would present the greatest threat to violate the water quality standards. Therefore, to appropriately protect the stream environment from excessive DO depletion, the ability to determine the critical location deserves great attention.

Moreover, great potential savings in water quality monitoring costs can be achieved if the critical location can be identified or somehow established within a narrow section of the stream system. Knowing the whereabout of the critical location within stream reach, a more intensive monitoring effort

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could be given to locations within this region rather than to those outside the region which present a lesser threat to violate water quality standards. Consequently, more effective monitoring programs and cost saving would be possible.

Finding the critical location within a reach for stream system describable by deterministic water quality models is generally straight forward. However, the task to identify the critical location in a stochastic stream environment is no longer a simple deterministic calculation. There have been some researches made in attempting to estimate the uncertainty in calculating the DO concentration and to derive the probability distribution for the DO concentration. However, to the author's knowledge, no article has been published addressing the issue on how one could locate the critical point in a stochastic stream environment. This is perhaps because no definition is given to what consititutes a critical location in a stochastic stream environment. The intent of this paper is to discuss four potentially usable forms of the critical location in a stochastic stream environment and to present methodologies by which such locations can be determined. The four critical locations examined in this study are:

(1) the location with the lowest expected DO concentration;

(2) the location at which the variance of deficit is the largest;

(3) the location with the highest probability of violating the water quality standards; and

(4) the location most likely to be critical.

Detailed discussions of the significance and rationale of these four critical locations will be given later.

## BASIC WATER QUALITY MODEL

In determining the critical location, some types of water quality model describing the interaction between the physical and biological processes occurring within the stream are generally employed. To demonstrate the main theme of this paper, the well-known Streeter–Phelps equation (Streeter and Phelps, 1925) is adopted herein. The following mathematics of Streeter–Phelps model can be found in many water quality textbook (e.g. Rich, 1973; Peavy et al., 1985). They are briefly presented to preserve the continuity of the paper and to define notations.

In differential form, the mass balance equation for the DO deficit, $D$, is given as:

$$\frac{\mathrm{d}D}{\mathrm{d}t} = K_\mathrm{d} L - K_\mathrm{a} D \tag{1}$$

The steady-state solution to equation (1), replacing $t$ by $x/U$, with appropriate initial condition is:

$$D_x = \frac{K_d L_0}{K_a - K_d}\left[\exp\left(-\frac{K_d x}{U}\right) - \exp\left(-\frac{K_a x}{U}\right)\right] + D_0 \exp\left(-\frac{K_a x}{U}\right) \qquad (2)$$

where $K_d$ is the deoxygenation coefficient (day$^{-1}$), $K_a$ is the reaeration coefficient (day$^{-1}$), $x$ is the distance downstream from the source of biochemical oxygen demand (BOD) in miles (1 mile ≈ 1609 m), $U$ is the average stream velocity in miles per day, $D_x$ is the DO deficit concentration (mg/L) at a distance $x$ downstream of discharge point, $D_0$ and $L_0$ are the initial DO deficit and instream BOD concentrations (both in mg/L), respectively.

The concentration of DO at any downstream location is computed as:

$$C_x = C_s - D_x \qquad (3)$$

in which $C_s$ is the saturated DO concentration. The downstream location, $X_c$ (miles), where the maximum DO deficit occurs can be found by differentiat-

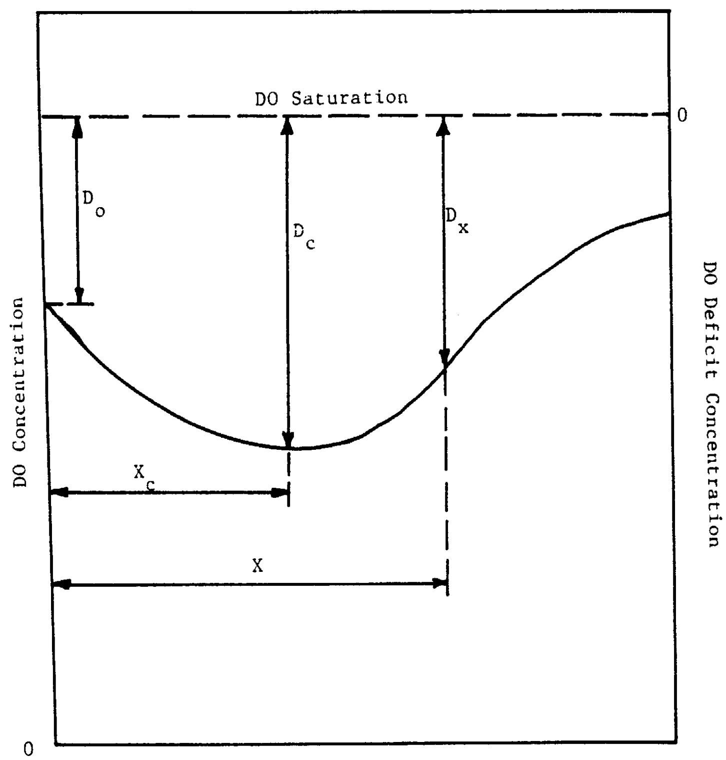


Fig. 1. Typical dissolved oxygen sag curve.

ing equation (2) with respect to $x$ and solving for the value of $x$ where the derivative is equal to zero:

$$X_c = \left(\frac{U}{K_a - K_d}\right) \ln\left\{\frac{K_a}{K_d}\left[1 - \frac{(K_a - K_d)D_0}{K_d L_0}\right]\right\} \tag{4}$$

The point $X_c$ herein is referred to as the 'critical location'.

Since its conception, the Streeter–Phelps equation has been modified to account for discrepancies between analytical estimations, computed from equation (2), and actual data collected in the field. These discrepancies have arises as a result of the exclusion of a number of possible oxygen sources and sinks in the original equation. Dobbins (1964) pointed out that there are eight other possible factors which could contribute to in-stream BOD and DO variations. Several studies have been conducted in which one or more of the processes have been included in the model formulation in attempting to improve model predictability (Camp, 1963; Dobbins, 1964; Hornberger,

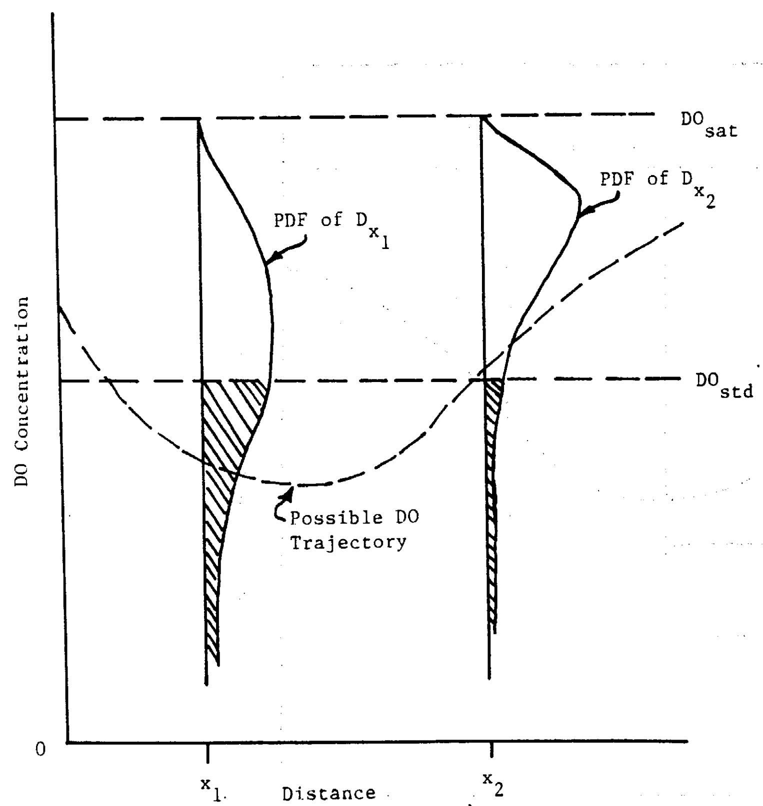


Fig. 2. Schematic diagram of the probability density function for the DO deficit.

1980; Krenkel and Novotny, 1980). However, to simplify the algebraic manipulations, the original Streeter–Phelps equation will be utilized throughout this paper. A typical DO profile for a single reach under a given set of stream water quality parameters is shown in Fig. 1.

## UNCERTAINTY IN THE WATER QUALITY MODEL

The basic water quality model presented previously is a function of several stream parameters, such as the reaeration and deoxygenation coefficients, the average stream velocity, etc. In reality, the stream environment to which this model is applied is extremely variable, both spatially and temporally. The stream system represents a dynamic environment in which the physical and biological characteristics are constantly changing. Given such facts, it is quite obvious that the parameters utilized in the water quality model cannot be quantified or measured with exact certainty. The inherent random nature of the system leads to uncertainties in estimating model parameters.

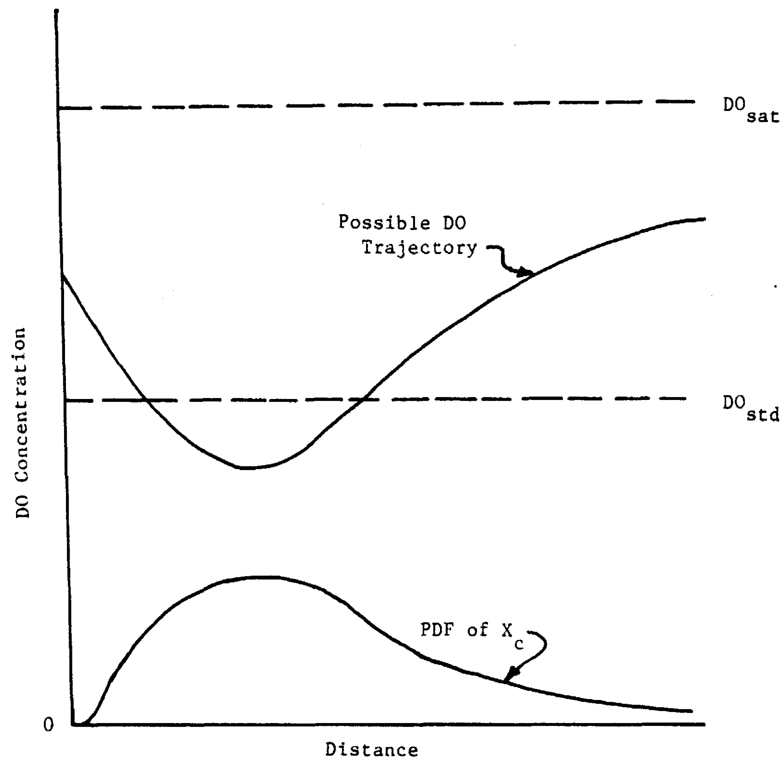


Fig. 3. Schematic diagram of the probability density function for the critical location.

The uncertainty associated with equations (2) and (4), can basically be divided into three categories: inherent, parameter, and model uncertainties. Inherent uncertainties are associated with the natural randomness exhibited by the physical and biological processes. This inherent uncertainty is the product of temporal and spatial variations, for example, in-stream biological composition (Churchill et al., 1962; Bansal, 1973; Wright and McDonnell, 1979). In addition, the availability of limited sample data describing the characteristics of the stream system results in insufficient information to estimate the model parameters with accuracy. The model uncertainty araises from the use of a simplified and idealized model in describing a rather complicated real-life system behavior.

In light of the existence of uncertainties, the stochastic nature of the stream system should be included in the model formulation if accurate assessment of DO characteristics at any location in the stream and other related questions are to be attained. The general approach for describing these uncertainties has been to appropriately assign statistical properties, probability distributions, and correlations to each of the parameters in equations (2) and (4). Schematic diagrams for illustrating uncertainty features of DO deficit and the critical location are shown, respectively, in Figs. 2 and 3.

## CRITICAL LOCATION IN A STOCHASTIC STREAM ENVIRONMENT

As discussed above, the stream environment may be regarded as inherently random by nature, and as such, it should be treated accordingly in the modeling of its components. That is, the model parameters in equation (2) for computing the DO deficit and in equation (4) for computing the critical location should be treated as random variables. It has been shown by Tung and Hathhorn (1988a) that the uncertainty associated with the critical location computer by equation (4) under such stochastic conditions is quite significant.

In order to provide an effective means of monitoring the effects of waste discharge on the DO profile within any reach, the critical location in a stochastic stream environment must be defined. In this paper, the potentially useful critical locations are defined according to the following four criteria:
(1) the location determined by equation (4) using the mean values of water quality parameters, $X_c^{(1)}$;
(2) the location at which the variance of the DO deficit is the largest, $X_c^{(2)}$;
(3) the location where the probability of violating a specified DO standard is maximum, $X_c^{(3)}$; and
(4) the location 'most likely' to be critical, $X_c^{(4)}$, according to the probability distribution assumed for the critical location computed by equation (4).

The significance and rationales of each of these criteria in defining the critical location in a stochastic stream environment are discussed in the following subsections:

*Critical location determined by using the mean-valued water quality parameters.* Basically, this is a deterministic approach for finding the critical location in which the mean values of the water quality parameters are utilized in (4). The critical location so determined can be regarded, approximately, as the averaged critical location.

*Critical location associated with the maximum variance of do deficit.* Referring to the DO profile shown in Fig. 1, one should realize that such a figure exists only when a stream environment behaves deterministically. In a stochastic system, however, the DO deficit at any point in the stream system is no longer a fixed, unique value (see Fig. 2). The value of the DO deficit at any location is uncertain.

The location with a maximum variance of the DO deficit is where the uncertainty of DO deficit is the largest. The rationale for considering such a location as the critical point is based on the argument that this point may possess significant potential for violating a specified DO standard. Although the point of minimum expected DO might be estimated, it may not necessarily represent a point posing the greatest threat to water quality violation in terms of its frequency. For instance, consider a point upstream and downstream of the location with the minimum expected DO. If the variance of the DO deficit at either of these points is larger than that at the point of the minimum expected DO, these other points may pose a greater threat for possible violations of DO standard.

Several researches have already attempted to analyze the variance associated with the DO profile in a stochastic stream environment. In review of those articles, conflicting results have been reported. Thayer and Krutchkoff (1967) and Padgett (1978) have reported that the location of the maximum DO variance coincides with the point of minimum expected DO. On the other hand, Burges and Lettenmaier (1975) and Esen and Rathbun (1976) have contradicted such finding and have reported that the point of the maximum variance is located at a downstream distance approximately twice that of the location with the minimum expected DO.

Although its true location remains unresolved, the significance of knowing the point with the largest variance in DO deficit prediction is clear. This point represents the location in the stream system where the uncertainty in DO prediction is the largest.

*Critical location associated with the maximum probability of violating water quality standard.* The location associated with the maximum probability of violating a specified DO standard represents the point having the highest frequency causing water quality transgression. If one assumes the notion that damage to aquatic biota would occur when the DO standard is violated. It is this location, amongst all others in the stream environment, at which the potential for damaging aquatic biota is the greatest. Therefore, the knowledge of the location associated with the maximum probability of violating a specifying DO standard can play an important role in the overall management of stream water quality.

*Location most likely to be critical.* Acknowledging the existence of uncertainties in the stream environment, the computation of the critical location, using equation (4), is no longer a deterministic exercise. Instead, the critical location associated with the maximum DO deficit is a random variable characterized by its associated probability distribution (see Fig. 3).

Based on the recent study, Tung and Hathhorn (1988b) have examined the appropriatenes of some commonly used univariate distributions including normal, lognormal, gamma, and Weibull for the critical location computed by equation (4). It was observed that a two-parameter gamma distribution provides the best fit to the simulated results in the majority of the cases investigated in which water quality parameters were assigned with various combinations of unimodal distributions.

As with any unimodal distribution, the value most likely to occur is commonly known as the mode. Thus, when considering the distribution of the critical location, it is this point, amongst all others, that the maximum DO deficit occurs most frequently.

## DETERMINATION OF THE CRITICAL LOCATIONS

With the exception of finding the critical location using the mean values of the water quality parameters, each of the remaining criteria seeks to find the location (the sole decision variable) associated with the maximum value of their respective functions (i.e., the variance of the DO deficit, the probability of violating a specified DO standards, and the ordinate of probability density function of the critical location). In theory, each of these locations could be determined analytically by the principle of calculus of extremes. However, this would require the specification of the corresponding objective functions and their first derivatives. For the problem considered herein, such procedures are analytically formidable and impractical. Furthermore, the nature of the problem is a univariate optimization problem with the location as the only decision variable. As an alternative to the analytical approach for solving the maximization of these criteria, the Fibonacci search technique (Beveridge and Schechter, 1970; Sivazlian and Stanfel, 1974) which requires no information about the derivatives was chosen to perform the tasks of maximization. Naturally, other sequential optimum seeking algorithms such as the golden section method can be applied.

*Finding the critical location associated with the maximum variance of DO deficit.* To compute this location, an expression for the variance of the DO deficit as function of the distance from the discharge point must be derived. Approximation of the variance of the DO deficit can be made by using first-order analysis (Benjamin and Cornell, 1970). Considering the correlation between the water quality parameters $K_a$ and $U$, the expression approximating the variance of DO deficit can be obtained as:

$$\begin{aligned}\mathrm{Var}[D_x] \approx {} & (P'_{K_d})^2\,\mathrm{Var}[K_d] + (P'_{K_a})^2\,\mathrm{Var}[K_a] \\ & + (P'_U)^2\,\mathrm{Var}[U] + (P'_{L_0})^2\,\mathrm{Var}[L_0] \\ & + (P'_{D_0})^2\,\mathrm{Var}[D_0] + 2P'_{K_a}P'_U\,\mathrm{Cov}[K_a, U] \qquad (5)\end{aligned}$$

in which $\mathrm{Var}[X]$ represents statistical variance of random variable $X$, $\mathrm{Cov}[X, Y]$ denotes the covariance between two correlated random variables $X$ and $Y$, and $P'_Y$ represents the first-order partial derivative of the DO deficit with respect to water quality parameter $Y$ which is a function of the decision variable $x$. Expressions for the partial derivatives $P'_Y$ based on equation (2) can be found in Chadderton et al. (1982) and Hathhorn and Tung (1987). For this case, equation (5) is a univariate function of the downstream location, $x$, with the values of the statistical properties of the stream parameters ($K_d$, $K_a$, $U$, $L_0$, $D_0$) being known. Thus, the essence of this approach is to find a critical location such that the variance of the DO deficit, given by equation (5), is maximized. Applying the Fibonacci search technique with equation (5) as the objective function, the location at which the variance of the DO deficit is maximum can be found.

Some investigators have considered a positive correlation between $K_a$ and $K_d$ in their statistical analysis of water quality modeling (Esen and Rathbun, 1976; Padgett, 1978). Although statistical analysis of a given field data set may reveal a correlation between these parameters, it does not necessarily imply such a correlation has any meaningful physical representation of the system behavior. It is known that $K_a$ is a function of the physical characteristics of stream, while $K_d$ is characterized by the biological composition of the waste discharge and stream environment. It is the opinion of the authors that the correlation between $K_a$ and $K_d$ is spurious and, thus, is not considered in the study.

*Finding the location associated with the maximum probability of violating DO standard* . Although the results obtained in the recent Monte Carlo simulation study (Tung and Hathhorn, 1988b) support the use of a two-parameter log–normal distribution to describe the random DO deficit at any downstream location, several probability distributions were used in this study for the purpose of examining the sensitivity of the critical location of this kind the various distribution utilized. Specifically, the DO deficit was assumed to follow one of the four distributions: normal, lognormal, gamma, and Edgeworth asymptotic expansion.

Evaluating the probability of violating a specified DO standard at any downstream location $x$ by using normal, lognormal, or gamma probability models is straightforward given that the first two statistical moments of $D_x$ are known or estimated. In this study, statistical moments of $D_x$ are estimated by first order analysis (Benjamin and Cornell, 1970). In addition, Edgeworth asymptotic expansion was also employed to provide a means for computing the probability of a given quantile without having to assume or adopt any parametric distribution of any specific form (Abramowitz and Stegun, 1972; Kendall et al., 1987). This, however, requires the knowledge of higher order moments of the random variable under investigation. By estimating the moments of the DO deficit up to the fourth order using first-order analysis (Hathhorn and Tung, 1987), Edgeworth asymptotic expansion was truncated to give the following approximation:

$$F(w) \approx \Phi(w) - \left[\Upsilon_X\, \phi^{(2)}(w)/6\right] + \left[\eta'_X\, \phi^{(3)}(w)/24\right] + \left[\Upsilon_X^2\, \phi^{(5)}(w)/72\right] \tag{6}$$

where $F(w)$ is the cumulative probability for the standardized random variable $W$; $\Phi(w)$ is the cumulative standard normal probability; $\Upsilon_X$ and $\eta'_x$ are the skewness and coefficient of excess (kurtosis minus 3) of the random variable $X$, respectively; and $\phi^{(r)}(w)$ is the $r$th derivative of the standard normal probability density function, $\phi(w)$, whose expression can be found in Abramowitz and Stegun (1972, p. 934).

In the present study the random variable under study is the DO deficit at any downstream location $x$ from the discharge point. The standardized Do deficit, $W$, can be obtained as:

$$W = \left[D_x - \mathrm{E}(D_x)\right] / \sqrt{\mathrm{Var}(D_x)} \tag{7}$$

in which $\mathrm{Var}(D_x)$ is the variance of the DO deficit at any downstream location $x$ from the discharge point which can be estimated by equation (5).

$E(D_x)$ is the expectation of DO deficit which, similarly by the first-order analysis, can be estimated as:

$$\begin{aligned}
E[D_x] \approx D_x(\bar{K}_d, \bar{K}_a, \bar{U}, \bar{L}_0, \bar{D}_0) &+ \tfrac{1}{2}\mathrm{Var}[K_d]\, P''_{K_d,K_d} \\
+ \tfrac{1}{2}\mathrm{Var}[K_a]\, P''_{K_a,K_a} &+ \tfrac{1}{2}\mathrm{Var}[U]\, P''_{U,U} \\
+ \tfrac{1}{2}\mathrm{Var}[L_0]\, P''_{L_0,L_0} &+ \tfrac{1}{2}\mathrm{Var}[D_0]\, P''_{D_0,D_0} \\
+ \mathrm{Cov}[K_a, U]\, P''_{K_a,U} &
\end{aligned} \tag{8}$$

in which $P''_{X,Y} = \partial^2 D_x / \partial X\, \partial Y$, a second-order partial derivative of $D_x$ with respect to water quality parameters $X$ and $Y$. The probability of violating DO standard at any location $x$ can be found as:

$$\Pr[D_x > D_{\mathrm{std}}] = 1 - F(w_{\mathrm{std}}) \tag{9}$$

in which

$$w_{\mathrm{std}} = [D_{\mathrm{std}} - E(D_x)] / \sqrt{\mathrm{Var}(D_x)} \tag{10}$$

where $D_{\mathrm{std}}$ is the allowable DO deficit.

Based on this criterion the task is to determine the critical location, $X_c^{(3)}$, at which the probability of violating the required DO standard, $C_{\mathrm{std}}$, is maximum. To do this, appropriate probability distributions for the DO deficit at the downstream location $x$ must be assumed along with the statistical properties of the stream parameters ($K_d$, $K_a$, $U$, $L_0$, and $D_0$). With equation (11) as the objective function, the critical location, $X_c^{(3)}$, for each of the distribution assumed for the DO deficit was found using the Fibonacci search technique.

*Finding the location most likely to be critical.* Again, several distributions were assumed for the critical location computed by equation (4): normal, lognormal, gamma, and Edgeworth asymptotic expansion. Although the recent study (Tung and Hathhorn, 1988a) found that a two-parameter gamma distribution best describes the random behavior of the critical location associated with the maximum DO deficit, other probability models were considered with the intent to examine the sensitivity of its determination with respect to the use of different distributions.

As pointed out earlier, the most likely point to be critical, $X_c^{(4)}$, is the mode of the distribution assumed for the critical location computed by equation (4). Thus to find the mode for each of the parametric distributions considered, formula relating the mode and the first two statistical moments from Patel et al. (1976) and Haan (1977) can be utilized. The mean and variance of $X_c$ can be estimated by first-order analysis as:

$$\mathrm{E}(X_c) \approx X_c(\bar{K}_d, \bar{K}_a, \bar{U}, \bar{L}_0, \bar{D}_0) \tag{11}$$

$$\begin{aligned}\mathrm{Var}(X) \approx\ & F_{K_d}'^2 \mathrm{Var}(K_d) + F_{K_a}'^2 \mathrm{Var}(K_a) + F_U'^2 \mathrm{Var}(U) \\ & + F_{L_0}'^2 \mathrm{Var}(L_0) + F_{D_0}'^2 \mathrm{Var}(D_0)\end{aligned} \tag{12}$$

in which $F_Y' = \partial X_c / \partial Y$. Expression for the first-order partial derivatives of $X_c$ with respect to water quality parameter were derived by Hathhorn and Tung (1987).

Finally, the mode using Edgeworth asymptotic expansion can be found by locating the point at which the ordinate of the density function of $X_c$ is maximum. This can be done using Fibonacci search technique with the following objective function:

$$f(y) \approx \phi(y) - \left[\Upsilon_{X_c}\, \phi^{(3)}(y)/6\right] + \left[\eta_{X_c}'\, \phi^{(4)}(y)/24\right] + \left[\Upsilon_{X_c}^2\, \phi^{(6)}(y)/72\right] \tag{13}$$

where $f(y)$ is the probability density function for the standardized critical location using Edgeworth expansion (Abramowitz and Stegun, 1972), $\Upsilon_{X_c}$ and $\eta_{X_c}'$ are skew and excess coefficients of $X_c$, respectively, and their expressions approximated by first-order analysis were derived by Hathhorn and Tung (1987); $Y$ is the standardized $X_c$ defined as:

$$Y = [X_c - \mathrm{E}(X_c)] / \sqrt{\mathrm{Var}(X_c)} \tag{14}$$

## NUMERICAL EXAMPLE AND DISCUSSIONS OF RESULTS

In order to compute the critical locations based on each of the four criteria, knowledge of the mean, standard deviation (or variance), skewness, and kurtosis of the stream water parameters ($K_d$, $K_a$, $U$, $L_0$, $D_0$) is required. The mean and standard deviation of the model parameters used in the numerical study based on data from published articles Burges and Lettermair, 1975; Hornberger, 1980; Chadderton et al., 1982) are shown in Table 1. To examine the sensitivity of the statistical moments on the critical location determination, 15 cases of various skewness, Kurtosis, and correlation (between $K_a$ and $U$) listed in Table 2, along with the mean and standard deviation in Table 1, were considered. The critical locations in a stochastic stream environment using each of the four criteria are computed for all the 15 cases. The results of the numerical computations displayed in Tables 3 through 6.

TABLE 1

Mean and standard deviation of water quality parameters used

| Model parameter | Mean | Standard deviation | Unit |
|---|---|---|---|
| $K_d$ | 0.35 | 0.10 | $day^{-1}$ |
| $K_a$ | 0.70 | 0.20 | $day^{-1}$ |
| $U$ | 10.00 | 3.00 | miles/day |
| $L_0$ | 18.00 | 5.00 | mg/L |
| $D_0$ | 1.00 | 0.30 | mg/L |

In addition to finding the critical locations, $X_c^{(i)}(i = 1, 2, 3, 4)$ using each of the four criteria, the probabilities of violating the minimum DO standard of 4 mg/1 at the four types of critical locations were calculated based on the assumption of a normal, log–normal, and gamma distribution as well as the Edgeworth approximation for the DO deficit. This information is important in assessing the risk of potential damaging effects to be suffered by the stream environment under various distribution assumptions for the DO deficit.

It should be pointed out that the probability evaluation using the Edgeworth expansion is dependent on the skewness and kurtosis of the DO deficit, which in turn are dependent on the skewness of kurtosis of each water quality parameter. Thus, a unique value for the critical locations defined by the maximum probability and the most likely criteria (see Tables 5 and 6) was obtained for each of the 15 cases presented in Table 1. Also, the probability of violation computed by the Edgeworth expansion would be different for all 15 cases. While using the three parametric distributions, the resulting critical locations and the associated probability of violation would depend only on the first two statistical moments, i.e. the mean and variance, which, in turn, was dependend on the inclusion of the correlation between $K_a$ and $U$ in the first-order analysis. To shorten the tables, the critical locations and associated probability of violating the minimum DO standard using Edgeworth expansion for the distribution of DO deficit were grouped according to those cases which consider correlation between $K_a$ and $U$ and those which do not. The range of values for each case computed by Edgeworth expansion is presented in the last columns of Tables 3–6.

Examining the results obtained for the critical locations, it is not that the calculation of the critical location using the first criteria is independent of the correlation between parameters $K_a$ and $U$. This is because, when analyzing equation (4), only the mean values of water quality parameters were used in the computation. Correlations between model parameters were not used. The same reason can be applied to explain the results shown in

TABLE 2

Combinations of skew, kurtosis, and correlations considered

| Case No. | $\rho(K_a, U)$ | $K_d$ | | $K_a$ | | $U$ | | $L_0$ | | $D_0$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $\gamma$ | $K$ | $\gamma$ | $K$ | $\gamma$ | $K$ | $\gamma$ | $K$ | $\gamma$ | $K$ |
| 1 | 0.0 | 0.0 | 3.0 | 0.0 | 3.0 | 0.0 | 3.0 | 0.0 | 3.0 | 0.0 | 3.0 |
| 2 | 0.8 | 0.0 | 3.0 | 0.0 | 3.0 | 0.0 | 3.0 | 0.0 | 3.0 | 0.0 | 3.0 |
| 3 | 0.0 | 0.0 | 2.0 | 0.0 | 2.0 | 0.0 | 2.0 | 0.0 | 2.0 | 0.0 | 2.0 |
| 4 | 0.0 | 0.0 | 4.0 | 0.0 | 4.0 | 0.0 | 4.0 | 0.0 | 4.0 | 0.0 | 4.0 |
| 6 | 0.0 | −0.5 | 2.0 | −0.5 | 2.0 | −0.5 | 2.0 | −0.5 | 2.0 | −0.5 | 2.0 |
| 6 | 0.0 | −0.5 | 3.0 | −0.5 | 3.0 | −0.5 | 3.0 | −0.5 | 3.0 | −0.5 | 3.0 |
| 7 | 0.0 | −0.5 | 4.0 | −0.5 | 4.0 | −0.5 | 4.0 | −0.5 | 4.0 | −0.5 | 4.0 |
| 8 | 0,8 | 0.0 | 2.0 | 0.0 | 2.0 | 0.0 | 2.0 | 0.0 | 2.0 | 0.0 | 2.0 |
| 9 | 0.8 | 0.0 | 4.0 | 0.0 | 4.0 | 0.0 | 4.0 | 0.0 | 4.0 | 0.0 | 4.0 |
| 10 | 0.8 | 0.5 | 2.0 | 0.5 | 2.0 | 0.5 | 0.2 | 0.5 | 2.0 | 0.5 | 2.0 |
| 11 | 0.8 | 0.5 | 3.0 | 0.5 | 3.0 | 0.5 | 3.0 | 0.5 | 3.0 | 0.5 | 3.0 |
| 12 | 0.8 | 0.5 | 4.0 | 0.5 | 4.0 | 0.5 | 4.0 | 0.5 | 4.0 | 0.5 | 4.0 |
| 13 | 0.8 | −0.5 | 2.0 | −0.5 | 2.0 | −0.5 | 2.0 | −0.5 | 2.0 | −0.5 | 2.0 |
| 14 | 0.8 | −0.5 | 3.0 | −0.5 | 3.0 | −0.5 | 3.0 | −0.5 | 2.0 | −0.5 | 3.0 |
| 15 | 0.8 | −0.5 | 4.0 | −0.5 | 4.0 | −0.5 | 4.0 | −0.5 | 4.0 | −0.5 | 4.0 |

$\gamma$, skew coefficient.
$K$, kurtosis.

TABLE 3

Critical locations found using mean valued water quality parameters

| $\rho(K_a, U)$ | $X_c^{(1)}$ (miles) | Probability of violating 4 mg/L DO standard | | | |
|---|---|---|---|---|---|
| | | Normal | Lognormal | Gamma | Edgeworth [a] |
| 0.0 | 18.2 | 0.284 | 0.242 | 0.258 | 0.284–0.301 |
| 0.8 | 18.2 | 0.317 | 0.269 | 0.278 | 0.302–0.335 |

These values represent the range of probabilities for all cases in Table 2.

TABLE 4

Critical locations associated with maximum variance of DO deficit

| $\rho(K_a, U)$ | $X_c^{(2)}$ (miles) | Probability of violating 4 mg/L DO standard | | | |
|---|---|---|---|---|---|
| | | Normal | Lognormal | Gamma | Edgeworth [a] |
| 0.0 | 31.9 | 0.106 | 0.105 | 0.112 | 0.106–0.111 |
| 0.8 | 15.5 | 0.333 | 0.283 | 0.301 | 0.325–0.357 |

These values represent the range of probabilities for all cases in Table 2.

TABLE 5

Critical locations associated with the maximum probability of violating the minimum do standard (4 mg/L)

| $\rho(K_a, U)$ | Critical location, $X_c^{(3)}$ (miles) | | | |
|---|---|---|---|---|
| | Normal | Log-Normal | Gamma | Edgeworth |
| 0.0 | 15.81 [a] | 15.65 | 15.71 | 15.67–15.81 |
| | (0.294) [b] | (0.250) | (0.267) | (0.294–0.312) |
| 0.8 | 15.42 | 15.42 | 15.42 | 14.97–15.07 |
| | (0.333) | (0.283) | (0.301) | (0.326–0.358) |

[a] Critical location (miles).
[b] Probability of violation associated with $X_c^{(3)}$.

TABLE 6

Locations most likely to be critical

| $\rho(K_a, U)$ | Critical location, $X_c^{(4)}$ (miles) | | | |
|---|---|---|---|---|
| | Normal | Log-Normal | Gamma | Edgeworth |
| 0.0 | 18.17 [a] | 15.05 | 15.74 | 18.17–19.11 |
| | (0.284) [b] | (0.250) | (0.267) | (0.275–0.293) |
| 0.8 | 18.17 | 15.05 | 15.74 | 17.23–19.11 |
| | (0.317) | (0.282) | (0.301) | (0.309–0.328) |

[a] Critical location (miles).
[b] Probability of violation association with $X_c^{(3)}$.

Table 6 for the critical location determined on the basis of the most likely criterion using normal, lognormal, and gamma distributions.

Interestingly, the critical locations associated with the maximum variance of the DO deficit, when $K_a$ and $U$ were assumed independent (see Table 4), are about twice as large as the critical locations computed using the remaining criteria. These results agree closely with those obtained by Burges and Lettenmaier (1975) and Esen and Rathbun (1976). However, when $\rho(K_a, U) = 0.8$, the critical location associated with maximum variance is close to those critical locations computed by the other criteria. This result is consistent with those obtained by Thayer and Krutchkoff (1967) and Padgett (1978).

In Table 5, the values of the critical location, $X_c^{(3)}$, associated with the maximum probability of violating the minimum DO standard based on the various probability distribution are presented. One can see that the differences in critical locations computed under the various distributional assumptions for the DO deficit concentration and the various combinations of skewness and kurtosis in Edgeworth expansion seem relatively small for a given correlation coefficient between $K_a$ and $U$. However, it should be noted that these distances are in terms of miles; small changes, such as two- or three-tenths, actually represent several hundreds, possible thousands of feet difference between these values. This might, in fact, become a significant factor in establishing an adequate monitoring system to control water quality conditions at the critical location, while attempting to simultaneously reduce the cost of the instrumentation and labor required to accomplish these tasks.

In Table 6, the critical locations based on most likely criterion, $X_c^{(4)}$, are displayed. Since the correlation between $K_a$ and $U$ is not considered in first-order uncertainty analysis of equation (4), and hence, it has no effect on the outcome in computing the most likely critical point for the three parametric distributions considered. In contrast, the critical locations found using Edgeworth expansion showed larger differences for the variety of the combinations of skewness and kurtosis selected. Among the distributions considered, the resulting critical locations $X_c^{(4)}$ also differ quite significantly.

However, the associated probabilities of violation are relatively insensitive to the distribution of $X_c^{(4)}$ as well as the skewness and kurtosis of the water quality parameters.

It is observed from Tables 3–6 that the probability of violation increases when a positive correlation of 0.8 between $K_a$ and $U$ is considered. To explain these results, refer to equations (6) and (9) for computing the expectation and variance of the DO deficit, respectively. When considering a positive correlation between $K_a$ and $U$, the expectation and variance for the DO deficit at a given location are increased. Thus, the magnitude of standardized DO deficit computed by equation (10) is reduced. Hence, the probability of violation is increased. It is also observed that the critical locations $X_c^{(2)}$ and $X_c^{(3)}$ shown in Tables 4 and 5 are closer to the waste discharge point for the case when a positive correlation between $K_a$ and $U$ is considered.

From Tables 3–6, it is revealed that, for a given critical location criterion, the probability of violation is relatively insensitive to changes in the skewness ($-0.5$ to 0.5) and kurtosis (2.0 to 4.0) used for the water quality parameter in this study. Furthermore, the probability of violation is also relatively insensitive to change in probability distributions assumed for the DO deficit concentration. However, the consideration of a positive correlation between model parameters $K_a$ and $U$ has a more pronounced impact on evaluating the probability of violation.

## CONCLUSION

This paper has indicated four potentially useable criteria for determining the critical locations in a stochastic stream environment. Amongst the four, it would seem that the critical location determined by the criteria of the maximum probability of violating a minimum water quality standard or the most likely point to be critical would be the most appealing from a practical viewpoint. Observing that the critical locations determined by the most likely criterion were quite sensitive to the distribution of $X_c$ computed by equation (4), more effort should be given in attempting to identify an accurate distribution for $X_c$ if such a criterion is to be used. It should be pointed out that in a limited study made by Tung and Hathhorn (1988a), a two-parameter gamma distribution, in a majority of the cases considered therein, best described the random characteristics of $X_c$.

Although $X_c^{(1)}$ differs from $X_c^{(3)}$ by more than 2 miles, its simplicity in computation as compared with other criteria would make its use competitive in comparison with the other criteria presented here. Besides, the probability of violation at $X_c^{(1)}$ is less than the maximum probability by only about one percent in the cases considered. The location with maximum variance, when correlation between $K_a$ and $U$ is considered, is very close to $X_c^{(3)}$. However, without considering $\rho(K_a, U)$, the result is much less desirable.

It is the authors' opinion that the method of determining the critical locations, $X_c^{(3)}$, associated with the maximum probability of violating a minimum water quality standard would be the best criteria both in the theory and practicality. It is this point, $X_c^{(3)}$, which poses the greatest threat to water quality violation, by definition, in stream environments under uncertainty.

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