MULTI-OBJECTIVE DETENTION BASIN DESIGN IN URBAN DRAINAGE SYSTEMS--TRADEOFF BETWEEN RISK AND COST

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Research Note:

Multi-Objective Detention Basin Design in Urban Drainage Systems – Tradeoff Between Risk and Cost

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1. Introduction

An appropriate design for the detention basin storage facility plays an important role in the control of pollution caused by combined urban stormwater and sewer overflow. Due to the random characteristics of runoff magnitude, duration, and inter-arrival time, a detention basin of a given size is always subject to the potential possibility of overflow. In an urban drainage system involving treatment and detention facilities, the risk of a possible occurrence of overflow is inversely related to the treatment capacity and size of the detention basin. Therefore, the design of the treatment capacity and size of the detention basin is multi-objective in nature. The intention of this note is to establish a framework exploring the trade-off between the risk of overflow and the cost of treatment and storage facilities, so that a more realistic design decision can be made.

2. Overflow Risk in Urban Drainage Systems

This section primarily digests the pertinent theoretical results recently derived by Loganathan *et al.* (1985). The results are applicable to an urban drainage system schematically shown in Figure 1. Three random elements are involved: volume of runoff event (X_1, in basin inches); duration of runoff event (X_2, in hours), and inter-arrival time between events (X_3, in hours).

Three assumptions were made in the derivation of the overflow risk: (1) $X_1^{(i)}$, $i = 1, 2, \ldots, n$ are independent, identically distributed (i.i.d.) random variables ($X_2^{(i)}$ and $X_3^{(i)}$ are also i.i.d. random variables); (2) X_1, X_2, and X_3 are statistically independent; and (3) X_1, X_2, and X_3 are exponentially distributed with parameters α, β, and γ, respectively. Based on the above three assumptions, the distribution function of the storage volume available in a detention basin of size b (in basin inches) along

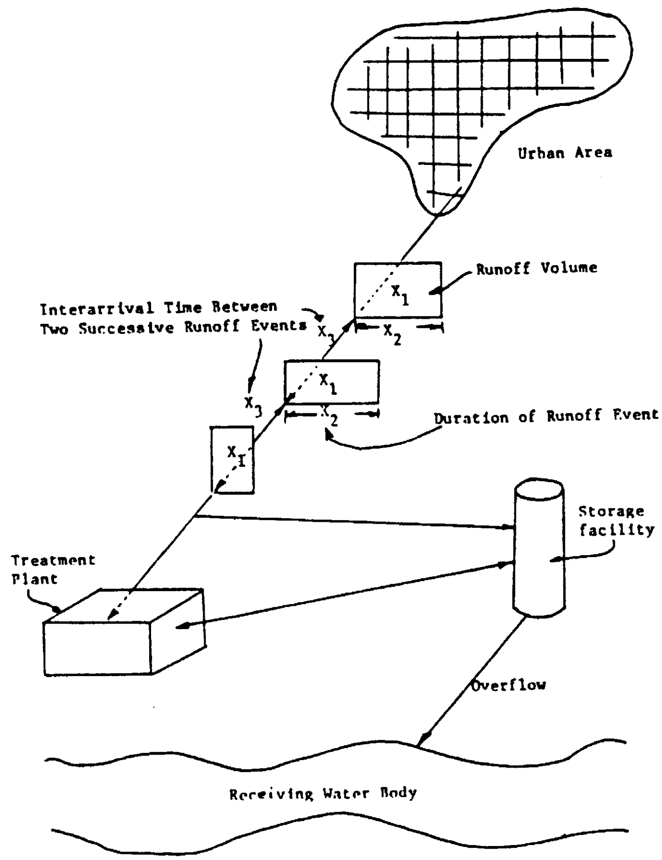

Fig. 1. Schematic representation of urban stormwater runoff process (Loganathan and Delleur, 1982).

with a treatment capacity a (in basin inches per hour) after n storm events, can be derived for the following two cases:

(i) *case 1*: $0 \leqslant c \leqslant s < b$

$$\begin{aligned}&\Pr[S(n) \leqslant s \mid S(n-1) = c] \\ &\quad = 1 - m\left\{\exp\left[-\frac{\beta}{a}(s-c)\right] - \exp\left[-\frac{\gamma}{a}(s-c)\right]\right\} - \\ &\qquad - (1-k)\exp\left[-\frac{\gamma}{a}(s-c)\right] + k\frac{\alpha a}{\gamma}\exp\left[-\alpha(b-s) - \frac{\gamma}{a}(b-c)\right], \quad (1)\end{aligned}$$

in which

$$m = \frac{\alpha\gamma a}{(\alpha a + \beta)(\gamma - \beta)}, \qquad k = \frac{\beta\gamma}{(\alpha a + \beta)(\alpha a + \gamma)}. \qquad (2)$$

(ii) *case 2*: $s \leqslant c < b$

$$\Pr[S(n) \leqslant s \mid S(n-1) = c] = k\left\{\exp[-\alpha(c-s)] + \frac{\alpha a}{\gamma}\exp\left[-\alpha(b-s) - \frac{\gamma}{a}(b-c)\right]\right\}. \tag{3}$$

Of special interest in the detention basin design is the risk of overflow as related to the design variables: treatment capacity and detention basin storage. This overflow risk can be obtained easily from Equation (3) by letting $s = 0$

$$R_c' = \Pr[S(n) \leqslant 0 \mid S(n-1) = c] = k\left\{\exp(-\alpha c) + \frac{\alpha a}{\gamma}\exp\left[-\alpha b - \frac{\gamma}{a}(b-c)\right]\right\}. \tag{4}$$

Equations (1) and (3) define the transition probabilities from one available storage space at the end of the previous storm event to that of the current storm event. It enables us to construct a transition probability matrix P of available storage volume in a detention basin of a specified size, along with a given treatment capacity. If the process is ergodic, then there exists a steady-state or unconditional probability for each of the available storage spaces in the detention basin. The steady-state column vector $\boldsymbol{\lambda}$ can be obtained by solving

$$P\boldsymbol{\lambda} = \boldsymbol{\lambda}, \tag{5}$$

subject to $\mathbf{I}'\boldsymbol{\lambda} = 1$ where

$$\mathbf{I}' = (1, 1, \ldots, 1).$$

3. Multi-Objective Models for Detention Basin Design

Multi-objective optimization models, in general, involve simultaneous consideration of several conflicting, noncommensurable objectives subject to a set of constraints as

$$\text{minimize } \{Z_1(\mathbf{X}), Z_2(\mathbf{X}), \ldots, Z_k(\mathbf{X})\}, \tag{6a}$$

so that

$$g_i(\mathbf{X}) \geqslant 0, \quad i = 1, 2, \ldots, m, \tag{6b}$$

in which $\mathbf{X}$ is a nonnegative n-dimensional vector of decision variables, the $g_i(\mathbf{X})$'s are the constraint equations, and the $Z_k(X)$'s are the different objective functions under consideration.

The solution procedures for multi-objective optimization problems can be classified into two categories: generating techniques and techniques incorporating prior preference (Cohen, 1978). The generating techniques primarily develop trade-off information among objective functions. The trade-off information defines the so-called noninferior solution set from which the best compromised solution can be achieved once the decision-maker's preference function is specified. Among the various generating techniques developed, the constraint method will be the one adopted here. This method for generating a noninferior solution set of a multi-objective optimization problem, is to make all objective functions but one become constraints in the original problem. The resulting 'single-objective' problem has the form

$$\min Z_1(\mathbf{X}), \tag{7}$$

so that

$$Z_k(\mathbf{X}) \geqslant \varepsilon_k, \quad k = 2, 3, \ldots, K, \tag{8}$$

$$g_i(\mathbf{X}) \geqslant 0, \qquad i = 1, 2, \ldots, m, \tag{9}$$

in which the ε_k's are to be varied parametrically. When the problem has a linear programming format, the problem can be solved by the parametric linear programming procedure.

Two bi-objective detention basin design models are considered in this note. The first model considers minimizing (i) the total cost of providing treatment and storage facilities and (ii) the steady-state probability of overflow. The model can be formulated as

$$\min\{C(a, b), \lambda_0(a, b)\}, \tag{10}$$

subject to

$$a_l \leqslant a \leqslant a_u, \qquad b_l \leqslant b \leqslant b_u, \tag{11}$$

where $C(a, b)$ is the cost function including both treatment and storage costs and $\lambda_0(a, b) = \lim \Pr[S_n(0)]$, and subscripts u and l represent the upper and lower bounds, respectively, for the appropriate decision variables.

The second model involves the minimization of the total cost of treatment and storage while, at the same time, minimizing the largest risk of overflow during the current storm event. That is

$$\min\{C(a, b), R'_{\max}\}, \tag{12}$$

subject to

$$R'_c - R'_{\max} \leqslant 0, \quad \text{for all } 0 \leqslant c \leqslant b, \tag{13}$$

$$a_l \leqslant a \leqslant a_u, \qquad b_l \leqslant b \leqslant b_u, \tag{11}$$

in which $R'_{\max}$ is the largest risk of overflow and R'_c can be calculated by Equation (4). By intuition, constraint equation (13) will be binding only when $c = 0$. In other words, a constraint equation of the form Equation (13) for $c \neq 0$ will be redundant and have no effect on the optimal solution of a and b. Therefore, these constraint equations can be replaced by a single constraint, where $c = 0$, as

$$k\left\{1 + \frac{\alpha a}{\gamma} \exp\left[-b\left(\alpha + \frac{\gamma}{a}\right)\right]\right\} - R'_{\max} \leqslant 0. \tag{14}$$

To generate the noninferior solution set for the two bi-objective detention basin design problems, the constraint method is adopted. Using this method, the first model is converted into

$$\text{minimize } C(a, b), \tag{15}$$

so that

$$\lambda_0(a, b) \leqslant \lambda_0^*, \tag{16}$$

$$a_l \leqslant a \leqslant a_u, \qquad b_l \leqslant b \leqslant b_u, \tag{11}$$

and the second model is reformulated as

$$\text{minimize } C(a, b) \tag{15}$$

so that

$$k\left\{1 + \frac{\alpha a}{\gamma} \exp\left[-b\left(\alpha + \frac{\gamma}{a}\right)\right]\right\} - R'_{\max} \leqslant 0, \tag{14}$$

$$R'_{\max} \leqslant R^*_{\max}, \tag{16}$$

$$a_l \leqslant a \leqslant a_u, \qquad b_l \leqslant b \leqslant b_u. \tag{11}$$

The first model deals with the steady-state probability of overflow which cannot be analytically expressed as a function of the decision variables (treatment capacity and storage size of detention basin). Therefore, nonlinear optimization techniques using gradient methods cannot be applied. The first model is solved parametrically by a direct search technique called the Hooke–Jeeve algorithm (Hooke and Jeeves, 1961). As to the second model, the generalized reduced gradient technique (Lasdon and Warren, 1983) is applied parametrically with different $R^*_{\max}$ for generating the noninferior solution set.

4. Application

Model applications were made to 3052 acres of the West Lafayette area (the same study area as chosen by Loganathan and Delleur (1982)). The means of runoff volume, runoff duration, and inter-arrival time and their corresponding probability model parameters are shown in Table I. The cost function for treatment and storage capacities is also taken from Laganathan and Delleur (1982)

$$C(a, b) = 3.45 \times 10^7 a + 9.50 \times 10^6 b. \tag{17}$$

Results of the two bi-objective models are shown in Figure 2. One thing that is common to the two models is that the cost of providing treatment and storage

Table I. Runoff data (Loganathan and Delleur, 1985)

Runoff volume (X_1)	0.06 in	= 16.7 l/in
Runoff duration (X_2)	2.1 h	= 0.4761 l/h
Inter-arrival time (X_3)	70 h	= 0.0141 l/h

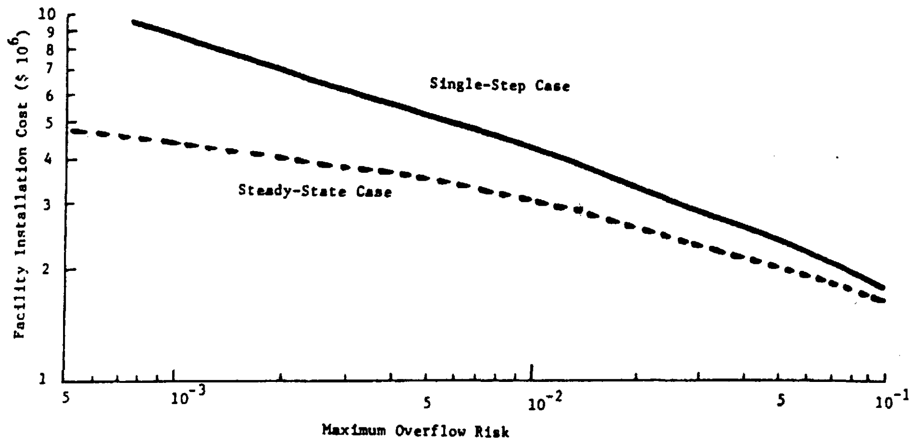

Fig. 2. Tradeoff between facility installation costs and maximum overflow risk.

facilities decreases as the tolerance of maximum risk (steady-state or single-step transition) increases. In comparison between the two figures, it is observed that at a given budget level the steady-state overflow risk is smaller than the best single-step overflow risk under the worst conditions.

The curves show the trade-off between the overflow risk of a detention basin and the corresponding installation cost. This trade-off information would assist the decision-maker in reaching an appropriate design decision once the preference function can be specified.

5. Summary and Conclusions

This note incorporates some of the previous research results given by Loganathan *et al.* (1985) to construct bi-objective detention basin design models considering the trade-off between the cost of installing treatment-storage facilities and the risk of detention basin overflow (both steady-state and single-step transition). The trade-off information is useful in helping the decision-maker make appropriate decisions in detention basin design. The use of a multi-objective framework enlarges one's scope which would help in making realistic decisions.

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