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Y.K. Tung

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Y.K. Tung
Wyoming Water Research Center
University of Wyoming
Laramie, Wyoming

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by Yeou-Koung Tung[1], A.M. ASCE

ABSTRACT

The practice of waste load allocation in water quality management involves a number of noncommensurate and conflicting objectives. In particular, the objectives considered in this multiobjective stochastic waste load allocation study were (1) the maximization of total waste discharge, (2) the maximization of instream dissolved oxygen concentration, (3) the minimization of difference in equity measures, and (4) the maximization of reliability of water quality compliance. The model was illustrated through a hypothetical example involving six waste dischargers.

INTRODUCTION

The issues involved in many environmental problems facing water quality professionals today are becoming more complex. The necessity for improved environmental protection has not precluded the problem of waste load allocation (WLA) from increasing governmental and societal demands on water quality assurance. As society progresses with time, the demand placed on water quality will continue to grow. In fact, the decision-making process in most environmental problems is cultivated by the desire to achieve several goals simultaneously. The problem of optimal WLA is without exception to these aspirations. Therefore, in the course of searching for effective and efficient management decision for protecting and preserving water quality in the WLA

[1]Associate Professor, Wyoming Water Research Center and Department of Statistics, University of Wyoming, Laramie, Wyoming, 82071.

process, several management objectives or goals, which may very possibly be conflicting, must be considered simultaneously. In other words, the most problems in water quality management are multiple-objective in nature.

In the past, majority of the researches performed for solving the optimal WLA problems have been centered around the problem with a single goal or objective, i.e., the minimization of treatment cost or the maximization of waste discharge. Due to the multiobjective nature of the problem, an "optimum" solution to a WLA problem can only be obtained by carefully deliberating the tradeoff among the various physical, legal, and economic aspects in the problem. It is unlikely that a "true" optimum solution to such problems could be obtained by considering only a single objective in the decision process. The use of a single objective formulation to obtain an optimum solution to the WLA problem is not necessary realistic.

The importance of considering a multiobjective approach in the area of water resources has been cited in a number of previous works (Cohen and Marks, 1975; Taylor et al., 1976). By incorporating multiobjective procedures in the decision-making process, three major improvements are accomplished: (1) the role of the analyst and decision-maker are more clearly defined, (2) the results from the multiobjective approach provide a greater number of alternatives to the decision-making process, and (3) models utilizing such techniques are generally more realistic. The use of multiobjective procedures possess the distinct advantage of allowing a variety of problems to be solved, while simultaneously considering several noncommensurable and conflicting objectives (Cohen, 1978).

It is the purpose of this paper to present an analysis for multiobjective WLA problem in a stochastic stream environment in which uncertainties in water quality parameters are explicitly considered. Given the rising demands placed on water quality assurance by government and society, the utilization of multiobjective procedures can only lead to improved water quality protection and control.

MULTIOBJECTIVE MODELING

In a multiobjective problem, it involves a number of scalar objective functions. The problem is sometimes referred to as the vector optimization. The general framework of a multiple-objective model can be expressed as

$$\text{Max} \quad Z(X) = [\ Z_1(X),\ Z_2(X),\ \ldots\ ,\ Z_K(X)\ ] \tag{1}$$

subject to

$$g(X) \leq 0 \tag{2}$$

where Z(X) is a K-dimensional vector of the objective functions, X is an n-dimensional vector containing the decision variables, and g(X) is an m-dimensional vector of constraints.

In the context of multiobjective modeling, the ideological theme of "optimality" that prevails in the single-objective problems is no longer appropriate because there normally exists several objectives which are noncommensurable and conflicting each other. Without a prior knowledge of the preference function among the different objectives, the solution to a multiobjective problem would result in a set of points defining the tradeoff among objectives. Consequently, the concept of "noninferior solution" in the multiobjective analysis replaces the concept of "optimum solution" in the single-objective framework. Cohen (1978) defined the noninferiority in the following passage: "A feasibility solution to a multiobjective programming problem is noninferior if there exists no other feasible solution that will yield an improvement in one objective without causing a degradation in at least one other objective."

The noninferior solution set, in general, is defined by a unique continuous curve or surface depicting the tradeoffs between the various objectives. In theory, an infinite number of noninferior solutions may exist to a multiobjective problem. It is not until the decision-maker provides the characterization of preference among objectives that a best compromising solution can be identified. The "best-compromising" solution to the multiobjective problems is then an alternative which possess the property of maximum combined utility and are elements in both the noninferior solutions set and indifference curve. Such an alternative only exists at the point where the indifference curve and noninferior solution set are tangent (Cohen, 1978).

SINGLE-OBJECTIVE STOCHASTIC WLA MODEL

In all fields of science and engineering, the outcomes of a system on which decisions are based generally depend on a number of parameters and/or variables. More often than not one or more of these parameters cannot be assessed with certainty. This is particularly true in decision-making for environmental management problems. The environment in which decisions are to be made concerning instream water quality management are inherently subject to many uncertainties (Ward and Loftis, 1983). The stream system itself, through nature, is an animate environment abundant with ever-changing processes, both physically and biologically.

In this study, the natural inherent uncertainties of water quality parameters in a stochastic stream system were incorporated in the WLA model through the chance-constrained framework (Charnes and Cooper, 1963; Kolbin, 1977). There have been several articles recently utilizing chance-constrained model for water quality management (Lohani and Thanh, 1979; Yaron, 1979; Burn and McBean, 1985; Fujiwara et al., 1986; Ellis, 1987). In this study the single-objective stochastic WLA model, which serves as the basic model for the multiobjective formulation, is expressed as the followings.

Objective Function. - The objective function adopted was

$$\text{Maximize} \sum_{j=1}^{N} (B_j + D_j) \tag{3}$$

where B_j and D_j are the biochemical oxygen demand (BOD) concentration (mg/l) and dissolved oxygen (DO) deficit concentration (mg/l) in the effluent at discharge location j, respectively, and N is the total number of waste dischargers. This objective function was chosen for its simplicity and its economical equivalence to the minimization of treatment cost. Both effluent waste discharge and DO deficit were chosen in attempting to replicate actual design condition because they were controllable. By reducing the DO deficit in the effluent through an induced reaeration process, greater quantity of BOD waste could be discharged without violating the minimum DO requirements within the stream environment, hence, waste removal costs could be reduced. Of course, a price must be paid in order to provide this reaeration.

Constraints.- The constraints in an optimal stochastic WLA model basically involve the following types.

(i) Constraints on Water Quality.- The most common requirement of a WLA problem has been the assurance of minimum concentrations of DO throughout the river system in an attempt to maintain desired living environment for aquatic biota. In general, the constraint relating the response of DO to the addition of effluent waste can be defined by the Streeter-Phelps equation (Streeter and Phelps, 1925) or its variations (Dobbins, 1964; Krenkel and Novotny, 1980). In this study the original Streeter-Phelps equation was employed for deriving the water quality constraints. The reason of adopting the Streeter-Phelps equation herein is to demonstrate the proposed methodologies without over complicating the algebraic manipulations.

To ensure the compliance of water quality standard, a number of control points within each reach of the river system were selected. Constraint equations in the WLA model were established for each control location at which water quality condition was checked. A typical water quality constraint without considering uncertainties in water quality parameters could be expressed as the following:

$$a_{0i} + \sum_{j=1}^{n_i} \theta_{ij} B_j + \sum_{j=1}^{n_i} \Omega_{ij} D_j \leq DO_i^{sat} - DO_i^{std}, \text{ for } i=1,2,\ldots,M \quad (4)$$

where θ_{ij} and Ω_{ij} are the technological transfer coefficients indicating the relative impact on DO concentrations at downstream locations, i, resulting from a unit waste input at an upstream location, j. The technological transfer coefficients are functions of water quality parameters such as reaeration and deoxygenation rates, flow velocity, etc.. Also in Eq.(4), n_i is the number of the waste dischargers upstream of the control point i; DO_i^{std} and DO_i^{sat} represent the required DO standard and saturated DO concentration at control point i, respectively; a_{0i} is the transfer coefficient relating the DO deficit concentration at control point i as affected by the initial waste load at the upstream end of the entire stream system; M is the total number of control points. Expressions for θ_{ij} and Ω_{ij} based on the Streeter-Phelps equation can be found elsewhere (Hathhorn, 1986).

In reality, water quality parameters such as reaeration and deoxygenation coefficients, flow velocity, initial DO and BOD concentrations are random (Kothandaraman and Ewing, 1969; Esen and Rathbun, 1976; Hornberger, 1980; Chadderton et al., 1982; Ward and Loftis, 1983). Due to the existence of uncertainty within the stream environment, the compliance of water quality standard in the stream system cannot be assessed with certainty. Therefore, the water quality constraints given by Eq.(4) should be expressed probabilistically as

$$Pr\left\{a_{0i} + \sum_{j=1}^{n_i}\theta_{ij}\, B_j + \sum_{j=1}^{n_i}\Omega_{ij}\, D_j \le DO_i^{sat} - DO_i^{std}\right\} \ge \alpha_i \qquad (5)$$

where Pr{} represents the probability operator and α_i is the specified water quality compliance reliability at control point i.

However, the probabilistic statement given by Eq.(5) is not mathematically operational. It has to be transformed into its deterministic equivalent. The corresponding deterministic equivalent of Eq.(5) can be derived as

$$\sum_{j=1}^{n_i} E[\theta_{ij}]B_j + \sum_{j=1}^{n_i} E[\Omega_{ij}]D_j + Z_i(\alpha_i)\sqrt{(B,D)^t\ C(\theta_i,\Omega_i)\ (B,D)} \le R_i' \qquad (6)$$

in which $R_i' = DO_i^{sat} - DO_i^{std} - E[a_{0i}]$, (B,D) is the column vector of BOD and DO deficit concentrations in waste effluent, $C(\theta_i,\Omega_i)$ is the covariance matrix associated with the technological coefficients in the i-th water quality constraint, including a_{0i}; $Z_i(\alpha_i)$ is the α_i-th order quantile associated with the standardized variable Z_i

$$Z_i = \frac{R_i' - \left\{ \sum_j E[\theta_{ij}]B_j + \sum_j E[\Omega_{ij}]D_j \right\}}{\sqrt{(B,D)^t \; C(\theta_i,\Omega_i) \; (B,D)}} \tag{7}$$

As can be seen that the deterministic equivalent of chance-constrained water quality is nonlinear involving the squared root of a quadratic function of waste load decision variables.

Note that in order to solve the stochastic WLA model with chance constraints such as Eq.(6), the knowledge of covariance matrix of technological coefficients in water quality constraints must be known or estimated. Because of the nonlinearity of water quality model, the use of analytical techniques to determine the statistical properties of the random technological coefficients would be an extremely formidable task, if not impossible. The level of complexity increases rapidly as the control points at which water quality constraints are set move toward downstream. Furthermore, the existence of spatial correlation of water quality parameters and cross-correlation among the parameters makes such task even more difficult. Even if one is willing to assume that water quality parameters were uncorrelated spatially, the fact that the technological coefficients in the water quality constraints would not be uncorrelated because they are functions of the same water quality parameters. As a practical alternative, simulation procedures were used to estimate the mean and covariance structure of the random technological coefficients in a given water quality constraint. In particular, unconditional simulation developed in geostatistics were applied in this research to generate the random but spatially correlated water quality parameters. Detailed descriptions of the use of unconditional simulation for estimating statistical properties of the technological transfer coefficients in stochastic water quality constraints were given by Tung et al. (1988).

(ii) Constraints on Treatment Equity.- In addition to the constraints for complying water quality standard, constraints were also employed to define equity between the various dischargers along the river system. Without the inclusion of equity considerations in the WLA model, any attempts to maximize waste discharge (or to minimize treatment cost) would result in the allocation of large quantities of waste to the upstream users, while the downstream dischargers would be required to treat their effluent at levels of maximum possible efficiency. This is especially true for fast moving streams. There have been several articles discussing the importance of equity considerations in the WLA problem (Gross, 1965; Loucks et al., 1967; Miller and Gill, 1976; Brill et al., 1976).

Recognizing the importance of equity consideration in the WLA process, the choice must then be made as to the type of equity to be used. Based on the conclusion drawn by Chadderton et al. (1981), the type of equity measure considered in this study was the equal percent removal which can be expressed mathematically as

$$| (B_j/I_j) - (B_{j'}/I_{j'}) | \leq E_A \quad , \text{ for } j \neq j' \qquad (8)$$

where I_j is the influent raw waste concentration (mg/l BOD) at discharge location j, E_A is the specified allowable difference in equity measure between any two waste dischargers.

Additionally, it should be noted that for any given stream system, one or more of the waste dischargers considered might be influent tributaries. The waste discharge from a tributary should be excluded from the consideration of equity in order to prevent an undue restriction being placed on the required treatment levels assigned to other dischargers. Therefore, provisions should be included to account for tributary flows and their waste inputs in order to identify the entirety of potential waste sources.

(iii) Constraints on Treatment Efficiency.- This set of constraints defined the acceptable range of the treatment efficiency. A range between 35 and 90 percent removal of incoming raw waste at each discharge location was used in this study. The minimum requirement of 35 percent removal was to prevent floating solids from being discharged to the stream environment. The discharge of solids of this type is both socially and environmentally objectionable. On the other hand, the upper limit of 90 percent removal represents the maximum efficiency (assumed) attainable by practical treatment technology.

The treatment efficiency constraints for each discharge location can be expressed as

$$0.35 \leq B_j/I_j \leq 0.90 \quad , \text{ for } \quad j = 1,2,\ldots,N \qquad (9)$$

Certainly, readers might argue that the limits set on treatment efficiency were antiquated. Nonetheless, these limits were selected solely to illustrate the use of the model presented here. By changing these limits, only the size of the feasible region in which the optimum solution is sought would be affected, not the utility of the model.

Finally, non-negativity constraints on the decision variables should be included in the model.

MULTIOBJECTIVE STOCHASTIC WLA MODEL

In this paper model presentation and discussion are based on a four-objective stochastic WLA problem formulation. The objective functions considered are discussed in the followings.

As stated previously that it is incomplete in the WLA model without incorporating the idea of "fairness" into the model formulation. Without the consideration of equity among waste dischargers, the attempt to maximize waste discharge would result in an allocation of large quantities of waste to the upstream users while the downstream dischargers would be required to treat their influents at levels of maximum possible efficiency. Therefore, as the requirement of fairness measure is raised, the total waste load to the stream system would generally be reduced. Furthermore, from the perspective of preserving stream water quality, the higher the water quality standard is set the more desirable the water quality would be maintained. However, it is intuitively understandable that the waste treatment cost would be increased as the instream water quality standard is raised. Therefore, the objectives of preserving water quality and of enhancing economic efficiency are conflicting each other. Lastly, as the requirement of reliability for complying water quality standard in a stochastic stream environment is raised, the total waste load that can be discharged would be expected to reduce.

All the above intuitive arguments of tradeoff among objectives can be easily made for most of multiobjective problems. However, the exact tradeoff behavior generally cannot be made without going through the formalism of solving the problem by appropriate techniques.

The four objective functions considered for the stochastic WLA problem in this study are: (1) the maximization of the total waste load, (2) the minimization of the maximum difference in equity measure between various dischargers in the stream environment, (3) the maximization of the lowest allowable DO concentration level in the stream, and (4) the maximization of the lowest water quality compliance reliability.

The first objective function considered is formulated as the single-objective case as stated previously.

$$\text{Maximize} \quad Z_1 = \sum_{j=1}^{N} (B_j + D_j) \tag{3}$$

For a stream system involving multiple dischargers, the differences in equity measure would generally be varying. To collapse different values of equity measure into one single representative indicator, the worst case associated with the largest difference was adopted in the study. Hence, the second objective can be expressed as

$$\text{Minimize} \quad Z_2 = \delta E_{max} = \max \quad |(B_j/I_j)-(B_{j'}/I_{j'})|, \ \forall \ j \neq j' \tag{10}$$

where δE_{max} is a new decision variable representing the largest difference in equity measure between the various dischargers.

The third objective considered is the maximization of the lowest allowable DO concentration level that should be maintained in the stream environment. In the study, this third objective is expressed as

$$\text{Maximize} \quad Z_3 = DO_{min}^{std} \tag{11}$$

where the new decision variable DO_{min}^{std} is the minimum required DO standard in the stream.

Similar to the difference in equity measure, the water quality compliance reliability at different control points will not be uniform. To utilize a single representative measure of compliance reliability for the entire system, a conservative view of looking at the lowest reliability was adopted. The objective is to maximize this lowest compliance reliability, i.e.

$$\text{Maximize } \alpha_{min} = \min \{ \alpha_1, \alpha_2, \ldots, \alpha_M \} \tag{12}$$

By the definition of α_{min}, the chance constraints for water quality compliance, Eq.(5), would satisfy the following relation.

$$\Pr \left\{ a_{0i} + \sum_{j=1}^{n_i} \theta_{ij} B_j + \sum_{j=1}^{n_i} \Omega_{ij} D_j + DO_{min}^{std} \le DO_i^{sat} \right\} \ge \alpha_{min} \tag{13}$$

The corresponding deterministic equivalent of Eq.(13) can be expressed as

$$\sum_{j=1}^{n_i} E[\theta_{ij}]B_j + \sum_{j=1}^{n_i} E[\Omega_{ij}]D_j + DO_{min}^{std}$$

$$+ Z(\alpha_{min}) \sqrt{(B,D)^t \, C(\theta_i,\Omega_i) \, (B,D)} \le R'' \tag{14}$$

in which $R_i'' = DO_i^{sat} - E[a_{0i}]$.

Note that the original objective function in Eq.(12) was to maximize α_{min}. However, under the assumption that the standardized left-hand-sides of the water quality constraints, i.e. Z_i's, are continuous and unimodal random variables, the decision variable α_{min} would have a strictly increasing relation with $Z(\alpha_{min})$. Therefore, maximization of α_{min} is then equivalent to maximizing $Z(\alpha_{min})$. In the actual model solving, it is more convenient to replace Eq.(12) by

$$\text{Maximize } Z_4 = Z(\alpha_{min}) \tag{14}$$

Note that, now, the substituting decision variable $Z(\alpha_{min})$ is unrestricted-in-sign. The objective function of maximizing the lowest compliance reliability is equivalent to minimizing the largest water quality violation risk.

SOLUTION PROCEDURE TO MULTIOBJECTIVE STOCHASTIC WLA MODEL

There are various methods developed for solving multiobjective problems (Cohon, 1978; Geocoichea et al. 1980; Haimes, 1977). In general, the solution techniques can be categorized into one of the two types: (i) generating techniques and (ii) techniques incorporating preference information (Cohon, 1978). In this study, one of the generating techniques called the constraint method was employed.

The constraint method was first cited by Marglin in the book by Maass et al. (1962) and again by Marglin (1967). This approach enables an analyst to generate the noninferior solution set in entirety without regards to convexity. The computational simplicity is probably the most distinguished advantage of the constraint method. When using the constraint method, the multiobjective problem is solved by adopting only one objective in the objective function. The remaining objectives are simply transformed into constraints in the problem formulation.

Once the multiobjective problem has been formulated, the constraint method provides a relatively effortless computational methodology for generating the noninferior solution set. Moreover, if the multiobjective formulation follows a LP format, the constraint method can be solved by a parametric LP approach. For a detailed analysis of the attributes of the constraint method readers should consult Cohen and Marks (1975) and Cohen (1978).

In summary, the multiobjective stochastic WLA problem described above can be cast into the following format to be solved by the constraint method.

$$\text{Maximize} \quad Z(\alpha_{min}) \tag{16}$$

Subject to

$$\sum_{j=1}^{n_i} E[\theta_{ij}]\, B_j + \sum_{j=1}^{n_i} E[\Omega_{ij}] D_j + DO_{min}^{std} + Z\,(\alpha_{min}) \sqrt{(B,D)^t\; C(\theta_i,\Omega_i)\;(B,D)} \le R'' \tag{14}$$

$$0.35 \le B_j/I_j \le 0.90 \quad , \text{for} \quad j = 1,2,\ldots,N \tag{9}$$

$$|(\,B_j/I_j\,) - (\,B_{j'}/I_{j'}\,)| - \delta E_{max} \le 0 \quad , \text{for } j \ne j' \tag{17}$$

$$\sum_{j=1}^{N} (B_j + D_j) \geq Z_1^o \tag{18}$$

$$DO_{min}^{std} \geq Z_3^o \tag{19}$$

$$\delta E_{max} \leq Z_2^o \tag{20}$$

and non-negativity constraints for the decision variables except for $Z(\alpha_{min})$. In the above formulation, the right-hand-sides Z_1^o, Z_2^o, and Z_3^o are the values of objective functions 1, 2, and 3, respectively, which are to be varied parametrically.

MODEL SOLUTION TECHNIQUE

The deterministic equivalent transformation of chance-constrained water quality constraints resulted in the presence of nonlinearity as shown in Eq.(14). The problem became one of nonlinear optimization which could be solved by various nonlinear programming techniques such as the generalized reduced gradient technique (Lasdon and Warren, 1979) and others.

Alternatively, this study adopted a procedure to linearize the nonlinear terms of the water quality constraints in the stochastic WLA model and solved the linearized model by the LP technique iteratively.

Tung (1986) proposed an approach using the first-order Taylor's expansion to linearize a nonlinear terms involving the squared root of the variance which is a quadratic function of waste load decision variables. The linearization procedure required an initial guess of the solution to the optimization problem which was not known. As a result, the linearized problem had to be solved iteratively until the solution converges. Since the linearization process utilized by Tung (1986) was a cumbersome exercise in this case and the resulting linearized model still had to be solved iteratively. In this study, the assumed solutions to the stochastic WLA model were used to calculate the value of the squared root terms and were treated as a constant associated with the decision variable $Z(\alpha_{min})$. The resulting linearized water quality constraints in the stochastic WLA model could then be written as

$$\sum_{j=1}^{n_i} E[\theta_{ij}]B_j + \sum_{j=1}^{n_i} E[\Omega_{ij}]D_j + DO_{min}^{std} + Z(\alpha_{min}) \sqrt{(\hat{B},\hat{D})^t \, C(\theta_i,\Omega_i) \, (\hat{B},\hat{D})} \le R'' \qquad (21)$$

in which $\hat{B}$ and $\hat{D}$ are the assumed solution vectors to the stochastic WLA model.

Consequently, the linearized stochastic WLA model can then be solved by the LP technique iteratively, each time comparing the values of the current solutions with those obtained in the previous iteration. Then, the current solutions were used to compute the covariance of the left-hand-sides (LHS_i) in each of the stochastic water quality constraints

$$LHS_i = a_{0i} + \sum_{j=1}^{n_i} \theta_{ij} B_j + \sum_{j=1}^{n_i} \Omega_{ij} D_j \tag{22}$$

until convergence criteria were met between any two successive iterations. A flow chart depicting the procedures is shown in Figure 1. Of course, alternative stopping rules could be incorporated in the algorithm to prevent excessive iteration during the computation.

To solve the multiobjective stochastic WLA model as formulated above requires no knowledge about the distribution of random LHS_i's. However, in order to assess the minimum compliance reliability of water quality constraints, the probability distribution for the LHS_i must be known or assumed. Once such distributional assumption is made, the minimum probability compliance can be made when the solution technique converges at which time the means and variances of LHS_i can be evaluated.

It should be noted that the decision variable $Z(\alpha_{min})$ is not without upper bound. The highest value possible for $Z_i(\alpha_i)$, as can be observed form Eq.(14), could be achieved only when there is no waste discharged into the stream system, i.e. B=0 and D=0,

$$Z_i^* = \frac{R_i''}{\sqrt{Var(\ a_{0i}\ )}} \tag{23}$$

where Var() is a variance operator. Therefore, the upper bound of $Z(\alpha_{min})$ is equal to $Z^*_{min} = \min \{ Z^*_1, Z^*_2, \ldots , Z^*_M \}$. As the solution iteration proceeds, the upper bound for $Z(\alpha_{min})$ needs to be updated accordingly.

Under the normality assumption for the LHS_i's in Eq.(22), the highest minimum compliance reliability can be easily computed by utilizing the standard normal distribution. However, when lognormal distribution was assumed, the same value for Z_i's in different water quality constraints does not necessarily indicate the same compliance reliability because the higher moments may not be the same. In this case, the procedure is, first, to identify the binding water quality constraints and, then, calculate the associated compliance reliability. The smallest reliability from the binding constraints will be the largest minimum compliance reliability achievable by the stream system.

Due to the nonlinear nature of the stochastic WLA model, it should also be pointed out that, in general, the optimum solution obtained cannot be assured to be the global optimum. Thus, it was suggested that a few runs of the solution procedure with different initial solutions should be carried out to ensure that model solution converges to the overall optimum. Other suggestions such as how to select proper initial solutions for the iterative procedure, particularly for the optimal WLA problems, can be found elsewhere (Tung et al., 1988).

EXAMPLE APPLICATION

The means and standard deviations for the stream water quality parameters are shown in Tables 1 and 2. An illustration of the six-reach example is given in Figure 2. To assess the statistical properties (i.e. the mean and covariance matrix) of the technological transfer coefficients in the water quality constraints for this example, 200 sets of technological coefficients were generated by the unconditional simulation approach under the condition that all stream water quality parameters are normally distributed. It was found numerically in the previous study (Tung et al. 1988) that the statistical properties of θ_{ij} and Ω_{ij} reached a very stable values based on 200 sets of simulated parameters. The mean and covariance matrix of the technological coefficients computed from the simulated results were used in this four-objective stochastic WLA model. However, for purpose of illustration, spatial independence of water quality parameters was considered in estimating the means and covariance matrices of the technological coefficients in water quality constraints.

Based on the study by Tung and Hathhorn (1988), a two-parameter lognormal distribution was found to be the best parametric distribution for describing the DO deficit concentration computed by the Streeter-Phelps equation regardless of the probability distribution of water quality parameters and the correlation between reaeration coefficient and average flow velocity. Therefore, an adoption of a lognormal distribution for the random left-hand-side, LHS_i, given in Eq.(22) were made to compute the minimum water quality compliance reliability once the model is solved.

The tradeoff curves among the various objectives considered with a given minimum DO standard concentration are shown in Figures 3-5. As can be seen that, for a specified minimum DO standard and total waste loading, the largest water quality violation risk decreases as the largest difference in equity measure increases. Increase in equity measure implies a larger tolerance for the "unfairness" among waste dischargers. As the level of minimum required DO standard is raised, the set of tradeoff curves move upward. To show the tradeoff for different minimum DO standard, Tables 6 and 7 were plotted for risk of water quality violation, equity measurement, and water quality standard while the total waste load were fixed at specified levels.

SUMMARY AND CONCLUSIONS

Most environmental management problems, including waste load allocation, are multiobjective by nature and should be treated accordingly. Thus, the continued reliance upon a single-objective optimization framework to manage a variety of environmental systems seems unreasonable.

In an attempt to improve river water quality management practice, this paper presented a methodology to analyze a four-objective stochastic WLA problem using the constraint method.

The model developed considered explicitly the uncertainties in water quality parameters. The multiobjective model presented here was applied to a multiple-discharger river system in which the goals of maximization of total waste discharge, minimization of the largest differences in equity measure among waste dischargers, maximization of minimum DO standard, and maximization of lowest water quality compliance reliability were considered. The relevance of this multiobjective approach to the problem is that a more realistic solution to the problem of WLA could be identified by specifying the tradeoffs (given by the noninferior solution set) among the four objectives. This information can then be passed on to the decision-making entity where the ultimate responsibility of management policy lies. The information provided by this approach will likely enhance the decision-maker's ability to select a "best-compromising" solution given the set of alternatives to the problem of optimal river water quality management.

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Table 1. The Mean Values of Physical Stream Parameters Used in the Example of WLA Model

(a) Mean Stream Characteristics for Each Reach

Reach i	Deoxygenation Coefficient (K_d)	Reaeration Coefficient (K_a)	Average Stream Velocity (U)	Raw Waste Concentration (I)	Effluent Flow Rate (q)
1	0.6	1.84	16.4	1370	0.13
2	0.6	2.13	16.4	6	44.00
3	0.6	1.98	16.4	665	4.62
4	0.6	1.64	16.4	910	35.91
5	0.6	1.64	16.4	1500	3.20
6	0.6	1.48	16.4	410	0.78
Units	1/day	1/day	miles/day	mg/l BOD	ft^3/sec

(b) Background Characteristics

Upstream Waste Concentration L_o	Upstream Flow Rate Q_o	Upstream DO Deficit D_o
5.0	115.0	1.0
mg/l BOD	ft^3/sec	mg/l

Table 2. Standard Deviations Selected For The Physical Stream Characteristics

(a) For Each Reach

Reach	Deoxygenation Coefficient (k_d)	Reaeration Coefficient (k_a)	Average Stream Velocity (U)
1-6	0.2	0.4	4.0
Units	1/day	1/day	ft^3/sec

(b) Background Characteristics

Upstream Waste Concentration (L_o)	Upstream Flow Rate (Q_o)	Upstream DO Deficit (D_o)
1.0	20.0	0.3
mg/l BOD	ft^3/sec	mg/l

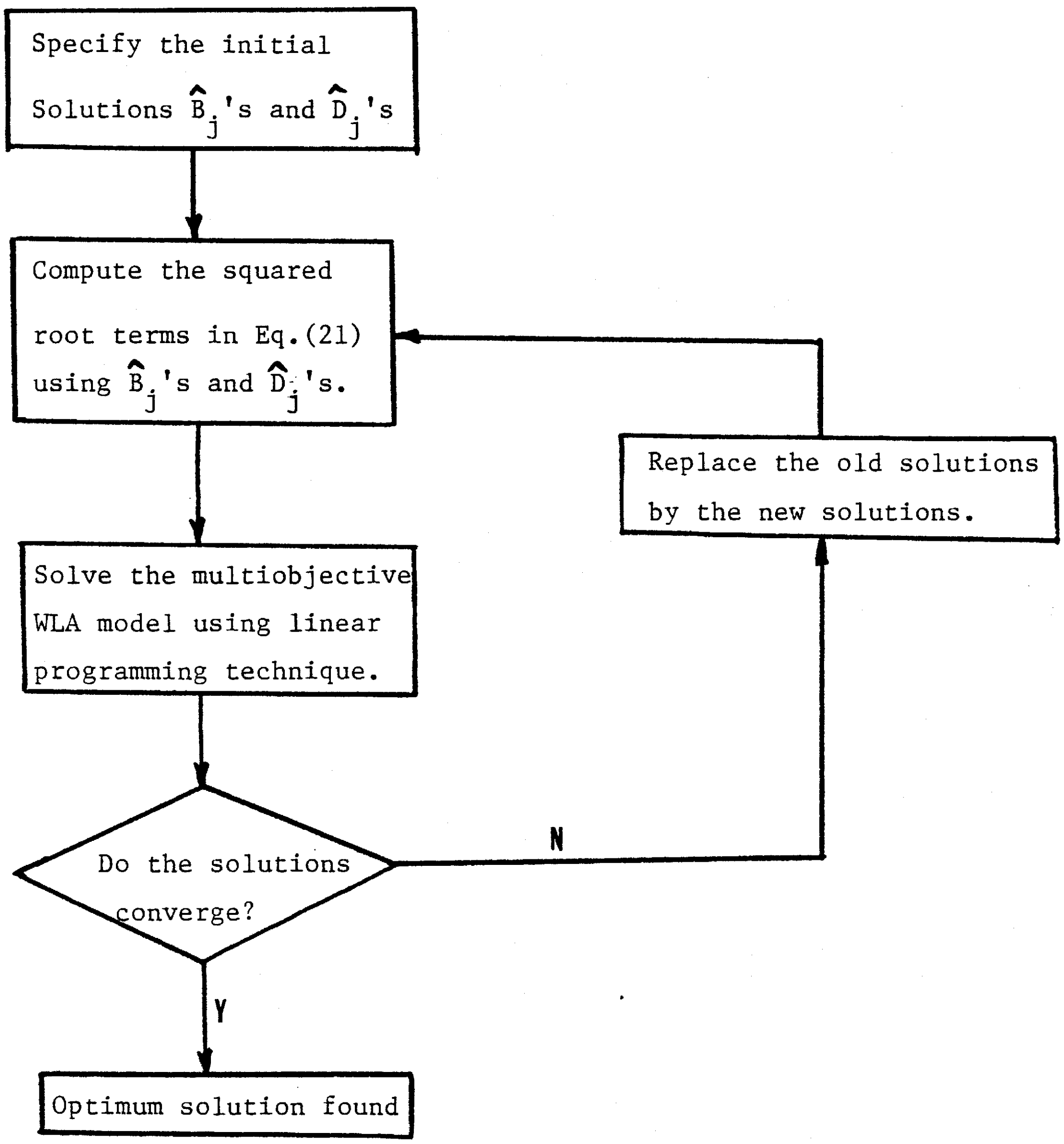

Figure 1. Flow Chart for Solving the Linear Multiobjective Stochastic Waste Load Allocation Model.

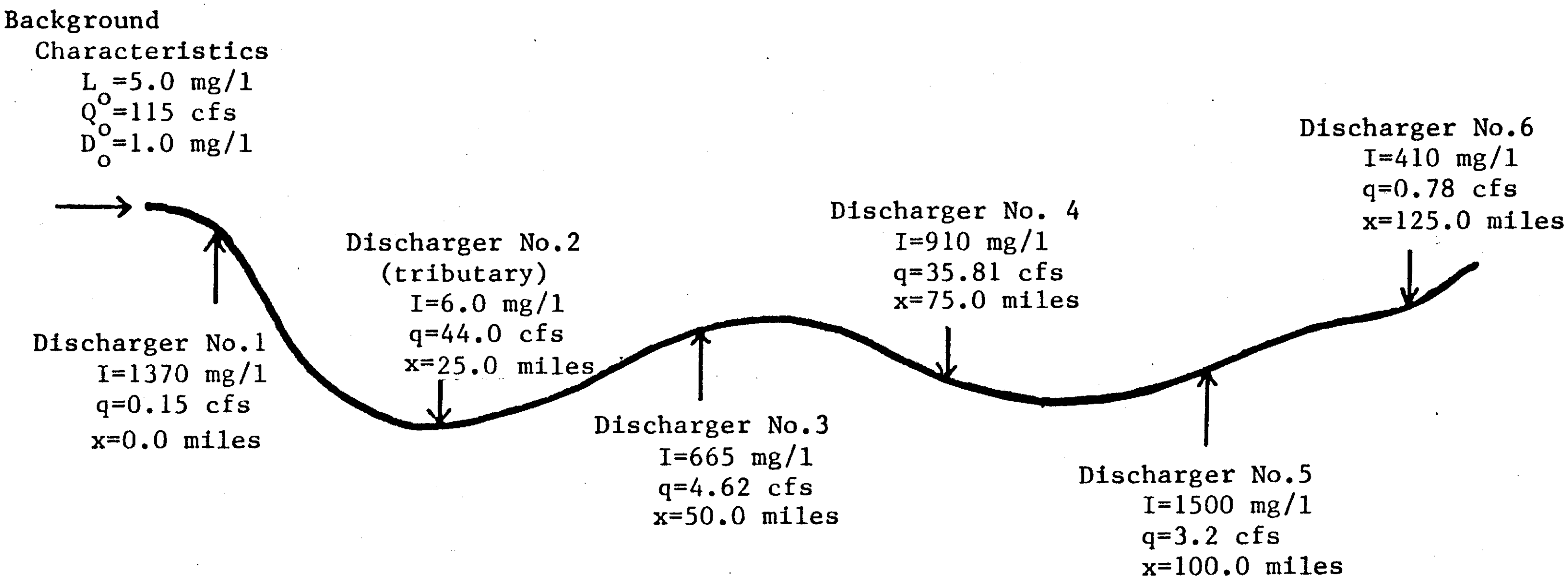

Figure 2 Schematic Sketch Of The Example System In WLA Problem

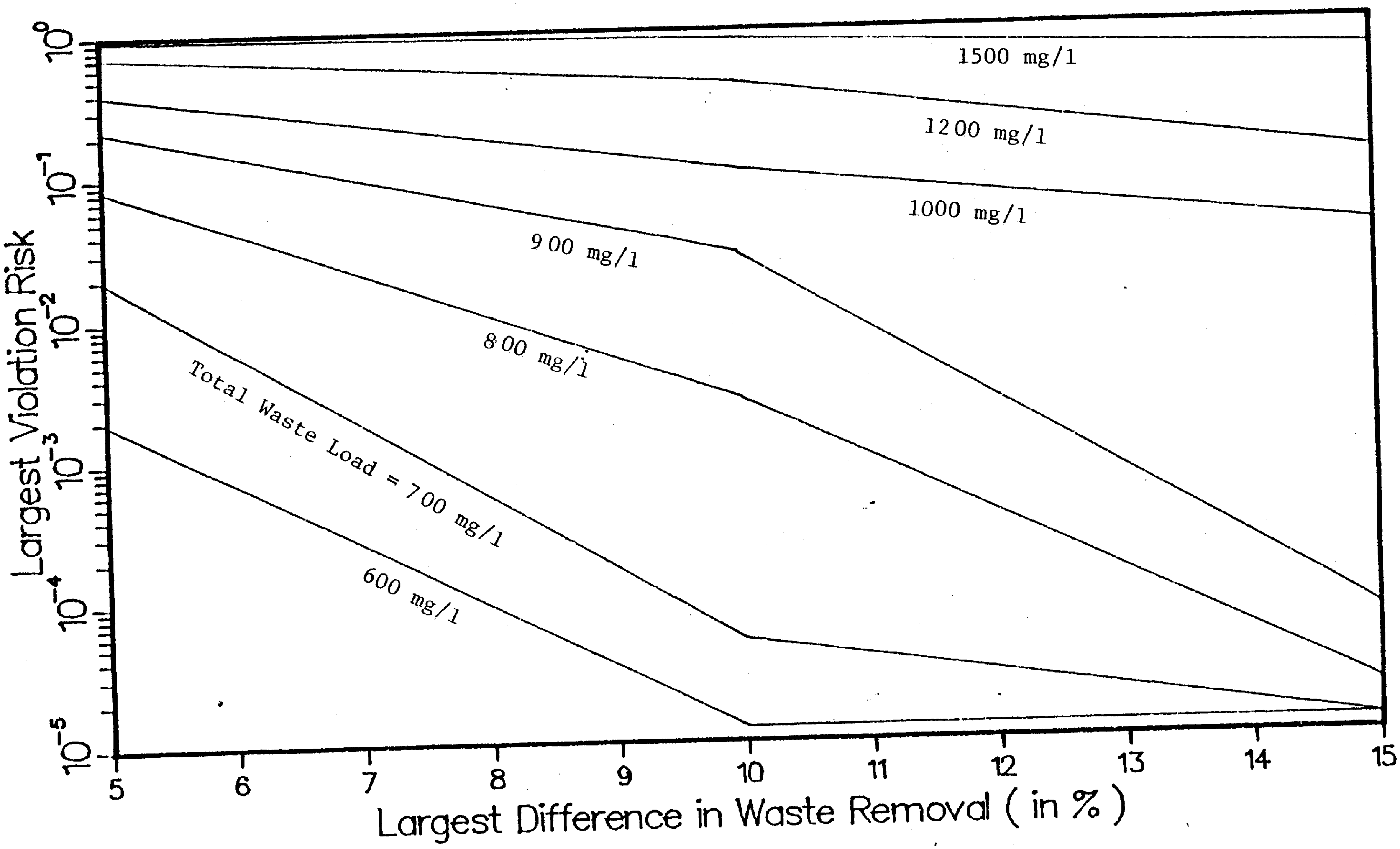

Figure 3. Tradeoff Curves of the Various Objectives in Stochastic WLA Problem with 4mg/l Minimum DO Standard.

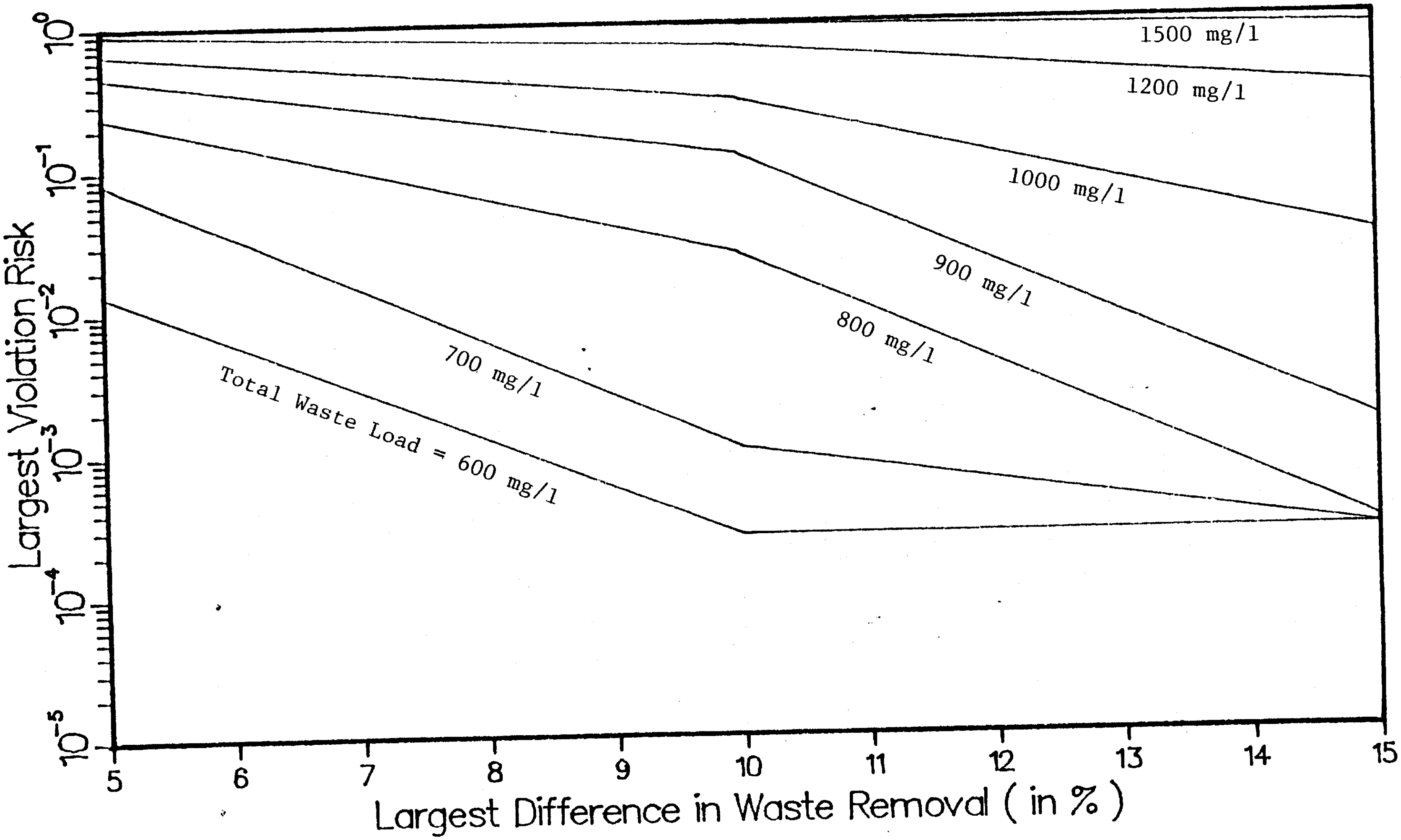

Figure 4. Tradeoff Curves of the Various Objectives in Stochastic WLA Problem with 5 mg/l Minimum DO Standard

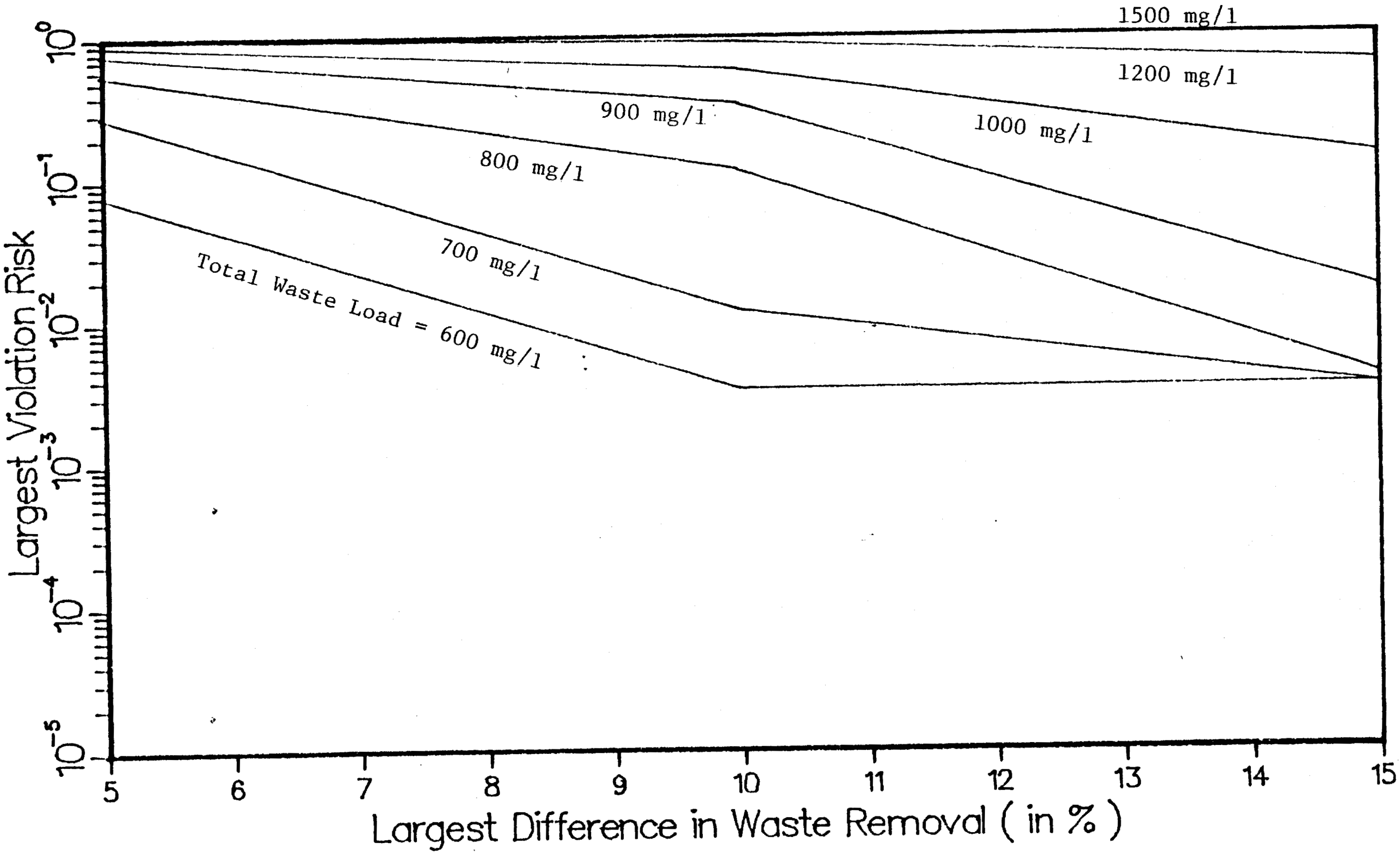

Figure 5. Tradeoff Curves of the Various Objectives in Stochastic WLA Problem with 6 mg/1 Minimum DO Standard.

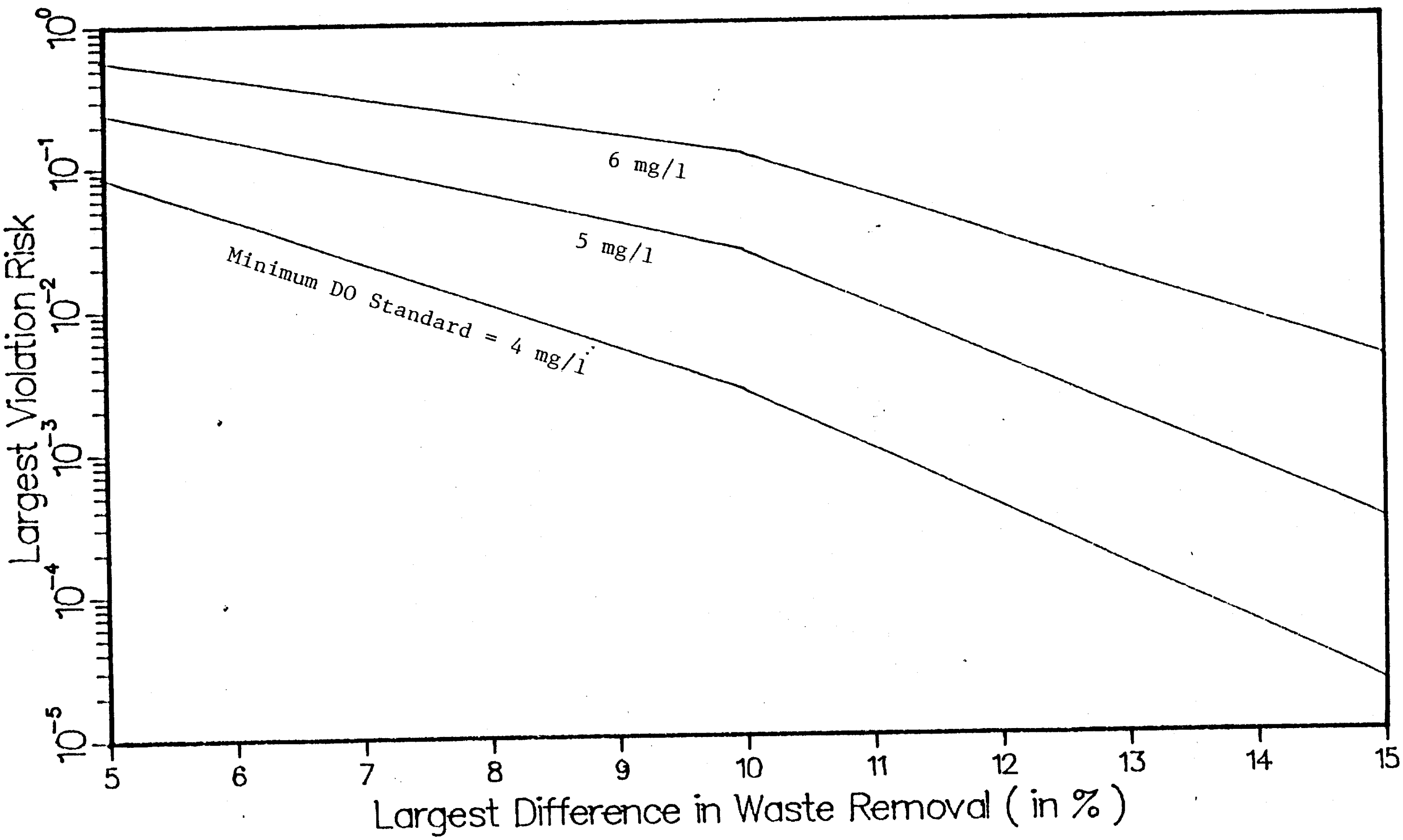

Figure 6. Tradeoff Curves of the Various Objectives With Total Waste Load Fixed at 800 mg/l

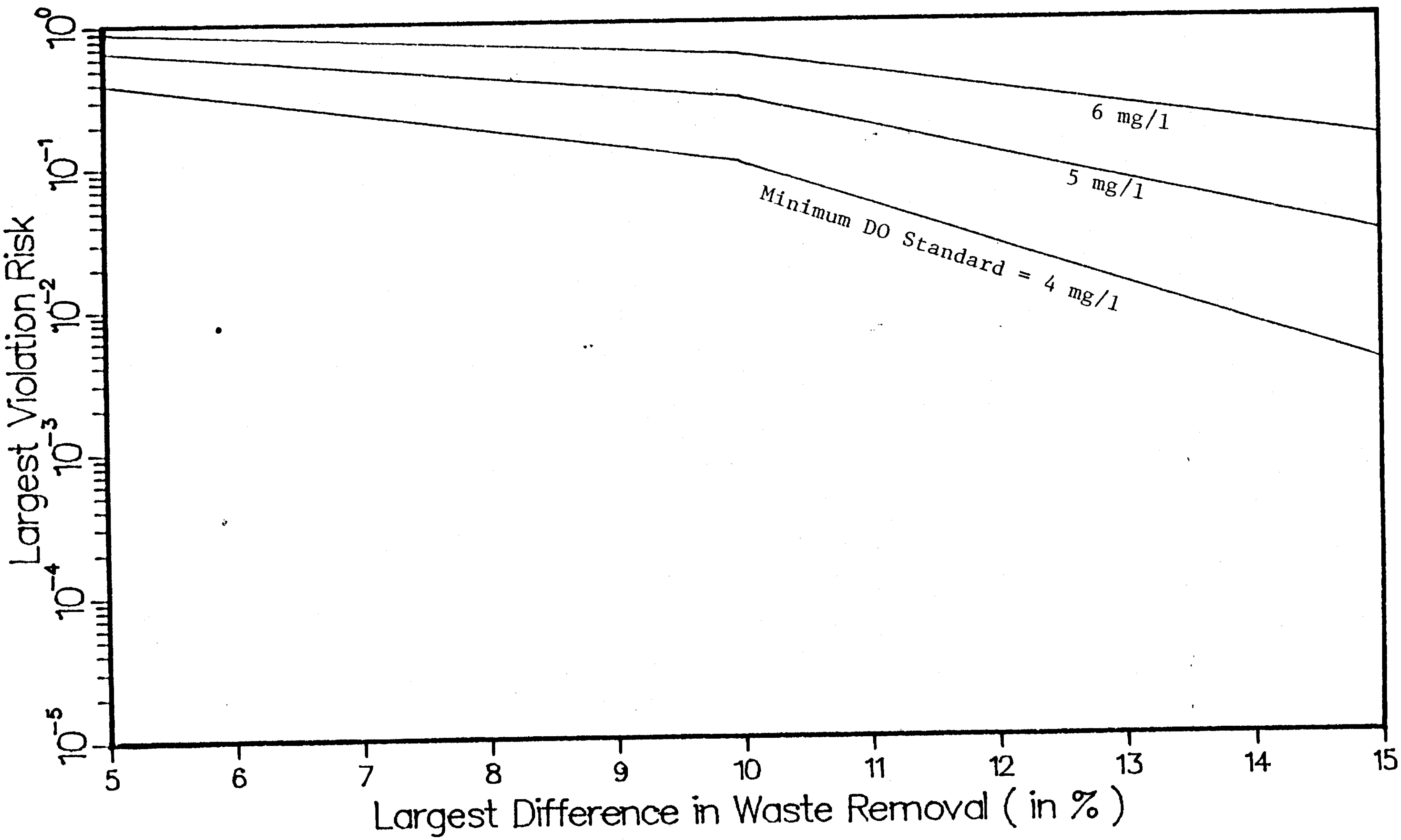

Figure 7. Tradeoff Curves of the Various Objectives in Stochastic WLA Problem With the Total Waste Load Fixed at 1000 mg/l