MULTI-OBJECTIVE STOCHASTIC GROUNDWATER MANAGEMENT OF NONUNIFORM, HOMOGENEOUS AQUIFERS

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Multi-Objective Stochastic Groundwater Management of Nonuniform, Homogeneous Aquifers

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Abstract. Like any other resources planning and management, groundwater management is performed in a stochastic environment in which the system itself involves a number of random elements. Consequences as a result of decisions made based on analyses are not certain. This paper presents a management model using the chance-constrained framework which explicitly considers the random nature of aquifer properties. The model enables the derivation of an optimal groundwater management policy that would satisfy required operation performance reliability. Furthermore, the chance-constrained model is extended to the multi-objective optimization framework in which a tradeoff between total water supply pumpage and system performance reliability is explicitly considered. The models are applied to a hypothetical example of a steady, nonuniform, homogeneous confined aquifer.

Key words. Groundwater management, optimization, uncertainty analysis, reliability, multiple-objective analysis.

1. Introduction

As the result of improper planning and management of groundwater aquifers, undesirable consequences have occurred including, among others, depletion and contamination of aquifers, and land subsidence. Enhanced by the advancement of computing capability and the development of efficient numerical techniques, many groundwater (simulation and/or optimization) models have been developed to assist water-resource managers in making effective decisions.

Literature on optimal groundwater management has been extensive. Previous publications have reported that the development of models has a wide spectrum of complexity and sophistication. Basically, the methodologies employed in groundwater management can be divided into two categories: simulation and optimization. The simulation approach employs numerical groundwater models in which the analyst successively adjusts management policy until the responses of the aquifer system under study becomes both feasible and acceptable. A typical example of using the simulation approach in groundwater management can be found in Bredehoeft and Young (1970).

As an alternative to simulation, solutions to groundwater management have also been derived by direct optimization. Depending on the details of the system behavior to be considered, the models can further be classified into lumped-system and distributed-parameter models. Lumped-system models are primarily concerned with the temporal allocation of water and ignore the spatial variation of system states. However, lumped-system models are, generally, computationally simpler, thus serving the purpose of analysis in the early stages of planning and development. Examples of using lumped-system models for optimal groundwater and conjunctive use managements can be found elsewhere (Buras, 1963; Domenico *et al.*, 1968).

If groundwater management decisions are concerned with both temporal and spatial aspects of aquifer responses, a distributed-parameter model should be employed. Optimal groundwater management models with distributed-parameter capability are commonly approached either by the embedding technique or the response matrix technique. The embedding technique directly and explicitly incorporates the governing groundwater flow equation as a constraint in the optimization framework. The technique is general, but it suffers severe computational difficulties due mainly to its requirement of tremendous computer storage and to computational instability (Tung and Koltermann, 1985). As an alternative, the response matrix technique, which utilizes the concept of influence function and linear system theory, becomes a practical tool for real-world groundwater management practice (Atwood and Gorelick, 1985; Gorelick and Remson, 1982; Heidari, 1982; Morel-Seytoux, 1978). The so-called algebraic technologic function developed by Maddock (1972) and the discrete kernel function developed by Morel-Seytoux and Daly (1975) are of this type. However, the response matrix technique is not without its disadvantages. Recently, Wanakule *et al.* (1986) linked a groundwater simulation model with an optimization algorithm for determining the optimal management policy of regional groundwater systems. Their work, thus far, can be considered to be the most advanced state-of-the-art technique in distributed-parameter groundwater management optimization.

Like any other resources management, groundwater management is performed under the stochastic environment. There exist many elements, such as aquifer properties, boundary conditions, etc., which are random and/or uncertain in nature. Therefore, the response of a stochastic aquifer system to the external stresses cannot be predicted with certainty. In addition to the inherently natural variability of subsurface flow systems, data on most groundwater basins are generally lacking and, possibly, contaminated by error (Gelhar, 1974; Freeze, 1975). This is especially true during the early stages of the planning process for aquifer development. Thus, it is argued that the random and uncertain features of subsurface flow systems, if possible, should be taken into account in deriving the optimal management solution.

Although the capabilities of sophisticated distributed-parameter subsurface flow models are well understood, meaningful results can only be generated if there are sufficient amounts of good, quality data available. In light of the stochastic environment in groundwater, the effects of uncertainty and random features of subsurface flow systems on management decisions have been examined through sensitivity analysis when distributed-parameter simulation models are employed (Young and Bredehoeft, 1972; Maddock, 1974). To explicitly incorporate the random nature of system parameters into a distributed-parameter optimization model remains a challenging task.

Direct incorporation of the random characteristics of an aquifer system, so far, has only been made to lumped-system models (Flores *et al.*, 1978; Tung, 1986). Lumped-system models are appropriate for use in the early stages of planning because they are computationally simpler, thus providing decision-makers with quick and reasonable solutions. Besides, they do not demand extensive data information, which is very likely to be lacking. Several sensitivity analyses made by researchers (Maddock, 1974; McElwee and Yukler, 1978; Young and Bredehoeft, 1972) have indicated that the sophistication of a groundwater model is not a very crucial element in management problems and suggested that a simpler model would be adequate.

In groundwater resource management under uncertainty, the decision-makers concern not only the attainment of an optimal management strategy but also the associated risk of violating the specified system performance criteria, because the state of the system cannot be predicted with certainty. As a result, stochastic groundwater management is multiple-objective in nature. This paper basically consists of three parts. The first part describes the concept of influence function and its utility in formulating groundwater management models. The second part describes the derivation of a stochastic groundwater management model in chance-constrained format (Loucks *et al.*, 1981) and the analysis of uncertainties. Application of the chance-constrained model is made to a hypothetical homogeneous but nonuniform aquifer in a single-objective framework. The last part extends the previous model formulation to bi-objective groundwater management where the trade-off between constraint compliance reliability and water supply is considered. A general discussion of the stochastic programming as applied to groundwater management is given by Willis and Yeh (1987).

2. Influence Function

The influence function of a system, generally speaking, is a function that relates the response of the system to an imposed stimulus or stress. In the groundwater management models considered herein, the responses are the drawdowns at various control points and the stimuli are the pumpages at various potential production wells. Sometimes, the influence function is also called the unit response function. Heidari (1982) presents a procedure for generating the influence function for management models by numerical groundwater simulation. Under certain idealized assumptions, the influence function can also be derived from an analytical or semi-analytical solution to the groundwater flow problems.

For purposes of illustrating the concept, a simplified system of confined and homogeneous aquifers is adopted. Further assumptions of the aquifer system are: (1) the aquifer is infinite in horizontal extent; (2) a radial flow pattern; (3) wells fully penetrate the entire aquifer thickness; (4) the aquifer is nonleaky; and (5) the piezometric head is uniform throughout the entire aquifer prior to pumping. Incorporating these assumptions, the drawdown equation of steady radial flow for a confined aquifer can be expressed as

$$s_{ij} \frac{\ln(r_{0j}/r_{ij})}{2\pi T} Q_j \tag{1}$$

in which s_{ij} = drawdown at control point i resulting from a pumpage of Q_j at the potential production well j; r_{0j} = radius of influence of potential production well j; r_{ij} = distance between control point i and production well j; T = aquifer transmissivity. Equation (1) is the well-known Thiem equation.

Referring to Equation (1), we notice that the aquifer response, i.e., drawdown, is linearly related to the stimulus, i.e., pumpage, imposed on the system. From the characteristics of the influence function stated previously, the influence function (C_{ij}) for a steady confined radial flow aquifer system can be immediately identified as

$$C_{ij} = \frac{\ln(r_{0j}/r_{ij})}{2\pi T}. \tag{2}$$

If more than one well is operating in a field, the overall effect of aquifer drawdown at any control point i can be obtained, by the principle of linear superposition, as the sum of responses caused by all production wells in the field (Viessman *et al.*, 1977),

$$s_i = \sum_{j=1}^{N} C_{ij} Q_j \tag{3}$$

where N = total number of potential production wells under consideration.

3. Deterministic Groundwater Management Model

Consider the management problem where the objective is to obtain the optimal pumpage and a pumping pattern subject to the constraint that undesirable consequences are not created. The adverse consequences are generally avoided by controlling the drawdowns or hydraulic responses in the aquifer system.

Since the influence function characterizes the relationship between aquifer and pumpage, groundwater management models can be formulated very easily once the influence function is defined. Without considering the random nature of aquifer properties, a deterministic groundwater management model for a steady-state confined aquifer can be formulated as follows

Maximize

$$\sum_{j=1}^{N} Q_j. \tag{4}$$

Subject to

$$\sum_{j=1}^{N} C_{ij} Q_j \leqslant s_i^*, \quad \text{for all } i = 1, \ldots, K, \tag{5}$$

$$\sum_{j=1}^{N} Q_j \geqslant D \tag{6}$$

and

$$Q_j^L \leqslant Q_j \leqslant Q_j^U, \quad \text{for all } j = 1, \ldots, N \tag{7}$$

where Q_j = pumpage at production well j, a nonnegative decision variable, s_i^* = specified maximum allowable drawdown at control point i resulting from pumping operations over the entire well field, K = total number of control points, D = demand, and Q_j^U and Q_j^L are the upper and lower bounds of pumpage for production well j, respectively. The first constraint, Equation (5), ensures that the total drawdown at all control points do not exceed the allowable limits. The second constraint, Equation (6), ensures that demand is satisfied. The last constraint, Equation (7), limits the pumpage within the individual well capacity.

The groundwater management model described above is deterministic if aquifer properties, i.e., transmissivity, are treated as deterministic. The model formulation is simple and follows the well-known linear programming (LP) format which can be easily solved by the simplex algorithm. The problem size basically depends on the number of production wells (decision variables) and the number of control points (constraints).

4. Derivation of Stochastic Groundwater Management Model

In general, the value of transmissivity is derived from a pump well test. This test provides *in-situ* values of aquifer parameters averaged over a large and representative aquifer volume (Freeze and Cherry, 1979). Hence, transmissivity should be treated as a random variable instead of being deterministic. For such a circumstance, the influence functions, C_{ij}'s in constraint equation (5), become random variables because they are functions of the aquifer transmissivity, which is now considered as a random variable. Consequently, the left-hand side of the aquifer response constraints, leading to the calculation of drawdowns in the groundwater management model, also become random variables. This implies that the compliance of the constraints at each control point cannot be assured with certainty. Under such circumstances, it is more appropriate and realistic to examine the constraint performance probabilistically.

In a stochastic environment, it is operationally feasible to specify limitations on allowable risk or required reliability of the constraint performance. For a confined aquifer, if we imposed a restriction that the total drawdown at any control point i cannot exceed a predetermined value s_i^* with a reliability requirement α_i, the constraints can then be expressed as

$$\Pr\left\{\sum_{j=1}^{N} C_{ij} Q_j \leqslant s_i^*\right\} \geqslant \alpha_i, \quad \text{for all } i = 1, \ldots, K \tag{8}$$

in which Pr { } represents the probability.

After replacing Equation (5) in the management model by Equation (8), a stochastic management model in a chance-constrained format is developed. However, it should be noted that the current chance-constrained formulation, in the form of Equation (8), is not mathematically operational. It requires further modifications. To transform the probabilistic statement in the form of the chance-constrained equation into its deterministic equivalent for mathematical tractibility, it is necessary to assess the statistical properties of the overall drawdown at the various control points.

5. Analysis of Uncertainty

The statistical properties of a random variable commonly used are the mean and variance. To quantify or estimate the statistical properties of an overall drawdown at each control point, both the approximated solution or exact solution approaches can be employed. The exact solution approach requires the probability distribution of the overall drawdown at each control point be known or analytically obtainable which, in general, is a rare situation. In most situations, the response of an aquifer system is a complex function of the random aquifer parameters. Even though the probability distribution of the individual aquifer parameter can be assessed, the distribution of the combined effect is generally difficult to estimate. In general, a practical way to approach the problem is to estimate the statistical moments of a function involving random variables and then to make an assumption about the distribution of such a function. One method that is commonly applied is the first-order analysis (Cornell, 1972) in which the function involving random variables is expanded in Taylor series about the mean values of random variables as

$$f(\mathbf{x}) = f(\boldsymbol{\mu}) + \sum_{m=1}^{P} \left.\frac{\partial f(x)}{\partial x_m}\right|_{\mathbf{x}=\boldsymbol{\mu}} (x_m - \mu_m) + \varepsilon \tag{9}$$

in which $f(\mathbf{x})$ is a function involving P random variables, $\boldsymbol{\mu}$ is a vector of mean values of P random variables, and ε is the higher-order terms in the Taylor expansion. Neglecting the higher-order terms in Equation (9) and assuming independency of the random variables involved, the mean and variance of the function $f(\mathbf{x})$ can be approximated as

$$\mathrm{E}[f(\mathbf{x})] \simeq f(\boldsymbol{\mu}), \tag{10}$$

$$\mathrm{Var}[f(\mathbf{x})] \simeq \sum_{m=1}^{P} \left(\left.\frac{\partial f(x)}{\partial x_m}\right|_{\mathbf{x}=\boldsymbol{\mu}} \right)^2 \sigma_m^2 \tag{11}$$

in which E[ ] and Var[ ] represent the expectation and variance, respectively, and σ_m^2, is the variance of the mth random variable. Application of first-order analysis to stochastic aquifer management using the Cooper–Jacob equation, where the transmissivity and storage coefficient are considered random, has been made by Tung (1986).

In the problem considered herein, the only random variable is the aquifer transmissivity. There have been numerous field investigations (cited in Freeze, 1975) indicating that the hydraulic conductivity has a log-normal distribution. From this, it is reasonable to assume that the transmissivity also has a log-normal distribution because transmissivity is the product of the aquifer thickness and hydraulic conductivity. As a result, the exact distribution of the aquifer drawdown under steady-state conditions can be assessed. This exact solution approach is adopted for the model development in this paper.

The random variable, i.e., transmissivity, in Equation (3) can be factored out and the drawdown equation can be rewritten as

$$s_i = \frac{1}{T}\left(\sum_{j=1}^{N} A_{ij} Q_j\right) \tag{12}$$

where $A_{ij} = \ln(r_{0j}/r_{ij})/2$. The term in the parenthesis of Equation (12) is a constant. Therefore, the random variable $s_i' = \ln(s_i)$ has a normal distribution with the mean

$$\bar{s}_i' = \ln\left(\sum_{j=1}^{N} A_{ij} Q_j\right) - \mu_{T'} \tag{13}$$

and the variance $\sigma_{T'}^2$ in which $\mu_{T'}$ and $\sigma_{T'}^2$ are the mean and variance, respectively, of the log-transformed transmissivity, $T' = \ln(T)$. Since T has a log-normal distribution, the mean and variance of T' can be obtained as (Haan, 1977)

$$\mu_{T'} = \tfrac{1}{2} \ln\left(\frac{\bar{T}^2}{1 + CV_T^2}\right), \tag{14}$$

$$\sigma_{T'}^2 = \ln(CV_T^2 + 1) \tag{15}$$

in which CV_T is the coefficient of variation of the transmissivity, i.e., σ_T/μ_T.

6. Resulting Stochastic Groundwater Management Model

The stochastic groundwater management model formulated previously contains chance-constraints which are not mathematically operational. After the analysis of uncertainty, the statistical properties of the random terms in the chance-constraints can be quantified. This further enhances the transformation of the chance-constrained equations into their deterministic equivalents which would become mathematically operational. Under the log-normal assumption of aquifer transmissivity, the statistical characteristics of aquifer responses can be derived. The original chance-constrained statement, Equation (8), is equivalent to

$$\Pr\{s_i' \leqslant \ln s_i^*\} \geqslant \alpha_i, \quad \text{for all } i = 1, 2, \ldots, K. \tag{16}$$

Since s_i' has a normal distribution with the mean as given in Equation (13) and variance $\sigma_{T'}^2$, Equation (16) can be written as

$$\Phi\left(\frac{(\ln s_i^* + \mu_{T'}) - \ln(\Sigma_{j=1}^{N} A_{ij} Q_j)}{\sigma_{T'}}\right) \geqslant \alpha_i \tag{17}$$

in which $\Phi[\ ]$ is the cumulative normal probability. As a result, the deterministic equivalent of Equation (8) using the exact solution approach can be easily derived as

$$\sum_{j=1}^{N} A_{ij} Q_j \leqslant \exp[(\ln s_i^* + \mu_{T'}) - \sigma_{T'} \Phi^{-1}(\alpha_i)], \quad \text{for all } i \tag{18}$$

where exp [ ] is an exponential operator. Equation (18) can be used to substitute Equation (5) in the deterministic management model described previously and the resulting stochastic management model is formulated. Note that Equation (18) is linear, the resulting stochastic model can again be solved by the LP technique.

7. Application No. 1

7.1. EXAMPLE AQUIFER

The stochastic groundwater management model developed is applied to a hypothetical aquifer containing three production wells and five control points. The locations of the production wells and control points are shown in Figure 1. The radius of influence of all pumping wells is assumed to be 213 m. The maximum allowable drawdown at each control point is also shown in Figure 1. The lower and upper bounds of pumpage for each production well are 0 and 0.008762 cubic meters per second (CMS), respectively. The mean transmissivity is 0.0007187 CMS/m. The distances between pumping wells and control of points are given in Table I.

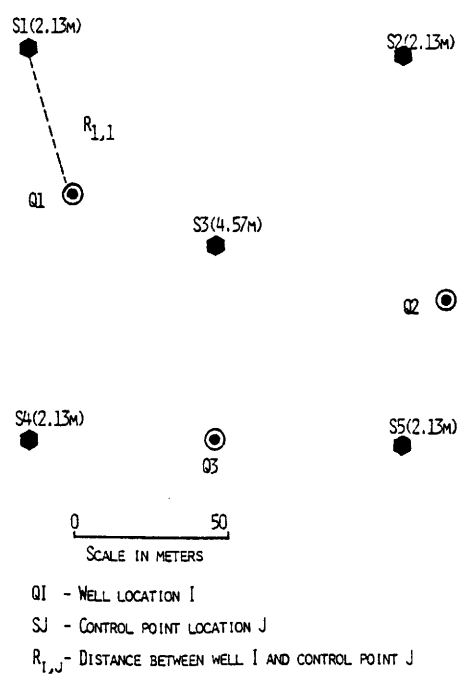

Fig. 1. Locations of wells and control points for the hypothetical example.

Table I. Distance (in meters) between potential pumping wells and control point in hypothetic example

Pump well	Control points				
	1	2	3	4	5
1	48.16	116.13	48.16	77.72	131.06
2	156.97	77.72	89.00	144.48	48.16
3	136.25	136.25	60.96	60.96	60.96

7.2. SOLUTIONS

To examine the effects of reliability levels and aquifer property uncertainties on the optimal solutions, problems with different reliability requirements and various levels of uncertainty in aquifer parameters are solved. The results including pumping pattern and total pumpage are shown in Table II. The degree of uncertainty of aquifer transmissivity is expressed in terms of its coefficient of variation (CV_T). In all cases, the maximum total allowable pumpage, as expected, decreases as reliability requirements increase when the level of uncertainty in the aquifer is fixed. For a given reliability requirement, the maximum total pumpage from the aquifer decreases as the value of CV_T gets larger. Logically, this can be explained by the fact that pumping rates would be less from the aquifer to meet reliability constraints when the variability of the aquifer properties is large or less information is available about the system.

The use of mean values of the system parameters is not an uncommon practice in water resources planning and analysis. It should be recognized that the deficiency of such use is that the mean values are unable to consider the degree of uncertainty of the system parameters. When the mean values of the aquifer parameters are used, this roughly corresponds to a 50% performance reliability as required to meet the specified operation constraints. However, if the distribution of system responses is known to be nonnormal and positively skewed, such as log-normal distribution, the use of mean aquifer parameter values would lead to less than 50% system performance reliability.

Table II. Optimal pumping pattern and maximum total allowable pumpage (in 10^{-4} CMS) for different uncertainty levels of transmissivity and reliability requirements for steady confined flow (mean transmissivity = 0.0007187 CMS/m)

	Reliability = 0.975				Reliability = 0.950				Reliability = 0.900			
	CV_T				CV_T				CV_T			
Well	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8	0.2	0.4	0.6	0.8
1	6.816	5.769	4.474	3.244	9.398	8.099	6.460	4.873	13.936	11.936	9.830	7.727
2	3.245	2.746	2.130	1.550	4.474	3.856	3.075	2.320	6.458	5.682	4.679	3.679
3	3.715	3.145	2.439	1.774	5.123	4.415	3.521	2.656	7.395	6.519	5.358	4.212
Total	13.776	11.660	9.042	6.579	19.000	16.370	13.056	9.849	27.421	24.123	19.867	15.618

7.3. POST-OPTIMALITY SIMULATION

In addition to the examination of the effects of the reliability requirements and levels of parameter uncertainty on model results, it is also interesting to examine how close the model outputs comply with the required system reliability at various locations. To do that, a post-optimality simulation study can be conducted. One thousand log-normally distributed random samples for transmissivity were generated based on its statistical properties used in the hypothetical example. Optimal pumpages determined from the chance-constrained model with various reliability requirements and uncertainty levels were input to the simulation study. It was found that no violation of drawdown constraints were observed in the simulation study for all conditions except that about 0.8% of violation for control points 1, 4, and 5 under reliability requirement of 90% with $CV_T = 0.8$. This implies that the solutions determined from the chance-constrained model are conservative with respect to the performance reliability specified. That is, the actual performance reliability is higher than the specified performance reliability.

8. Multiple-Objective Stochastic Groundwater Management Model

In groundwater management under uncertainty, decision-makers are basically concerned with two aspects; (1) water supply from the aquifer system and (2) the risk of violating specified performance criteria at various control points. From the example study presented, it is realized that water supply and system performance reliability are noncommensurable and mutually conflict with each other. The problem is a bicriterion optimization problem in which the decision-maker(s) would like to maximize the total pumpage for water supply while, at the same time, minimize the risk of violating specified performance criteria.

The general formulation of multiple-objective (vector optimization) problem can be expressed as

$$\underset{\mathbf{x} \in X}{\text{Min}} \; [f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_n(\mathbf{x})], \tag{19}$$

$$\text{s.t.} \quad g_i(\mathbf{x}) \leqslant 0, \quad i = 1, 2, \ldots, m \tag{20}$$

where $f_j(\mathbf{x})$ is the jth objective function, $j = 1, \ldots, n$; X is the set of feasible solutions defined by the set of constraint equations, Equation (20); and $\mathbf{x}$ is the vector of decision variables. Several optimization (Haimes, 1977; Cohen, 1978). The method to be applied herein is the ε-constraint method which generates a noninferior solution set for the problem.

Once the drawdown limitations are specified for each control point, it is general that the probability of violating (or complying) the specified drawdown constraint at each control point will not be identical under a given pumping pattern. That is, the actual values of the right-hand side of chance constraints, α_j's in Equation (8) are different. One common measure of the different risks (or reliabilities) is that of the largest risk (or the smallest reliability) which is to be minimized (or maximized). Hence, adopting this common measure,

$$\beta_{max} = \max\{\beta_1, \beta_2, \ldots, \beta_K\} \tag{21}$$

or

$$\alpha_{min} = \min\{\alpha_1, \alpha_2, \ldots, \alpha_K\} \tag{22}$$

where $\beta_i = 1 - \alpha_i$ = the risk of violating the specified drawdown at control point i.

Under the assumption of log-normal distribution for the transmissivity, the chance-constrained formulation using β_{max} as a risk measure for the system performance can be written as

$$\Pr\left\{\frac{1}{T}\sum_{j=1}^{N} A_{ij}Q_j \geqslant s_i^*\right\} \leqslant \beta_{max} \tag{23}$$

in which β_{max} currently is a decision variable to be minimized. The deterministic equivalent of Equation (23) can be derived as

$$\sum_{j=1}^{N} A_{ij}Q_j - \exp[\ln s_i^* + \mu_{T'}]\exp[\sigma_{T'}\Phi^{-1}(\beta_{max})] \leqslant 0. \tag{24}$$

Since $\Phi^{-1}(\beta_{max})$ is a monotonically increasing function of β_{max} and $\sigma_{T'} > 0$, the term $\exp[\sigma_{T'}\Phi^{-1}(\beta_{max})]$ is a monotonically increasing function of β_{max}. Therefore, minimization of $\exp[\sigma_{T'}\Phi^{-1}(\beta_{max})]$ would also lead to the minimization of β_{max}. We now let $W = \exp[\sigma_{T'}\Phi^{-1}(\beta_{max})]$ and Equation (24) becomes

$$\sum_{j=1}^{N} A_{ij}Q_j - A_0 W \leqslant 0, \quad \text{for all } i = 1, 2, \ldots, K \tag{25}$$

where $A_0 = \exp[\ln s_i^* + \mu_{T'}]$ = a constant.

To solve this two-objective optimization problem, the ε-constraint method is used to generate noninferior solutions. The model to be solved is summarized as follows

$$\text{Min } W, \tag{26}$$

$$\text{s.t.} \quad \sum_{j=1}^{N} A_{ij}Q_j - A_0 W \leqslant 0, \quad \text{for } i = 1, \ldots, K. \tag{27}$$

$$\sum_{j=1}^{N} Q_j \geqslant D^*, \tag{28}$$

$$Q_j^L \leqslant Q_j \leqslant Q_j^U, \quad \text{for all } j = 1, 2, \ldots, N \tag{7}$$

where D^* is the demanded quantity of pumpage to be varied parametrically.

9. Application No. 2

The ε-constraint method is applied for solving the above two-objective groundwater management model. The same hypothetical aquifer system used previously is again

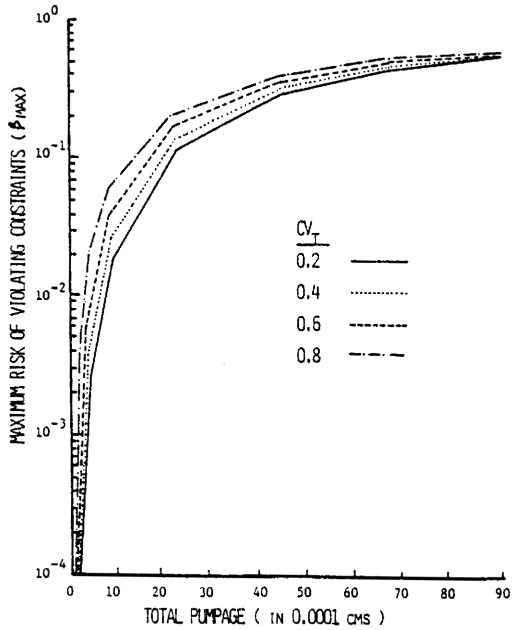

Fig. 2. Noninferior solution set between the total pumpage and maximum performance risk for various uncertainty levels of transmissivity.

considered. The resulting noninferior solutions for various levels of uncertainty of the transmissivity are shown in Figure 2. The tradeoffs between total water pumpage and risk of violating the specified drawdown limits can be obtained by computing the slope of noninferior solution curves. The decision-makers can incorporate the noninferior solution information generated by the model, along with their preferences, to reach a best compromised solution for the management model.

10. Summary and Conclusions

The concept of a stochastic groundwater management using the chance-constrained formulation is presented. The model is developed utilizing the concept of the influence function based on the well-known Thiem equation for steady, homogeneous, but nonuniform, confined aquifers. The model later is extended to multiple-objective optimization framework in which the tradeoff between the total aquifer pumpage and risk of violating the specified drawdown limitations is considered. In the derivation of the model, the aquifer transmissivity is assumed to have a log-normal distribution according to the observations from many field experiments. The chance-constrained model and its multiple-objective version are applied to a hypothetical aquifer for examining model behavior.

As expected, the total maximum allowable pumpage increases as the performance reliability requirement decreases under a given uncertainty level. The decreasing performance reliability requirement of the system is equivalent to an increase in the decision-maker's tolerance on the frequency of violating the system performance criteria. An increase in the level of uncertainty of the aquifer parameters results in a decrease in maximum total allowable pumpage at a given reliability level. The results are intuitive, noting that decision-making tends to be more cautious and conservative in cases when less knowledge is available about the system behavior. Results from the multiple-objective optimization model reveal these same facts. The noninferior solution curve generated clearly demonstrates that more pumpage can only be made at the expense of increasing the risk of violating the system's specified drawdown limitations.

The stochastic groundwater management models developed are illustrated through an application to steady confined aquifers of nonuniform, homogeneous medium. Their applications seem quite restricted in the sense that real groundwater aquifer systems are heterogeneous. However, because information on most aquifers is lacking, especially for undeveloped groundwater basins, it does not warrant the use of sophisticated distributed-parameter models. Under such circumstances, the use of simple analytical formula would be valid and justifiable in order to provide quick solutions to assist decision-making in the early stages of planning. Prickett (1981) advocated the development of groundwater models of various levels of sophistication as management tools for different situations. He also warned (1979) that, "In any event, choosing an overly sophisticated model which doesn't fit the problem is a case of applying the wrong model." Of course, after an aquifer is developed and more data are collected, the use of more sophisticated models would be appropriate. Of course, the concept of formulation illustrated in this paper can also be applied to steady or unsteady management of confined or unconfined aquifer systems.

Finally, readers are reminded of the practical limitations of the chance-constrained models. As pointed out by Loucks *et al.* (1981), as well as by Willis and Yeh (1987), that the chance-constrained formulation neither explicitly accounts for the consequences of constraint violations nor provides recourse action to correct such violations. The performance reliabilities have to be specified prior to model solution and it is rarely obvious what they should be.

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