WASTE LOAD ALLOCATION IN STOCHASTIC STREAM ENVIRONMENTS

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ABSTRACT

The waste load allocation (WLA) process is an important but complex part of decision-making in regional water quality management. The results of WLA have profound implications on regional environmental protection. It requires a sound understanding of the physical, chemical and biological processes of the aquatic environment and an appreciation for the legal, social, and environmental impacts of such decisions.

Most WLA processes are performed using either pollutant transport simulation models or deterministic optimization models. Only a few stochastic WLA models have been developed which treat either streamflow or background pollution as random variables to avoid nonlinearity in the model formulation so as to simplify the solution procedures.

This research is directed toward the development of an optimal stochastic waste load allocation model considering the uncertainty associated with physical, chemical and biological parameters in the transport equation for a complex multiple discharge setting. A general solution technique for solving the nonlinear WLA model, as a result of considering the overall uncertainty, will be developed. The result of this investigation is to expand the capability of performing WLA in a stochastic environment and to enhance the understanding of the tradeoffs between risk and economics, ultimately in order to develop a more effective and realistic decision-making process for the future.

FORWARD

This report is the completion report for the Wyoming Water Research Center on the project, "Stochastic Waste Load Allocations with Explicit Considerations of Embedded Uncertainties." The support from the Wyoming Water Research Center is gratefully acknowledged. This report is essentially the same as the M.S. thesis of Wade E. Hathhorn entitled, "Stochastic Optimal Waste Load Allocation in a Stream Environment Under Uncertainty." The authors would like to thank Dr. Robert Brocksen, former director of the Water Center, for his encouragement and support for the study. Appreciation is extended to Drs. Victor Hasfurther, Michael J. Humenick, and Leon E. Borgman of the University of Wyoming, all of whom have reviewed this material and given their critical comments. Special thanks are due to Mrs. Ruth Daniels of the Wyoming Water Research Center for her patient, painstaking typing of this report.

This research study resulted in one journal publication and several others that are under review:

1. Hathhorn, W.E. and Tung, Y.K., "Assessing the Risk of Violating Stream Water Quality Standards," accepted by J. of Environmental Management, 1986.

2. Hathhorn, W.E. and Tung, Y.K., "Optimal Waste Load Allocation: A Moving Control Approach," submitted to the J. of Environmental Engineering, ASCE, 1987.

3. Hathhorn, W.E. and Tung, Y.K., "Optimal Waste Load Allocation: A Multiobjective Approach," submitted to the J. of Water Resources Planning and Management, ASCE, 1987.

4. Hathhorn, W.E. and Tung, Y.K., "Optimal Waste Load Allocation by Fuzzy Linear Programming," submitted to Water Resources Research, AGU, 1987.

5. Tung, Y.K., and Hathhorn, W.E. "Assessment of Probability Distribution of Dissolved Oxygen Deficit," submitted to the J. of Environmental Engineering, ASCE, 1987.

6. Tung, Y.K. and Hathhorn, W.E., "Assessment of Probability Distribution of Dissolved Oxygen Deficit," submitted to the J. of Ecological Modeling, 1987.

7. Tung, Y.K. and Hathhorn, W.E., "Determination of CRitical Locations in Stochastic Stream Environments," submitted to Water Resources Research, AGU, 1987.

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CHAPTER 1

INTRODUCTION

1.1 UNDERSTANDING THE SIGNIFICANCE OF WASTE LOAD ALLOCATION

Water is the fundamental element, either directly or indirectly, in all human activity. Thriving species, especially "man", can neither progress nor survive without water resources of various quantities and qualities. Our needs for water range from drinking water supplies and hydroelectric power production to irrigation of crop lands and recreational uses. Although more than two-thirds of the surface of the earth is covered with water, less than one percent is fresh water on land (Krenkel and Novotny, 1980). These figures reveal that such important sources of water are, in fact, limited. Unfortunately, only in relatively recent historical times has the development of society begun to recognize the importance of preserving the quality of this invaluable resource.

No country has seen growth, either exploitative or planned, of the magnitude of that experienced by the United States in the past century. Such growth, in terms of economics, industry, and cultural activities, has left this country with a standard of living higher than all others. However, the prosperity enjoyed by this country has not materialized exempt from its own ill side effects. Specifically, the uncontrolled plight of this country into the future has had serious effects on its surrounding environment. This fact could not have been more apparent than that experienced by the quality of this nation's

rivers in the 1960's. For after World War II, the technological growth of this country was phenomenal, however, its rivers were naively left to suffer the brunt of the industrial and municipal wastewater disposal resulting from such growth.

As a result of the severe water quality conditions in many of the rivers in the United States during that time, the recognition of problems concerning surface water quality could no longer be avoided. Such factors led legislators to introduce several regulatory water pollution control measures, including the Water Pollution Control Act Amendments of 1961, the Water Quality Act of 1965, and the Clean Water Restoration Act of 1966. Legislation of this type culminated in 1972 with the introduction of the Water Pollution Control Act Amendments, more commonly referred to as Public Law (PL) 92-500.

PL 92-500 provided impetus for three essential tasks: (1) regulation of waste discharge from point sources (i.e., industrial plants, municipal sewage treatment facilities, and livestock feedlots); (2) regulation of oil spills and hazardous substances; and (3) assistance, in the form of financial aid, for the construction of wastewater treatment facilities (Ispen and Raisch, 1974; Krenkel and Novotny, 1980). This Act, in particular, set deadlines and goals specifying effluent limitations based on the "best practical control technology (BPT)" currently available and future guidelines for discharges from point sources that were to meet effluent standards based on the "best available technology (BAT)." Moreover, the 1972 Amendments established a national goal of eliminating all waste discharges by 1985. However,

although technologically achievable, the goal of zero waste discharge, subdued by various economic and political pressures, has yet to be met as of the end of 1986.

More importantly, the intent of this legislation was to require treatment levels which technology could achieve rather than those simply required for acceptable water quality. This was done, in effect, to force dischargers to install or upgrade treatment facilities which were commensurate with current technology. In theory, such plants would then be capable of producing effluents with a quality significantly better than the existing quality of the water body to which these effluents were to be discharged.

On the other hand, if the water assimilative capacity for a particular stream segment is below the total waste discharge authorized by the 1972 Amendments, more stringent controls may be required to ensure the protection of the natural instream biota. Stream segments in which these conditions exist are referred to as "water quality limited."

It is this type of stream segment to which WLA is most applicable under current law in the United States. For streams under water quality limited conditions or where effluent standards are not implemented, the question then becomes "how to effectively allocate the existing assimilative capacity of the receiving stream amongst several wastewater dischargers without detrimental effects to the aquatic environment?" This question, in essence, defines the role of WLA methodologies in preserving the quality of various water resources.

However, the procedures of WLA, combined with the water quality laws of this country, can become a relatively complicated task. Figure 1.1 provides some insight into the overall WLA process.

In reference to Figure 1.1, it should be noted that a major component of the total waste load is that of nonpoint source pollution such as that from agricultural and erosion runoff. This factor plays a significant role in the overall WLA process, however, accurate quantification of this variable can be a difficult task (Miller and Gill, 1976). Thus, given the nature of nonpoint source pollution and the need to limit the scope of this study, such factors are excluded in any subsequent discussions presented here. However if the entire WLA process is to be analyzed properly, the existence and importance of estimating nonpoint source pollution should not be ignored.

As an overview, with the passage of Public Law 92-500 and its subsequent implementation, the methodologies of WLA procedures is among the forerunners in current water quality management interest. Through such interests, research in this field has and will continue to play an important part in protecting the quality of water resources in this country for future generations.

1.2 STREAM ASSIMILATIVE CAPACITY AND ITS EFFECT ON INSTREAM DISSOLVED OXYGEN

The stream environment itself is home for a number of plant and animal species; to support much of this life, required levels of dissolved oxygen (DO) must be present. However, through the addition

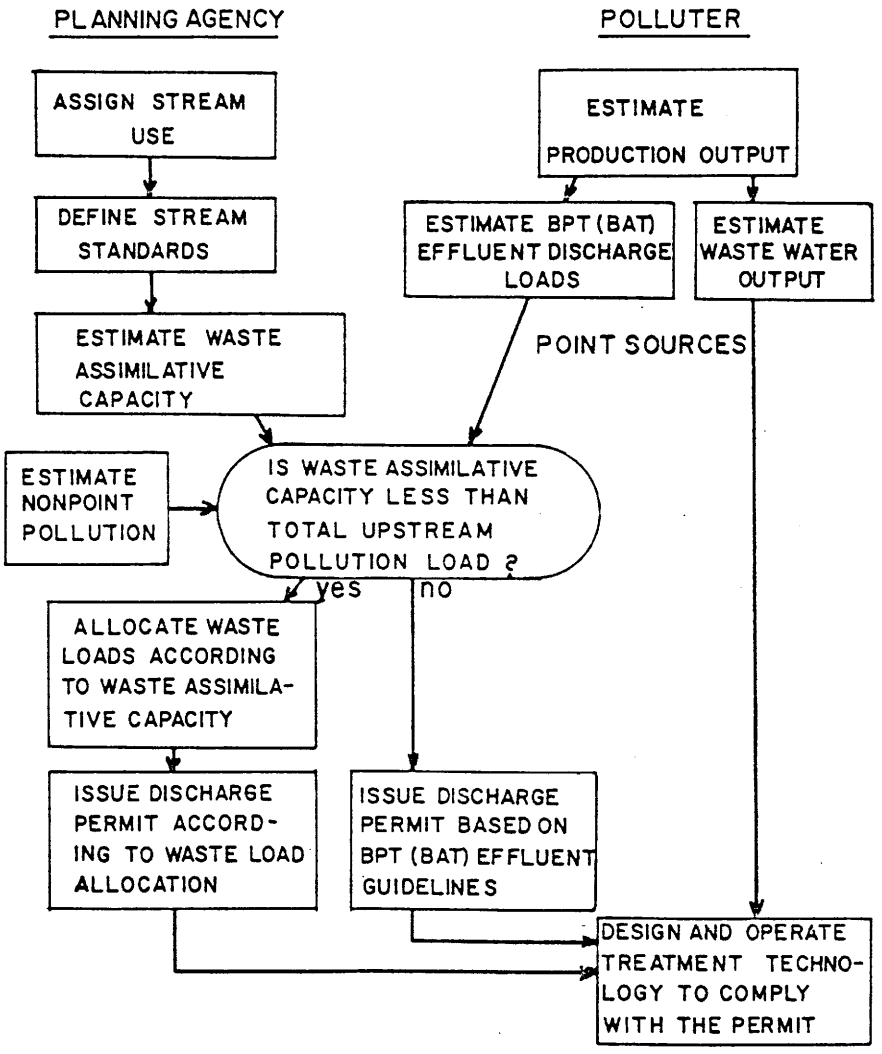

Figure 1.1 The Waste Load Allocation Process
(Krenkel and Novotny, 1980)

of waste discharge, the ability of the stream to maintain these levels of DO is threatened.

Moreover, surface waters, in general, contain a variety of microorganisms. When organic waste is discharged into these waters, the organic matter is utilized for heterotrophic microorganism growth at the expense of a certain quantity of instream DO. This process can be better understood through the following equation (Tchobanoglons and Schroeder, 1985):

$$\text{Organic Matter} + O_2 + \text{nutrients} \xrightarrow{\text{microorganims}} CO_2 + H_2 + \text{new cells} + \text{nutrients} + \text{energy} \qquad (1.1)$$

The amount of oxygen utilized in the metabolism of the organic matter is called biochemical oxygen demand (BOD).

In measuring the relative organic pollution in surface waters, it is a common practice to identify the rate at which oxygen is consumed in the metabolism (biological decomposition) of instream organic matter. Upon the introduction of organic waste to the stream environment, dissolved oxygen is utilized in a two-stage process: (1) first-stage BOD resulting from the decomposition of organic carbon matter (carbonaceous BOD) and (2) second-stage BOD resulting from the oxidation of nitrogen compounds (nitrogeneous BOD). Standard procedures, based on a five-day test, to determine the BOD of waste discharges have been developed and utilized for a number of years. However, in general, due to the length of time lapsed before the onset of nitrification, second-stage BOD in many cases is considered to have little

practical significance in the standard test and is simply ignored (Rich, 1973).

Based on these arguments, the rate of oxygen uptake for the metabolism of waste can be expressed by the following first-order rate reaction

$$\frac{dL}{dt} = -K_d L \tag{1.2}$$

where L is the concentration of first-stage BOD (mg/1) and K_d is the deoxygenation coefficient (generally expressed as 1/days).

To index the water quality of surface waters, the concentration of DO is often used. The basis for such an index lies in the fact that biota within the stream are quite sensitive to the amount of DO present, which is seldom in excess of 10 mg/1. On the other hand, it is generally accepted that at least 4 mg/1 of DO are required to maintain a balance of desirable aquatic species. As conditions fall below this minimum, the existence of many aquatic species is threatened. Moreover, at a DO concentration below 1.0 mg/1, aneorobic conditions are established resulting in the production of hydrogen sulfide, discoloration of the water, and the destruction of fish and other aquatic organisms (Rich, 1973).

As the concentration of dissolved oxygen falls below its natural saturation concentration, C_s, the equilibrium (between the atmosphere and the stream) is left unbalanced. From this, a driving force to transfer atmospheric oxygen to the stream is established. Water undersaturated with respect to DO is said to undergo atmospheric

reaeration. The time rate of exchange of DO through the process of reaeration is based on Henry's Law and can be expressed as

$$\frac{dC}{dt} = K_a(C_s - C) \qquad (1.3)$$

where C is the instream DO concentration (mg/l) and K_a is the reaeration coefficient (generally expressed as 1/days). It should also be noted that the term $(C_s - C)$ represents that portion of the stream which is undersaturated with respect to DO. This term is more commonly referred to as the DO deficit, D.

More importantly, when combined, as in nature, the processes of biological decomposition and natural reaeration allow the stream to treat a certain quantity of waste discharge. This combined effort is more commonly referred to as the assimilative capacity of the stream.

The concept of an assimilative capacity for the stream environment has been utilized by industries and municipalities for a number of years. By taking advantage of this process, waste dischargers have been able to treat less of their influents, relying on the stream to do some of the work. This was done, obviously, in an attempt to reduce overall treatment costs. Unfortunately, if unregulated, such a process can be exploited by the users, resulting in large quantities of waste being discharged to the stream environment. Unable to treat these quantities, the biological consumption of DO with the stream surpasses the natural reaeration rate, ultimately leading to an anaerobic environment in which stream biota cannot survive. This process, like many in nature, is a delicately balanced operation. Thus, regulatory

management plays an important role in protecting the stream environment from unwarranted and needless waste discharge.

1.2.1 The Basic Water Quality Model

To better understand the aquatic environment, several mathematical models have been developed to describe the interaction between the physical and biological processes occurring within the stream. The most well known expression of this type is the Streeter-Phelps equation (Streeter and Phelps, 1925). In differential form, the equation is given as:

$$dD/dt = K_d L - K_a D \tag{1.4}$$

The solution to Eq. (1.4), replacing t by x/U, is:

$$D_x = \frac{K_d L_o}{K_a - K_d} \left[e^{-K_d x/U} - e^{-K_a x/U} \right] + D_o e^{-K_a x/U} \tag{1.5}$$

where K_d is the deoxygenation coefficient (1/days), K_a is the reaeration coefficient (1/days), x is the distance downstream from the source of BOD (miles), U is the average stream velocity (miles/day), D_x is the DO deficit concentration (mg/l) within a unique reach at a downstream distance x, D_o is the initial DO deficit (at distance x = 0), and L_o is the initial in-stream BOD concentration (both in mg/l).

From Eq. (1.5), it is evident that the Streeter-Phelps equation is limited to only two instream processes: (1) deoxygenation of the

water due to bacterial decomposition of carbonaceous organic matter, and (2) reaeration directly proportional to the DO deficit.

The concentration of DO at any downstream location is given as:

$$C_x = C_s - D_x \tag{1.6}$$

in which C_s is the saturated DO concentration. The downstream location, X_c (miles), where the maximum DO deficit occurs can be found by differentiating Eq. (1.5) and solving for x:

$$X_c = \frac{U}{K_a - K_d} \ln \left[\frac{K_a}{K_d} \left(1 - \frac{(K_a - K_d)D_o}{K_d L_o} \right) \right] \tag{1.7}$$

The point X_c will herein be referred to as the "critical location". The resulting maximum DO deficit is computed using X_c and Eq. (1.5):

$$D_{max} = (L_o K_d / K_a) \exp (-K_d X_c / U) \tag{1.8}$$

It should also be noted that several assumptions have been made in the development of the "Streeter-Phelps" equation: (a) steady, uniform flow; (b) DO deficits predicted by Eq. (1.5) are one-dimensional (functions only of the position downstream from a discharge point); and (c) rate of biodegradation and reaeration, expressed by K_d and K_a, are described by first-order kinetics. A typical DO profile for a single reach is shown in Figure 1.2.

Moreover, Eq. (1.5) describes the response of DO in a single reach of stream as a result of the addition of a "point-source" loading of waste at the upstream end of the reach. This equation can be used

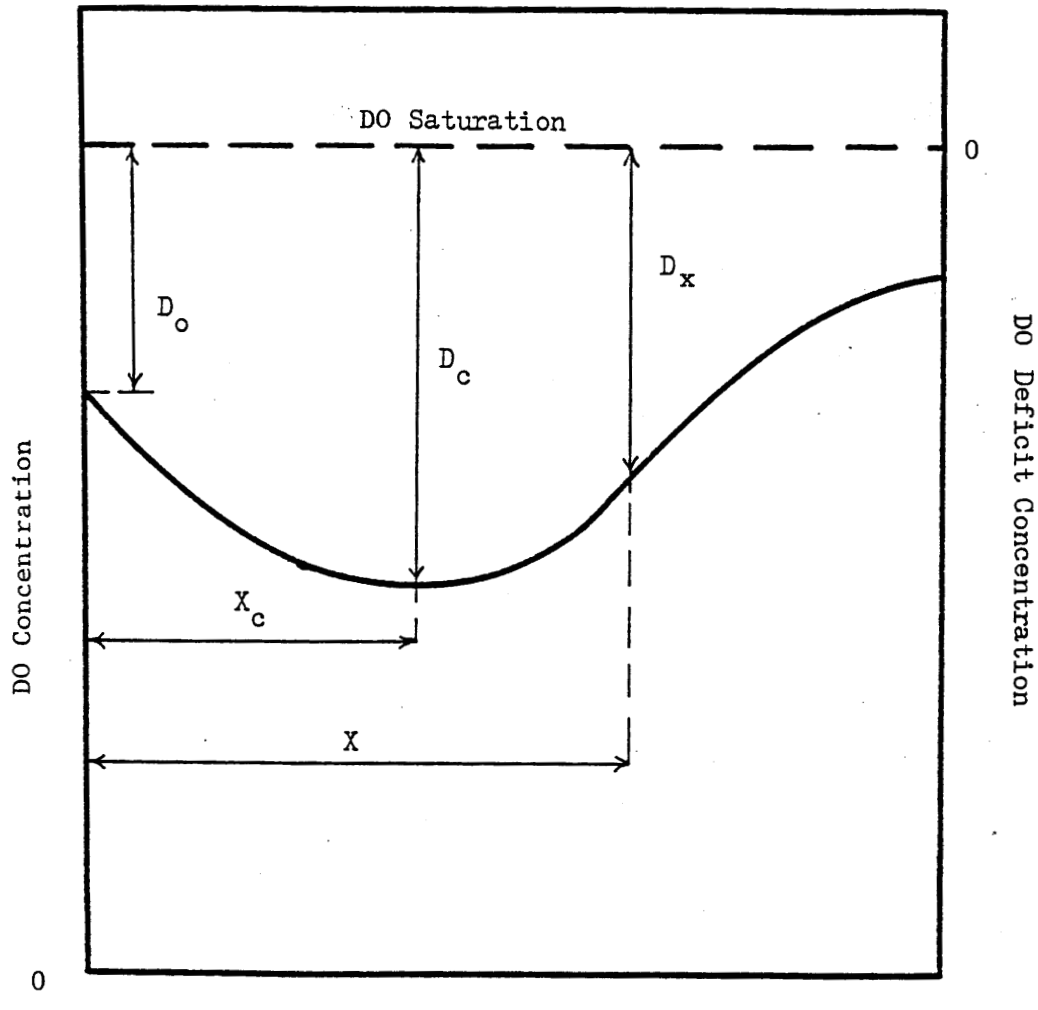

Figure 1.2 Typical Dissolved Oxygen Sag Curve
(Clark et al., 1977)

to determine the DO concentration in several successive reaches by applying the deficit at the downstream end of one reach as the initial deficit of the succeeding reach. Thus, Eq. (1.5) can be applied iteratively to determine the DO profile of an entire stream system (Liebman and Lynn, 1966).

1.2.2 Modification of the Original Streeter-Phelps Equation

Since its conception, the "Streeter-Phelps" equation has been modified to account for discrepancies between analytical estimations, computed from Eq. (1.5), and actual data collected in the field. These discrepancies have arisen as a result of the exclusion of a number of oxygen sources and sinks in the original equation. Dobbins (1964) pointed out several other possible factors which could contribute to instream BOD and DO variations: (1) removal of BOD by sedimentation or absorption; (2) addition of BOD along the stretch by the scour of bottom deposits or by the diffusion of partly decomposed organic products from the benthal layer into the water above; (3) addition of BOD along the stretch by local runoff; (4) removal of oxygen from the water by diffusion into the benthal layer to satisfy the oxygen demand in the aerobic zone of this layer; (5) removal of oxygen from the water by purging action of gases rising from the benthal layer; (6) addition of oxygen by photosynthetic action of plankton and fixed plants; (7) removal of oxygen by the respiration of plankton and attached plants and (8) continuous redistribution of both BOD and oxygen by longitudinal dispersion.

There have been several studies conducted in which one or more of the processes have been included in the model formulation in an attempt to improve model predictability (Dobbins 1964; Hornberger, 1980; Krenkel and Novotny, 1980). In general, these modifications can be made by simply adding terms to Eq. (1.5) to account for the various additional factors listed above. However, in order to simplify the algebraic manipulations, the original Streeter-Phelps equation will be utilized in this and all remaining chapters where appropriate. It is simply the author's intention at this point to note the improvements made to the original formulation by various other researchers.

1.3 APPROACHES TO WASTE LOAD ALLOCATION

As was pointed out in earlier sections of this chapter, the process of WLA can become quite complex. In attempts to optimize the stream's waste assimilative capacity and beneficial use, several methodologies have been developed which utilize a variety of approaches: ad hoc procedures, mathematical programming, and simulation. Each of these procedures is discussed in the following subsections.

1.3.1 Ad Hoc Procedures

Although sophisticated models which attempt to optimize the allocation of the waste assimilative capacity of streams are quite rational, their complexities, in many instances, lead some regulatory agencies to incorporate much simpler approaches to the problem. For example, the total allowable waste discharge may be determined by

simply dividing it among several users in some arbitrary fashion, or each discharger may be required to treat their effluents based on a proportion of the raw wastewater characteristics.

These procedures may afford the regulatory agency relief from obvious computational burdens and/or allow a greater flexibility in the decision-making process. However, such an approach is neither scientifically nor economically justifiable. From this viewpoint, such procedures would be virtually impossible to defend, and as such, would most likely be discarded in a legal sense if implemented. Hence, the ability to enforce such regulations would be essentially nonexistent.

1.3.2 Water Quality Simulation

The achievement of various water quality goals often requires substantial capital investment and cooperation with public interests. Within this day and age, water quality control measures are, in general, costly. The brunt of these costs are incurred by the citizens of this country. Such costs, either direct through taxes or indirect through increased manufacturing costs passed along to the consumer, include the price of treatment facilities designed to reduce the discharge of contaminants, or to improve existing instream water quality. Those responsible for the formulation and adoption of water quality policies must have an appropriate means of evaluating the economic, environmental, and ecological impacts of these policies. Such a need has stimulated the development of a wide range of mathematical modeling techniques to investigate the impacts of various water quality plans (Loucks et al., 1981).

One approach of this type is that of "simulation". In fact, with the advent of high-speed computers, simulation has become a widely used technique in many areas of water resources planning and management. Simulation is not an optimization procedure; rather, simulation merely provides a means for evaluating the performance of the system under investigation.

In essence, simulation procedures can be characterized in two basic steps: (1) define and develop a model (most commonly a computer model) which best describes the physical, biological, and economic responses associated with the system under investigation (e.g., the stream environment itself); and (2) then specify desired input characteristics and iterate the model for a sufficient number of trials to obtain information which indicates the performance of the system subject to a variety of modeling parameter conditions.

There have been a number of simulation models developed solely for the purpose of water quality prediction. A list of commonly used water quality simulation models is given in Table 1.1. For an in-depth review of simulation approaches and their application to water resources planning and management (which include water quality applications), the reader is referred to Krenkel and Novotny (1980) and Loucks et al. (1981).

1.3.3 Water Quality Optimization Procedures

Another approach to mathematical modeling is that of constrained optimization. Unlike simulation, constrained optimization provides information revealing the "best" alternative amongst all those

TABLE 1.1 OVERVIEW OF SELECTED STREAM WATER QUALITY MODELS

MODEL	DEVELOPER AND/ OR SOURCE	MODEL CHARACTERIZATION	PARAMETERS MODELED	INPUT DATA AND COMPUTER REQUIREMENTS
DOSAG	Texas W. Dev. Board[3]	steady state	DO, nitrogen	small
QUAL-II	EPA	semi-dynamic	DO, temperature, most of water quality parameters	medium
SWMM-RECEIV	Wat. Res. Eng., EPA[2]	dynamic	DO, nitrogen, conservative pollutants	large
HSP-II CHANNEL	Hydrocomp International[1]	dynamic	DO, nitrogen, conservative pollutant transport	large
MIT Network	MIT International[4]	dynamic	DO, nitrogen, conservative pollutant temperature	large

[1]Hydrocomp International, Palo Alto, CA.
[2]Water Resources Engineers, Walnut Creek, CA.
[3]Texas Water Development Board, Austin, TX.
[4]Massachusetts Institute of Technology, Dept. of Civil Engineering, Cambridge, MA.

available which conform to the requirements specified by the model formulation. However, in general, optimization procedures are seldom able to deal with all the complexities and nonlinearities incorporated by the simulation model. On the other hand, when a constrained optimization model is developed properly, it provides an adequate approximation of the real problem. Moreover, optimization models can provide the decision maker with information pertaining to, for example, the overall least cost or most beneficial alternative amongst those possible.

The term "constrained optimization" is related to a general class of modeling techniques more commonly referred to as mathematical programming. A number of mathematical programming techniques have been developed: Lagrange multipliers, linear programming, dynamic programming, quadratic programming, and geometric programming. At one time or another, practically all these methods have been applied to the problem of WLA. For example, Loucks et al. (1967) and ReVelle et al. (1968) utilized linear programming techniques; Liebman and Lynn (1966) and Shih (1970) applied dynamic programming; and Ecker (1975) incorporated the use of geometric programming.

Although several techniques exist for solving the WLA problem, this study will concentrate the solution procedures based solely on those of linear programming. As will be shown in subsequent chapters, the type of objective function and constraints that will be utilized throughout this study (i.e., linear functions of the decision variables) make the use of linear programming the most appropriate solution

technique. In addition, linear programming is probably the most widely used mathematical programming technique. Linear programming packages are available at most scientific computing facilities worldwide. In fact, its popularity has led to such a vast array of available computer algorithms that one need only know how to use the computer programs available and to interpret their results to effectively apply the linear programming technique. However, this author feels that a short review of linear programming methodology is necessary before a true appreciation for its application can be understood.

1.4 BRIEF REVIEW OF THE LINEAR PROGRAMMING TECHNIQUE

The general linear programming (LP) model is expressed in terms of two major components: (1) a linear objective function expressing the goal of the model formulation which is to be optimized (i.e., maximized or minimized); and (2) the model constraints, also linear functions, describing the physical, legal, and economic behavior of the system under investigation. LP provides a means of comparing all possible solutions in order to identify the solution which optimizes the objective function, while simultaneously satisfying all the constraints. All LP problems can be expressed in standard form as

$$\text{Maximize } \underline{C}^T\underline{X} \tag{1.9}$$

subject to

$$\underline{\underline{A}}\,\underline{X} \leq \underline{B} \tag{1.10}$$

and

$$\underline{X} \geq \underline{0} \tag{1.11}$$

where $\underline{C}^T$ is a K-dimensional row vector; $\underline{\underline{A}}$ is a J x K matrix; $\underline{X}$ is a K-dimensional column vector; and $\underline{B}$ is a J-dimensional column vector. For further information concerning the development of linear programming models, the reader is referred to Taha (1982).

1.5 UNCERTAINTIES IN WASTE LOAD ALLOCATION

Having acknowledged the necessity of incorporating mathematical modeling into the WLA process, the proper management of the quality of various water resources depends on an understanding of the system to be modeled. In the case of stream water quality management, the system to be modeled is that of the stream itself. The processes occurring within such a system are dictated by nature, and as such, are in many instances, inherently random. The stream itself is an agglomeration of many physical, biological, and ecological processes which, in general, cannot be predicted with certainty.

If water quality management is to develop a methodology to manage the stream environment, techniques must be developed which accurately reflect the stochasticity of the stream environment. Ward and Loftis (1983) have recently summarized the importance of uncertainty considerations in water quality management:

> "In terms of the current scientific understanding, water quality is a mix of deterministic and stochastic components, and its management can only be truly effective when both components are properly balanced."

From a regulatory perspective, in order to properly acknowledge the inherent stochastic nature of the stream environment, some means

within the management process must by provided to account for the concept of risk. Risk, in a water quality sense, defines the probability that a given stream standard will be violated. One of the earliest works to recognize the concept of risk in water quality management was that of Loucks and Lynn (1966). Quoting from the conclusion of their paper, Loucks and Lynn state:

> "Thus a more realistic approach for establishing dissolved oxygen standards would be to introduce the concept of a maximum allowable probability of the dissolved oxygen concentrations dropping below a specified concentration for a given number of consecutive days."

The risk of violating a standard would be exactly known if the probability distribution of water quality were known. However, uncertainty is the result of one's inability to determine the exact properties of the population for the various water quality parameters. To add to the problem, data which is often deficient or aliased is commonly used to estimate these properties in models which do not exactly reflect the character of the system being modeled. This leads to errors and loss of information that causes one to be uncertain with respect to the true population properties. Thus, uncertainty can be thought of as the difference between population properties and their respective estimates computed from sample data (Ward and Loftis, 1983).

From Eqs. (1.5) through (1.8), it is evident that the mathematical modeling of the WLA process is dependent on knowing the value of several water quality parameters (i.e., K_d, K_a, U, L_o and D_o). To do this, either experimental and/or stream sampling data are utilized in

an attempt to estimate these parameters. It is quite obvious that such a process is subject to a number of inherent uncertainties as the result of sampling errors and the inherent random nature of the stream environment itself. Thus, in order to accurately develop an effective management policy based upon modeling techniques, the uncertainty associated with the WLA should be incorporated into the mathematical model.

1.6 NATURE, SCOPE, AND OBJECTIVES OF RESEARCH

The significance of WLA and its role in managing the quality of various water resources has already been presented at the outset of this chapter. In this discussion, the complexities of such procedures, arising from legal, economic, and bureaucratic barriers, are pointed out. Moreover, such a task is further complicated by the multiobjective and interdisciplinary nature of the problem, the lack of sufficient information about the system (i.e., physical and chemical data of the stream itself), and the existence of inherent random components of the physical and biological processes involved.

By acknowledging the existence of various uncertainties within the stream environment and the modeling process of this system, the prediction of water quality responses to effluent waste discharges is no longer a simple deterministic calculation as that given by the Streeter-Phelps equation. Specifically, the WLA modeling process is dependent upon knowing or estimating the assimilative capacity of the stream which, in turn, is a function of water quality standards, flow

rates, reaction coefficients, and modeling techniques; all of which are subject to a variety of uncertainties.

Unfortunately, to date, the conventional approach to WLA analysis is performed by considering the most critical environmental conditions (where the streamflow is low and the temperature is high) in a deterministic model formulation. This generally leads to a highly conservative result which, many times, requires increased levels of treatment and concomitant higher costs.

Like any other resources planning and management, regional water quality control and management involves risk. Decision-making in WLA requires knowledge and understanding of the processes occurring when pollutants are discharged into receiving waters. Modeling activities play an important role in the process in as much as they reflect our knowledge of the phenomena occurring. However, the problem becomes complicated because of the existence of uncertainties mainly due to the lack of sufficient information(e.g., inadequacy or incompleteness of the models) and the inherent randomness of the processes involved (e.g., variations of streamflow, temperature, channel geometry, reaction rates, etc.). As a result, the WLA is a decision-making process to be performed in the stochastic environment. It would be ideal and realistic to develop WLA models which explicitly incorporate the random and uncertain features of the processes involved.

The decision-making process in WLA is affected by many factors. For example, the system configuration and physical characteristics, the legal, social, economical, and environmental setting, and the type

of pollutant. Moreover, a number of uncertainties exist within the chemical and biological processes involved. However, all these factors cannot be reasonably investigated within a single study. Hence, the scope of the proposed study is as follows: (1) only the effects of uncertainties in physical, chemical and biological processes on the WLA decision-making are considered; (2) the system includes stream channels with multiple dischargers in which convective transport in the longitudinal direction prevails; (3) carbonaceous biochemical oxygen demand is the pollutant to be modeled; (4) steady state pollutant transport models will be employed in deriving optimal stochastic WLA models.

A number of pollutant transport models have been developed with various levels of sophistication (Krenkel and Novotny, 1980). The selection of the model to be used depends on the quality and availability of data, the accuracy of results desired, and the level of analysis, etc. It is not the intention of the proposed research to develop a new pollutant transport model. Instead, various existing transport models will be evaluated and the most appropriate one selected for constructing the stochastic optimal WLA models.

To date, the number of research articles in which the risk of water quality violations and/or the uncertain nature of the stream environment is incorporated into the management of the quality of water resources is limited. Furthermore, among the articles already published, many of the results and methodologies are contradictory. Hence, the need for improvements in this area of research are virtually unbounded at this point in time.

Thus the main objectives of this study are as follows: (1) to develop improved deterministic linear programming methodologies for solving the problem of effective WLA by updating existing approaches and investigating the multiobjective nature of the problem; (2) to develop a systematic and consistent approach for evaluating the effects of uncertainty levels associated with system parameters on model results. (Specifically, consideration will be given to the analysis of the risk of various water quality violations and the probability distributions of dissolved oxygen and critical locations within a given reach of stream); (3) to develop a stochastic WLA model considering explicitly the model parameters with uncertainty. (The model will be specifically directed toward the consideration of complex multiple discharge situations, including various concepts of economic and equity measures between dischargers); (4) to devise a solution technique for the optimal stochastic WLA model; and (5) to study the sensitivity of various outputs from all model formulations, regarding varying levels of model parameter uncertainty and different management objectives.

This, like many other research studies, cannot begin to answer or address all the questions pertaining to the problem of effective WLA. Some of the notable limitations of this study are: (1) spatial correlation of the stream parameters (i.e., between successive reaches) in the WLA models presented here are not considered; (2) a simplified transport model (i.e., the original Streeter-Phelps equation) is utilized throughout this study in which a number of oxygen sources and sinks, proven to exist, are excluded; (3) the costs of treatment are

not directly considered in the WLA model formulation. However, indirect measures of various related economic considerations are discussed throughout this study.

1.7 ORGANIZATION OF STUDY

There are eight chapters presented in this text; the first is an introduction and the last is an overall summary and conclusion of study results. Each of the remaining chapters are essentially unique investigations into a separate aspect of water quality management and/or uncertainty analysis of the stream environment. It should be pointed out that a separate literature review is not provided within this text. Instead, review of important articles pertaining to each topic are cited in the introductions and text of each of the chapters where appropriate. The organization of this study and a brief description of the content of each chapter are given in the following paragraphs.

In Chapter 2, a unique property of the typical dissolved oxygen sag curve is utilized in developing an improved deterministic WLA model, whereby, the number of water quality constraints are significantly reduced. Such an approach will be shown to reveal considerable savings in computer storage and exact solutions to the WLA problem.

Chapter 3 presents a deterministic multiobjective approach to WLA using two methodologies: (1) constraint methods and (2) fuzzy linear programming. From this, it is believed that a more realistic approach to WLA is presented.

Chapter 4 presents methodologies for evaluating the risk of violating various assumed water quality standards using Monte Carlo

simulation in conjunction with the dissolved oxygen deficit computed by the original Streeter-Phelps equation. In this chapter, plots of contours of equal risk are presented for a variety of water quality conditions. Such results are believed to provide important information in the water quality regulatory process.

In Chapter 5, attention is placed on the uncertainty analysis of stream dissolved oxygen and the critical location. By incorporating Monte Carlo simulation and first-order uncertainty analysis, a "best" estimate for the probability distribution of dissolved oxygen at any location and the critical location within any reach are determined. Additionally, confidence intervals for this information are investigated.

Acknowledging the importance of estimating the location of the critical point, from both an economic and monitoring viewpoint, several methods are developed in Chapter 6 for estimating its position in a stream environment under uncertainty using Monte Carlo simulation and Fibonacci search techniques. These methods are based on locating the critical point associated with: (1) average water quality model parameters, (2) maximum variance in predicted dissolved oxygen concentration, (3) maximum probability of violating a given water quality standard, and (4) the point most likely to be critical (i.e., the mode of the distribution assumed for the critical location.)

Finally, in Chapter 7, having accepted the existence of uncertainty in the WLA process, an optimal stochastic WLA model is developed using chance-constrained optimization techniques. In addition,

estimates of the uncertainty in the technological coefficients (i.e., the mean and variance) are computed using Monte Carlo simulation procedures. A sensitivity analysis of this approach for computing the technological coefficients is also conducted.

CHAPTER 2

DETERMINISTIC OPTIMAL WASTE LOAD ALLOCATION MODELS: A SINGLE OBJECTIVE FRAMEWORK

2.1 INTRODUCTION

Realizing the importance of water quality management, the need for developing a methodology allowing the environmental decision-maker, for example a governmental regulatory agency, to, optimally, solve the waste load allocation (WLA) problem is in order. The ultimate goal of such an analysis is the implementation of a policy which simultaneously seeks to maximize the benefits while ensuring that the utilization of natural resources does not exceed allowable levels mandated by law. Given these objectives, the utilization of mathematical programming techniques to solve the WLA problem become quite attractive. In fact, mathematical programming techniques have been used quite extensively to solve problems concerning optimum water quality management (Rich, 1973; Loucks et al., 1981).

Moreover, the broad use of linear programming (LP) techniques, both commercially and academically, have resulted in the availability of a large number of computer codes to solve this type of model formulation. As a consequence, there have been several previous research studies utilizing LP techniques in attempts to solve the problem of effective WLA (Thomann and Soble, 1964; Loucks et al., 1967; ReVelle et al., 1968). The most common approach has been that of using a deterministic LP model formulation in which the random effects of system

behavior are ignored and a single, fixed value is assigned to the physical parameters describing the stream environment (Sobel, 1964; Converse, 1972; Graves et al., 1972). Although the use of such procedures has been criticized for its oversimplification of the system behavior, the development of a deterministic LP model to solve various optimization problems offers the analyst the advantages of reduced complexities in both computational analysis and problem formulation. Hence, the use of LP techniques to solve problems of optimal environmental water quality management has been and will continue to be quite popular.

As an alternative to the use of LP, several other types of mathematical programming techniques have been applied to the problem of optimal WLA. The first of these approaches is that of dynamic programming. Dynamic programming is a very useful tool for optimizing both linear and nonlinear problems, especially those involving sequencing and scheduling. The inherent qualities of dynamic programming makes it an attractive technique to solve the sequential format of the optimal WLA problem. Several researchers have already employed dynamic programming in attempts to solve this problem (Liebman and Lynn, 1966; Shih, 1970). Other techniques which have been utilized include both integer programming and geometric programming (Liebman and Marks, 1968; Ecker, 1975). In each approach, the analysts have taken advantage of the identity of the individual methodologies and model characteristics in attempts to uncover improved computational methods and model predictability. Although several techniques have been developed to solve the problem of

optimal WLA, this and all remaining chapters will be confined to use LP procedures where appropriate.

In order to control water quality in the WLA model formulation, past research studies have utilized a method in which several control points are selected within each reach of the river system. Herein, a reach is simply defined as the river segment between two successive point discharge locations. In the LP model, a constraint is formulated for each control point defining the DO concentration at that location, while ensuring that the utilization of this limited resource does not exceed a minimum specified level. Thus, a number of control points are required within each reach to ensure minimum DO concentrations throughout the entire river system. By increasing the number of control points per reach, the possibility of violating the water quality standards at any location is reduced. Theoretically, this approach would require an infinite number of control points per reach to reduce the possibility of such violations to zero.

When using LP techniques, it is generally known that computational efforts to solve an optimization problem increase exponentially as the number of constraints are increased. Thus, for the approach incorporating a number of fixed control points in the LP model, a trade-off exists between the number of constraints to be used to ensure the overall compliance of minimum water quality standards and the computational effort required to solve the WLA problem.

Alternatively, a methodology will be presented in this chapter to solve the optimal WLA problem utilizing a unique property of the DO

profile within each reach. This new technique utilizes a single control point per reach while simultaneously ensuring that the possibility of water quality violations at any location does not exist. The essence of this new approach is based on a procedure in which the LP formulation is solved iteratively, each time updating a new, single control point defined by the "critical location" within each reach. The results from this new approach will be shown to be both computationally efficient and more exact in its solution of the optimal WLA problem when compared against the fixed control point approach commonly used in the past.

2.2 OBJECTIVE FUNCTION

There are two major components of any LP formulation: (1) the objective function and (2) the model constraints. To begin, the first such component to be examined is that of the objective function. Given the general discussion in Chapter 1, one should remember that the most important characteristic of LP formulation is that both the objective function and constraints are linear functions of the decision variables in the problem. In general, the objective function can be expressed as

$$\text{Minimize } \underline{C}^T\underline{X} \tag{2.1}$$

Where $\underline{C}^T$ is an n-dimensional row vector containing the values of the unit costs associated with a given level of treatment, $\underline{X}$ is an n-dimensional decision vector containing the specified level of treatment for each discharger, and n is the number of dischargers within the stream system.

The most common application of LP to the optimal WLA problem has been one in which the objective function and decision variables are defined as the minimization of treatment cost and the effluent waste concentration at each discharge location, respectively (Loucks et al., 1967; ReVelle et al., 1968). The values contained in vector $\underline{C}$ of an objective function can be regarded as weighting factors in the overall outcome of the LP problem. By defining the objective function as the minimization of treatment cost in which vector $\underline{C}$ contains the unit cost of treating each of the elements of vector $\underline{X}$, the solution algorithm (most commonly the simplex method) will be inclined to use less units of X_1, compared to those of X_2, given C_1 is the greater than C_2. Thus, by simply manipulating the cost values specified in vector $\underline{C}$, the optimal solution of the LP problem can be greatly varied.

In order to avoid the manipulation of cost values in the objective function and the effects of such procedures on the optimal solution, an objective function is defined in this study as the maximization of total waste discharge. In addition, the decision variables are selected as the effluent waste concentration and DO deficit at each discharge location. In using this approach, each of the decision variables in the problem are assigned an equal weight in the objective function as follows:

$$\text{Maximize} \sum_{j=1}^{N} (L_j + D_j) \tag{2.2}$$

where L_j and D_j are the waste concentration (mg/1 BOD) and DO deficit (mg/1) in the effluent at each discharge location j, and N is the total number of discharge locations.

The decision variables, effluent waste discharge and DO deficit at each point source location are chosen in an attempt to replicate actual design conditions. In the design and operation of industrial and municipal treatment plants, both waste concentration and DO deficit in the effluent are controllable. Obviously, various levels of treatment can be incorporated into the design of most industrial plants, whereby, the quality of the effluent discharged from these plants can be controlled. Furthermore, the DO deficit in the effluent can also be varied by incorporating a reaeration system at the end of the treatment process train. Thus, by utilizing both the DO deficit and waste concentration in the effluent as the decision variables, a more realistic setting to the problem of WLA is constructed.

In review of treatment plant operations, a trade-off exists between the allowable waste discharge and the DO deficit in each plant effluent. By reducing the DO deficit in the effluent through an induced reaeration process, greater quantities of waste can be discharged without violating the minimum DO requirements within the stream environment, hence, waste removal costs are reduced. Of course, a price must be paid in order to provide this reaeration. Given this formulation, an analogy can be drawn between the maximization of waste discharge and minimization of treatment cost, in fact, both goals are economically quite similar. By maximizing waste output, the associated overall

treatment costs are generally reduced. Though not identical, the economic parallelism between these objectives is evident. It should also be noted that by using the objective function of maximized waste discharge, the optimal values of the decision variables will not be identical to the optimal solutions when minimizing total cost, unless uniform cost coefficients are assigned to each of the decision variables.

2.3 CONSTRAINTS

The second component of the LP formulation is the set of constraints which define the physical, biological, legal, and economic limitations of the system itself. The constraints are simply a set of mathematical relationships describing both the system behavior and availability of limited resources as functions of the decision variables in the problem formulation. The purpose of the constraints is to provide a restriction on the use of limited resources when attempting to optimize a given objective. For example, when maximizing profits from the sale of a product, the constraints would describe both the utilization of material for the production of this product, while simultaneously ensuring that the allocation of raw materials does not exceed some available limit.

In this study, the objective of the WLA problem is to maximize waste discharge, however, this action is not without its own limitations. Obviously, unrestricted waste discharge to a stream environment will pose detrimental effects to the aquatic biota, eventually producing an anaerobic environment in which all forms of desired life cease to exist. Hence, the inclusion of constraints which

properly defined and protect the use of limited resources within the stream environment are essential in the WLA problem formulation.

2.3.1 Constraints on Water Quality

The most common requirement of the WLA problem has been the assurance of minimum concentrations of DO throughout the river system in an attempt to maintain desired levels of aquatic biota. Specifically, the constraint relating the response of DO to the addition of in-stream waste is generally defined by the Streeter-Phelps equation or a variation of this equation (ReVelle, et al., 1968; Bathala et al., 1979). In the past, attempts to incorporate water quality constraints into the model formulation, researchers have placed a number of control points within each reach of the river system under investigation. By utilizing the Streeter-Phelps equation, each control point and discharge location becomes a constraint in the LP model providing a check on water quality at that location. In a generalized framework, a typical water quality constraint would be as follows:

$$\sum_{j=1}^{n_i} \theta_{ij} L_j + \sum_{j=1}^{n_i} \psi_{ij} D_j \leq R_j \qquad (2.3)$$

where

$$\theta_{ij} = \frac{q_i}{Q_o + \sum_{m=1}^{n_i} q_m} \left[\left(\prod_{\ell=j}^{n_i-1} b_{\ell,\ell+1} \right) d_{n_i,i} + \right.$$

$$b^{a}_{n_i,i} \sum_{p=1}^{n_i-1} \left(\prod_{\ell=j}^{n_i-p-1} b_{\ell,\ell+1} \right) d_{n_i-p,n_i-p+1} \left(\prod_{k=n_i-p+1}^{n_i-1} b^{a}_{k,k+1} \right) \Bigg] \tag{2.4}$$

$$\psi_{ij} = \frac{q_i}{Q_o + \sum_{m=1}^{n_i} q_m} \left(\prod_{\ell=j}^{n_i-1} b_{\ell,\ell+1} \right) b^{a}_{n_i,i} \tag{2.5}$$

$$R_i = DO_i^{sat} - DO_i^{std} - \frac{q_i}{Q_o + \sum_{m=1}^{n_i} q_m} \Bigg[\left(L_o Q_o \prod_{\ell=1}^{n_i-1} b_{\ell,\ell+1} \right) +$$

$$L_o Q_o \sum_{p=2}^{n_i-1} \left(\prod_{\ell=1}^{n_i-p} b_{\ell,\ell+1} \right) d_{n_i-p+1,n_i-p+2} \left(\prod_{k=n_i-p+1}^{n_i-1} b^{a}_{k,k+1} \right) +$$

$$\left(D_o Q_o \prod_{k=1}^{n_i-1} b^{a}_{k,k+1} \right) \Bigg] \tag{2.6}$$

$$d_{n_i,i} = \frac{K^{d}_{n_i}}{K^{a}_{n_i} - K^{d}_{n_i}} \left(b_{n_i,i} - d_{n_i,i} \right) \tag{2.7}$$

and

$$b_{n_i,n_i+1} = \exp \left[- K^{d}_{n_i} x_{n_i,n_i+1} / U_{n_i} \right] \tag{2.8}$$

$$b^{a}_{n_i,n_i+1} = \exp\left[- K^{a}_{n_i} \, x_{n_i,n_i+1} \, / \, U_{n_i} \right] \qquad (2.9)$$

M is the total number of control points, n_i is the number of the dischargers upstream of the control point i, $K^{a}_{n_i}$ and $K^{d}_{n_i}$, are the reareation and deoxygenation coefficient ($days^{-1}$) in reach, L_o, Q_o, and D_o are the upstream waste concentrations (mg/l BOD), flow rate (cfs), and DO deficit (mg/l), respectively; D_{n_i}, L_{n_i}, and q_{n_i} are the DO deficit (mg/l), waste concentration (mg/l BOD), and effluent flow rate (cfs) from each discharge location, respectively; $x_{n_i,i}$ is the distance (miles) between discharge location and control point i, and U_{n_i} is the average stream velocity (miles/day) in reach n_i; R_i represents the allowable DO deficit at the control point i, available for the utilization of waste discharge (mg/l). It should also be noted that, in addition to each control point i, water quality is also checked at each discharge location n_i. Thus the total number of control points, M, and, hence, the total number of water quality constraints are

$$M = N + \sum_{j=1}^{n} nc_i \qquad (2.10)$$

where nc_i are the number of control points in each reach i; and N is the total number of dischargers. Lastly, in Eq. (2.6), values of 4 and 10 mg/l were used throughout this chapter as the required minimum level of in-stream DO (DO_i^{std}) and DO saturation concentration (DO_i^{sat}) at each

control point i. The latter concentration is selected based on its general acceptance as the lower limit required to support various levels of aquatic biota within the stream environment.

2.3.2 Constraints on Treatment Equity

In addition to the constraints satisfying water quality, constraints are also required which define equity between the various dischargers along the river system. Without the inclusion of equity considerations in the WLA model, any attempts to maximize waste discharge would result in the allocation of large quantities of waste to the upstream users, while the downstream dischargers would be required to treat their effluents at levels of maximum possible efficiency. There have been several articles citing the importance of equity considerations in the WLA problem (Gross, 1965; Loucks et al., 1967; Miller and Gill, 1976).

From a decision making viewpoint, the objective of the WLA problem is to obtain an optimum solution from a model formulation which has incorporated as many factors as possible concerning actual system behavior. By doing so, the execution of such a model will result in an optimum solution attaining the highest degree of consciousness. Hence, any attempts by a legislative body to mandate the compliance of a WLA policy where large equitable differences existed between the various dischargers would unquestionably be tried in both social and legal arenas. The implementation or regulatory enforcement of an optimum policy derived from the solution of any WLA model, in which equity is not considered, is neither acceptable nor justifiable.

Recognizing the importance of such consideration in the WLA process, the choice must then be made as to the type of equity to be selected. Based on the conclusions drawn by Chadderton et al. (1981), two types of equity were considered in this study: (1) equal percent removal and (2) equal effluent concentrations. In mathematical form, constraints for equity can be generally expressed as

$$\left| E_j - E_{j'} \right| \leq E_A, \text{ for } j \neq j' \tag{2.11}$$

where E_j represents the equity considered for discharge j, E_A is the allowable difference in equity between the two dischargers j and j'. In order to incorporate these constraints into an LP model, they must be expressed as linear functions of the decision variables (i.e., effluent waste concentration at each discharge location, L_j). In doing so, the constraints for equity when considering equal percent removal between the dischargers can be written as

$$\left| \frac{L_j}{I_j} - \frac{L_{j'}}{I_{j'}} \right| \leq E_A, \text{ for } j \neq j' \tag{2.12}$$

and when considering equal effluent concentrations

$$\left| L_j - L_{j'} \right| \leq E_A, \text{ for } j \neq j' \tag{2.13}$$

where I_j is the influent raw waste concentration (mg/l BOD) at discharge location j.

Additionally, it should be noted that for any given stream system, one or more the discharges considered may be an influent tributary. Special provisions should be included in the model formulation to exclude any tributary inflows from treatment and equity considerations. In general, the water quality of the discharge associated with a tributary is much less polluted (in terms of BOD) than the effluents from a normal industrial user of the stream environment. Therefore, the discharge from a tributary should be excluded from the consideration of equity in order to prevent the occurrence of undue restrictions being placed on the required treatment levels assigned to other dischargers. Furthermore, provision to exclude tributary flows from treatment should also be included because such a task is both practically and economically unrealistic.

2.3.3 Constraints on Treatment Efficiency

The final set of constraints to consider are those defining the acceptable range of the treatment level efficiencies. Specifically, a range between 35 and 90 percent removal of raw waste at each discharge location is considered in this study. The minimum requirement of 35 percent removal is to prevent floating solids from being discharged to the stream environment. The discharge of solids of this type is socially and environmentally objectionable. On the other hand, the upper limit of 90 percent removal represents the maximum efficiency (assumed) attainable by practical treatment technology (Loucks, et al., 1967). The constraints on treatment efficiency may be expressed as

$$0.35 \leq \frac{L_j}{I_j} \leq 0.90, \text{ for all } j=1,2,\ldots,N \qquad (2.14)$$

2.4 OPTIMAL WASTE LOAD ALLOCATION MODEL

Having comprehensively examined each of the components of the LP model, this section summarizes the formulation of the entire WLA problem utilized in this study. In LP format, the optimal WLA model can be expressed as

$$\text{Max} \sum_{j=1}^{N} (L_j + D_j) \qquad (2.2)$$

subject to

$$\sum_{j=1}^{n_i} \theta_{ij} L_j + \sum_{j=1}^{n_i} \psi_{ij} D_j \leq R_i, \quad \text{for all } i=1,2,\ldots,M \qquad (2.3)$$

$$\left| E_j - E_{j'} \right| \leq E_A, \quad \text{for all } j \neq j' \quad j'=2,\ldots,N \qquad (2.11)$$

$$0.35 \leq \frac{L_j}{I_j} \leq 0.90, \quad \text{for all } j=1,2,\ldots,N \qquad (2.14)$$

and non-negativity constraints

$$L_j \geq 0, \; D_j \geq 0, \quad \text{for all } j=1,2,\ldots,N$$

where M is the total number of dischargers and control point locations.

2.5 MODEL SOLUTION TECHNIQUES

2.5.1 Fixed Control Point Approach

The use of a fixed control point (FCP) approach to control water quality in the WLA problem is based on the preselection of several control points within each reach. Then, by utilizing the Streeter-Phelps equation, the water quality requirement at each control point location is transformed into a constraint in the model formulation. In order to ensure minimum DO concentrations for the entire system, the number of control points and, hence, the number of constraints can become quite large, even for a moderately sized WLA problem considering six to ten dischargers and ten control points per reach. Such a problem would require 60 to 100 constraints on water quality alone. Furthermore, there is no guarantee that any of the control points selected will, in fact, ensure the compliance of water quality standards at the worst or "critical" location within each reach. Because prior knowledge of such a location is not known, the selection of control points is essentially a random process. Thus, in an attempt to minimize the chance of violating the water quality standards, the most common approach is to select a sufficiently large number of control points (generally with equal spacing) throughout the entire stream system.

It is obvious that this type of an approach is both computationally and reliably inefficient. As noted earlier, a trade-off exists between the number of constraints required to ensure water quality and the computational effort required to solve the WLA problem.

Theoretically, an infinite number of control points would be required by the FCP approach to ensure ubiquitous compliance of the water quality standards throughout the river system. Obviously, the inclusion of an infinite number of control points in the model is impossible and impractical. Therefore, the possibility of water quality violations will always exist when a WLA model is formulated utilizing the FCP approach.

2.5.2 Moving Control Point Approach

To by-pass the shortcomings of the FCP approach, a simple property of the general DO profile in each reach can be utilized. Specifically, the typical DO profile is convex with a single point defining the critical location within each reach (refer to Figure 1.2). This implies that only one or few constraints in the FCP model formulation are active. The remaining constraints are simply redundant. Unlike the FCP approach utilizing a large number of control points and solving the WLA model once, a refined approach based on an iterative procedure can be implemented in such a way that only one control point per reach is required during each iteration of the WLA model. This new methodology will herein be referred to as the moving control point (MCP) approach.

The essence of this approach is to define a single moving control point using the critical location within each reach of the river system. Then, by utilizing the Streeter-Phelps equation, a constraint on the water quality is provided for each reach in the WLA problem formulation. The problem is solved iteratively until the critical locations and

optimum solution set between successive iterations converge. The WLA problem is then solved iteratively, each time updating the position of the single control point within each reach. The procedures are terminated when convergence criteria for the critical locations and optimum solutions between two successive iterations are met. The procedures of this new approach are discussed in greater detail in the paragraphs below.

During the first iteration of the MCP model formulation, a single control point in each reach is arbitrarily selected. The WLA model is then solved using water quality constraints defined for each control point and discharge location. The solutions of the current iteration are stored, and the critical locations within each reach are computed using Eq. (1.7) according to the current "optimal" solutions.

Once this information is obtained, the control point within each reach is updated to the current critical location, unless the computed X_c is beyond the geographical bounds defining the beginning and ending points of the respective reach. If so, the control points are simply moved to the reach boundary nearest the computed X_c. A schematic diagram of the MCP approach is shown in Figure 2.1.

2.6 APPLICATION OF MODELS

To illustrate the use of both the FCP and MCP approaches for solving a WLA problem, data describing the physical characteristics of an actual stream system containing six reaches were selected from a previous study conducted by Chadderton et al. (1981). A schematic diagram of the example system is shown in Figure 2.2. Note that

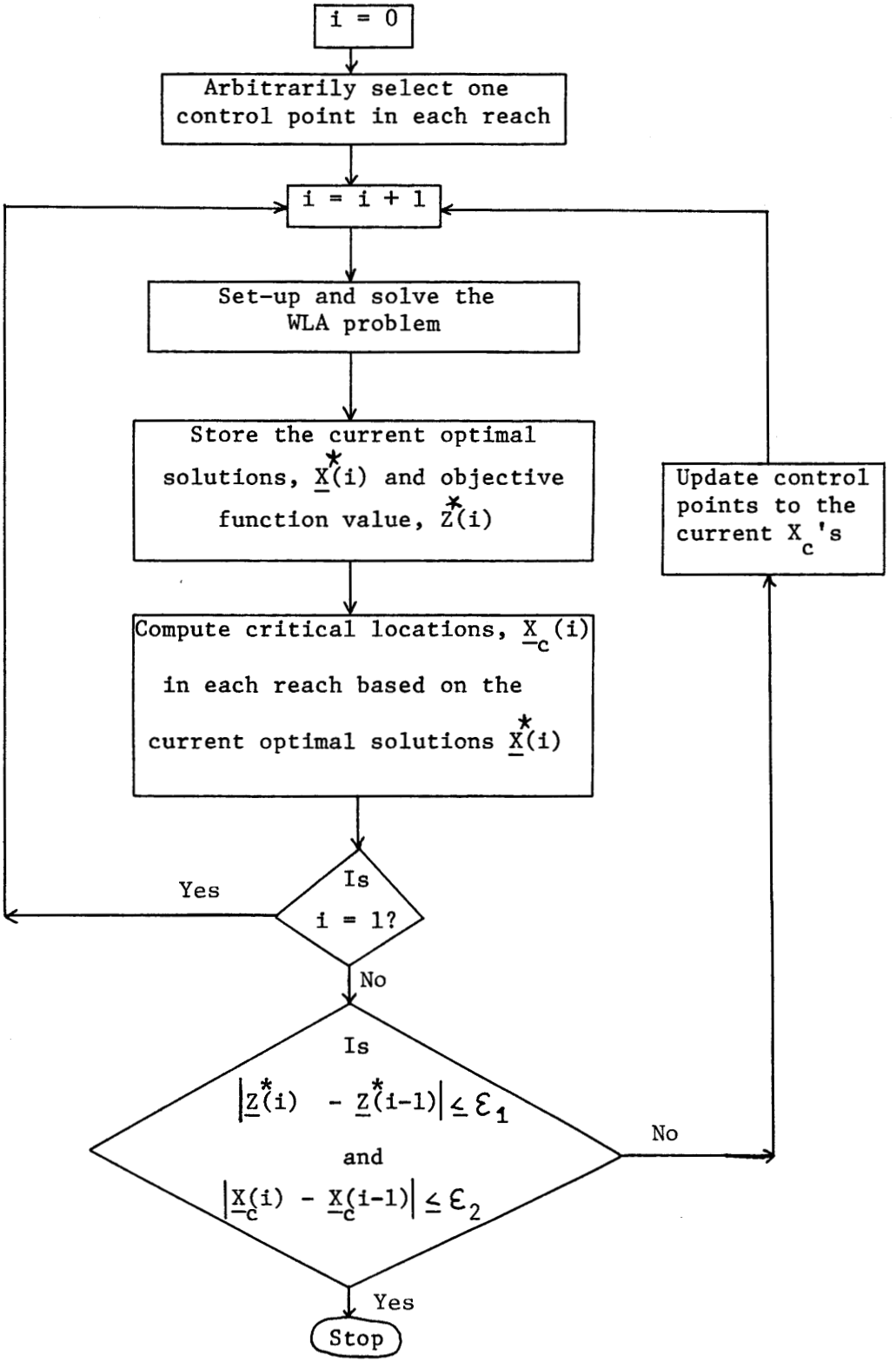

Figure 2.1 Flow Diagram of Moving Control Point Approach

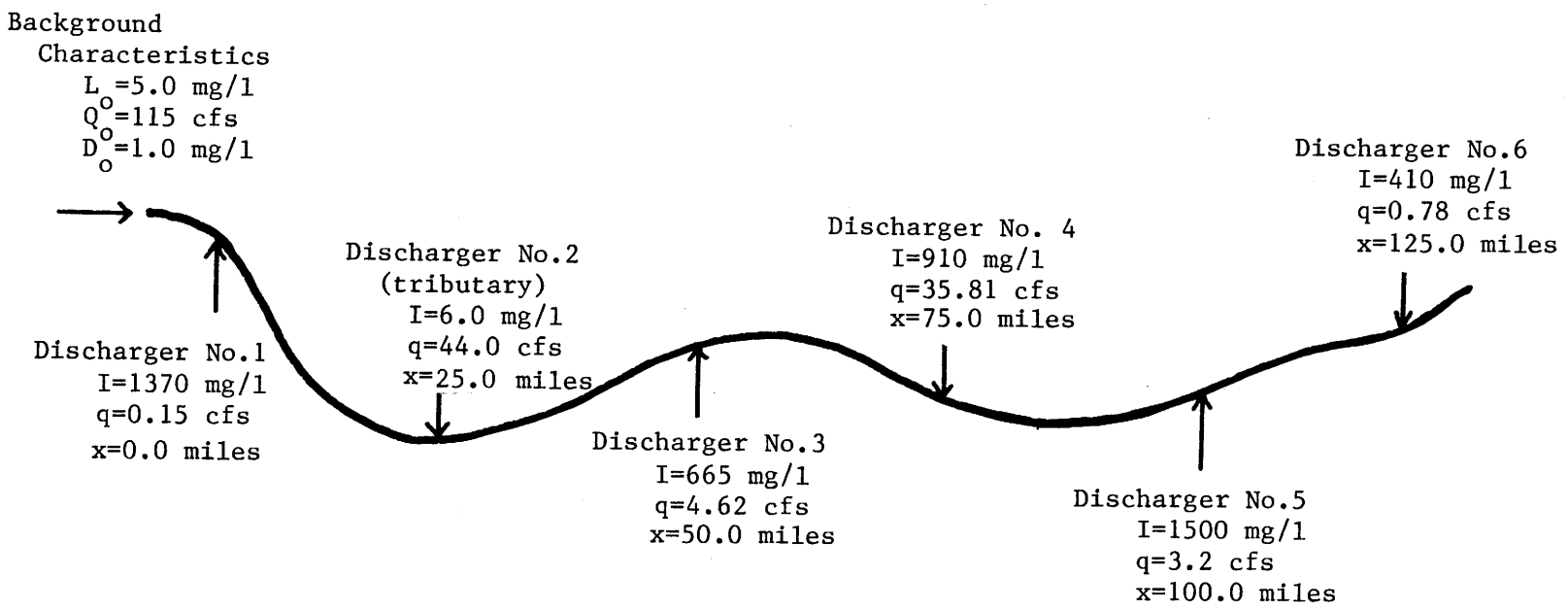

Figure 2.2 Schematic Sketch Of The Example System In WLA Problem

discharger 2 is a tributary for which no treatment will be provided. It should be pointed out that slight modifications were made on the original data set in order to enhance the results obtained in the solution of the LP problem presented in this study. A complete tabulation of the data used in this study is listed in Tables 2.1(a) and 2.1(b).

To execute the LP models developed in this study, computing services were obtained from a Control Data Corporation Cyber 760 digital computer and a mathematical programming solution package called XMP. XMP is a master library of computer subroutines containing algorithms for solving a variety of LP problems utilizing the simplex solution methodology. When coupled with a driving program, XMP becomes an effective and efficient technique for solving various types of mathematical programming problems (Marsten, 1981). In addition to generating the solution to the LP problem, XMP also provides information describing the relative computer storage required in the solution of each problem.

To compare the sensitivity of the solution of the WLA problem to variations in the number of control points selected in each reach, the model is solved by the FCP approach using a total of 1,3,5, and 7 equally spaced control points per reach. The MCP approach is also applied to provide a basis for comparison of the solutions and storage requirements between the two types of approaches. The execution of each of the models is performed using the equity consideration of equal percent removal, where the maximum allowable equity difference between each discharger was set at five percent. Furthermore, investigations of the sensitivity of the model results to changes in the measure of equity

TABLE 2.1 DATA OF PHYSICAL STREAM CHARACTERISTICS USED IN THE EXAMPLE OF WLA MODELS

(a) Stream Characteristics for Each Reach

Reach i	Deoxygenation Coefficient (K_i^d)	Reaeation Coefficient (K_i^a)	Average Stream Velocity (U_i)	Raw Waste Concentration (I_i)	Effluent Flow Rate (q_i)
1	0.6	1.84	16.4	1370	0.15
2	0.6	2.13	16.4	6.0	44.0
3	0.6	1.98	16.4	665	4.62
4	0.6	1.64	16.4	910	35.81
5	0.6	1.64	16.4	1500	3.2
6	0.6	1.48	16.4	410	0.78
UNITS	1/days	1/days	miles/day	mg/1 BOD	ft^3/sec

(b) Background Characteristics

Upstream Waste Concentration L_o	Upstream Flow Rate Q_o	Upstream DO Deficit D_o
5.0	115.0	1.0
mg/1 BOD	ft^3/sec	mg/1

were also made. Each of the procedures were reexecuted using a second type of equity, equal effluent concentrations, where the allowable equity difference between each of the discharger was set at 20 mg/1 BOD.

2.7 DISCUSSION OF MODEL PERFORMANCE

Tables 2.2 and 2.3 show the execution time, computer storage requirements, and solutions obtained by the FCP approach to the hypothetical WLA problem using the equity considerations of equal percent removal and equal effluent concentrations, respectively. The results from these two tables show that the effluent waste concentrations at each discharge location and the total waste discharged to the system decrease as the number of control points in each reach increases. By increasing the number of control points per reach, greater restrictions and controls are placed on the system to ensure that the 4 mg/1 minimum DO requirement is not violated at any location, thus, the allowable waste discharge for each user and the entire system is reduced. Furthermore, these two tables reveal that the difference in the model solutions obtained by using one or three control points is negligible and that changes in the amount of allowable waste discharge do not occur until at least five control points per reach are specified. This can be explained by the fact that, given any significant length of reach or stream system, there exist a lower limit on the number of control points which effectively control water quality within each reach. By selecting the number of control points per reach below this lower limit, checks on water quality, provided by the constraints, are simply "too few and far between."

TABLE 2.2 RESULTS FROM THE EXECUTION OF EXAMPLE WLA PROBLEM FORMULATED BY THE FCP APPROACH WITH EQUITY CONSIDERATION OF EQUAL PERCENT REMOVAL

No. of Control Points Per Reach	Computer Execution Time (Seconds)	Computer Storage Requirements (Words)	Effluent Waste Concentrations (mg/1 BOD) Discharger #1	#2	#3	#4	#5	#6	Total Waste Discharge (mg/1 BOD)
1	1.52	1882	301.2	6.0	145.5	153.5	328.1	89.7	1024.0
3	2.01	3694	301.2	6.0	145.5	153.5	328.1	89.7	1024.0
5	2.97	6082	298.2	6.0	144.0	151.6	324.8	88.8	1013.4
7	3.40	9064	297.5	6.0	143.7	151.1	324.0	88.6	1010.9

TABLE 2.3 RESULTS FROM THE EXECUTION OF EXAMPLE WLA PROBLEM FORMULATED BY THE FCP APPROACH WITH EQUITY CONSIDERATION OF EQUAL EFFLUENT CONCENTRATION

No. of Control Points Per Reach	Computer Execution Time (Seconds)	Computer Storage Requirements (Words)	Effluent Waste Concentrations (mg/1 BOD) Discharger #1	#2	#3	#4	#5	#6	Total Waste Discharge (mg/1 BOD)
1	1.61	1882	171.5	6.0	171.5	151.5	171.5	171.5	843.5
3	2.10	3964	171.5	6.0	171.5	151.5	171.5	171.5	843.5
5	3.05	6082	169.8	6.0	169.8	149.8	169.8	169.8	835.0
7	3.41	9046	169.3	6.0	169.3	149.3	169.3	169.3	832.5

Additionally, Tables 2.2 and 2.3 provide an interesting comparison for the computer storage requirements utilized as the number of control points in each reach are increased. Both tables reveal rapid consumption of computer storage as the number of control points are increased. Interestingly, when executing the WLA model formulated by the FCP approach on the CDC Cyber 760, the maximum number of control points per reach that could be specified for this six reach example was seven. Any attempts to increase the number of control points per reach to nine or ten resulted in computer storage requirements exceeding the maximum available limit on the CDC Cyber 760 computing system. From this, it is obvious that severe computer storage requirements can be imposed on a computing system when implementing the FCP approach with only a moderate number of control points selected per reach.

The use of the FCP approach cannot ensure that the DO standard can be met at every point within the stream environment. To illustrate this fact, DO profiles based on each of the solutions for the WLA model formulated by the FCP approach using 1,3,5, and 7 control points per reach are plotted and shown in Figures 2.3 through 2.10. Figures 2.3-2.6 correspond to the equity consideration of equal percent removal and Figure 2.7-2.10 correspond to equal effluent concentrations. Examining these figures, it is evident that noticeable violations of the water quality standard exist in reach 4, especially for those solutions in which a smaller number of control points per reach were used. By implementing the FCP approach in attempts to solve the WLA problem, there is no guarantee that the requirements of minimum DO will be

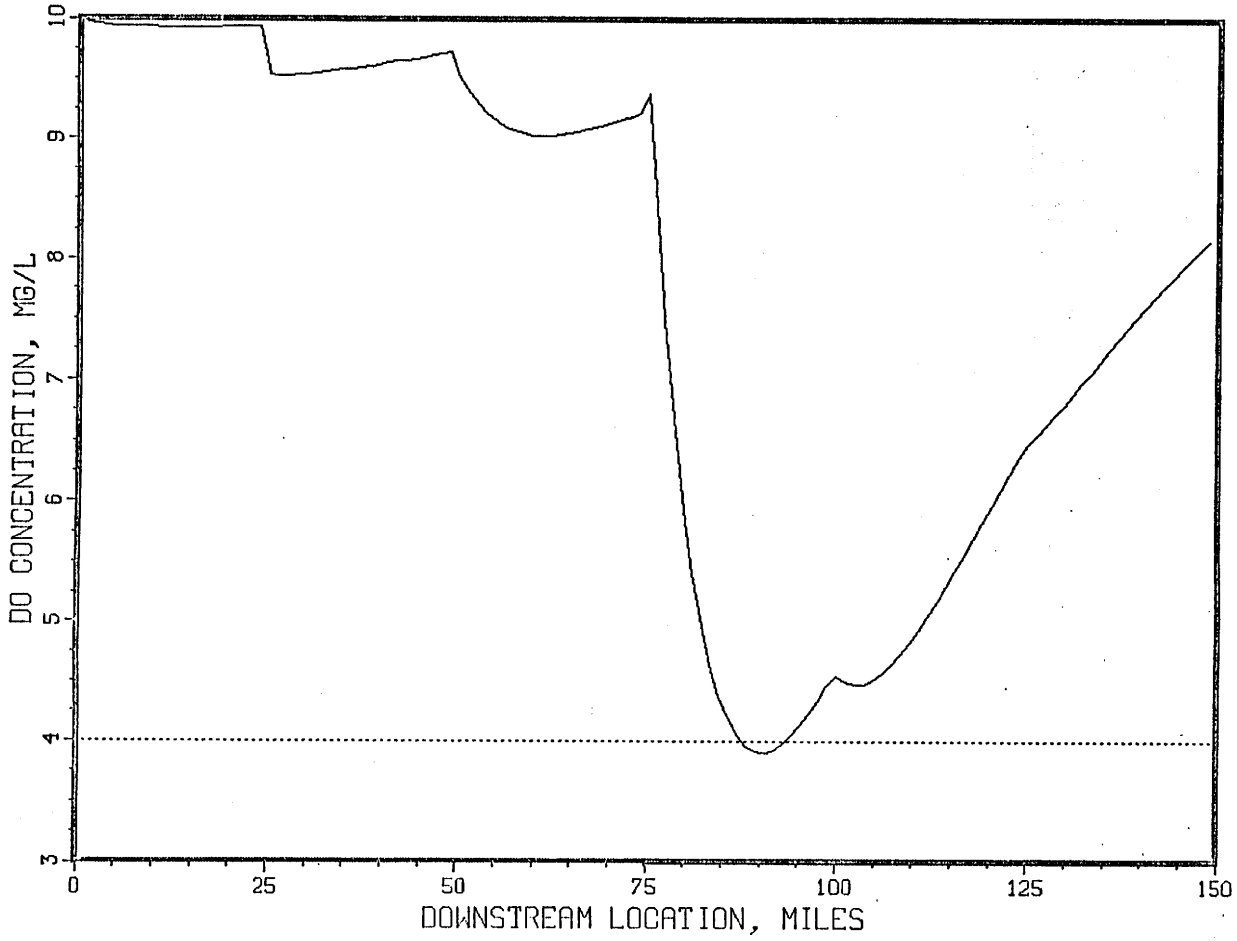

Figure 2.3 DO Profile Corresponding To FCP Approach With One Control Point Per Reach And Considering The Equity Of Equal Percent Removal

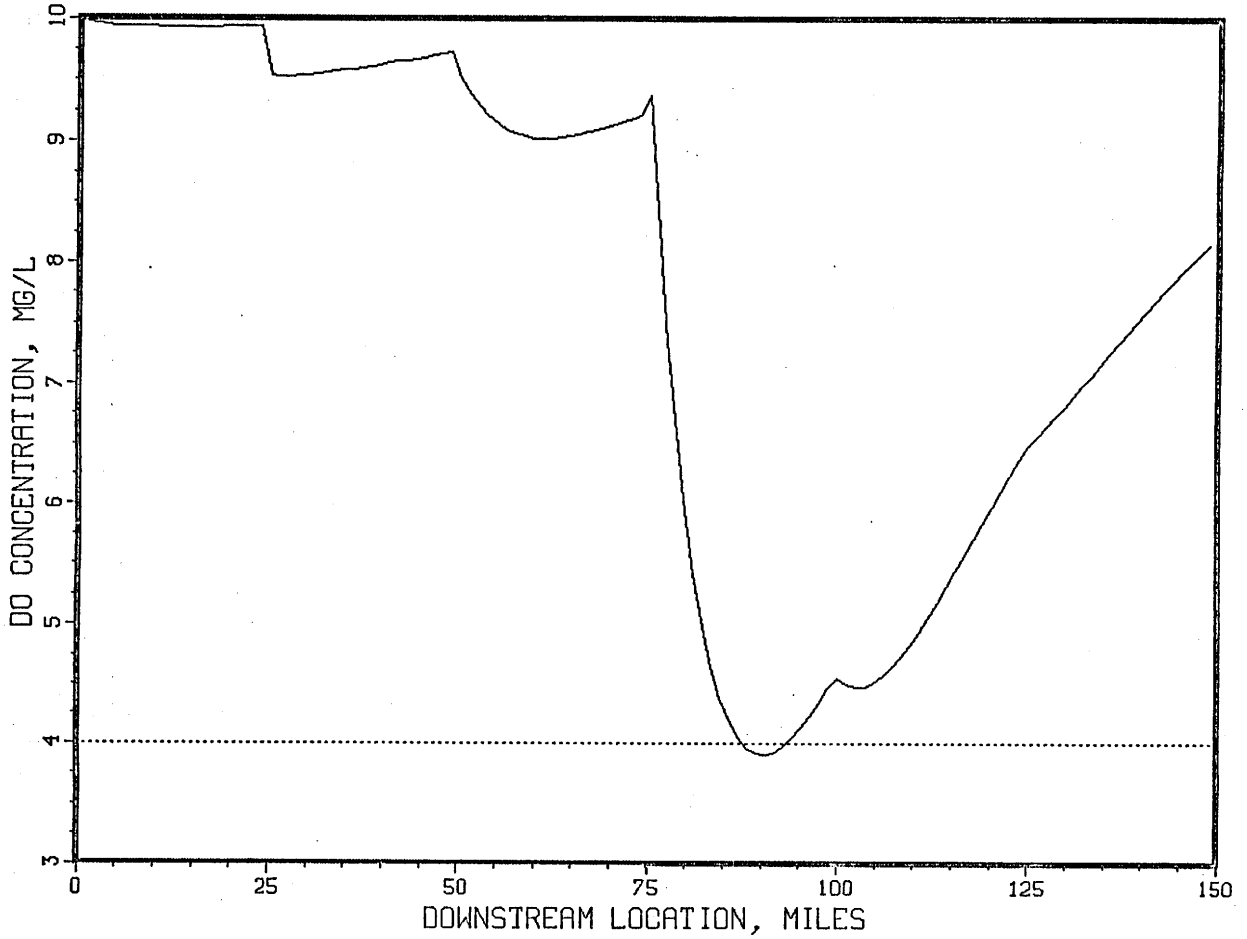

Figure 2.4 DO Profile Corresponding To FCP Approach With Three Control Points Per Reach And Considering The Equity Of Equal Percent Removal

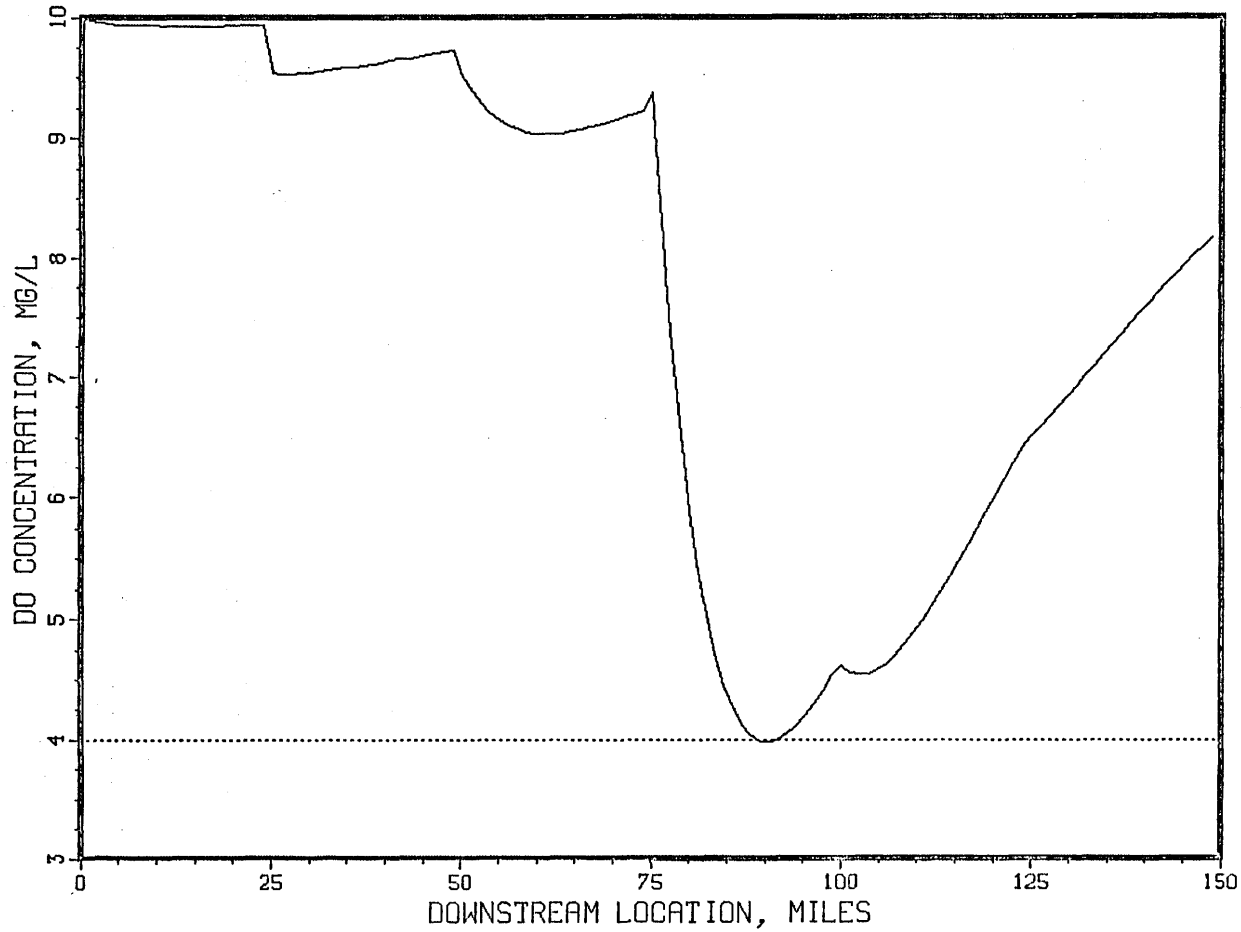

Figure 2.5 DO Profile Corresponding To FCP Approach With Five Control Ponts Per Reach And Considering The Equity Of Equal Percent Removal

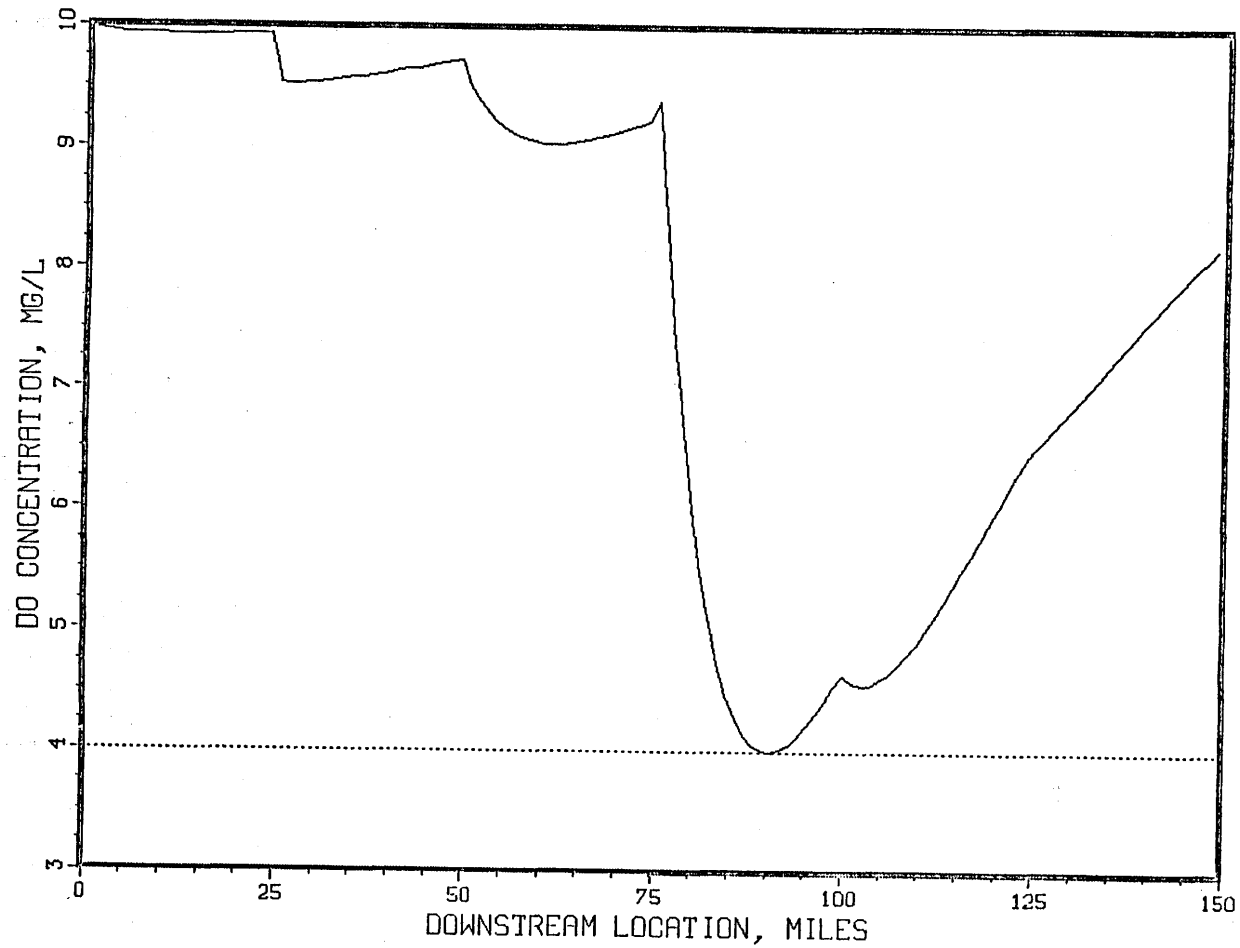

Figure 2.6 DO Profile Corresponding To FCP Approach With Seven Control Points Per Reach And Considering The Equity Of Equal Percent Removal

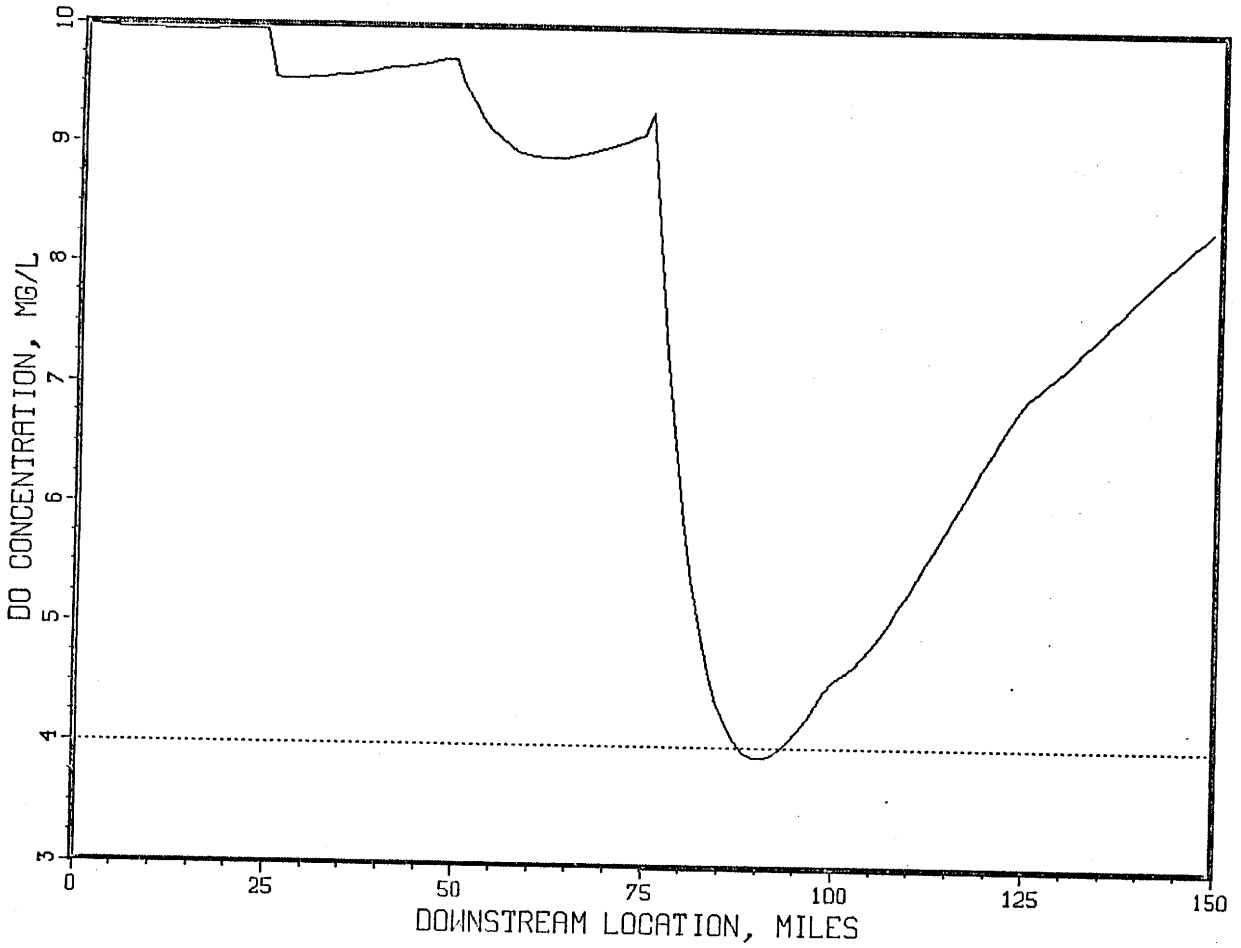

Figure 2.7 DO Profile Corresponding To FCP Approach With One Control Point Per Reach And Considering The Equity Of Equal Effluent Concentrations

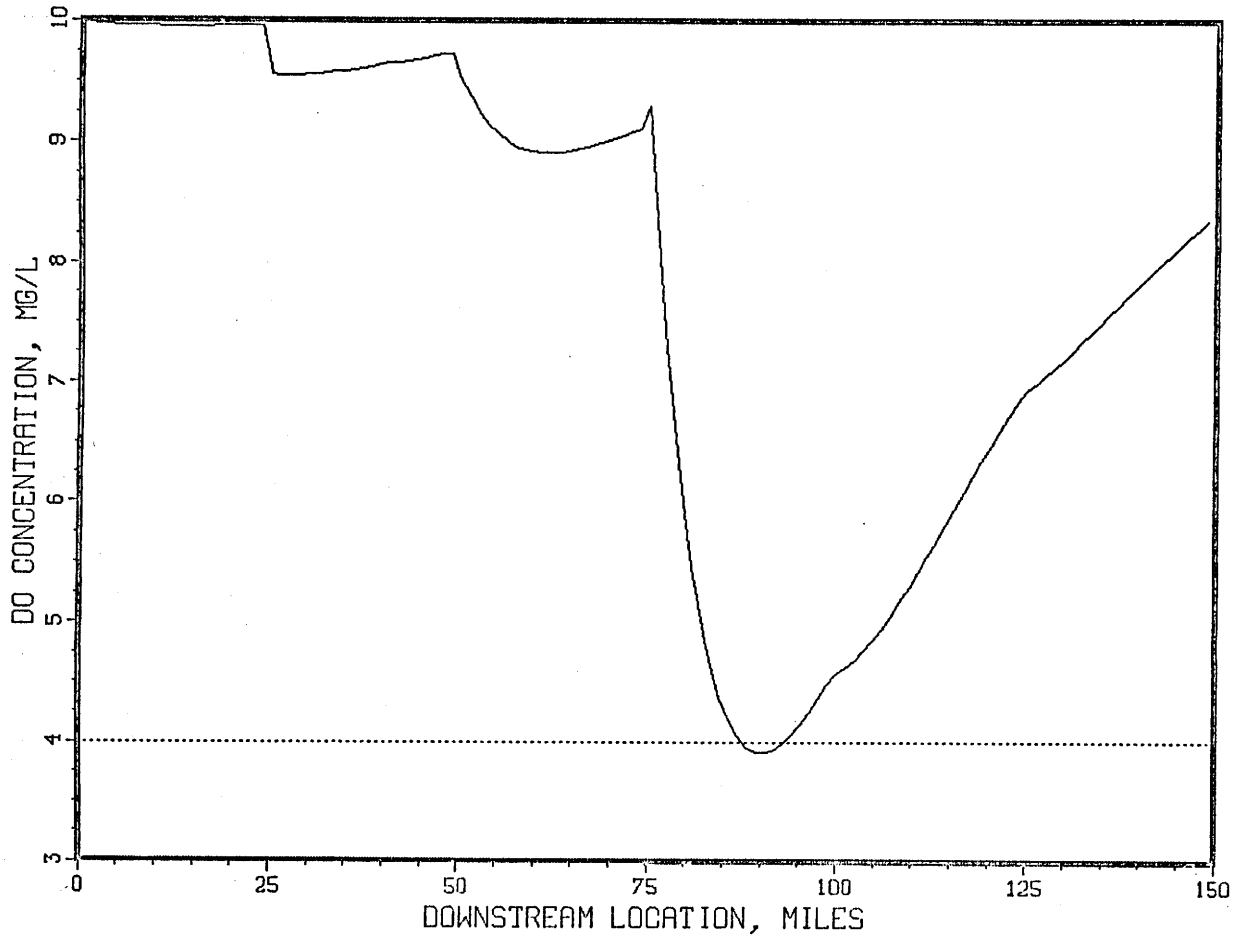

Figure 2.8 DO Profile Corresponding To FCP Approach With Three Control Points Per Reach And Considering The Equity Of Equal Effluent Concentrations

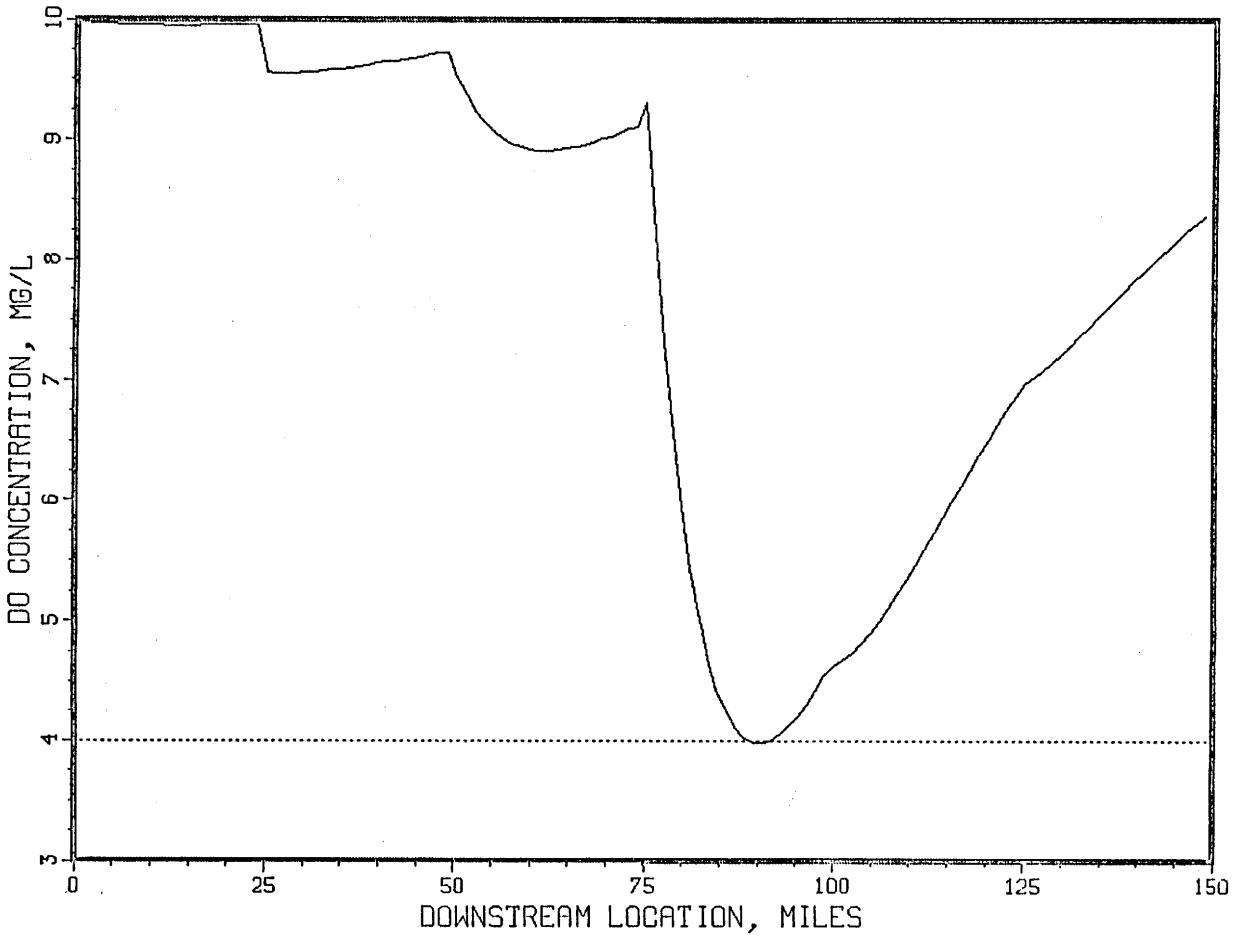

Figure 2.9 DO Profile Corresponding To FCP Approach With Five Control Points Per Reach And Considering The Equity Of Equal Effluent Concentrations

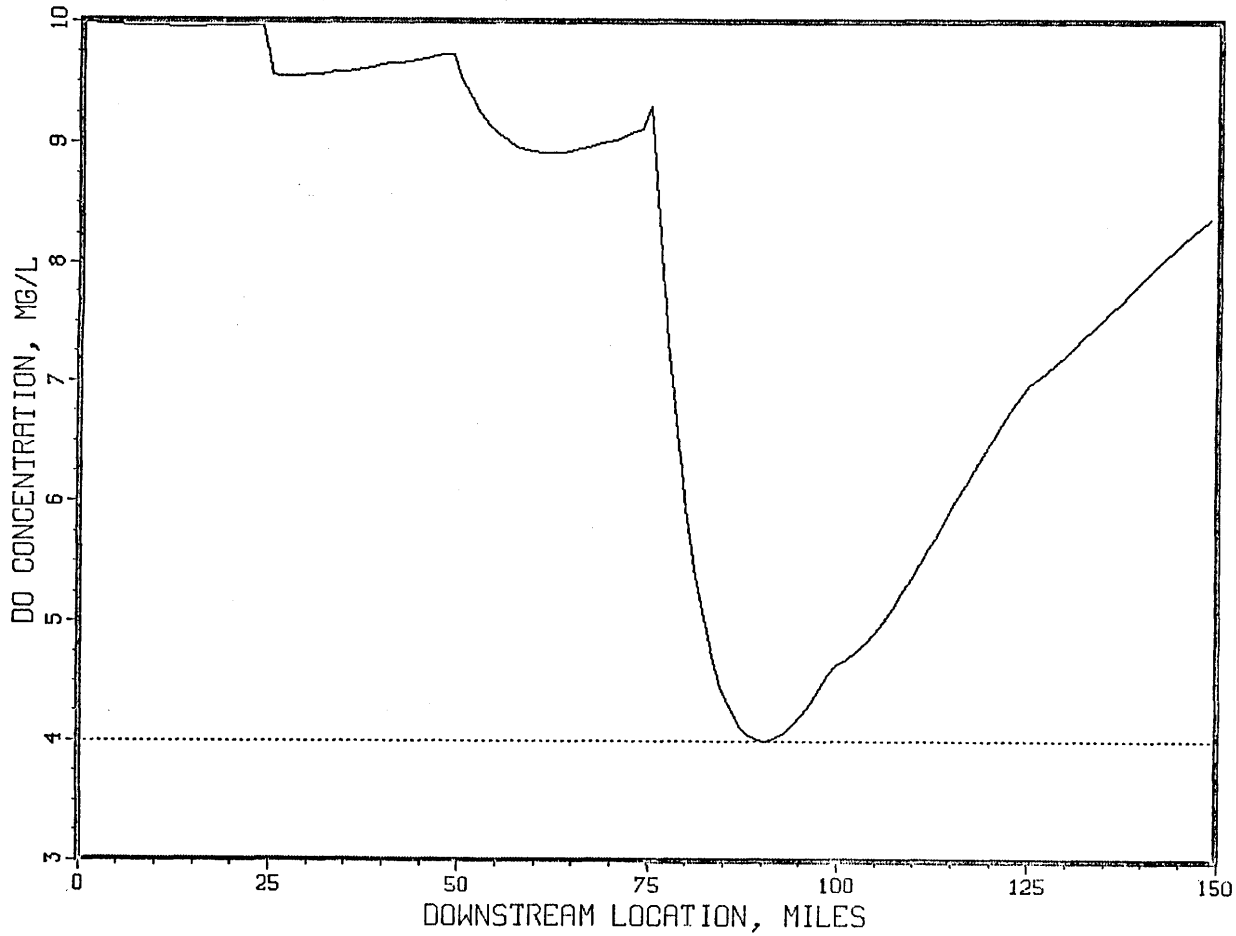

Figure 2.10 DO Profile Corresponding To FCP Approach With Seven Control Points Per Reach And Considering The Equity Of Equal Effluent Concentrations

satisfied at every location in the stream environment. Remember that water quality checks are only provided at each control point and discharge location when using the FCP approach.

Recalling the generalized DO profile (see Figure 1.2), there exists a unique point, the critical location, where the DO concentration is at a minimum. However, to compute the critical locations in each reach, the solution to the WLA must be obtained first. Information pertaining to the location of the critical points cannot be obtained prior to solving the WLA model. Therefore, there is no effective means, unless by chance, to select a control point in each reach prior to model execution that will coincide with or be near the critical point location in the river system. This leads to the possibility of water quality violations within the river system at one or several locations as can be seen in Figures 2.3, 2.4, 2.7, and 2.8. Obviously, by increasing the number of control points per reach, the possibility of such violations within the system is reduced, though the detrimental effects on computer storage requirements of such an approach have been evidenced. Thus, when utilizing the FCP approach, an appropriate balance between water quality assurance and the consumption of valuable computer storage should be kept in mind.

As an alternative to the FCP approach, the results presented in Tables 2.4 and 2.5 show that the use of the MCP technique offers the analyst of the WLA problem an opportunity to reduce computer storage requirements while ubiquitously assuring minimum levels of DO throughout the river system under investigation. By employing the MCP approach,

TABLE 2.4 RESULTS FROM THE EXECUTION OF EXAMPLE WLA PROBLEM FORMULATED BY MCP APPROACH WITH EQUITY CONSIDERATION OF EQUAL PERCENT REMOVAL

Computer Execution Time (Seconds)	Computer Storage Requirements (Words)	Effluent Waste Concentrations (mg/1 BOD) Discharger #1	#2	#3	#4	#5	#6	Total Waste Discharge (mg/1 BOD)
4.32	1882	294.3	6.0	142.8	150.0	322.2	88.1	1003.4

TABLE 2.5 RESULTS FROM THE EXECUTION OF EXAMPLE WLA PROBLEM FORMULATED BY MCP APPROACH WITH EQUITY CONSIDERATION OF EQUAL EFFLUENT CONCENTRATION

Computer Execution Time (Seconds)	Computer Storage Requirements (Words)	Effluent Waste Concentrations (mg/l BOD)						
		Discharger						Total Waste Discharge (mg/l BOD)
		#1	#2	#3	#4	#5	#6	
4.29	1882	168.2	6.0	168.2	148.2	168.2	168.2	827.0

the results show that the computer storage requirements are identical to that of the FCP approach using one control point per reach. It should be pointed out that this is only true during the first iteration of the MCP approach. Successive iterating may lead to computer storage requirements which are less than or equal to those utilized in the first iteration. To explain this, recollection must be made to an earlier discussion in this chapter indicating that the foundation of the control point model is an iterative procedure in which a single control point per reach is used during the first iteration. Then, control points are updated to coincide with the critical locations computed within each reach during the succeeding iterations. It was also pointed out that if the computation of the critical points in each reach led to locations that were beyond either the upper or lower geographical bounds of their respective reaches, then the control point within that reach was removed and water quality checks were provided only at the discharge locations for that reach. Thus, during the first iteration of the MCP approach, the number of constraints and computer storage requirements would be identical to that of the FCP approach using one control point per reach. Then, in succeeding iterations, it would be possible to remove some of the control points which were not needed, thus reducing the number of constraints and total computer storage requirements. From this, it is evident that the computer storage requirements for the MCP approach will only be as large as that required by the FCP approach using one control point per reach during the first iteration and possibly less during succeeding iterations.

Because the MCP approach is an iterative procedure relying on a convergence criteria, there might be a concern that the number of iterations to provide an acceptable convergence would be quite large or that the model, as formulated, might never converge. On the contrary, once the WLA model formulated by the MCP approach was executed, it took an average only three to four iterations to converge.

The results in Tables 2.4 through 2.5 show nominal increases in computer execution time when implementing the MCP procedures as compared to the execution times obtained using the FCP approach. However, it is important to note that implementing the MCP methodology can save large quantities of computer storage while increasing execution time only slightly when compared to the FCP approach.

Finally, the DO profiles based on the solutions for each execution of the MCP model considering the different types of equity are shown in Figures 2.11 and 2.12, where Figure 2.11 and 2.12 are based on the equity considerations of equal percent removal and equal effluent concentrations, respectively. In examining these figures, it is evident that no water quality violations are found at any point in the river system although an identical set of data was used in the execution of the WLA model formulated by the FCP approach where several violations occurred. This can be explained by the fact that the water quality requirements of DO in the MCP formulation are only checked at discharge locations and critical points. These points possess the greatest susceptibility to violation and are checked accordingly. All other points in the river system contain DO concentrations which are higher

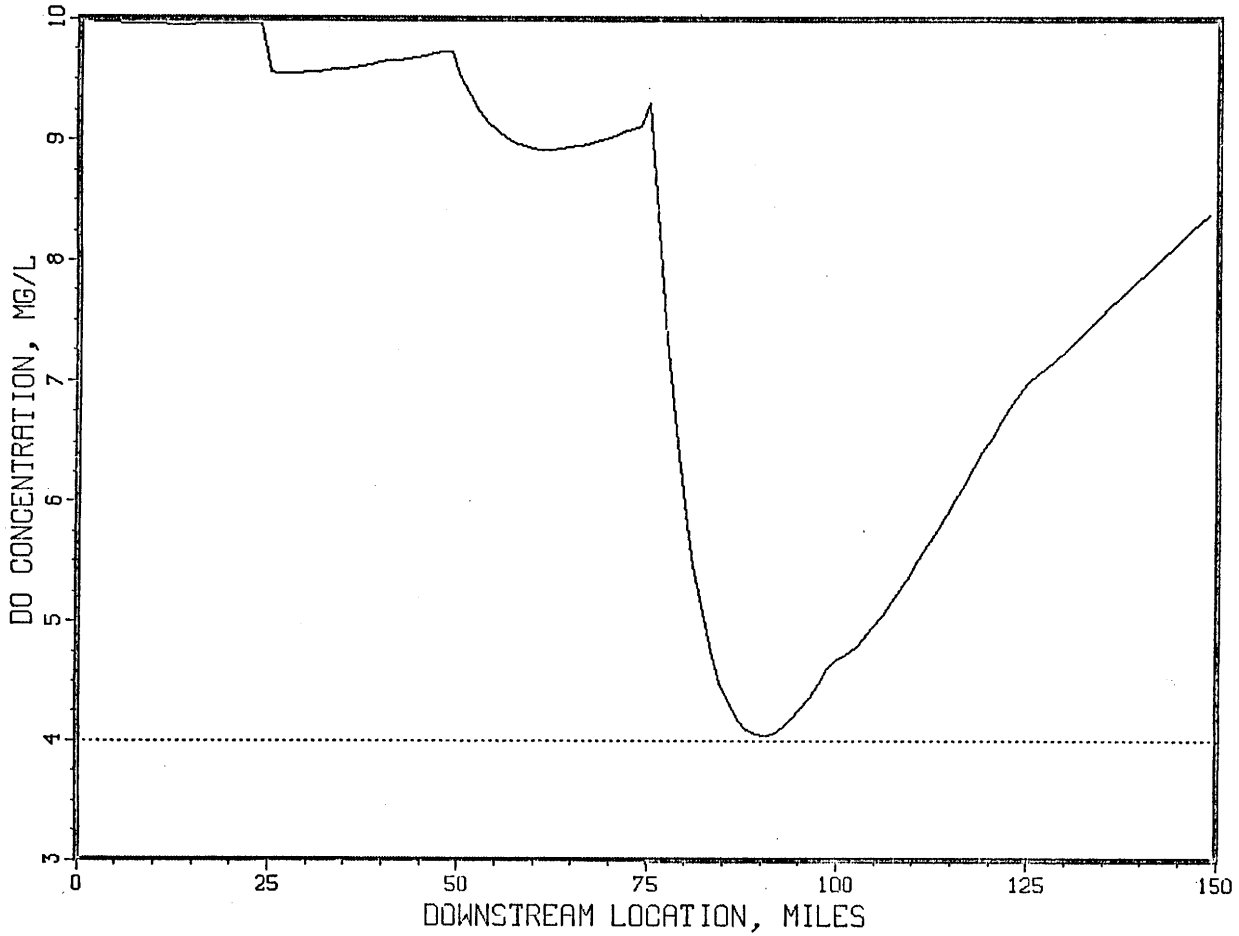

Figure 2.11 DO Profile Corresponding To MCP Approach And The Equity Of Equal Percent Removal

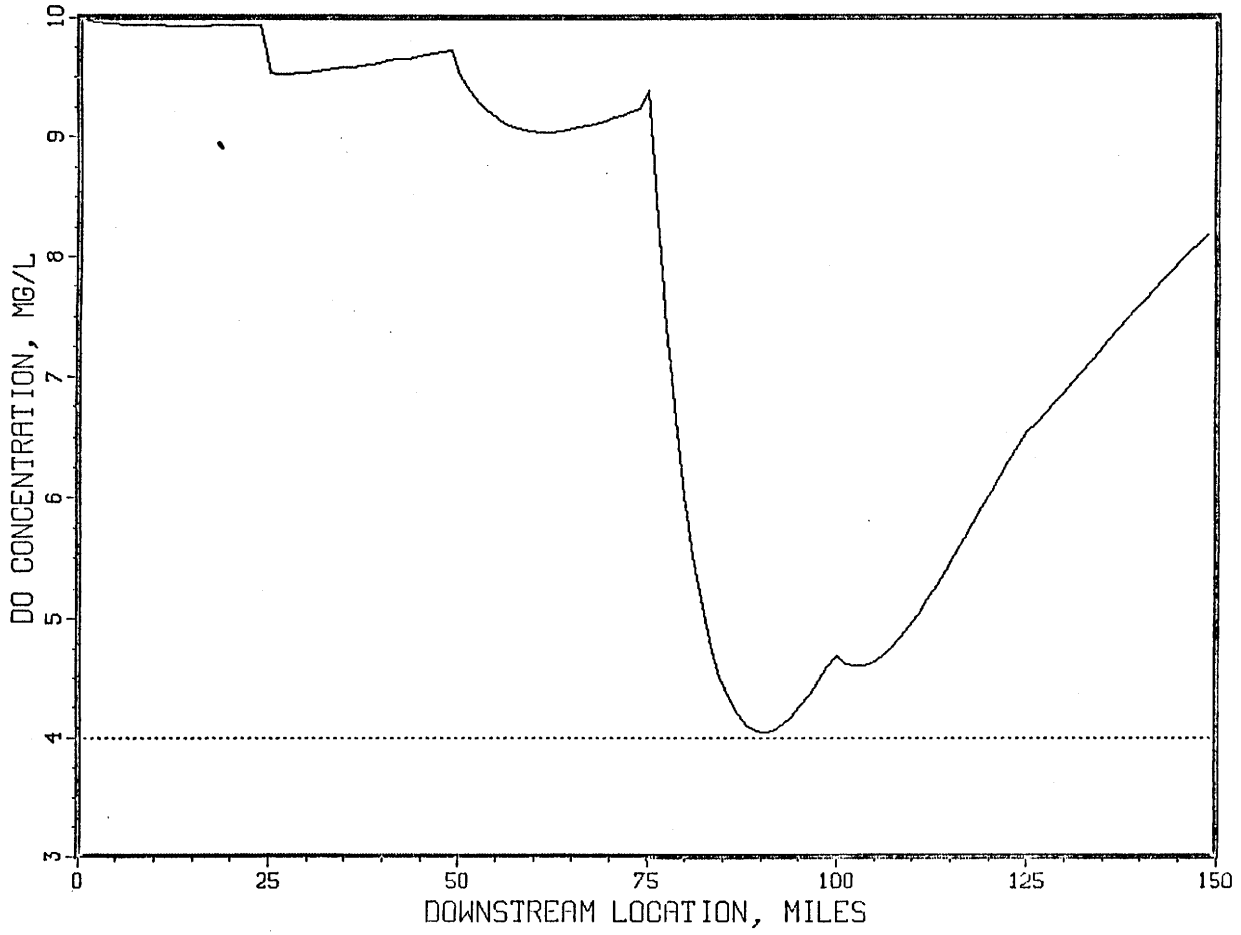

Figure 2.12 DO Profile Corresponding To MCP Approach And The Equity Of Equal Effluent Concentrations

than those at the critical points and discharge locations. Thus, by ensuring water quality at these points, the possibility of water quality violations within the river system is nullified.

The solutions obtained for waste discharge in the WLA problem using the MCP approach can be considered "exact" in comparison to the results obtained for the FCP formulation. By this, it is meant that as the number of control points per reach in the FCP model gets very large, the solution obtained for optimal waste discharge using the FCP approach will, in fact, become identical to the solution obtained from the implementation of the MCP procedure. Thus, it is obvious that the advantages in using the MCP approach, especially those of savings in computer storage and water quality assurance, make it a very attractive methodology for solving the WLA problem.

2.8 SUMMARY AND CONCLUSIONS

This chapter has presented two approaches to solve the optimal WLA problem. To date the most widely used methodology to ensure water quality requirements of minimum DO within a stream environment was the inclusion of water quality constraints in the model formulation which were derived by selecting several fixed control points within each reach of the river system. The FCP approach has been shown by this study to be inefficient in both computer storage consumption and in assuring that there are no water quality violations at any point in the stream environment.

In an attempt to circumvent the inherent inadequacies of the FCP formulation, a new technique utilizing the concept of moving control

points within each reach has been unveiled. This new approach (MCP) possesses the advantages of considerable savings in computer storage requirements and solutions, when generated deterministically, that contain no possibility of violating the water quality standards at any point in the river system. The efficiency and model predictability of the MCP procedure has proven that the implementation of such an approach is superior in comparison to that of the FCP approach so widely used in the past. In the author's opinion, the advantage of computer storage savings alone will make the MCP approach a very attractive alternative to solving the optimal WLA problem. This is particularly true when the WLA model is to be solved by microcomputers.

CHAPTER 3

DETERMINISTIC OPTIMAL WASTE LOAD ALLOCATION MODELS: A MULTIOBJECTIVE FRAMEWORK

3.1 INTRODUCTION

The solutions to a growing number of environmental problems facing water quality professionals today are becoming more complex. The necessity for improved environmental protection has not precluded the problem of optimal waste load allocation from increasing governmental and societal demands on water quality assurance. As society progresses with time, the demands placed on water quality requirements will continue to grow, resulting in the continued need for improved water quality prediction and protection techniques. Consequently, as demands grow, the solution to such problems will become ever increasingly complex.

Past research attempts to solve the optimal waste load allocation (WLA) problem have been centered around a single goal or objective to be attained in the problem formulation, i.e., the minimization of treatment cost or the maximization of waste discharge. From a decision-making viewpoint, an optimum solution to such a problem can only be obtained by including the entirety of possible physical, legal, and economic considerations in the problem formulation. In reality, most environmental problems, including optimal WLA, are multiobjective by nature. It is unlikely that the optimum solution to such problems are obtained by considering a single objective in the decision process. As in most environmental problems, the decision-making process is cultivated by the

desire to achieve several goals simultaneously. The problem of optimal WLA is without exception to these aspirations. The identification of a single objective to obtain a true optimum solution to the WLA problem, as so widely used in the past, is obviously unrealistic.

The importance of considering a multiobjective approach in the area of water resources has been cited in a number of previous works (Monarchi et al., 1973; Cohon and Marks, 1973; Taylor et al., 1975). By incorporating multiobjective procedures in the decision-making process, three major improvements are accomplished: (1) the role of the analyst and decision-maker are more clearly defined, (2) the results from the multiobjective approach provide a greater number of alternatives to the decision-making process, and (3) models utilizing such techniques are generally more realistic.

The traditional use of a single-objective function requires the model characteristics to be defined by a single measure of effectiveness. Moreover, it is solely the analyst's responsibility to define the individual impacts characterizing model performance. Because a unique optimum solution is obtained in the traditional single-objective problem setting, the majority of the decision-making process is left in the hands of the analyst.

On the other hand, the utilization of multiobjective procedures limits the role of the analyst to that of identifying the trade-offs between model objectives. A systematic evaluation of each of the objectives results in a greater number of feasible alternatives being defined. The selection of a "best" optimal solution is then incumbent

on the additional knowledge of preference provided by the decision-maker. Thus, in the multiobjective formulation, the ultimate responsibility of providing an optimal solution is given back to the decision-maker, where it belongs.

The most prominent support for the use of multiobjective analysis is that most environmental water quality problems are multiobjective by nature. In general, the traditional approach of utilizing a single-objective to identify the goal of water quality management problems is too restrictive and unrealistic. The use of multiobjective procedures possess the distinct advantage of allowing a variety of problems to be solved, while simultaneously considering several noncommensurable objectives (Cohon, 1978).

It is the intent of this chapter to present a methodology for formulating and solving the optimal WLA problem utilizing a multiobjective framework. Given the rising demands placed on water quality assurance by government and society, the utilization of multiobjective procedures can only lead to improved water quality prediction and control.

3.2 GENERAL FRAMEWORK OF THE MULTIOBJECTIVE OPTIMIZATION MODEL

3.2.1 Vector Optimization Model

In comparison to the traditional single-objective approach, the difference between the two approaches is that the multiobjective formulation consists of more than one scalar objective function. Once the utilization of a multiobjective approach has been accepted, the

problem becomes one of "vector optimization". In general, such a problem can be expressed as follows:

$$\text{Max } \underline{Z}(\underline{X}) = [Z_1(\underline{X}),\ Z_2(\underline{X}),\ldots,\ Z_K(\underline{X})] \tag{3.1}$$

subject to

$$\underline{\underline{A}}\ \underline{X} \leq \underline{b} \tag{3.2}$$

$$\underline{X} \leq \underline{0} \tag{3.3}$$

Where $\underline{Z}(\underline{X})$ is a K-dimensional vector of the objective functions, $\underline{X}$ is an n-dimensional vector containing the decision variables, $\underline{\underline{A}}$ is an m x n matrix containing the technological coefficients, and $\underline{b}$ is an m-dimensional vector specifying the resource limitations in the problem formulation.

It should also be noted that the statement defining the multiobjective problem in terms of "vector optimization" is somewhat misleading. In reality, a vector of objectives can only be optimized subsequent to the characterization of the preference between the objectives by the decision-maker. The vector optimization model is simply a convenient approach to mathematically formulate the multiobjective problem (Loucks et al., 1981).

3.2.2 Noninferior Solution Set

Recall that the "optimality" is the goal of the single-objective model formulation. In mathematical terms, the vector $\underline{X}^* \in \Omega$ is optimal, when maximizing $Z(\underline{X})$, if

$$Z(\underline{X}^*) \geq Z(\underline{X}) \quad \text{for all } \underline{X} \in \Omega \tag{3.4}$$

where Ω is a set containing all the feasible solutions to the problem (i.e., those solutions which simultaneously satisfy all model constraints).

In contrast, the ideological theme of "optimality" is no longer appropriate in the context of the multiobjective framework. Note, that within such a framework, there are normally several objectives which are noncommensurate and conflicting with each other. It is also important to realize that without prior knowledge of preference between the objectives (supplied by the decision-maker), the mathematical programming solution to the multiobjective problem results in a set of points defining the tradeoff between each objective. Here, the goal of "optimality" (in the single-objective framework) is replaced by the concept of "noninferiority" in the multiobjective analysis. The notion of "noninferiority" can be expressed by

$$Z_k(\underline{X}^o) > Z_k(\underline{X}) \text{ for all } k = 1,2,\ldots,K \tag{3.5}$$

where $\underline{X}^o$ is a noninferior solution such that $\underline{X}^o \in \Omega$ and $\underline{X} \in \Omega$, for all $\underline{X}$. (This relationship must be treated as a strict inequality.)

In order to illustrate the idea of a "noninferior" solution set, a plot of the trade-off between objectives in a two-dimensional problem is presented in Figure 3.1. Cohon (1978) defined the noninferiority in the following passage: "A feasibility solution to a multiobjective programming problem is noninferior if there exists no other feasible

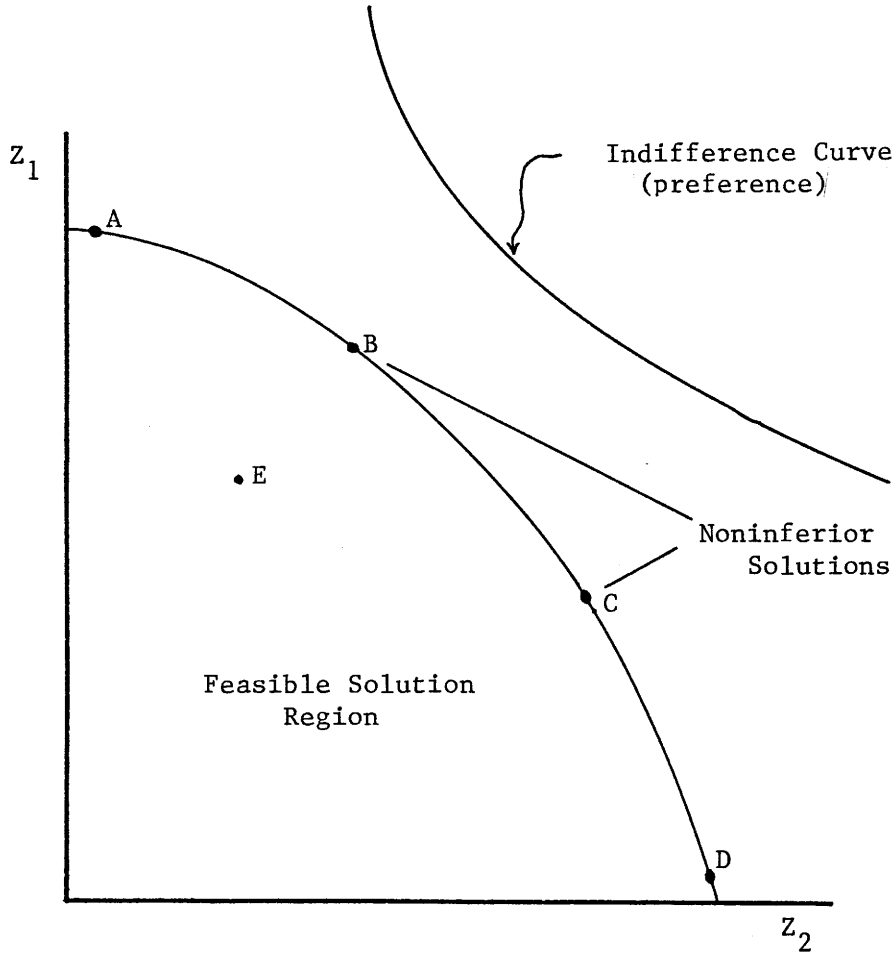

Figure 3.1 Illustration Of The Tradeoffs Between Objectives In A Two-Dimensional Problem Setting

solution that will yield an improvement in one objective without causing a degradation in at least one other objective."

Applying this definition to Figure 3.1, it is evident that all interior points, not elements on the curve ABCD, must represent inferior solutions to multiobjective problem. Hence, for every point in the interior of the curve ABCD, there exists at least one other feasible solution in which the measure of effectiveness for each objective can be improved simultaneously. For example, referring to points B and E in Figure 3.1, by moving the feasible solution at point E to the feasible solution at point B will lead to simultaneous improvements in both objectives Z_1 and Z_2. Following the definition, point E then represents an inferior solution. On the other hand, the solution at points B and C cannot be moved to any other points in the feasible region without degrading the measure of effectiveness in at least one of the objectives defining the utility of points B and C. Thus, points B and C represent "noninferior" solutions, by definition, to the multiobjective problem.

All feasible solutions which are elements of the curve ABCD are superior, in terms of utility, to any other point in the interior, therefore, the entirety of such points along this curve (ABCD) define the noninferior solution set. More importantly, when the objectives are considered simultaneously, the noninferior solution set simply represents the boundary of the feasible solution range, ultimately defining the maximum "frontier" of the solution alternatives. Each of the alternatives along this frontier are not comparable amongst themselves. For example, consider again points B and C in Figure 3.1. The utility,

measuring the effectiveness, of each alternative along the noninferior solution set, is a function of the values of each of the objectives Z_1 and Z_2. By moving from point B to point C along the frontier, a certain quantity of objective Z_1 must be sacrificed in order to trade for a certain gain in objective Z_2. This trade-off can be defined as the "marginal rate of substitution" between the objectives Z_1 and Z_2. Although the marginal rate of substitution is defined at this point, such information is not useful in comparing the utility of the individual alternatives. Once the noninferior solution is generated, the ultimate responsibility of defining a best compromised solution rests solely upon the preference characteristics provided by the decision-maker.

Interestingly, the noninferior solution set, in general, is defined by a unique continuous curve or surface depicting the trade-offs between the various alternatives. From this, it is obvious that, in theory, an infinite number of solutions exist to the multiobjective problem. It is not until the decision-maker provides the characterization of preference between each objective that a best compromised solution is identified. The information concerning the decision-maker's preference is most commonly depicted graphically by what is known as the "indifference curve." A typical indifference curve is also shown in Figure 3.1. Intuitively, the "best-compromised" solution to the multiobjective problems is a unique set of alternatives which possess the property of maximum combined utility and are elements in both the noninferior solution set and indifference curve. Such an alternative

only exists at the point where the indifference curve and noninferior solution set are tangent (Cohon, 1978).

3.3 MULTIOBJECTIVE PROGRAMMING SOLUTION TECHNIQUES

3.3.1 Generating Techniques

To clarify the structure of this type of an approach, generating techniques can be viewed as a "bottom-up" procedure for solving the multiobjective problem. The flow of information is from the analyst, at the bottom, to the decision-maker, on the top. By utilizing this category of technique, the analyst and decision-maker act separately, however, the identification of a best compromised solution is dependent upon the successful orderly completion of each of their tasks. Successively, the analyst first solves the multiobjective formulation without prior knowledge of the decision-maker's preference between each of the model objectives, thus producing the alternatives contained in the noninferior solution set. These alternatives are then passed along to the decision-maker where, once the preference between the objectives is characterized, the best compromise solution can be identified. Each of the tasks to be performed by the participants are clearly identified. The analyst contributes his analytical expertise in generating all possible alternatives, while the ultimate decision-making responsibility is left to the decision-maker where it belongs.

As subclasses of this category of technique, there have been several methodologies reported for solving the multiobjective problem: (1) weighting method, (2) constraint method, (3) adaptive search, and (4) functional derivation of the noninferior solution set (Loucks, 1975;

Cohon, 1978). Although a variety of techniques exist, the oldest and most appealing approaches are the weighting method and constraint method, respectively. Of the two candidate techniques considered here, the weighting method is operationally more cumbersome, and restricted only to problems having a convex solution of the noninferior set. Hence, the constraint method is selected as the preferred technique for generating the noninferior solution set in this study.

The constraint method was first cited by Marglin in the book by Maass et al. (1962) and again by Marglin (1967). This approach enables the analyst to generate the noninferior solution set in entirety, without regards to convexity. The computational simplicity is probably the most distinguished advantage of the constraint method, although, in general, such procedures are usually confined to multiobjective formulations containing fewer than four objectives. Using the constraint method, the multiobjective problem is solved by adopting only one objective in the objective function. The remaining objectives are simply transformed into constraints in the problem formulation. For example, recall the two-dimensional problem cited earlier in this chapter which considered objectives $Z_1(\underline{X})$ and $Z_2(\underline{X})$. The original vector optimization formulation given in Eqs. (3.1)-(3.3) are simply transformed into a single objective problem as:

$$\text{Max} \quad Z_2(\underline{X}) \tag{3.6}$$

subject to

$$\underline{\underline{A}}\ \underline{X} \leq \underline{b}$$

$$Z_1(\underline{X}) \geq c_1 \qquad (3.7)$$

$$\underline{X} \geq \underline{0}$$

where c_1 is the desired goal to be attained by objective Z_1. An initial value is assigned to the right-hand-side parameter, c_1. Then, the model formulation is solved iteratively, each time incrementally increasing the value for c_1 until the solution becomes infeasible. During the iterations, every pair of solution $Z_2(\underline{X}|c_1)$ and c_1 are recorded for constructing the noninferior solution set.

Once the multiobjective problem has been formulated, the constraint method provides a relatively effortless computational methodology for generating the noninferior solution set. Moreover, if the multiobjective formulation followed a linear programming format, the constrained method can be easily solved by a parametric linear programming approach. For a detailed comparison of the attributes for each of the generating techniques listed above, the reader should consult Cohon and Marks (1975) and Cohon (1978).

3.3.2 Techniques Incorporating Prior Knowledge of Preference

The basic structure of this category of techniques can be characterized as a "top-down" approach. The flow of information is from decision-maker to analyst, resulting in the direct solution of the best compromised alternative. The succession of solution procedures for this type of an approach is not as clearly defined as that in the category of generating techniques. The utilization of this general category of techniques to solve the multiobjective problem requires significant

interaction between the analyst and decision-maker. Essentially, the general procedures are based on the decision-maker providing the analyst with sufficient information to characterize the preference between each of the objectives prior to model development. In doing so, the analyst is then capable of incorporating this information into the multiobjective programming formulation. Once such a model has been derived, a direct solution identifying the best compromise solution, consistent with the decision-maker's preference, can be obtained.

In review of the literature on this subject, there have been several methodologies reported for obtaining a direct solution to this type of multiobjective problem: (1) sequential and multiphase linear goal programming, (2) surrogate worth trade-off method, (3) estimation of optimal weights, (4) electre method, and (5) step method (Loucks, 1975; Cohon, 1978; Ignizio, 1982). Each of the approaches cited above possess unique advantages and individual characteristics. Again, the reader should consult Cohon and Marks (1975) and Cohon (1978) for a detailed comparison of the attributes and the procedures associated with each of these techniques.

In addition to the methodologies mentioned above, there exists another technique for obtaining a best compromise alternative: "fuzzy linear programming (FLP)" (Kickert, 1978; Ignizio, 1982; Zimmerman, 1984). The use of FLP procedures has recently grown in both popularity and application in systems engineering. Because of the recent excitment surrounding this technique, FLP is selected as the methodology for obtaining a direct solution to multiobjective optimization formulation

in this study. The use of FLP will be discussed in detail in latter sections of this chapter.

3.4 THE MULTIOBJECTIVE WLA MODEL

The initial step in analyzing the optimal WLA problem in a multiobjective framework is to define the objectives to be considered in the model formulation. Throughout this chapter, the presentation of results and discussions are based on a two-dimensional multiple-objective problem formulation. The two objectives considered for the optimal WLA problem in this study are: (1) the maximization of waste discharge, where both BOD and DO deficits from each discharger are defined as the decision variables and (2) the minimization of the maximum difference in equity between the various users of the stream environment. The importance of such considerations have already been discussed in Chapter 2. With the exception to multiple-objective functions, the basic WLA model remains unchanged from that presented in Section 2.4. Therefore, the two-objective WLA model can be expressed as follows:

$$\text{(objective 1)} \qquad Z_1 = \text{Maximize} \sum_{j=1}^{N} (L_j + D_j) \tag{3.8}$$

$$\text{(objective 2)} \qquad Z_2 = \text{Minimize } E_{max} \tag{3.9}$$

subject to

$$\sum_{j=i}^{n_i} \theta_{ij} L_j + \sum_{j=i}^{n_i} \psi_{ij} D_j \leq R_i \qquad \text{for all } i = 1,2\ldots,M \tag{3.10}$$

$$\left| E_j - E_{j'} \right| \leq E_{max} \quad \text{for all } j \neq j' \tag{3.11}$$

$$0.35 \leq \frac{L_j}{I_j} \leq 0.90 \quad \text{for all } j = 1,2,\ldots,N \tag{3.12}$$

and

$$E_{max} \geq 0,\ L_j \geq 0,\ D_j \geq 0 \quad \text{for all } j = 1,2,\ldots N$$

where E_{max} is a new decision variable representing the maximum difference in equity between the various dischargers. Each of the remaining terms have been defined and described in Chapter 2. It should also be noted that the two forms of equity considered in Chapter 2, i.e., (1) equal percent removal and (2) equal effluent concentration, are again utilized in this chapter.

3.5 MULTIOBJECTIVE WLA USING THE CONSTRAINT METHOD

3.5.1 Formulation of Multiobjective WLA Model Using Constraint Method

Following the general procedures of the constraint method outlined in Section 3.3.1, the two-objective WLA model of this study must first be transformed into a single-objective model formulation. In doing so, the goal to maximize waste discharge is selected to be the same objective function as that in the constraint method approach. The objective to minimize the maximum difference in equity between the various dischargers is transformed into a constraint in the WLA model. Hence, the original two-objective formulation is reconstructed into a single-objective formulation as follows:

$$\text{Maximize} \sum_{j=1}^{N} (L_j + D_j)$$

subject to

$$\sum_{j=i}^{n_i} \theta_{ij} L_j + \sum_{j=i}^{n_i} \psi_{ij} D_j \leq R_i \qquad \text{for all } i = 1,2\ldots,M$$

$$\left| E_j - E_{j'} \right| \leq E_{max} \qquad \text{for all } j \neq j'$$

$$0.35 \leq \frac{L_j}{I_j} \leq 0.90 \qquad \text{for all } j = 1,2,\ldots,N$$

$$E_{max} \leq E^{\circ} \tag{3.13}$$

$$E_{max} \geq 0,\ L_j \geq 0,\ D_j \geq 0 \qquad \text{for all } j = 1,2,\ldots N$$

where E° is a pre-determined constant to control the maximum difference in equity, E_{max}. Depending on the type of equity considered, equal percent removal or equal effluent concentration, E° is specified in terms of percent removal (decimal fraction) or mg/l, respectively.

3.5.2 Application of Constraint Method to the Two-Objective WLA Problem

The hypothetical example of the six-reach stream system described in Section 2.6 is used. The data describing the physical parameters of the stream environment is given in Table 2.1. Once the two-objective WLA model using the constrained method is formulated, it simply becomes a matter of performing the iterative solutions procedures outlined

previously to generate the noninferior solution set. Initially, values of 0.05 and 5.0 mg/l were selected for E° for the two types of equity considered, i.e., equal percent removal and equal effluent concentrations, respectively. The individual model formulations, each uniquely considering one of the two types of equity, are solved iteratively with the primary objective of maximizing total waste discharge (measured in terms of BOD and DO deficits in mg/l).

During such interations, the right-hand-side value, E°, is incrementally increased by 0.05 and 5.0 mg/l for the respective types of equity considered. The solutions obtained for the maximization of waste discharge are stored during each iteration. In order to depict the noninferior solution set, the respective model formulations are solved, successively until the solution set became infeasible.

It should also be noted that the moving control point approach for controlling water quality in the model constraints, as detailed in Chapter 2, are incorporated into the constraint method model formulation of this study. Such provisions are placed in the formulation in order to take advantage of the savings in computer storage and improve model performance.

The solutions to the noninferior set for each of the types of equity considered are listed in Tables 3.1 and 3.2. These results are plotted and displayed graphically in Figures 3.2 and 3.3. Through these figures, the trade-off existing between the objectives, maximization of waste discharge and minimization of the maximum difference in equity, is clearly illustrated. Specifically, Figures 3.2 and 3.3 portray a linear

TABLE 3.1 NONINFERIOR SOLUTION SET CONSIDERING THE EQUITY OF EQUAL PERCENT REMOVAL(DIMENSIONLESS)

E_{max}	Total Waste Discharge, mg/l
0.05	1036
0.10	1222
0.15	1407
0.20	1590
0.25	1771
0.30	1947
0.35	2103
0.40	2257
0.45	2409

TABLE 3.2 NONINFERIOR SOLUTION SET CONSIDERING THE EQUITY OF EQUAL EFFLUENT CONCENTRATION (mg/l)

E_{max}	Total Waste Discharge, mg/l
5.0	806
10.0	824
15.0	842
20.0	860
25.0	879
30.0	897
35.0	915
40.0	934
45.0	952
50.0	970

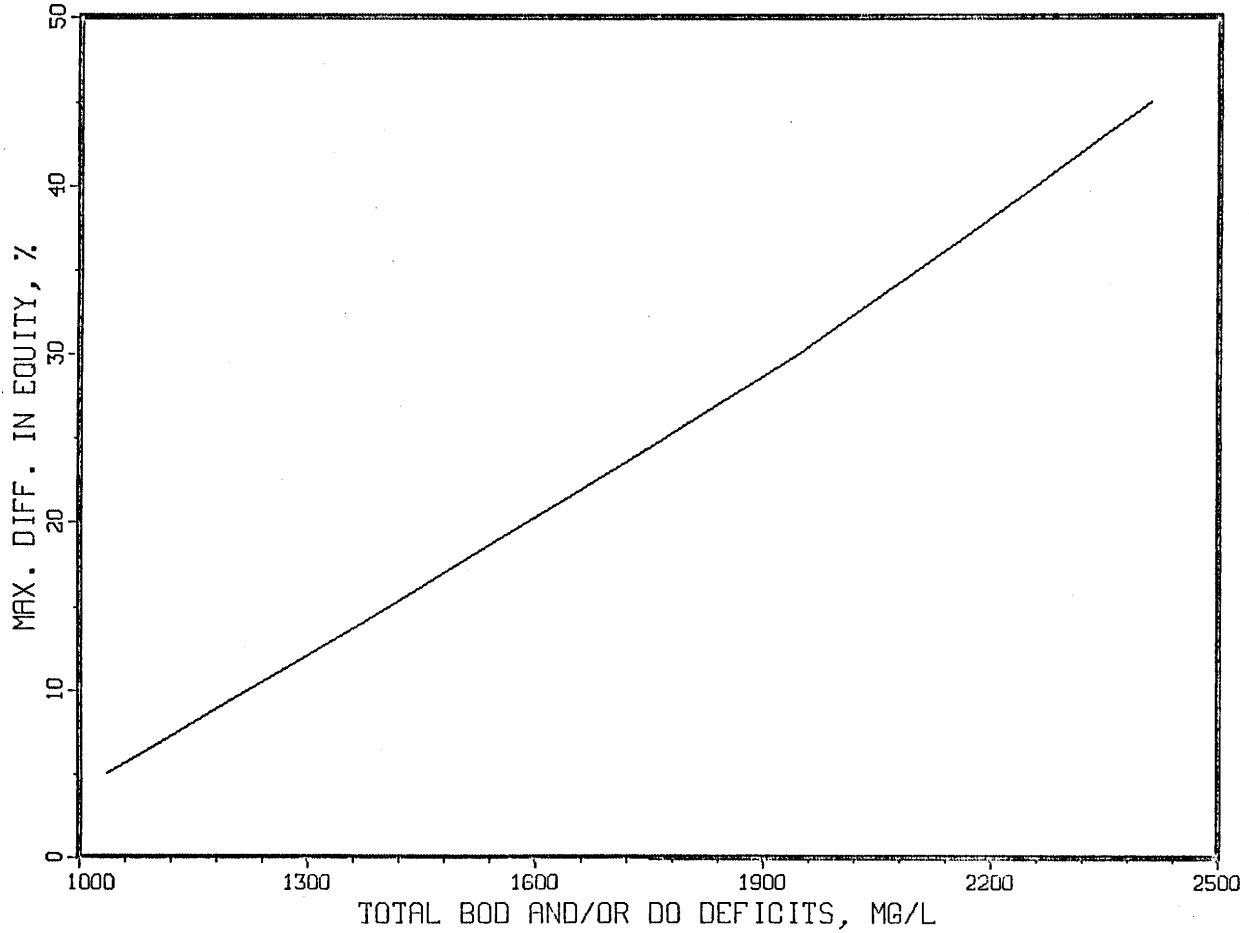

Figure 3.2 Noninferior Solution Set Corresponding To The Equity Of Equal Percent Removal

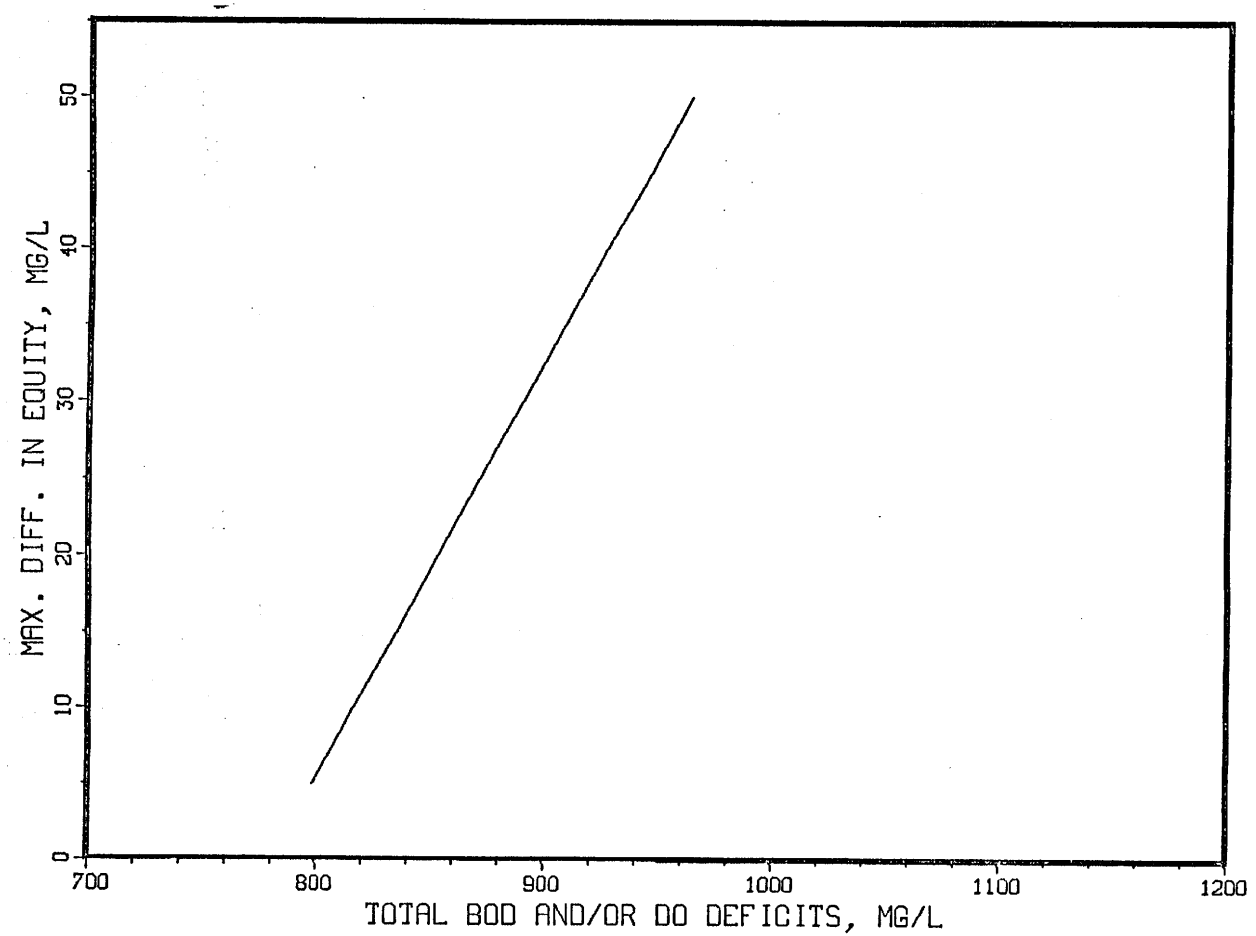

Figure 3.3 Noninferior Solution Set Corresponding To the Equity Of Equal Effluent Concentrations

marginal rate of substitution between the two objectives. From this, it is evident that for every unit of the maximum difference in equity that is given up, a uniform increase in the allowable waste discharge can be obtained. Intuitively, this increasing trend can be explained by the fact that as the constraints on equity between the various dischargers are relaxed, larger totals for the waste discharges to the stream environment are allowed, as long as the water quality requirements for dissolved oxygen are not violated. These results are only reasonable given the fact that the binding constraints in the model formulation must be those associated with the type of equity considered. These are, in fact, the conditions that occur when the constraint method is utilized.

3.6 FUZZY LINEAR PROGRAMMING IN MULTIOBJECTIVE OPTIMIZATION

The foundation for this methodology was born out of research introducing the theory and terminology associated with fuzzy set theory by Zadeh (1965). Zadeh's original studies were in search of improved decision analysis in the areas of expert systems and artificial intelligence. Since its conception, the application of fuzzy set theory to the field of mathematical programming were originally quite limited. Although this technique remains obscure to some extent, it has experienced a significant increase in popularity in recent years. The first extension of fuzzy theory in an LP formulation was presented by Zimmerman (1976). In order to completely grasp the use of these procedures, the methodologies associated with FLP can be divided into two central concepts: (1) defining the membership functions and

(2) outlining the FLP model formulation. Each of these concepts are discussed in detail below.

3.6.1 The Membership Function

The use of fuzzy theory to linear programming can be neither described as a deterministic nor probabilistic approach. Instead, such procedures are uniquely "fuzzy". To date, probably the most comprehensible and concise composition of the utility of fuzzy theory is that given by Bellman and Zadeh (1970):

> Much of the decision-making in the real world takes place in an environment in which the goals, the constraints and the consequences of possible actions are not known precisely. To deal quantitatively with imprecision, we usually employ the concepts and techniques of probability theory and, more particularly, the tools provided by decision theory, control theory and information theory. In so doing, we are tacitly accepting the premise that imprecision--whatever its nature--can be equated with randomness. This, in our view, is a questionable assumption. Specifically, our contention is that there is a need for differentiation between randomness and fuzziness, with the latter being a major source of imprecision in many decision processes. By fuzziness, we mean a type of imprecision which is associated with fuzzy sets, that is, classes in which there is no sharp transition from membership to nonmembership. For example, the class of green objects is a fuzzy set. So are the classes of objects characterized by such commonly used adjectives as large, small, significant, important, serious, simple, accurate, approximate, etc. Actually, in sharp contrast to the notion of a class or a set in mathematics, most of the classes in the real world do not have crisp boundaries which separate those objects which belong to a class from those which do not. In this connection, it is important to note that, in the discourse between humans, fuzzy statements such as "John is several inches taller than Jim," "x is much larger than y," "Corporation X has a bright future," "the stock market has suffered a sharp decline," convey information despite the imprecision...

Although fundamentally criticized by some, the use of FLP has genuinely proven to be an attractive technique for solving the ill-defined vector optimization problem formulation.

The most important point to note in a decision-making context, which attempts to incorporate the fuzzy environment, is that the objective function and system constraints are defined by a unique membership function. This membership function merely acts as a surrogate characterization of preference in determining the desired outcome for each of the objectives in the multiobjective framework. The process to appropriately define the membership function is performed in such a manner as to allow the function to take on values in the interval (0,1). The membership function, denoted μ_k for the k-th objective, should at least satisfy the following conditions:

$$\mu_k = \begin{cases} 0 & \text{if } Z_k(\underline{X}) \leq L_k \\ 0 < \mu_k < 1 & \text{if } L_k < Z_k(\underline{X}) < U_k \\ 1 & \text{if } Z_k(\underline{X}) \geq U_k \end{cases} \tag{3.14}$$

where $Z_k(\underline{X})$ is the outcome of k-th objective; L_k and U_k represent the least acceptable and most desirable outcome for $Z_k(\underline{X})$, respectively.

By defining the membership function in such a manner, the analyst and decision-maker, working interactively, can program a level of desirability for the various outcomes of each of the objectives into the model formulation. Once completed, the membership function acts as a scaling device, assigning a level of acceptance to each of the alternatives considered in the multiobjective formulation. Ultimately, the

best compromising solution can be identified as the alternative which attains the highest level of desirability while simultaneously satisfying the model constraints.

Several membership functions have been employed in FLP: (1) linear, (2) exponential, (3) hyperbolic, and (4) logistic. This list, by no means, is intended to represent the entirety of membership functions in existence. Although a variety of such functions are accessible, the linear and logistic membership functions are selected as the means of defining the level of desirability in this study. Through an appropriate transformation, the logistic membership function can be linearized preserving the linearity of LP formulation.

The linear form of the membership function, as shown in Figure 3.4, can be expressed as follows:

$$\mu_k(Z_k) = \begin{cases} 0 & \text{for } Z_k(\underline{X}) \leq L_k \\ \dfrac{Z_k(\underline{X}) - L_k}{d_k} & \text{for } {}_kL < {}_kZ\ (\underline{X}) < {}_kU \\ 1 & \text{for } Z_k(\underline{X}) \geq U_k \end{cases} \qquad (3.15)$$

where d_k is the range of outcomes for $Z_k(\underline{X})$ determined by $U_k - L_k$.

The logistic membership function is defined as:

$$\mu_k(Z_k) = \begin{cases} P_\ell & \text{for } Z_k(\underline{X}) \leq L_k \\ 1/\ 1 + \exp[-\alpha_k - \beta_k Z_k(\underline{X})] & \text{for } L_k < Z_k(\underline{X}) < U_k \\ P_u & \text{for } Z_k(\underline{X}) \geq U_k \end{cases} \qquad (3.16)$$

P_ℓ and P_u represent the degree of decision-maker's preference corresponding to the lowest and highest attainable values for the k-th objective,

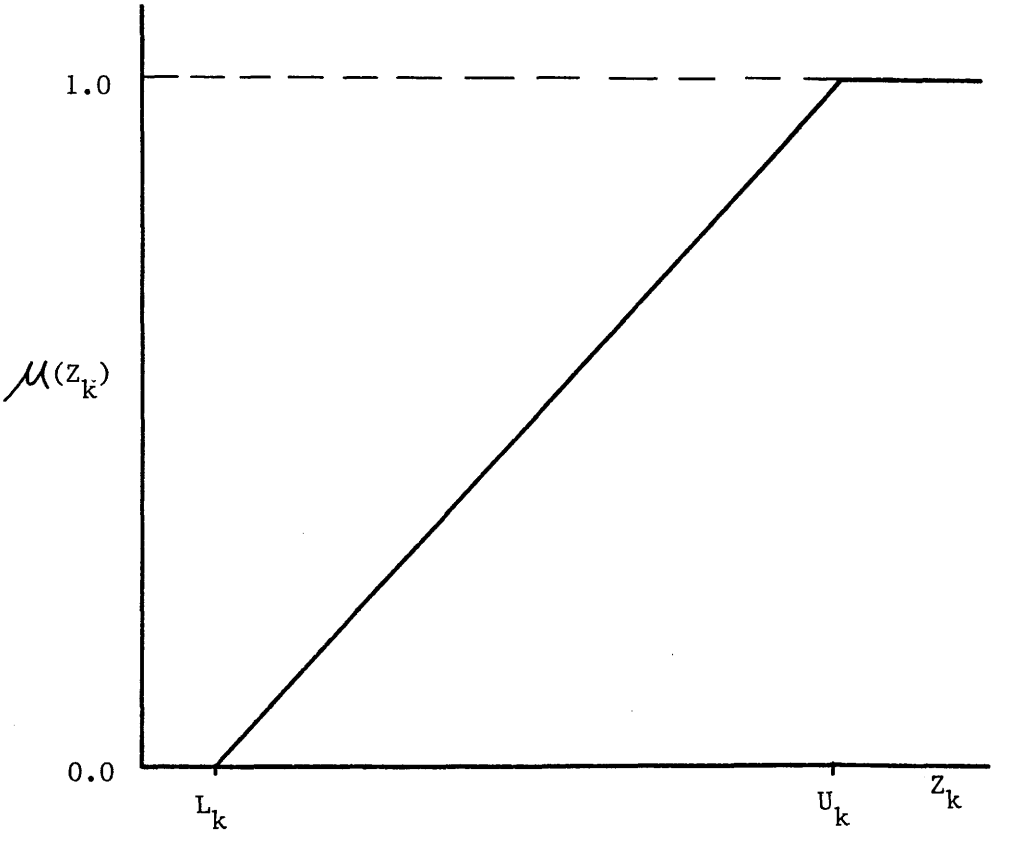

Figure 3.4 Linear Membership Function

where α_k and β_k are constants in the membership function which can be determined by

$$\alpha_k = (U_k/d_k)\ln\left(\frac{P_\ell}{1 - P_\ell}\right) - (L_k/d_k)\ln\left(\frac{P_u}{1 - P_u}\right) \tag{3.17}$$

$$\beta_k = (1/d_k)\left\{\ln\left(\frac{P_u}{1 - P_u}\right) - \ln\left(\frac{P_\ell}{1 - P_\ell}\right)\right\} \tag{3.18}$$

In general, values for P_u and P_ℓ are selected between 0.95-0.99 and 0.01-0.05, respectively. A representative configuration of a logistic membership function is shown in Figure 3.5.

3.6.2 The Generalized FLP Model Formulation

Given the theory behind the FLP model formulation, the goal of this technique is to obtain an optimal solution which maximizes the level of desirability for each of the objectives in the multiobjective problem. More precisely, the goal is to maximize the minimum attainable membership for each of the objectives. That is, the model adopts the max-min principal. This is accomplished by introducing a new decision variable, λ, representing the level of minimum of any of the objectives. The problem is then be formulated in a generalized LP format as follows:

$$\text{Maximize } \lambda \tag{3.19}$$

subject to

$$\underline{\underline{A}}\,\underline{X} \leq b$$

$$\mu_k[Z_k(\underline{X})] - \lambda \leq 0 \quad \text{for all } k = 1,2\ldots,K \tag{3.20}$$

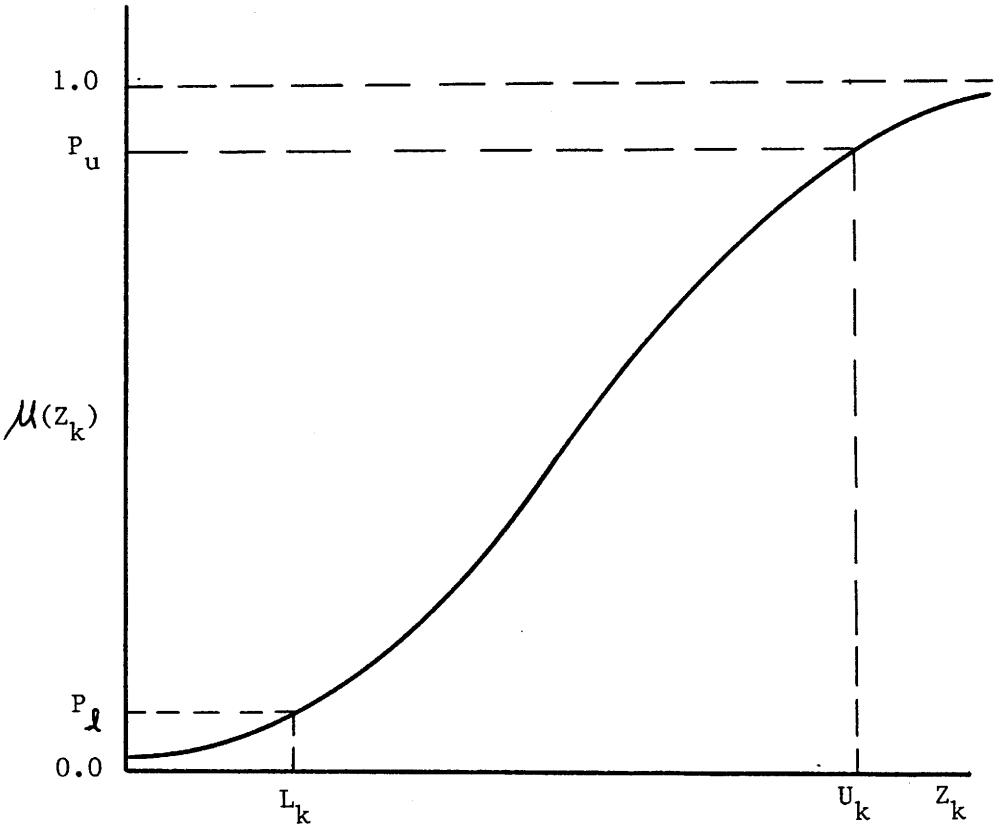

Figure 3.5 Logistic Membership Function

when K is the number of objectives considered in the problem formulation.

In solving the FLP model formulation, the procedures can be outlined in four basic steps:

1. Solve the vector optimization problem using only one of the objectives at a time, ignoring all others. Repeat the process until all objectives have been considered.
2. From the solutions in step 1, determine the best (U_k) and worst (L_k) outcomes for each of the objectives, k.
3. Define each of the membership functions, $\mu_k[Z_k(\underline{X})]$, from the results obtained for the objectives in step 2.
4. Redefine the objective function to maximize the minimum $\mu_k[Z_k(\underline{X})]$, include constraints (in addition to those controlling water quality, treatment, and equity), to control the membership functions, and solve the final formulation.

When performed correctly, these four steps provide an effective means of obtaining a direct solution to the optimal or best compromising alternative in the multiobjective model formulation (Ignizio, 1982).

3.7 MULTIOBJECTIVE WLA USING FUZZY LINEAR PROGRAMMING

3.7.1 The Linear Membership Model

As mentioned earlier, the multiobjective WLA problem considered herein has two objectives: (1) the maximization of total waste discharge and (2) the minimization of the maximum difference in equity. Referring

to the original two-objective WLA model presented in Section 3.4, the FLP formulation can be expressed as:

$$\text{Maximize } \lambda$$

subject to

1. original constraints in WLA model:

$$\sum_{j=i}^{n_i} \theta_{ij} L_j + \sum_{j=i}^{n_i} \psi_{ij} D_j \leq R_i \qquad \text{for all } i = 1,2\ldots,M$$

$$\left| E_j - E_{j'} \right| \leq E_{max} \qquad \text{for all } j \neq j'$$

$$0.35 \leq \frac{L_j}{I_j} \leq 0.90 \qquad \text{for all } j = 1,2,\ldots,N$$

2. linear membership constraints

   i) for maximization of total waste discharge

$$\frac{-1}{d_1} \sum_{j=1}^{N} (L_j + D_j) + \lambda \leq \frac{-L_1}{d_1} \tag{3.21}$$

   ii) for the minimization of maximum equity difference

$$\frac{1}{d_2} E_{max} + \lambda \leq \frac{L_2}{d_2} \tag{3.22}$$

where

$$\lambda = \min \left\{ \frac{\sum_{j=1}^{N} (L_j + D_j) - L_1}{d_1}, \frac{L_2 - E_{max}}{d_2} \right\} \tag{3.23}$$

3.7.2 The Logistic Membership Model

Referring to Eq. (3.16) we realize that a transformation of variables must be made in order to develop a linearized function for $Z_k(\underline{X})$ before the logistic function can be incorporated into a linear programming framework. Similar to the linear membership case, we define $\lambda = \min(\mu(Z_1), \mu(Z_2), \ldots, \mu(Z_k))$ and $0 \leq \mu \leq 1$. As a result,

$$\mu(Z_k) = \frac{1}{1 + e^{-[\alpha_k + \beta_k Z_k(\underline{X})]}} \geq \lambda \text{ for all } k = 1,2,\ldots,K \qquad (3.24)$$

After some simple algebraic manipulation, Eq. (3.24) can be rearranged into

$$\alpha_k + \beta_k Z_k(\underline{X}) \geq \ln\left(\frac{\lambda}{1 - \lambda}\right) \qquad (3.25)$$

Although λ is the decision variable to be maximized, the term $\ln[\lambda/(1-\lambda)]$ poses no difficulty since it is a strictly montonically increasing function of λ. To maximize λ will automatically maximize $\ln[\lambda/(1-\lambda)]$. With this property, we can define a new decision variable $\eta = \ln[\lambda/(1-\lambda)]$ and Eq. (3.25) can be reduced to linear form as

$$-\beta_k Z_k(\underline{X}) + \eta \leq \alpha_k \quad \text{for all } k=1,2,\ldots,K \qquad (3.26)$$

Notice that the value for η can be negative, zero, and positive (i.e., unrestricted in sign). When using the simplex algorithm developed for solving an LP model, a non-negativity requirement for decision variables is normally imposed. Thus, to satisfy this non-negativity requirement,

we can simply replace the original decision variable , which is unrestricted in sign, by the difference of two nonnegative decision variables as $\eta = \eta^+ - \eta^-$.

Using the two new nonnegative decision variables η^+ and η^-, a relationship utilizing the logistic membership can be incorporated into an LP format for solving the multiobjective WLA problem. The resulting FLP model can be expressed as:

$$\text{Maximize } (\eta^+ - \eta^-) \tag{3.27}$$

subject to

1. Original constraints including Eqs. (3.10), (3.11), (3.12)
2. Logistic membership constraints:

   i) for the maximization of total waste load

$$-\beta_1 \left(\sum_{j=1}^{N} L_j + D_j \right) + \eta^+ - \eta^- \leq \alpha_1 \tag{3.28}$$

   ii) for the minimization of maximum equity difference

$$-\beta_2 E_{max} + \eta^+ - \eta^- \leq -\alpha_2 \tag{3.29}$$

and iii) non-negativity constraints:

$$\eta^+ \geq 0,\ \eta^- \geq 0,\ E_{max} \geq 0, \text{ and } L_j \geq 0,\ D_j \geq 0 \text{ for all } j.$$

The coefficients α's and β's in Eqs. (3.28) and (3.29) can be computed by Eqs. (3.17) and (3.18), respectively.

3.8 APPLICATION OF FUZZY LINEAR PROGRAMMING TO EXAMPLE WLA PROBLEM

The example river system outlined in Figure 2.2. is again adopted here. Each of the FLP models were solved using the two-objective hypothetical model and the four basic steps outlined previously. In order to simplify the discussion, let the objective to maximize waste discharge be denoted Z_1, and that to minimize the maximum equity difference be Z_2.

During the first stage, the two-objective WLA problem is solved iteratively, with one objective at a time. The first iteration focusing Z_1 as the objective function and the second iteration Z_2. Although a single objective is considered during each of the first two iterations, the decision variables corresponding to the other objectives remain universal throughout the system of constraints. Thus, a solution to each of the objectives can be identified during any iteration. During the first iteration of this example a best solution (U_1) is obtained for objective Z_1, since it is the current focus of the optimization. On the other hand, a least desirable solution (L_2) is also identified for the objective Z_2. Conversely, the second iteration led to the identification of the best solution (U_2) for Z_2 and the worst solution (L_1) for Z_1. The values of U_1 and L_k for each of the objectives are given in Table 3.3 for the two types of equity considered in this study. It should also be noted that these values are independent of the type of membership function assumed in the problem formulation. Remember, the problem is being solved by separately considering the objectives Z_1 and

TABLE 3.3 THE BEST (U_k) AND WORST (L_k) SOLUTIONS FOR EACH OBJECTIVE WHEN CONSIDERING THE TWO TYPES OF EQUITY.

(a) Equity Type: Equal Percent Removal

Objectives	Bounds U_k	Bounds L_k
Z_1: Maximize Total Waste Discharge (mg/l BOD)	2691	493
Z_2: Minimize Maximum Difference in Equity (percent)	0.0	54.3

(b) Equity Type: Equal Effluent Concentration

Objectives	Bounds U_k	Bounds L_k
Z_1: Maximize Total Waste Discharge (mg/l BOD)	2691	758
Z_2: Minimize Maximum Difference in Equity (mg/l)	0	878

Z_2. The use of a membership function has not yet been considered at this stage in the solution process.

Once the values for U_k and L_k were identified, a unique linear membership function is derived for each of the objectives, Z_1 and Z_2. The linear membership function for objective Z_1, was simply obtained by substituting the values of U_1 and L_1 into Eq. (3.15), likewise for Z_2. Once completed, the problem is solved for the last time to obtain a direct solution to the multiobjective WLA problem using one type of equity and the FLP formulation outlined earlier. The entire process can be then repeated for the other types of equity.

The FLP solutions to the multiobjective WLA problem for the six-reach example using a linear membership function are displayed in Tables 3.4 and 3.5. Specifically, Table 3.4 contains the optimal WLA when the equity of equal percent removal between the dischargers is considered, while that in Table 3.5 is associated with the equity of equal effluent concentrations. When comparing the two sets of optimal allocations, the total allowable waste discharge for the equity of equal percent removal, 1700 mg/l BOD, is less than the total for the equity of equal effluent concentrations, 1837 mg/l BOD. This is the result of the unique characteristics possessed by each of the membership functions associated with the individual formulations. By considering the two different types of equity, two separate and distinct problems are formulated according to the FLP procedures. Once solved, the individual model formulations result in unique optimal solutions.

TABLE 3.4 OPTIMAL ALLOCATION OF WASTE FOR THE TWO-OBJECTIVE PROBLEM USING FLP, WITH THE LINEAR MEMBERSHIP FUNCTION, AND THE EQUITY OF EQUAL PERCENT REMOVAL

Discharger	No. 1	No. 2*	No. 3	No. 4	No. 5	No. 6
Allowable Waste Discharge (mg/1 BOD)	539	6	262	142	590	161
Required Percentage Raw Waste Removal	60.7	0	60.7	84.5	60.7	60.7

*Discharger No. 2 is a tributary.

TABLE 3.5 OPTIMAL ALLOCATION OF WASTE FOR THE MULTIOBJECTIVE PROBLEM USING FLP WITH THE LINEAR MEMBERSHIP FUNCTION, AND THE EQUITY OF EQUAL EFFLUENT CONCENTRATIONS.

Discharger	No. 1	No. 2*	No. 3	No. 4	No. 5	No. 6
Allowable Waste Discharge (mg/1 BOD)	502	6	432	129	502	266
Required Percentage Raw Waste Removal	63.4	0	35.0	85.8	66.5	35.0

*Discharger No. 2 is a tributary.

Additionally, the solution procedures were repeated, this time using the logistic membership function as reported in Eqs. (3.16) to (3.18). The optimal allocations for each type of equity utilizing a logistic membership function are identical to those obtained using a linear membership function. Because of the unique analytic expressions associated with the two unique membership functions, it was originally thought that such results were erroneous or coincidental to the example system chosen. Interestingly, the logistic membership problem was again solved, this time using significantly different assumed stream data. The optimum solutions for each of the membership functions were again identical. At this point, more than idle curiosity had been raised concerning these results. It turns out that, upon further analytical investigation, the identical results obtained for the linear and logistic membership functions can be proven to be continually true. A formal proof of this phenomena is provided in Appendix A.

In reviewing this proof, the arithmetic sum of the linear membership constraints given by Eqs. (3.21) and (3.22) are shown to be identical to the sum of the logistic membership constraints given by Eqs. (3.28) and (3.29). The physical inference of the conclusions of this proof is that the feasible domain described by each of the membership functions share an identical boundary containing the optimal solution. The difference between these feasible domains is related to the total volume of such space. Essentially, the planes of the feasible region described by the membership functions are rotated about a unique ridge,

containing the optimal solution, when either the membership according to the linear or logistic function is considered.

Clarification of these arguments can be made by relating to a schematic diagram shown in Figure 3.6. Figure 3.6 represents the feasible solution domain corresponding to the two-objective FLP problem when considering each of the membership functions, i.e., linear and logistic. Specifically, the domain bounded by points ABCD can be assumed to represent the feasible space for using the logistic membership and that of ABEF to be that feasible space for using the logistic membership, respectively. Additionally, point 0 lies along line AB, which is shared by each of the domains, and represents the optimal solution to the multiobjective WLA problem.

In this figure, the effects of considering each of the membership functions is portrayed. By changing the assumption of the membership function, the feasible space is changed, i.e., the domain of the linear membership function ABCD to that of the logistic membership ABEF. However, this change only occurs in the positioning of the vertical planes ABC to ABE and ABD to ABF. The position of the ridge boundary defined by line AB remains unaffected. Hence, by maximizing the minimum membership function, whether a linear or logistic member function is considered, the optimal solution of the FLP problem presented here remains unchanged. Moreover, these results should only be considered true for any two-objective model formulation. Until further research is conducted, these conclusions should not be extrapolated to problem formulations considering three or more objectives.

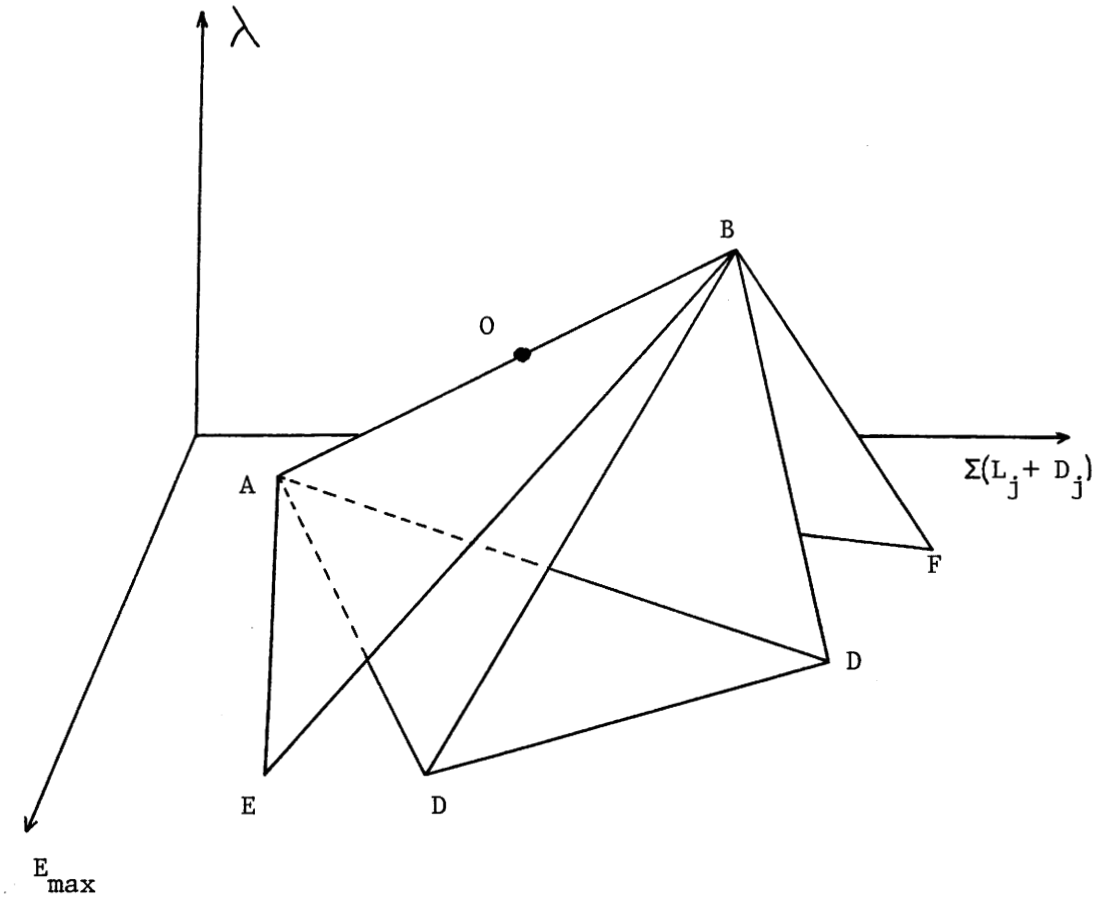

Figure 3.6 Feasible Region Defined By Membership Functions In WLA Model

In addition to the tabulated results, the dissolved oxygen profile resulting from the discharge of waste according to their respective optimal allocations are plotted for each type of equity. These plots are shown in Figures 3.7(a)-(c) and 3.8(a)-(c). The dashed line at 4 mg/l represents the assumed minimum requirement for dissolved oxygen (DO) to be maintained throughout the river system under investigation. The significance and historical selection of this value has already been discussed in Chapter 2. Figures 3.7(c) and 3.8(c) are an indication of the effects on in-stream DO concentration resulting from the optimal allocation of waste is illustrated.

3.9 SUMMARY AND CONCLUSION

As the demands on water quality continue to grow, the need for improved methodologies to protect aquatic environments from exploitative waste discharges increase. The problem of optimal waste load allocation is not new to this decade. For many years, regulatory agencies have utilized single objective optimization models to mandate and enforce waste load allocations for various river system throughout this country. Through continued research in the area of water quality management, the limitations of such an approach to solve the optimal waste load allocation problem is becoming ever apparent.

In general, environmental water quality management problems are multiobjective by nature. The problem of optimal waste load allocation is, without exception, included. In answer to the shortcomings of the single objective approach in WLA practice, this chapter has presented two methods for solving a two-objective optimal waste load allocation

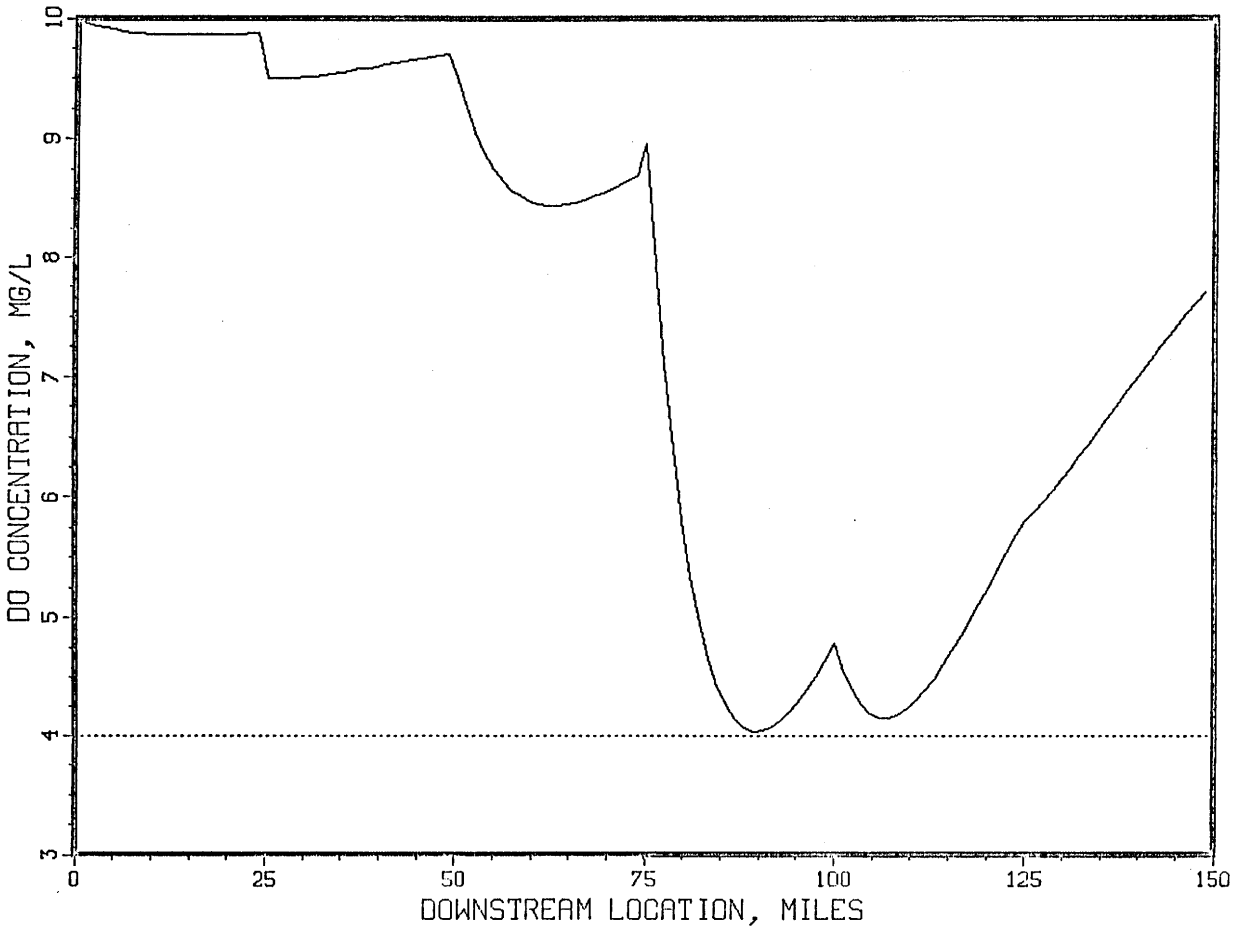

Figure 3.7(a) DO Profile Corresponding To Fuzzy Linear Programming Approach With The Objective To Maximize Total Waste Discharge And The Equity Of Equal Percent Removal

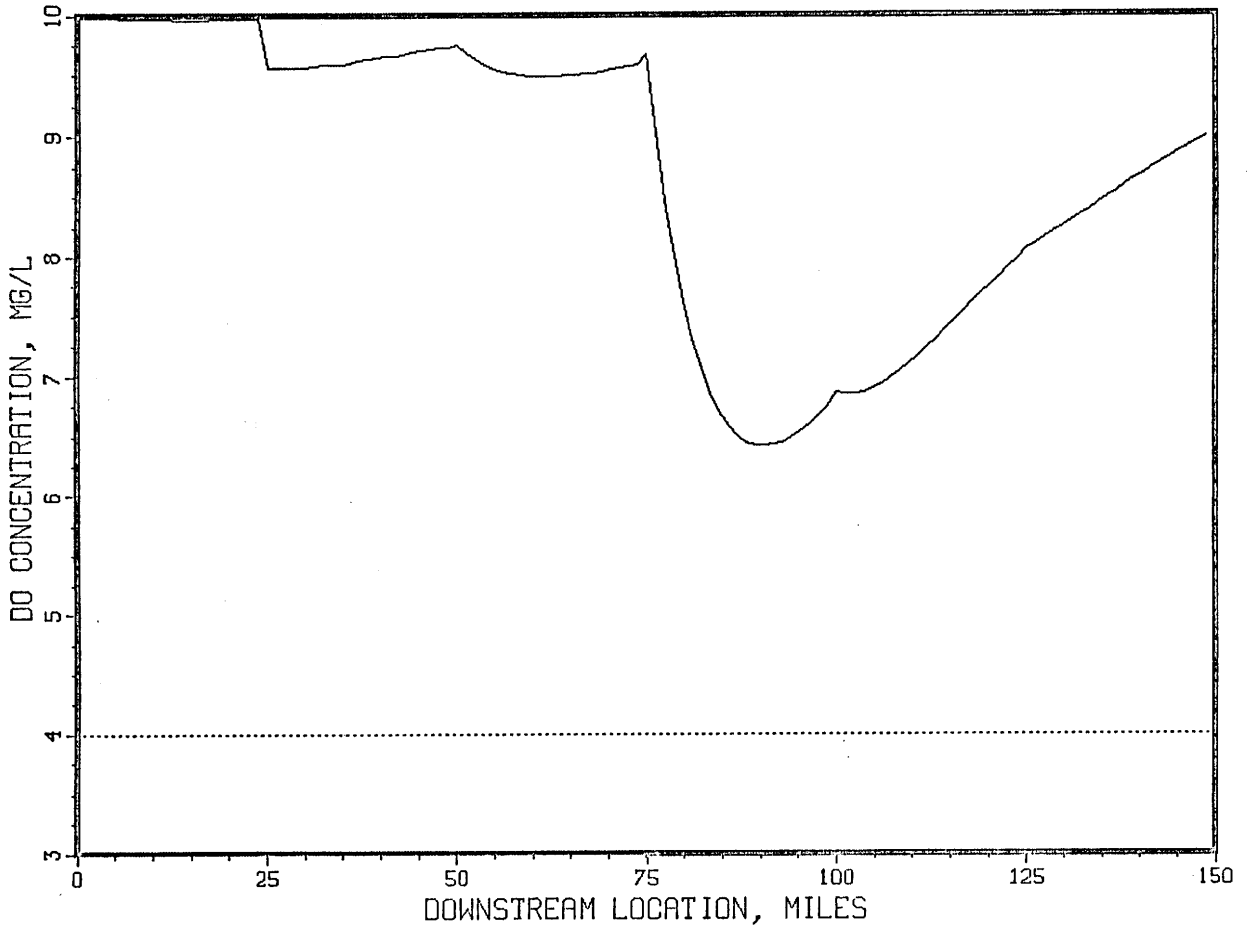

Figure 3.7(b) DO Profile Corresponding To Fuzzy Linear Programming Approach With The Objective To Minimize The Maximum Difference In Equity And Considering The Equity Of Equal Percent Removal

CHAPTER 4

RISK ASSESSMENT OF STREAM WATER QUALITY STANDARDS

4.1 INTRODUCTION

Although technology has greatly improved our ability to treat industrial and municipal wastes, it is still a common practice to discharge allowable quantities of pollution from these effluents into various watercourses. This practice is based on the principal that the receiving waters possess a natural ability to assimilate a specific quantity of pollutant. Given these conditions, the allowable waste concentrations and natural biota coexist within the dynamic environment of the stream system. Consequently, water quality officials have been given the arduous task of determining the socioeconomic tradeoffs between allowable waste load allocations and maintaining desired levels of aquatic life within the stream environment. In answer to these problems, water quality agencies have enacted regulations allowing the continuation of waste discharge to streams subject to a variety of water quality standards.

In contrast to the fact that each stream is highly variable by nature, the basis for the development of water quality standards continues to be a deterministic evaluation of the stream environment. As a result, many of the present water quality standards neglect the inherent stochastic nature of the system (i.e. rivers and streams) which they are supposed to protect. Several authors, noting the shortcomings associated with present water quality standards, have

criticized the ability of deterministic standards to provide adequate protection of the stream environment (Loucks and Lynn, 1966; Adams and Gemmel, 1975; Burn and McBean, 1985). Knowing the reality of the inherent random nature of these systems, deterministic standards should be amended to account for the stochastic processes present in the stream environment. In addition, most of the current standards do not differentiate between the various levels of exceedence nor the lengths of violation in the stream system. Given the deterministic structure of present water quality regulations, it is implied that all water quality violations are considered equal, irregardless of the effects on the stream environment. Presently, no emphasis is placed on the relative severity of the individual violations. For example, a small exceedence, resulting in minor damage, is treated in the same manner as a large exceedence, possibly resulting in significant damage. Both conditions are simply defined as "violations", thus neglecting the relative effects created by the specific violation conditions.

In an attempt to incorporate the random nature of the stream environment and the level of severity for various violation conditions into the water quality decision-making process, it is the objective of this chapter to present a methodology for evaluating the joint risk associated with a maximum dissolved oxygen deficit (beyond a specified standard) and the length of such violation within any given stream system. This chapter utilizes the simplified Streeter-Phelps equation and Monte Carlo simulation techniques to evaluate the risk based on several assumptions for the probability distributions assigned to each

parameter in the model formulation. In addition, a sensitivity analysis is performed to evaluate the effects of changes in the statistical characteristics of the model parameters on the risk. By evaluating the risks associated with water quality violations, it is believed a more realistic decision can be made between the economic and environmental questions facing water quality management agencies in the future.

4.2 UNCERTAINTY IN THE WATER QUALITY MODEL

The water quality model presented in Chapter 1 (Eqs. 1.5 to 1.8) is a function of several stream parameters, such as, the reareation and deoxygenation coefficients and the average stream velocity. In reality, the system (i.e., the stream environment) to which this model is applied is extremely variable, both spatially and temporally, by nature. Inherently, the stream system represents a dynamic environment in which the physical and biological characteristics are ever-changing. As with the unpretentious passage of time, continual changes occur in the character of the stream environment. Given such facts, it is quite obvious that the parameters utilized in the water quality model of Chapter 1 cannot be quantified with exact certainty. The inherent random nature of the system to be modeled leads to uncertainties in the prediction of model parameters. Thus, in order to accurately model such an environment, the uncertainties associated with the stream system must be included in the water quality model formulation.

The uncertainty linked with Eq. (1.5), for predicting DO levels in a stream system, can be divided into three categories: inherent, parameter, and model uncertainties. Inherent uncertainties are the

result of the natural randomness exhibited by the physical and biological processes described by Eq. (1.5). This inherent uncertainty is the product of temporal and spatial variations, for example, in streamflow, effluent waste concentration, temperature, and in-stream biological composition (Churchill et al., 1962; Bansal, 1973; Wright and McDonnell, 1979). In addition, the absence of unlimited data describing the characteristics of the stream system result in insufficient information to estimate the parameters of the model with absolute certainty. The combined effects of inherent randomness and imperfect data collection result in parameter uncertainty in the model formulation.

As previously mentioned, several researchers have modified the original Streeter-Phelps equation to account for discrepancies between DO deficits predicted by the model and collected field data. Such discrepancies were the result of the original model's exclusion of a number of oxygen sources and sinks. The inability of the model to accurately predict the DO deficits is known as model uncertainty. To account for this inadequacy, additional terms may be added to the model formulation to include the effects of the various oxygen sources and sinks. Alternatively, adjustment of the model may be accomplished by multiplying the original equation by a "model correction factor." This correction factor would simply be determined from an analysis of the differences between the predicted and field data collected. Accordingly, the model correction factor can also be treated as a random variable in the model formulation.

Given the fact that inherent, model, and parameter uncertainties exist, the stochastic nature of the stream system should be included in the model formulation if accurate DO predictability is to be attained. The general approach for describing these uncertainties has been to appropriately assign statistical properties, probability distributions, and correlations to each of the parameters in Eq. (1.5).

4.2.1 Selection of Statistical Properties for the Model Parameters

The selection of statistical properties include the appropriate determination of the mean, standard deviation, and possibly other higher moments for each of the model parameters. In order to accurately quantify the statistical properties, existing physical, chemical, and biological data are analyzed according to standard statistical procedures. Data used in the analysis should be obtained from the specific site under investigation in order to preserve the uniqueness associated with the various stream environments. Once the analysis has been performed, the resulting statistical properties become eligible for model applications.

4.2.2 Selection of Probability Distributions for the Model Parameters

Though several probability distributions are possible, the most common assumption is that each of the parameters in Eq. (1.5) follow a normal distribution (Kothandaramann and Ewing, 1969; Burgess and Lettenmaier, 1975; Esen and Rathbun, 1976). However, some investigators have utilized a variety of distributions to describe the random behavior of these parameters (Kothandaramann, 1970; Brutsaert, 1975).

Clearly, a universal agreement as to the type of distribution to use for each parameter in Eq. (1.5) does not exist. Given the uniqueness of each site under investigation, there is no reason to expect all the parameters to follow a given distribution for every location. Thus, it would seem reasonable to develop a procedure that will allow the model to be flexible with regards to the selection of the probability distribution for each parameter.

In following this idea, each parameter in the model of this study can be assigned one of five probability distributions: normal, log-normal, beta, gamma, and Weibull. By properly analyzing the recorded data describing the random nature of the stream environment for each site, an appropriate probability distribution can be selected for each parameter in the model.

4.2.3 Correlation Between Model Parameters

In general, the model parameters in Eq. (1.5) are considered to be independent. However, there has been extensive research in the development of mathematical functions directly relating the reaeration rate, K_a, to the physical characteristics of the stream such as average velocity, U (Bansal, 1973). These research results clearly demonstrate that a positive correlation exists between the model parameters K_a and U. Hence, procedures are provided in the model formulation of this study which allow for the inclusion of a correlation, between K_a and U.

Additionally, some investigators have proposed the existence of a positive correlation between K_d and K_a (Esen and Rathbun, 1976;

Padgett, 1978). Although the presence of a positive correlation between K_d and U may have resulted from the proper statistical analysis of a given set of stream data, intuitively, the inclusion of such a correlation is physically meaningless in the model formulation in this author's opinion. The author's reasonings are based on the fact that K_a is solely a function of the physical characteristics of the stream, while K_d is characterized by the biological composition of the waste discharge and stream environment. It is assumed that these processes act independently within the stream system. Consequently, the author feels that the correlation between K_d and K_a is spurious and, therefore, it is not considered in this study.

4.3 MEASUREMENT OF WATER QUALITY CONDITIONS

Presently, water quality standards are developed on the basis of maximum contaminant levels or minimum required concentrations, both of which are never to be violated. The concept of a deterministic water quality standard is plausible and feasible if the system has very little or no uncertainty involved. However, as previously discussed, it seems unreasonable to continue the enforcement of water quality requirements that neglect the probability of violating these standards. Because the stream system is inherently random and involves many elements subject to significant uncertainty, the risk of violating the required standards will always exist. Hence, in order to improve the basis for regulatory standards which recognize the stochastic nature of the stream environment, a measure of the probability

associated with the violation of water quality standards should be developed.

In following past procedures, the most widely used measure to indicate the water quality condition of a stream system is the extent to which BOD and/or DO deficit concentrations exceed existing water quality standards. The effects of such violations on the aquatic environment are related to the tolerance exhibited by the stream's biota to a given pollution concentration and length of stream (or time) subjected to these conditions. For instance, the stream system may be able to tolerate relatively large DO deficits for short lengths of violation or small DO deficits for much longer violation distances. In reality, a tradeoff exists between the level of DO deficit in violation and length of stream subjected to these violation conditions. Thus, in order to provide a more complete analysis of the stream environment under violation conditions, both the DO deficit in violation and the length of violation should be considered simultaneously.

In light of such facts, the joint probability of simultaneously violating a specified DO concentration and tolerable length of violation has been selected in this study as the measure of water quality in the stochastic stream environment. In doing so, both maximum and average DO violation conditions associated with a given length of stream violation distance are considered as follows:

$$\text{Risk} = \Pr\left(D'_{max} \geq D'_{tol} \text{ and } X_D \geq X_{tol}\right) \qquad (4.1)$$

or

$$\text{Risk} = \Pr\ (\bar{D}' \geq D'_{tol} \quad \text{and} \quad X_D \geq X_{tol}) \qquad (4.2)$$

in which Pr( ) represents the probability, D'_{max} and $\bar{D}'$ are the maximum and average DO deficits exceeding water quality standards (mg/1), respectively; X_D is the actual length of violation (miles), and D'_{tol} and X_{tol} are the specified tolerances for DO deficit beyond the standard (mg/1), and the length of violation in the stream system (miles), respectively (refer to Figure 4.1).

From this information, water quality management agencies could introduce regulatory measures that limit the maximum probability of violating the minimum dissolved oxygen standards. For example, an amended DO standard might read as follows: "the maximum probability of violating a minimum DO concentration by 1 mg/1 or less for a distance of 2 miles shall not exceed 0.05". Once the allowable level of risk associated with various violation conditions is quantified, water quality officials can then proceed with the determination of allowable waste load allocations for the various users of the stream environment.

4.4 QUANTIFICATION OF THE RISK OF VIOLATION

4.4.1 Determining the DO Deficit and Length of Violation

In reference to Figure 4.1, the length of violation is defined as the distance within the stream system where the DO profile drops below a specified minimum concentration (D_{std}) of 4.0 mg/1. (The significance of a minimum DO requirement of 4 mg/1 has already been

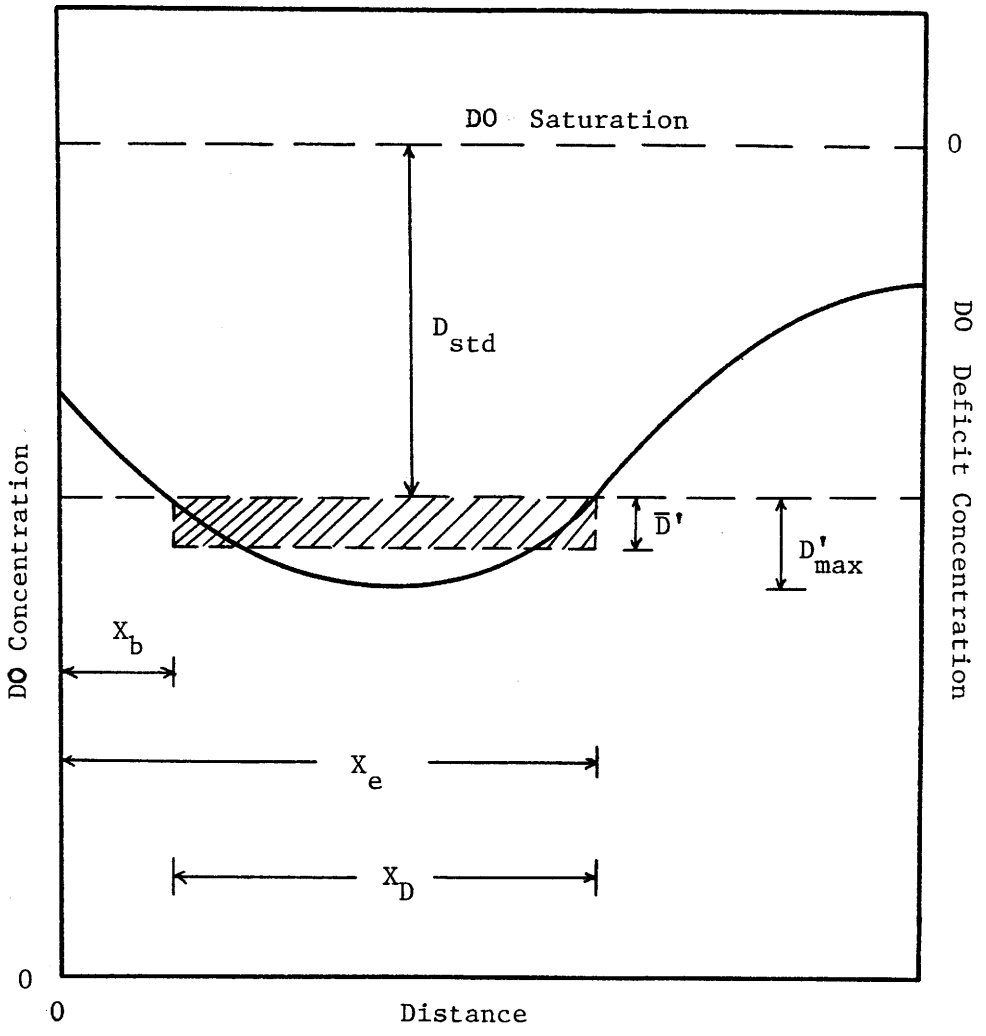

Figure 4.1 Illustration of Water Quality Violation Conditions

discussed in Chapter 1). Because of the non-linearity presented in Eq. (1.5), an analytical solution to determine the length of violation is impractical. To circumvent this problem, the Newton-Raphson numerical approximation technique was employed to solve the beginning and ending points of violation (Henrici, 1982; Rice, 1983).

The Newton-Raphson method is a commonly used numerical technique for finding the roots of a nonlinear equation. Thus, the beginning and ending points of violation were identified (using Newton-Raphson's method) by equating the DO deficit to zero and solving the following equation for the location x (in miles)

$$D_x - (C_s - C_{min}) = 0 \qquad (4.3)$$

where D_x is given by Eq. (1.5) and C_{min} is the minimum allowable DO concentration assumed to be 4 mg/l. Then, by taking the difference between these two end points, the length of violation is determined.

Once the maximum DO deficit is calculated using equation (1.5), the corresponding maximum DO violation is defined as the largest DO concentration deficit beyond the minimum standard of 4 mg/l (see Figure 4.1). In addition, the average DO deficit (within the length of violation) can be calculated by integrating Eq. (1.5) over the length of violation. Then, dividing this expression by the same length to obtain:

$$D_{avg} = \left\{ \frac{K_d L_o U}{K_a - K_d} \left[\frac{1}{K_d} \left(e^{-K_d X_b/U} - e^{-K_d X_e/U} \right) - \frac{1}{K_a} \left(e^{-K_a X_b/U} - e^{-K_a X_e/U} \right) \right] \right.$$

$$+ \frac{D_o U}{K_a} \left(e^{-K_a X_b/U} - e^{-K_a X_e/U} \right) \Bigg\} \Big/ X_D \qquad (4.4)$$

where X_b and X_e are the beginning and end points of violation, respectively; X_D is the length of violation in miles, $X_D = X_e - X_b$.

4.4.2 Monte Carlo Simulation

Monte Carlo methods are an extension of the branch of experimental mathematics which is concerned with experiments on random numbers. The use of Monte Carlo techniques can be classified into two general categories: (1) probablistic and (2) deterministic, depending on whether or not they are directly related to the behavior and outcome of a random process. When considering a probabilistic approach, the simplest Monte Carlo technique is to generate random numbers such that they simulate the physical random process of the system under investigation and to infer the desired solution from the behavior of these random numbers (Kothandaramann, 1968).

Monte Carlo simulation can be simply described as a sampling method used to approximate, through simulation, the solution of nonlinear formulation which would otherwise be extremely tedious to solve by direct analytical methods. The foundation for such an application lies in the large number of trials or iterations that are performed on the proposed model. By performing these iterations, a sufficiently large sample size can be generated, from which a relatively accurate solution to the model can be predicted.

Monte Carlo simulation techniques have found many applications in the modeling of stochastic processes. The essence of the technique is to develop a model that satisfactorily represents the random process to be analyzed. Then, through the use of a digital computer and random number generator, a large number of iterations are performed on the model formulation. During these iterations, input data is randomly generated according to selected probability distributions for each parameter in the model. Thus, successive iterations are performed on the proposed model formulation, each time using a completely new set of model parameters. Once the iterations are complete, the generated isolation set can be analyzed in entirety to determine its stochastic properties or the individual values may be used in further analysis (Brutsaert, 1975; Rubinstein, 1981). The application of Monte Carlo simulation has been made in succeeding sections of this and remaining chapters.

In this study a set of theoretical distributions including normal, log-normal, gamma, Weibull, and beta distributions are considered as candidates for each of the parameters in water quality models. In addition to assigning a distribution to each of the parameters in water quality models, the statistical properties including the mean and standard deviation of the parameters are specified. In cases that water quality parameters are assumed to be independent of each other, pseudo random realizations of each water quality parameter are generated independently according to their associated probability distributions and statistical properties specified.

When applying Monte Carlo simulation to Eqs. (1.5) and (1.6), it is possible to generate negative DO concentrations. Though the number of occurrences of such unrealistic values is low, provision should be included in the simulation procedures which constrain the DO concentrations to be greater than or equal to zero (Hornberger, 1980). In the methods utilized in this study, negative DO concentrations generated by simulation are simply ignored and replaced by another iteration until specified numbers of realistic conditions are established.

The probability density function (pdf), moment-parameter relations, and subroutines in International Mathematical Subroutine Library (IMSL) used for generating random numbers are listed in Table 4.1. For most probability distributions (except for the Weibull), the parameters in the distributions can be easily determined from the knowledge of the mean and standard deviation. For the Weibull distribution, Newton-Ralphson method is applied to solve for α as it is related to the coefficient of variation (Cv). Then the second parameter β can be calculated easily once α is computed.

When parameters K_a and U are considered correlated, a bivariate normal distribution is employed to model their log-transformed scale as well as the original scale. Generating bivariate normal random realizations for K_a and U with a correlation coefficient $\rho(K_a,U)$ by IMSL subroutine (GGNSM) is straightforward. However, when K_a and U each have a marginal log-normal distribution and are correlated with $\rho(K_a,U)$, it is necessary to compute the correlation coefficient for log-transformed

TABLE 4.1 LIST OF PROBABILITY DISTRIBUTION MODELS USED IN THE ANALYSIS

Distribution	pdf	Parameter-Moment Relations	IMSL Routine Employed	Remarks
Normal	$f(x \mid \alpha,\beta) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\alpha}{\beta}\right)^2}$ for $-\infty < x < \infty$	$\alpha = \mu_x$ $\beta = \sigma_x$ $C_v = \sigma_x/\mu_x$	GGNML	Generate normal (0,1) random deviate z. $x = \alpha + z\beta$.
Log-normal	$f(x \mid \alpha,\beta) = \frac{1}{\sqrt{2\pi}\,\beta x} e^{-\frac{1}{2}\left(\frac{\ln x-\alpha}{\beta}\right)^2}$ for $x > 0$	$\alpha = \frac{1}{2} \ln\left[\frac{\mu_x}{1 + C_v^2}\right]$ $\beta = \sqrt{\ln[1 + C_v^2]}$	GGNML	$x = \exp(y)$ where $y = \alpha + z\beta$.
Gamma	$f(x \mid \alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$ for $x > 0$	$\alpha = \mu_x^2/\sigma_x^2$ $\beta = \mu_x/\sigma_x^2$	GGMAR	Generate Gamma $(\alpha,\beta = 1)$ random deviate y. $x = y/\beta$

TABLE 4.1 (continued)

Distribution	pdf	Parameter-Moment Relations	IMSL Routine Employed	Remarks
Weibull	$f(x\|\alpha,\beta) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}}$ for $x > 0$	$\mu_x = \beta\,\Gamma(1 + \frac{1}{\alpha})$ $C_v^2 = 1 - \Gamma(1 + \frac{1}{\alpha})/\Gamma^2(1 + \frac{1}{\alpha})$	GGWIB	Generate Weibull $(\alpha,\beta = 1)$ random deviate y. $x = \beta y$
Beta	$f(x\|\alpha,\beta) = y^{\alpha-1}(1 - y)^{\beta-1}/B(\alpha,\beta)$ for $0 \leq y \leq 1$, where $y = \frac{x - x_\ell}{x_u - x_\ell}$	$\mu_y = (\mu_x - x_\ell)/(x_u - x_\ell)$ $\mu_y = \sigma_x/(x_u - x_\ell)$ $\alpha = \mu_y^2(1 - \mu_y)/\sigma_y^2 - \mu_y$ $\beta = \mu_y(1 - \mu_y)/\sigma_y^2 - (1 - \mu_y)$	GGBTR	Generate Beta (α,β) random deviate y. $x = x_\ell + y(x - x_\ell)$

TABLE 4.1 (continued)

Distribution	pdf	IMSL Routine Employed	Remarks
Bivariate Normal	$f(x,y/\mu_x,\mu_y,\sigma_x,\sigma_y,\rho_{xy})$ $= \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho_{xy}^2}} \exp\left[\frac{Q}{2(1-\rho_{xy}^2)}\right]$ where $Q = \left(\frac{x-\mu_x}{\sigma_x}\right)^2 - \frac{2\rho_{xy}(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y} + \left(\frac{y-\mu_y}{\sigma_y}\right)^2$	GGNSM	

K_a and U. By applying the concept of a moment generating function, the relation between $\rho(K_a,U)$ and $\rho(\ln K_a, \ln U)$ can be derived as

$$\rho(\ln K_a, \ln U) = \frac{\ln[1 + \rho(K_a,U) \cdot Cv(K_a) \cdot Cv(U)]}{\ln[1 + Cv^2(K_a)]^{0.5} \ln[1 + Cv^2(U)]^{0.5}} \qquad (4.5)$$

in which $Cv(K_a)$ and Cv(U) are the coefficients of variations of K_a and U, respectively. After the correlation coefficient between $\ln K_a$ and lnU is computed, the same subroutine GGNSM can be called to generate bivariate normally distributed values of $\ln K_a$ and lnU, from which a retransformation back to their original scale can be made.

4.4.3 Quantifying the Risk Associated with Various Violation Conditions

As previously noted, the joint risk is defined as the probability of occurrence for a given pair of violation conditions (i.e. a maximum or average exceeding DO deficit and length of violation). Direct analytical methods were shown to be infeasible as solution techniques to quantify these risks. Therefore, Monte Carlo simulation techniques are applied.

Various pairs of violation conditions are generated using Eqs. (1.5), (1.7), (1.8), and (4.4). In order to describe the random characteristics of the input data, each parameter in the water quality model is assigned one of the five probability distributions utilized in this study (normal, log-normal, gamma, beta, and Weibull), along with their associated statistical properties. Through the use of Monte Carlo simulation and Newton-Raphson's numerical technique, N pairs of violation conditions are generated for various DO deficits and lengths of

violation. The risk is then calculated by simply computing the ratio of the number of simulation pairs that jointly exceeded a specified deficit and length of violation, n, to the total random sample, N, generated:

$$\text{risk} = \frac{n}{N + 1} \tag{4.6}$$

In order to choose an appropriate sample size, random samples of various sizes were examined, finding that the joint probability of violating a specified pair of maximum deficit and length of violation differed only slightly for various sample sizes, between 500 and 2,000. Thus, an intermediate number of N = 999 is adopted as the satisfactory sample size in this study.

4.5 EXAMPLE OF APPLICATION

To illustrate the approach, an example is formulated using hypothetical data for each parameter in Eq. (1.5). The selection of the mean of the water quality model parameters is based on a general stream classification described as "low velocity" (Fair et al, 1968; Chadderton et al., 1982). In addition, the standard deviations for each of the model parameters are selected in accordance with the data presented by Chadderton et al. (1982). To complete the data set, a correlation coefficient, between K_a and U, of 0.8 is adopted on the basis of the experimental data tabulated in the article presented by Isaacs (1969). It should be again noted that when the correlation between K_a and U is specified, a bivariate normal or log-normal

distribution is used. A summary of the model input data for the parameters of this study is given in Table 4.2.

The joint probability of violation was evaluated for a combination of 15 maximum DO deficits, beyond the standard of 4.0 mg/l, (ranging from 0.0 to 1.5 mg/l) and 15 lengths of violation (ranging from 0.0 to 30.0 miles). The resulting 225 pairs of violation conditions were used to construct a contour map of the joint risk associated with the given combinations of maximum DO deficits and length of violation (see Figure 4.2 as an example). In order to illustrate the sensitivity of the risk to varying statistical characteristics, the procedure was iterated by assigning a variety of probability distributions to each of the parameters in the model. The entire process was repeated, this time developing risk contour maps based on various combinations of average exceeding deficits and lengths of violation.

4.6. DISCUSSION OF RESULTS

Figures 4.2 through 4.11 illustrate the contours of risk associated with the various assumptions for the probability distributions assigned to each parameter in Eq. (1.5) and the correlation between K_a and U. Each figure is documented with a heading providing information about the type of distribution, the mean, standard deviation, and correlation coefficient (ρ) assumed for the parameters in the risk assessment. Several combinations were explored for the various types of distribution utilized in this study. In order to analyze the results of this study, the discussion will focus on the sensitivity of the risk to variations with respect to the following factors: (a) the

TABLE 4.2 SUMMARY OF DATA FOR MODEL PARAMETERS

Parameter	Units	Mean	Standard Deviation	Remarks
K_d	$days^{-1}$	0.35	0.10	
K_a	$days^{-1}$	0.70	0.20	$\rho(K_a, U) = 0.8$
U	ft./sec.	0.61	0.18	
L_o	mg/1	18.00	1.00	
D_o	mg/1	1.00	0.30	

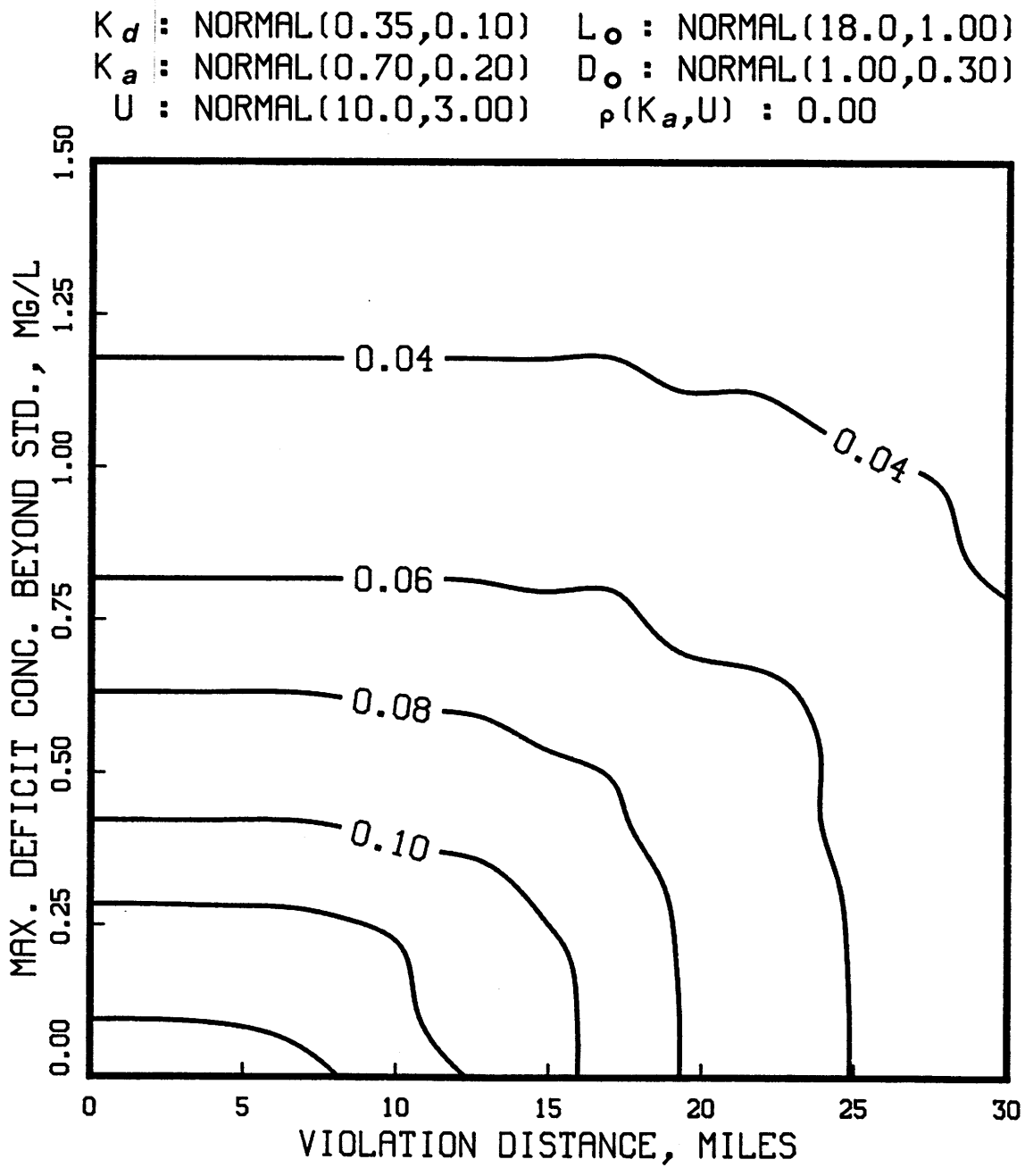

Figure 4.2 Contour Of Joint Risk Associated With Maximum Deficits And Length Of Violation For An All Normal Assumption Of The Model Parameters And Zero Correlation

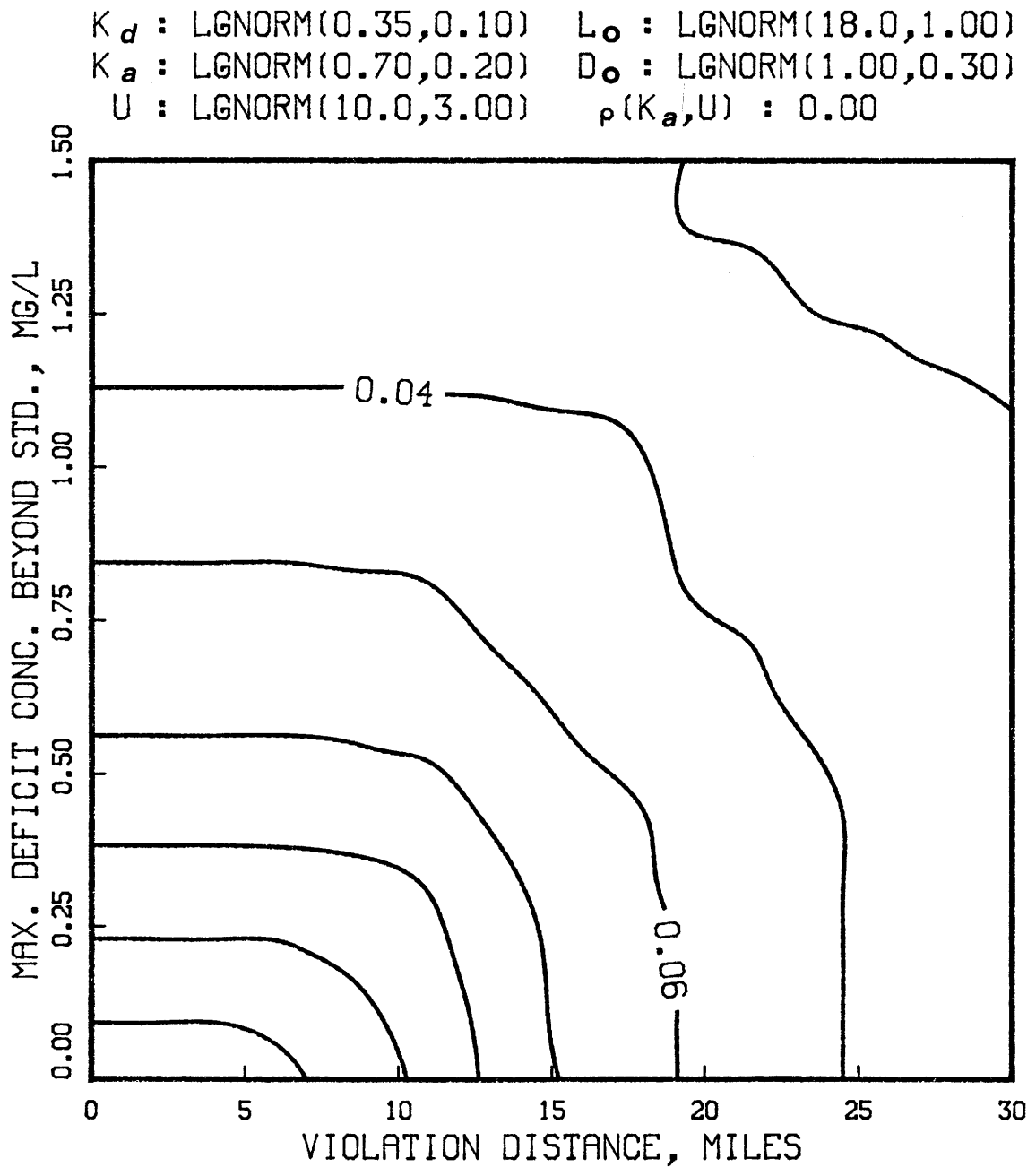

Figure 4.3 Contour Of Joint Risk Associated With Maximum Deficits And Length Of Violation For An All Lognormal Assumption Of The Model Parameters And Zero Correlation

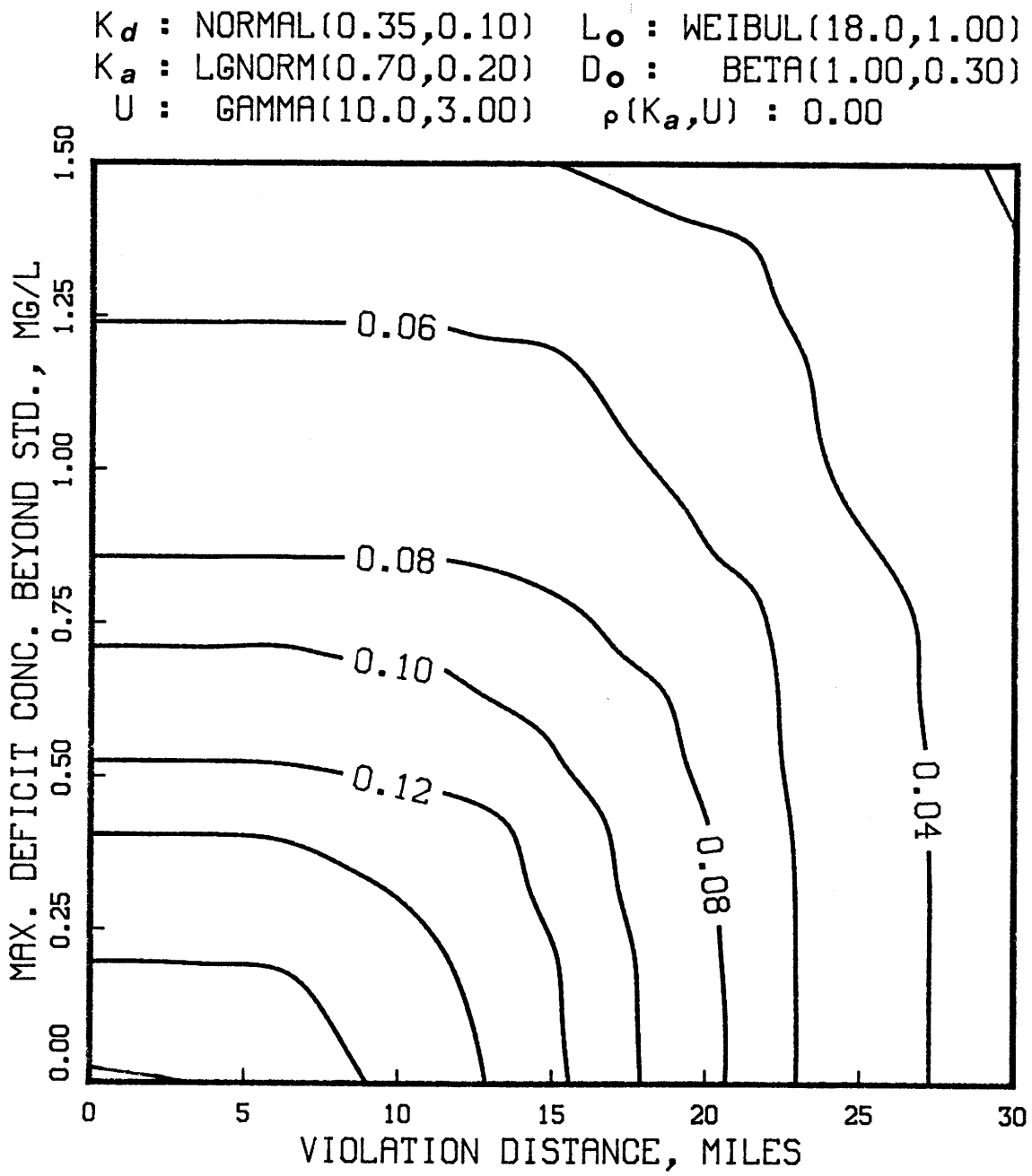

Figure 4.4 Contour Of Joint Risk Associated With Maximum Deficits And Length Of Violation For The Variety Of Distributions Assumed For The Model Parameters And Zero Correlation

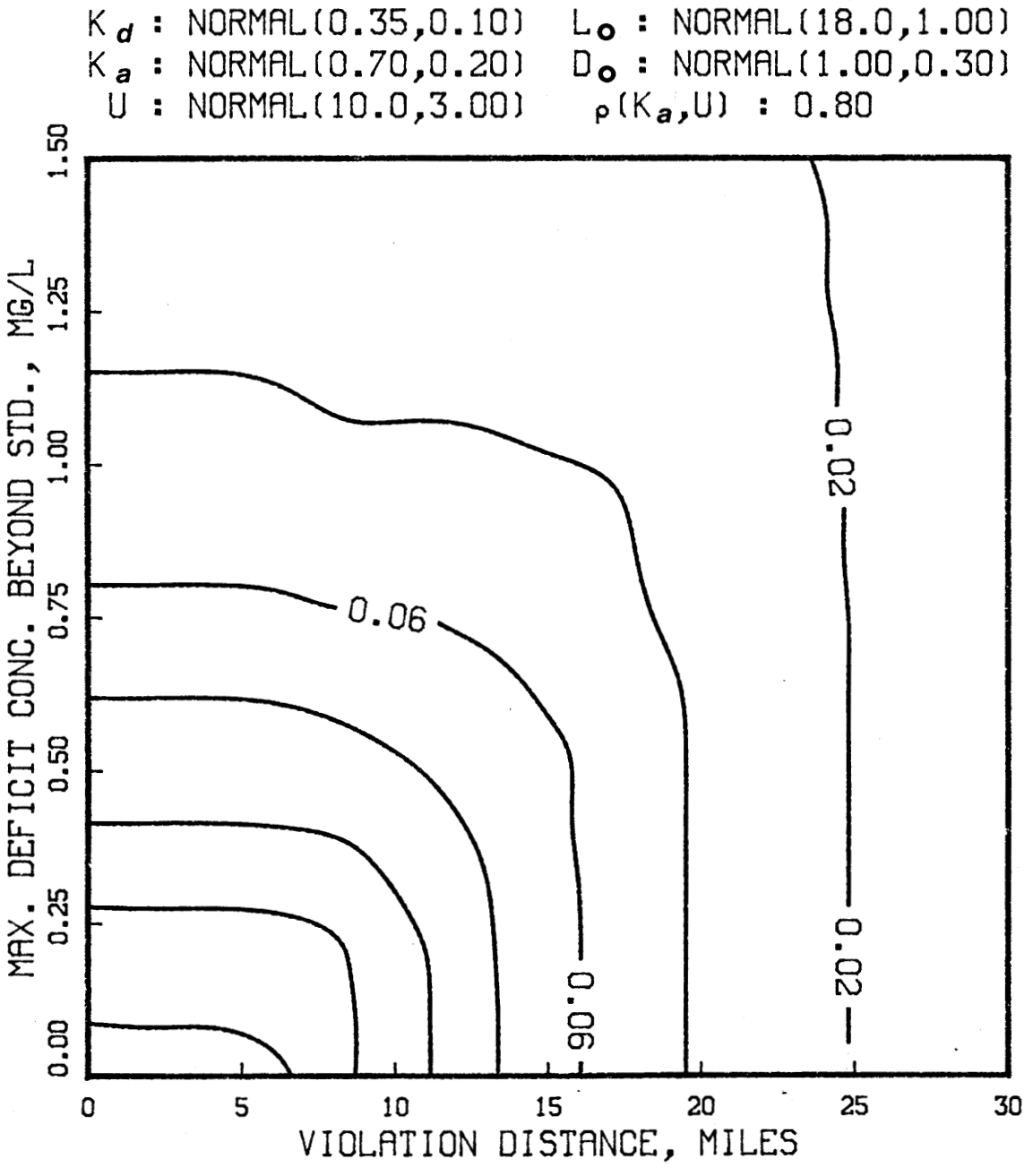

Figure 4.5 Contour Of Joint Risk Associated With Maximum Deficits And Length Of Violation For An All Normal Assumption Of The Model Parameters And Positive Correlation

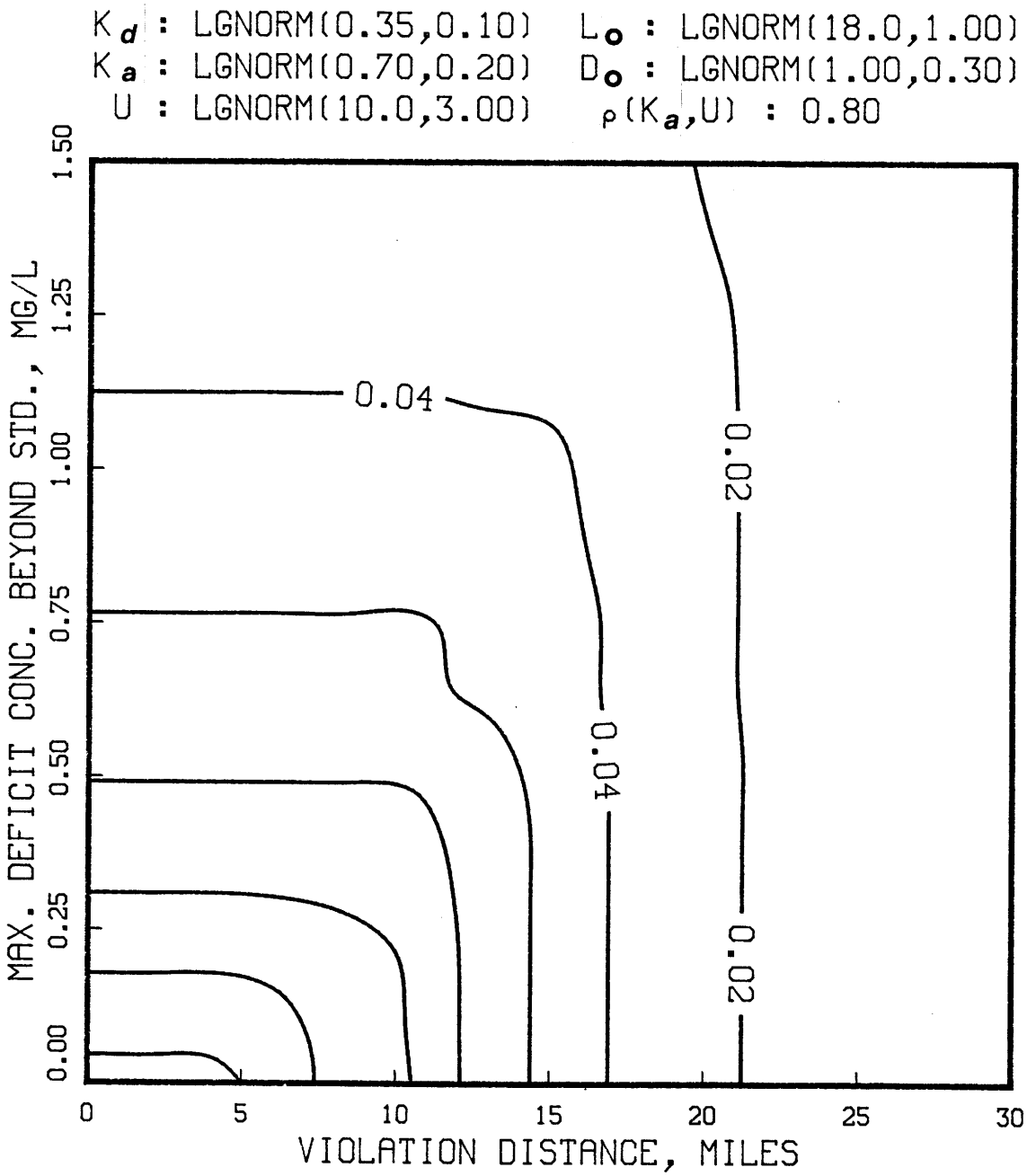

Figure 4.6 Contour Of Joint Risk Associated With Maximum Deficits And Length Of Violation For An All Lognormal Assumption Of The Model Parameters And Positive Correlation

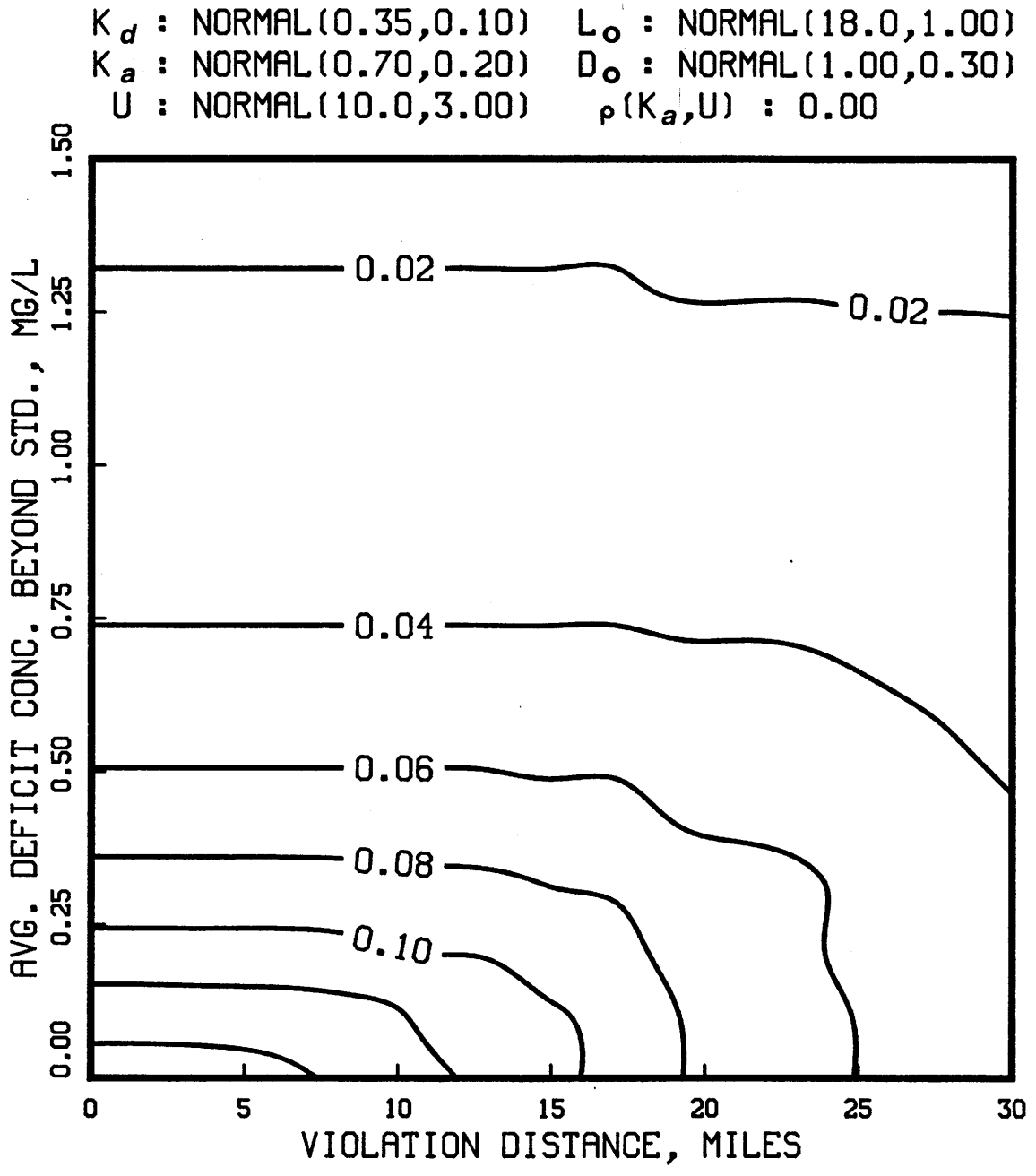

Figure 4.7 Contour Of Joint Risk Associated With Average Deficits And Length Of Violation For An All Normal Assumption Of The Model Parameters And Zero Correlation

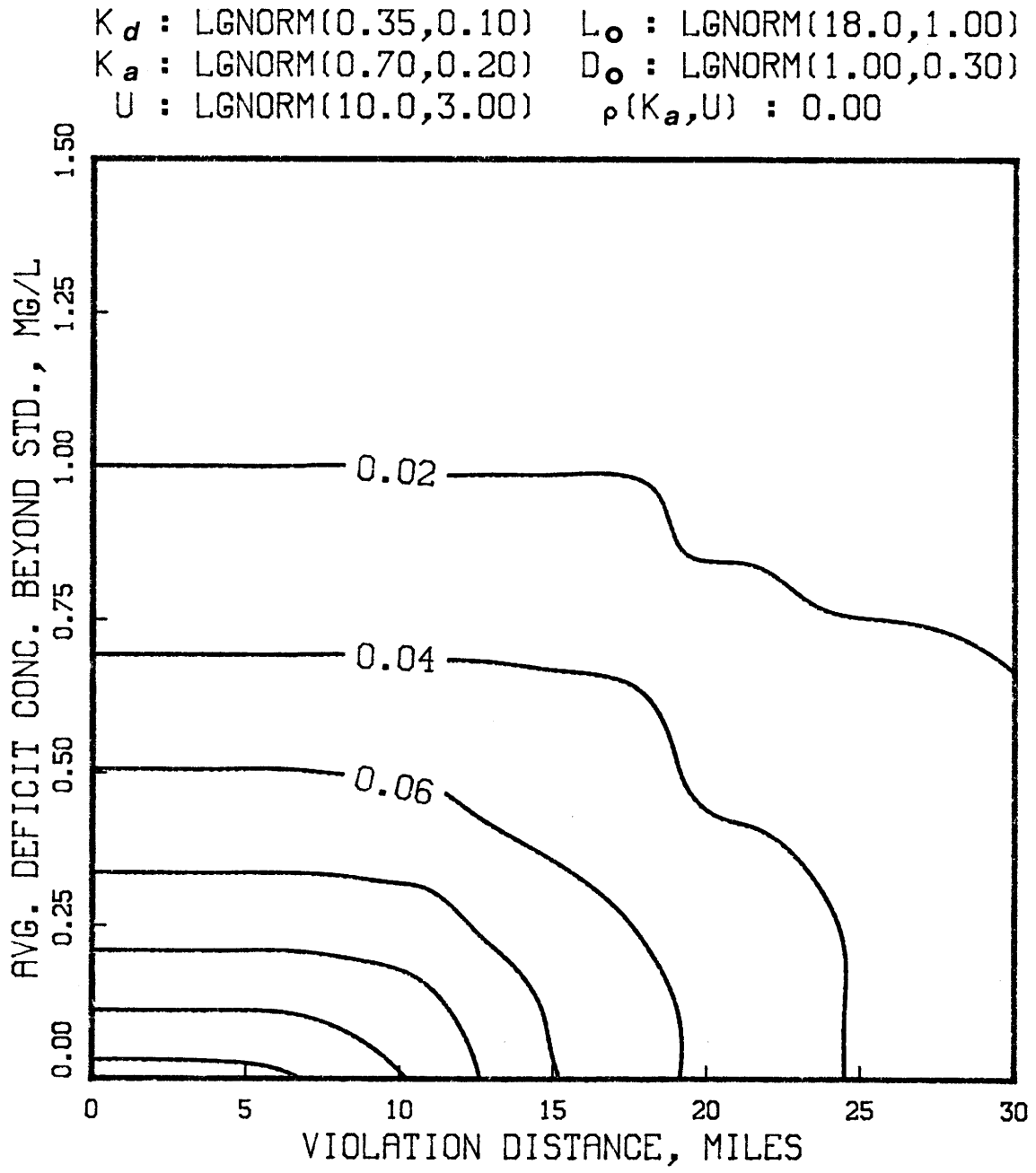

Figure 4.8 Contour Of Joint Risk Associated With Average Deficits And Length Of Violation For An All Lognormal Assumption Of The Model Parameters And Zero Correlation

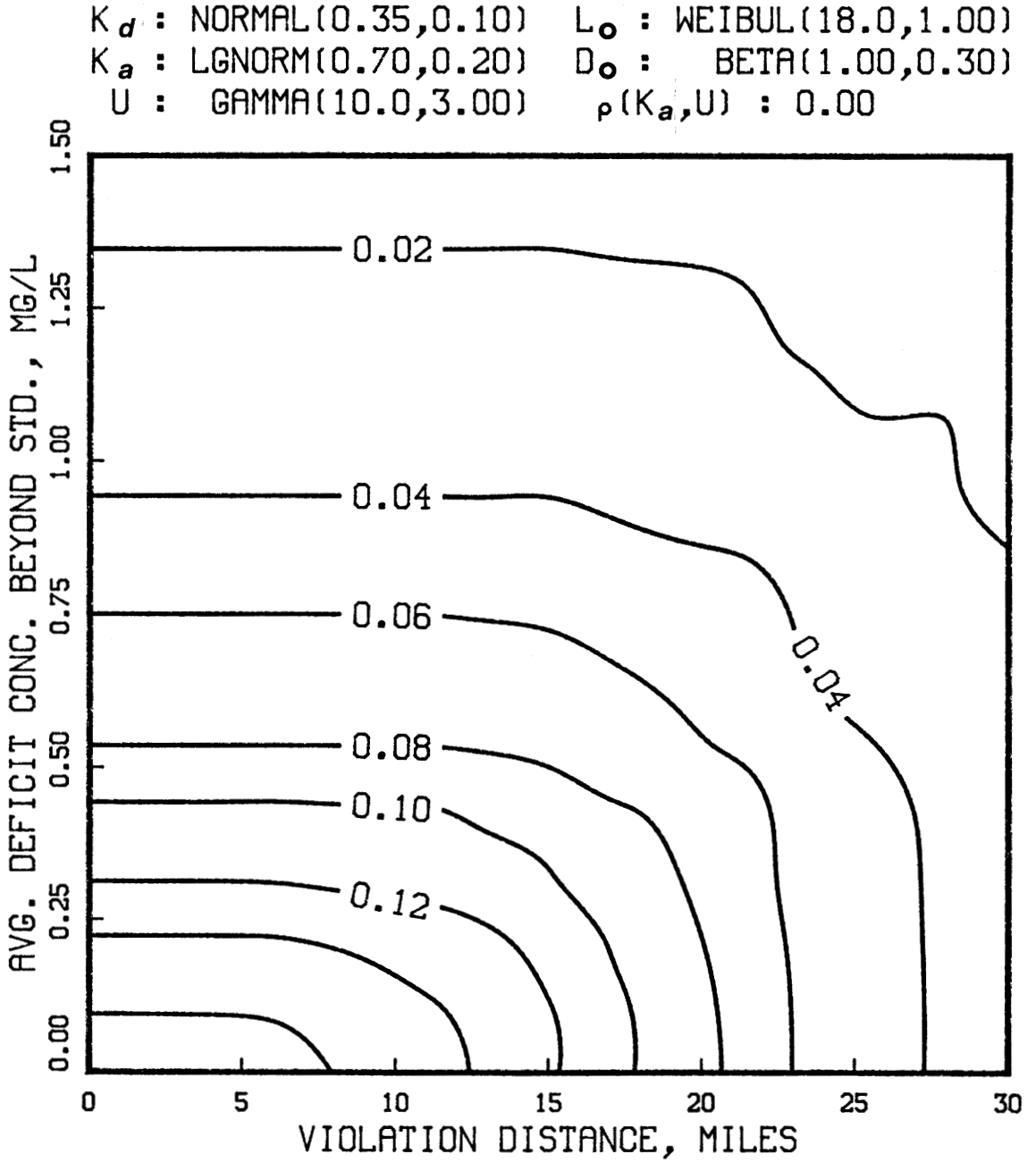

Figure 4.9 Contour Of Joint Risk Associated With Average Deficits And Length Of Violation For The Variety Of Distributions Assumed For The Model Parameters And Zero Correlation

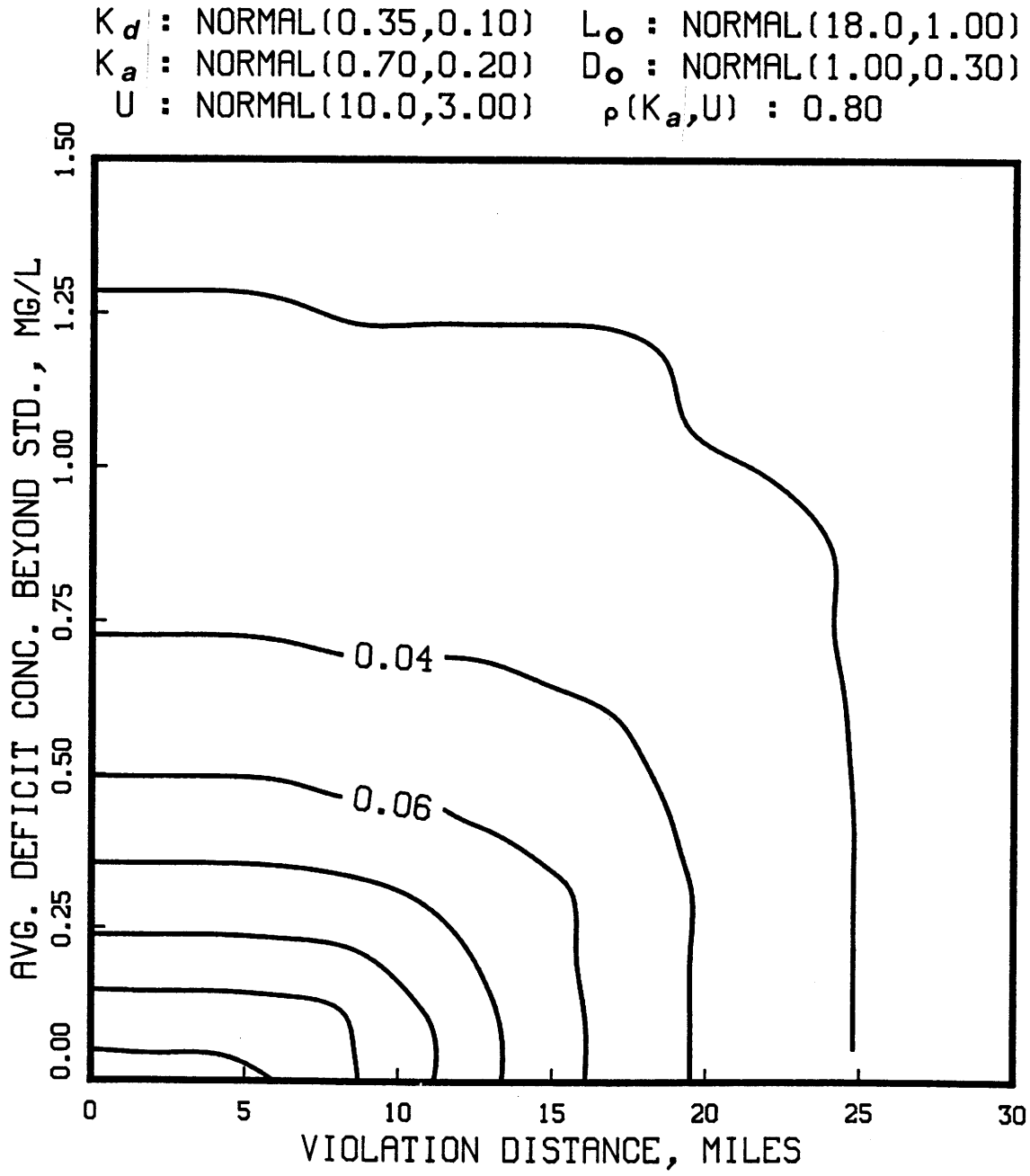

Figure 4.10 Contour Of Joint Risk Associated With Average Deficits And Length Of Violation For An All Normal Assumption Of The Model Parameters And Positive Correlation

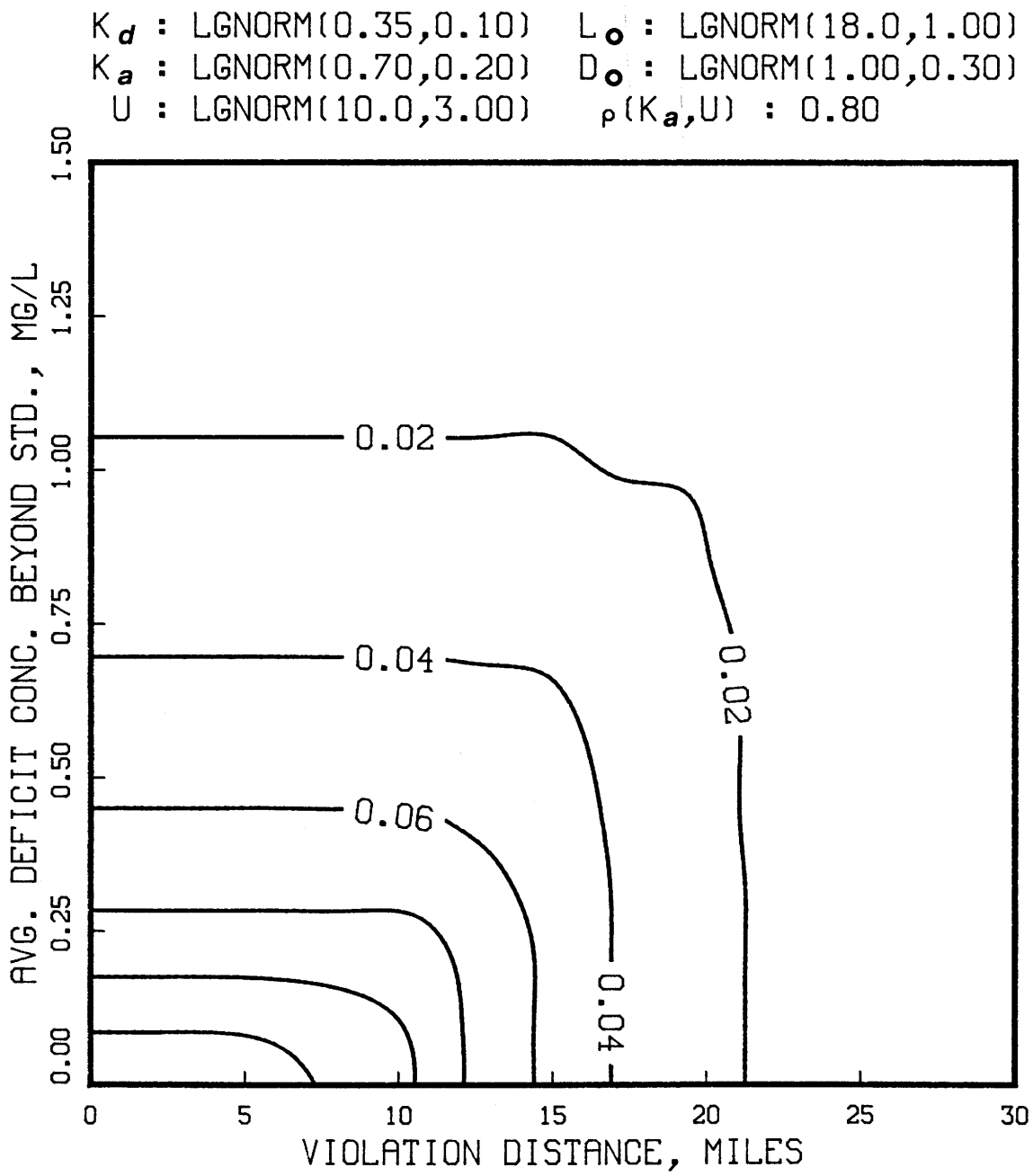

Figure 4.11 Contour Of Joint Risk Associated With Average Deficits And Length Of Violation For An All Lognormal Assumption Of The Model Parameters And Positive Correlation

probability distribution assigned to the model parameters; (b) the correlation between K_a and U; and (c) the statistical properties assigned to each parameter. In the following discussion, Figure 4.2 will be used as a basis for the comparison of other figures because a number of previous studies have utilized the assumptions of normal distribution and independency for all the model parameters. Though the sensitivity of DO response to changes in water quality parameters has been investigated by many researchers (Burgess and Lettenmaier, 1975; Esen and Rathbun, 1976; Hornberger, 1980; Chadderton et al., 1982), this study provides an attempt to evaluate the effects of the uncertainty of model parameters on the risk of violating water quality standards.

4.6.1 Sensitivity of the Risk to Variations in Probability Models

Initially, it is obvious from visual inspection of the figures presented that the type of distribution adopted for each model parameter has a significant impact on the resulting joint risk for both the maximum (see Figures 4.2-4.6) and average (see Figures 4.7-4.11) DO deficits. Using Figure 4.2 as a basis for comparison, closer examination of the results for the maximum deficits, presented in Figures 4.2 and 4.4, shows an average increase of about 30 percent in the risk for the variety of distributions selected in Figure 4.4. Conversely, a 20 percent average decrease is observed in the comparison between Figures 4.2 and 4.3. From these figures, it is evident that the risk is significantly affected by the distributions assumed for each parameter when considering maximum deficits of violation. Therefore, in order to

accurately assess the risk associated with specific violation conditions, the results of this portion of the study show that care should be given to the appropriate selection of the probability distribution for each parameter in the water quality model.

A comparison of the risk contour maps for the average deficits (Figures 4.7-4.11) with those of the maximum deficits (Figures 4.2-4.6) show an overall reduction in the risk associated with the average violation conditions. This would be expected since the average DO deficit beyond the specified standard over the length of violation is lower than that of the maximum deficit (see Figure 4.1). In addition, a comparison of the results among the average deficit conditions reveal the same general trends as those presented for the maximum deficits, thus reconfirming the sensitivity of the risk of violating water quality standards to the type of distribution assumed for each parameter in the water quality model.

In order to clearly summarize these conclusions and present additional combinations of the distributions selected for K_d, K_a, U, L_o and D_o, two tables have been constructed: (1) Table 4.3 contains the risk of violation for a variety of distributions assumed for the model parameters at select maximum violation conditions; and (2) Table 4.4 contains the difference in risk (percent) between the standard assumption of normality for the model parameters and the variety of distributions assumed.

TABLE 4.3 RISK OF VIOLATION FOR VARIOUS TYPES OF DISTRIBUTIONS ASSUMED FOR MODEL PARAMETERS WITH ZERO CORRELATION BETWEEN K_a AND U.

Case	Type of Distribution Assumed					(Max. Deficit Beyond Std., Distance of Violation)			
No.	K_d	K_a	U	L_o	D_o	(0.4,8.0)	(0.4,12.0)	(1.0,20.0)	(1.5,30.0)
1	N	N	N	N	N	.114	.065	.048	.028
2	LN	LN	LN	LN	LN	.107	.065	.038	.013
3	G	G	G	G	G	.104	.071	.052	.012
4	W	W	W	W	W	.122	.087	.068	.040
5	B	B	B	B	B	.100	.068	.044	.014
6	N	LN	LN	G	G	.101	.050	.031	.010
7	N	LN	G	W	B	.146	.095	.059	.018
8	B	N	N	LN	N	.121	.085	.057	.029
9	N	LN	LN	G	W	.104	.056	.037	.011
10	N	LN	LN	G	B	.094	.050	.021	.001

N = Normal; LN = Log-normal; G = Gamma; W = Weibull; B = Beta

TABLE 4.4 DIFFERENCE IN RISK (PERCENTAGE) BETWEEN THE STANDARD ASSUMPTION OF NORMALITY FOR THE MODEL PARAMETERS AND THE VARIETY OF DISTRIBUTIONS ASSUMED

Case	Type of Distribution Assumed					(Max. Deficit Beyond Std., Distance of Violation)			
No.	K_d	K_a	U	L_o	D_o	(0.4,8.0)	(0.4,12.0)	(1.0,20.0)	(1.5,30.0)
1	N	N	N	N	N	-	-	-	-
2	LN	LN	LN	LN	LN	-6.1	0.0	-20.8	-53.6
3	G	G	G	G	G	-8.8	9.2	8.3	-57.1
4	W	W	W	W	W	7.0	33.9	41.7	42.9
5	B	B	B	B	B	-12.3	4.6	-8.3	-50.0
6	N	LN	LN	G	G	-11.4	-23.1	-35.4	-64.3
7	N	LN	G	W	B	28.1	46.2	22.9	-35.7
8	B	N	N	LN	N	6.1	30.8	18.8	3.6
9	N	LN	LN	G	W	-8.8	-13.9	-22.9	-60.7
10	N	LN	LN	G	B	-17.5	-23.1	56.3	96.4

N = Normal; LN = Log-normal; G = Gamma; W = Weibull; B = Beta

4.6.2 Sensitivity of Risk to the Correlation Between K_a and U

Figures 4.5 and 4.6 illustrate a risk contour map when a positive correlation between K_a and U is considerated in risk assessment. The sensitivity of the risk to changes in the assumption of the correlation between K_a and U can be examined by comparing Figures 4.2 and 4.5. It can be seen from these figures that the inclusion of a positive correlation, between K_a and U, results in a significant reduction in the risk at large violation distances. A comparison of the results in these figures shows an average reduction of about 20 percent in the overall risk, and a reduction as high as 70 percent for large violation distances. These same conclusions can be emphasized in the comparison of Figures 4.3 and 4.6.

In order to explain this observation, it has been shown that an increase in the average stream velocity, U, results in an increase in the reaeration coefficient, K_a (Bansal, 1973). According to the physical process, an increase in K_a will lead to greater reaeration rates and reduced DO deficits at downstream locations. The overall effect can be seen in the reduction of the risk of violation at downstream locations. Therefore, a positive correlation between these parameters should be included in the model formulation in order to accurately describe the physical characteristics of the stream environment.

4.6.3 Sensitivity of Risk to Uncertainties in Statistical Properties

Given imperfect data collection, uncertainties arise in quantifying the statistical properties of the parameters in Eq. (1.5). The

sensitivity of the risk to these uncertainties was analyzed for changes in the mean and standard deviation associated with each parameter in the model. The results of this investigation are presented in Figures 4.12 through 4.16 for the mean, and Figures 4.17-4.21 for the standard deviation. As a basis for comparison, normal distributions were assigned to each parameter in this investigation, along with a positive correlation coefficient between K_a and U. The mean and standard deviation of each parameter was allowed to vary ±15 percent. In order to illustrate the sensitivity of the risk to these changes, figures were constructed showing the variations in a single contour of risk at six percent.

Figures 4.12-4.16 and 4.17-4.21 show that the variation of risk is more sensitive to equal percentage changes in the mean of K_d than that of equal changes in the standard deviation. This result was observed for all the parameters used in this study. These results imply that the accuracy in estimating the mean of the model parameters have a greater impact on the risk assessment than estimates for the standard deviations.

In comparing Figures 4.12-4.16, the results reveal that the risk is most sensitive to changes in the mean values of K_a and L_o, followed by K_d, U, and D_o, respectively. It is clear from these results that special attention should be given to the determination of the mean values for K_a, K_d and L_o if accurate DO predictability is to be attained. It is evident from this portion of the study that proper selection of the statistical properties is crucial in order to

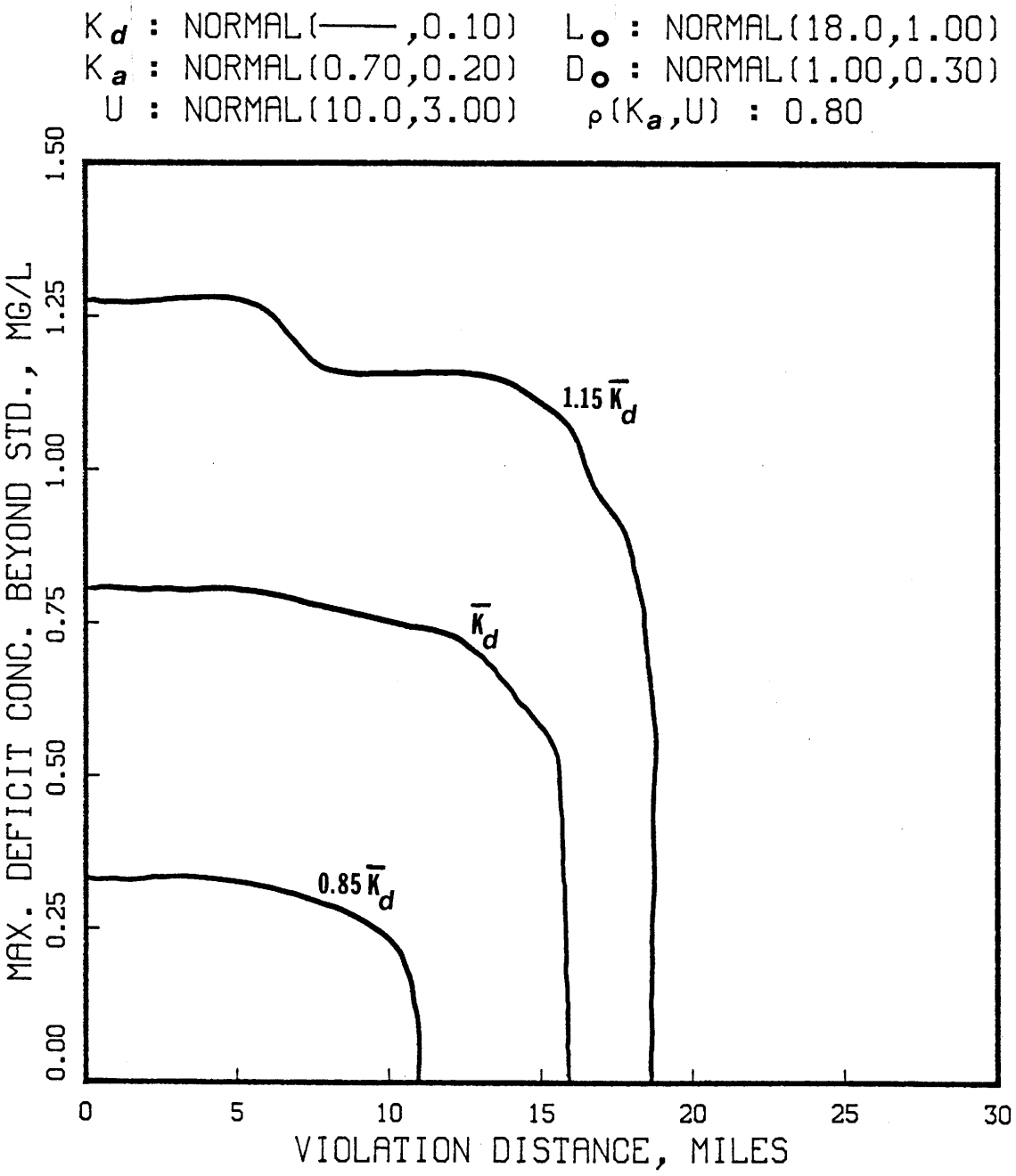

Figure 4.12 Sensitivity Of Six Percent Risk With Respect To The Mean Of K_d

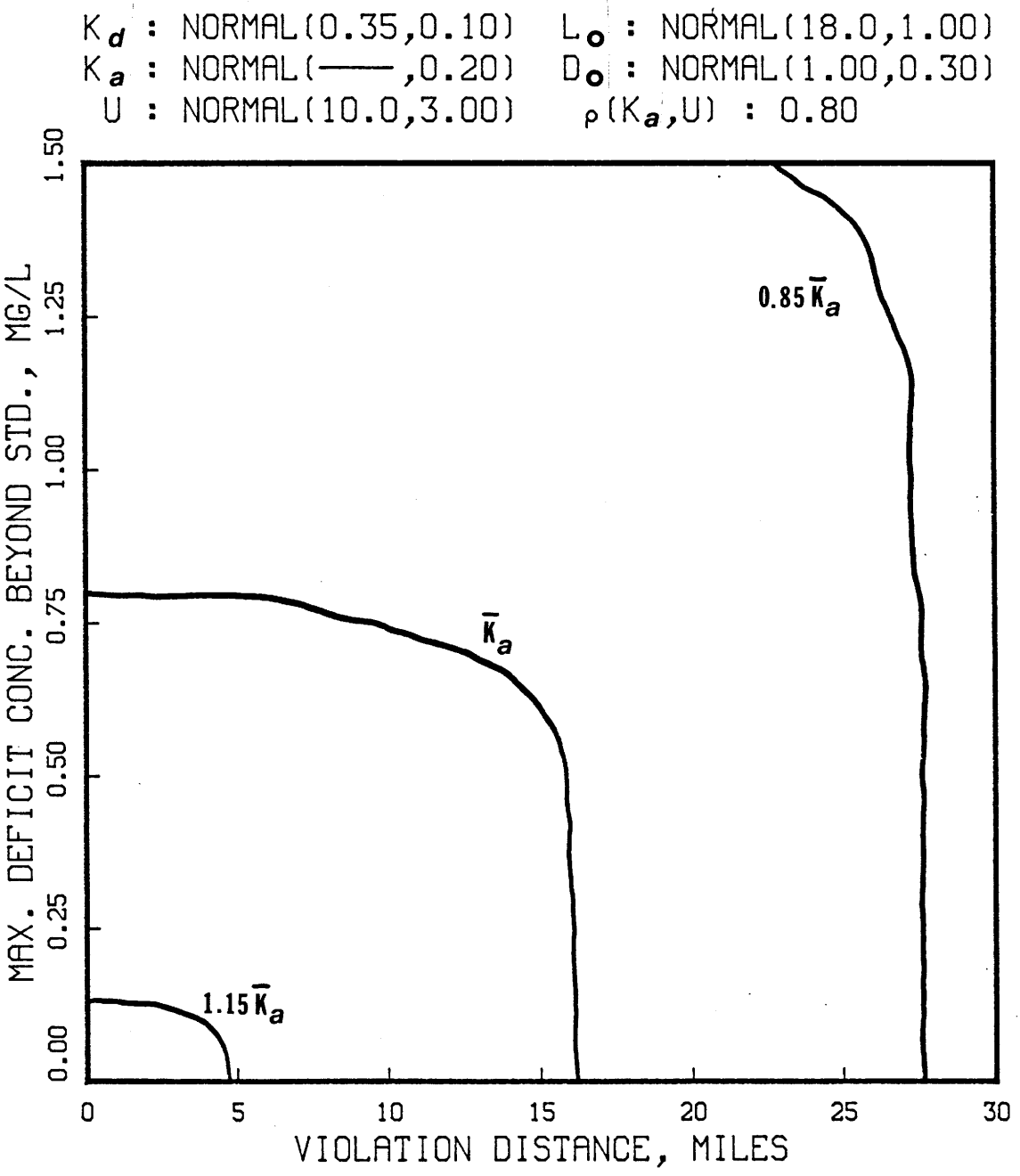

Figure 4.13 Sensitivity Of Six Percent Risk With Respect To The Mean Of K_a

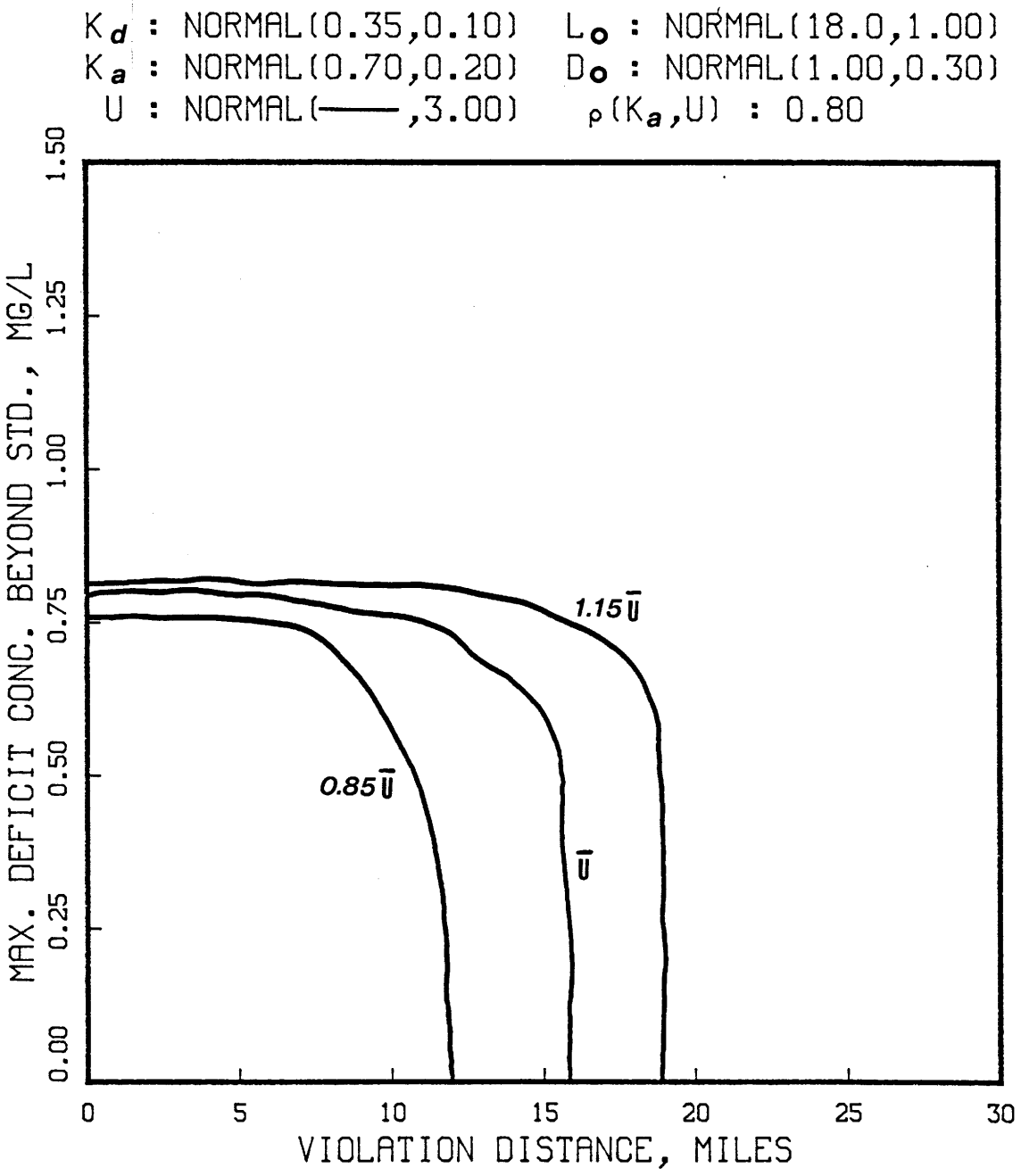

Figure 4.14 Sensitivity Of Six Percent Risk With Respect To The Mean Of U

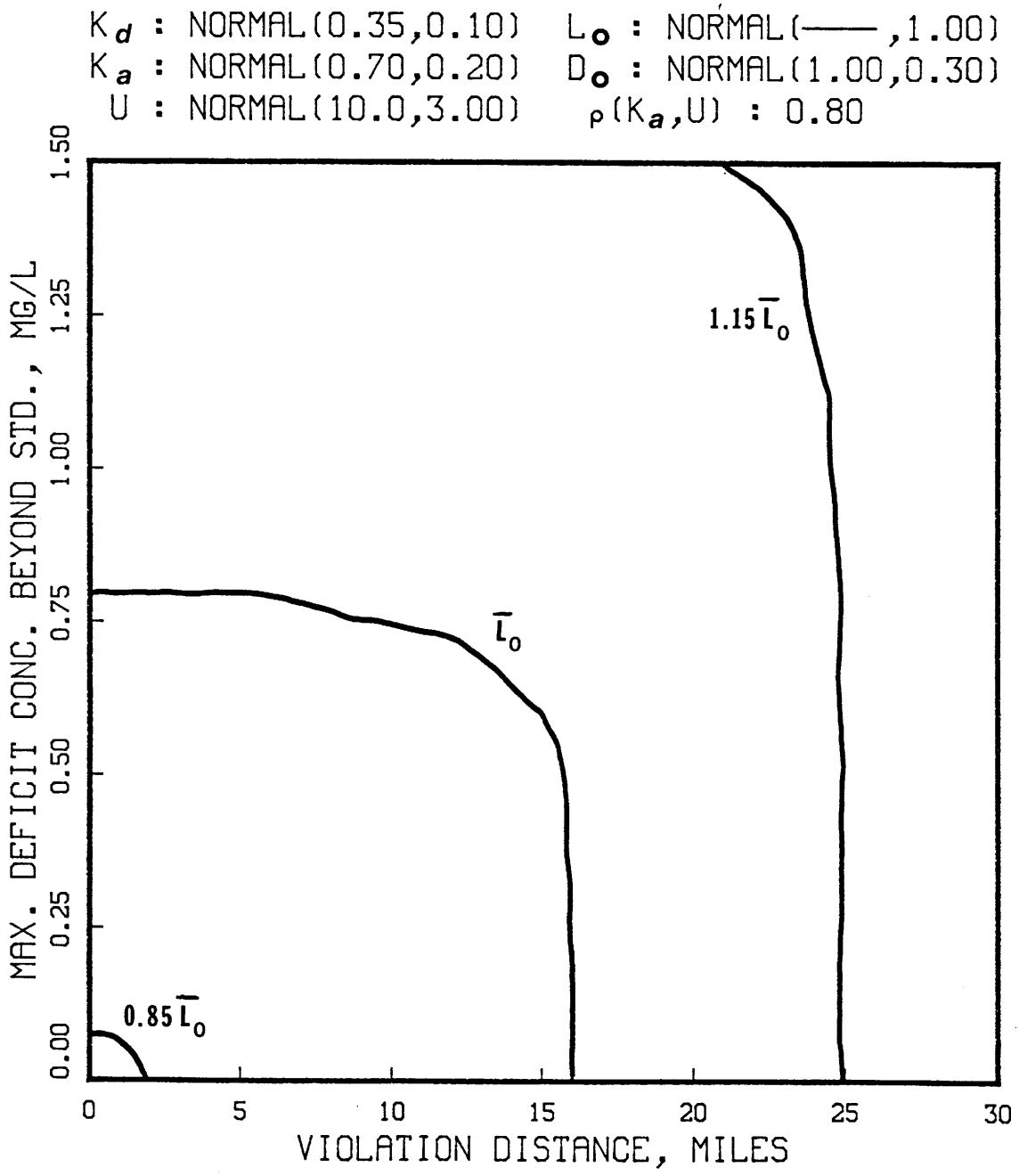

Figure 4.15 Sensitivity Of Six Percent Risk With Respect To The Mean Of L_o

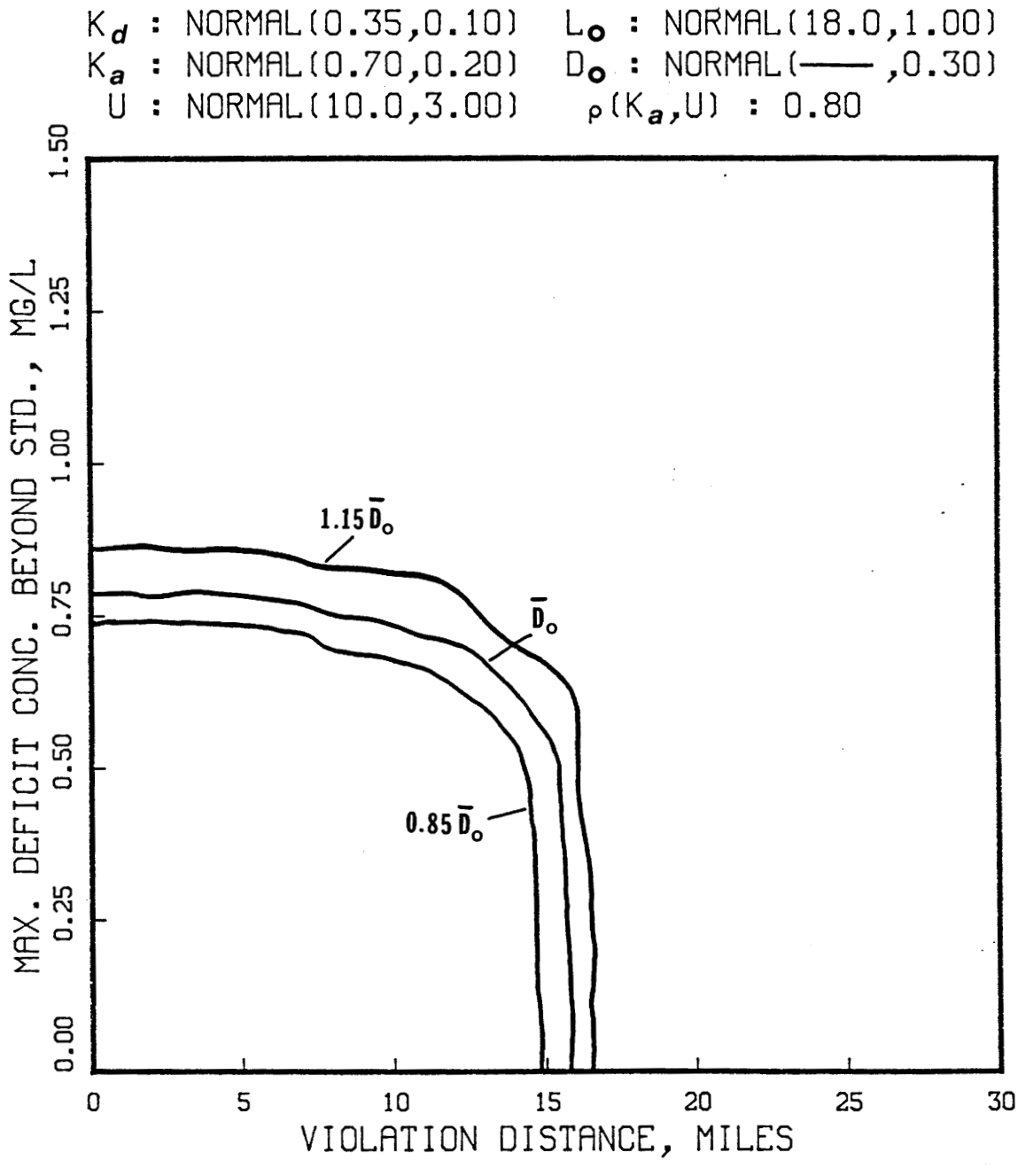

Figure 4.16 Sensitivity Of Six Percent Risk With Respect To The Mean Of D_o

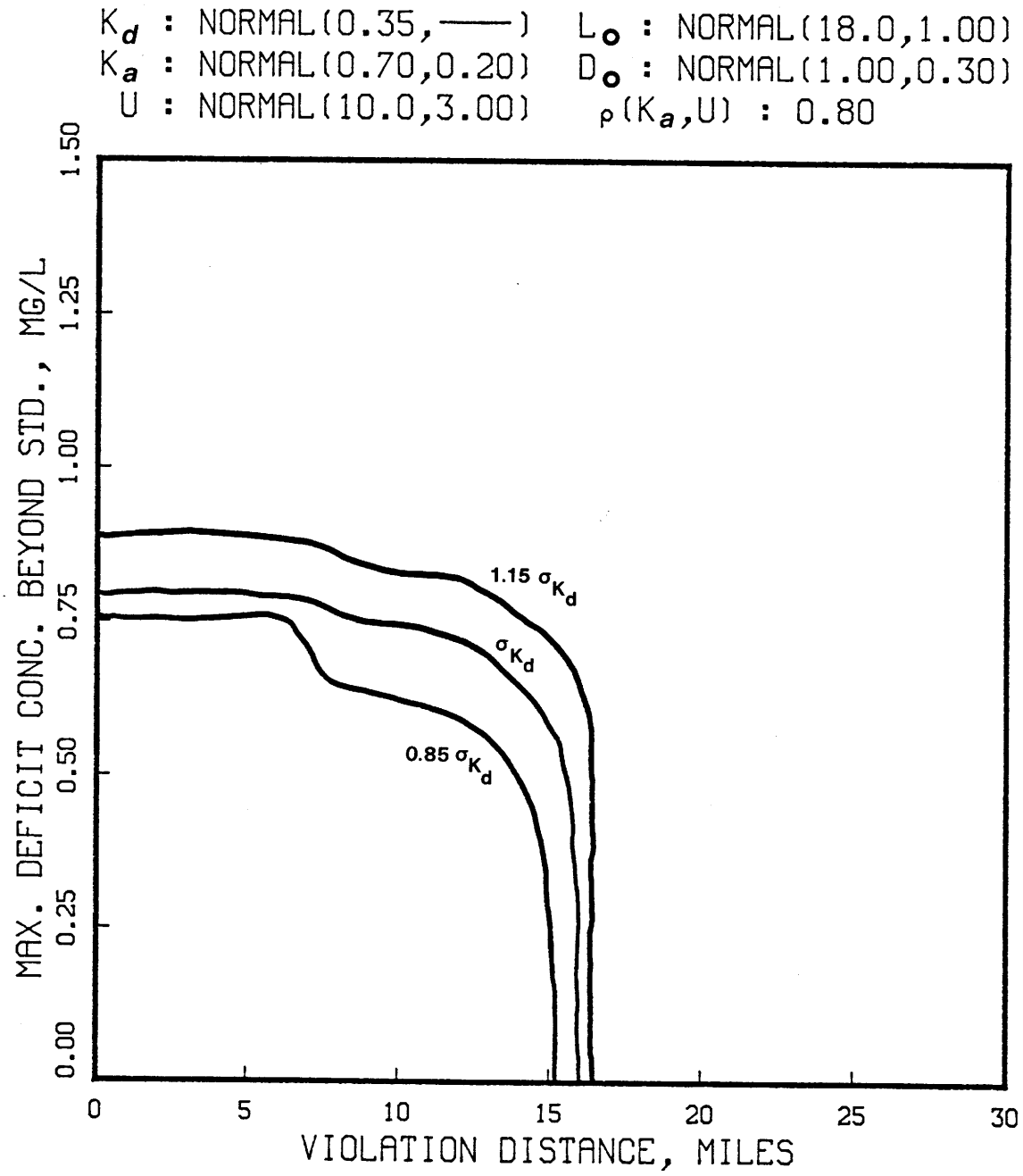

Figure 4.17 Sensitivity Of Six Percent Risk With Respect To The Standard Deviation Of K_d

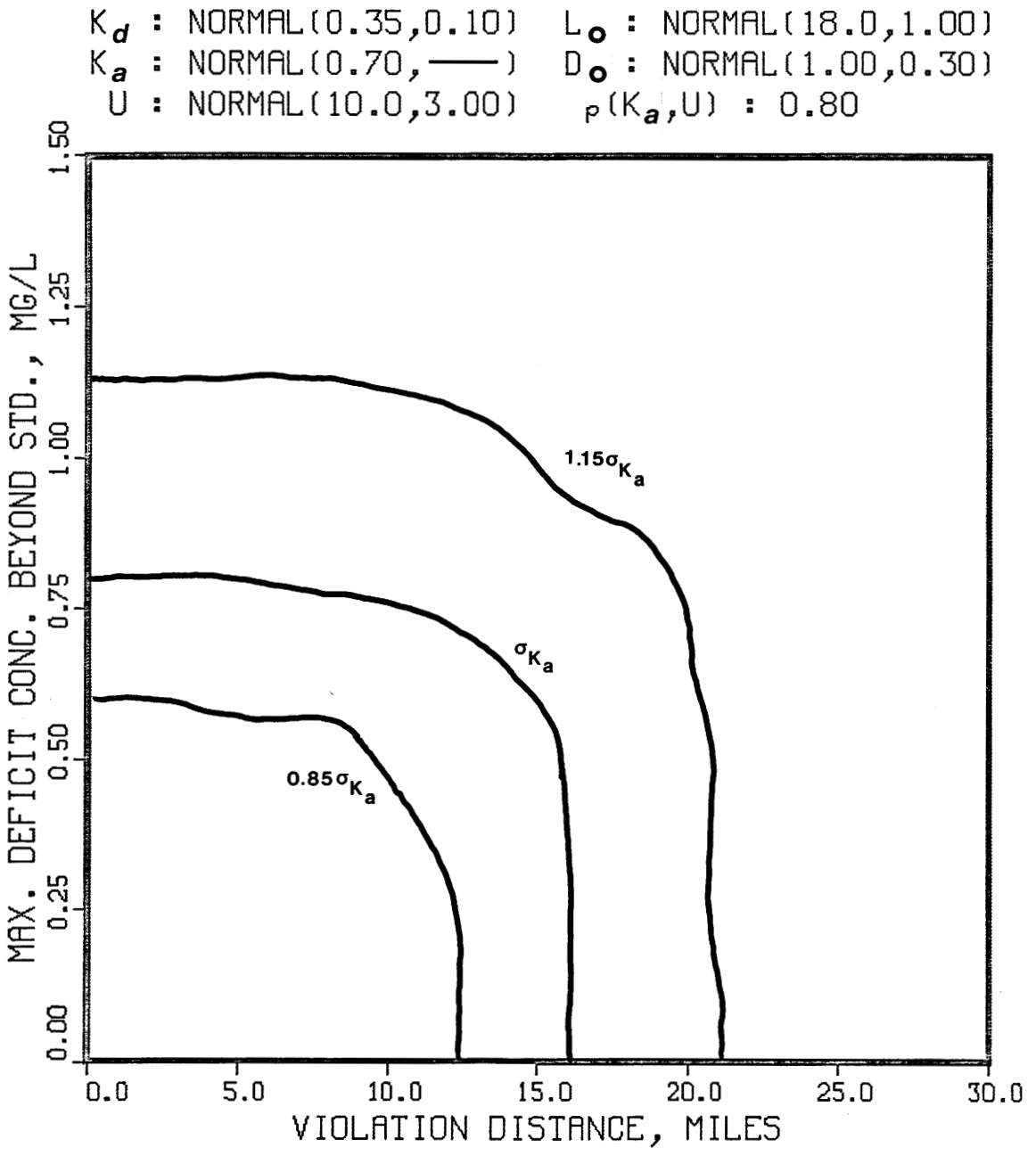

Figure 4.18 Sensitivity Of Six Percent Risk With Respect To The Standard Deviation Of K_a

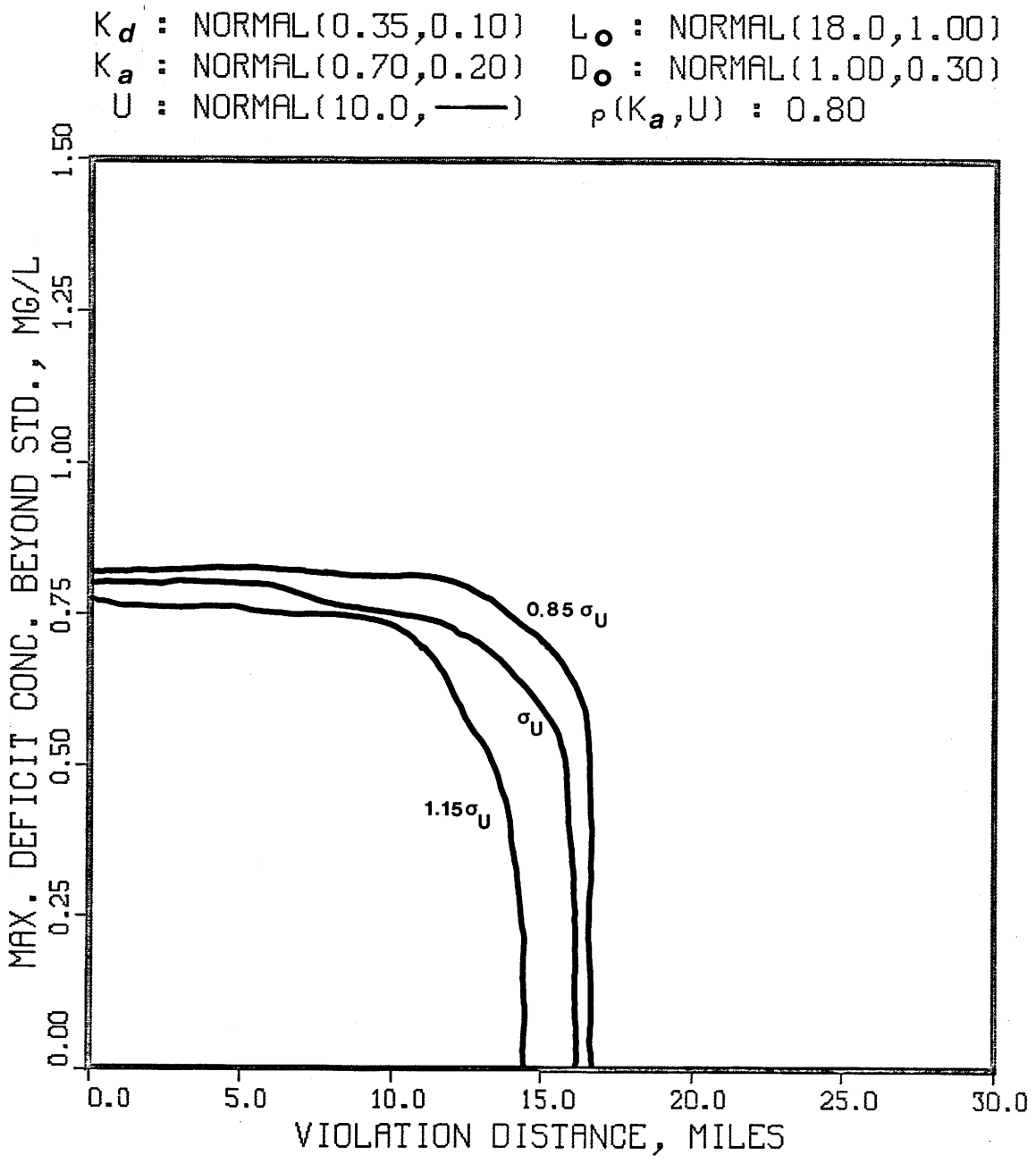

Figure 4.19 Sensitivity Of Six Percent Risk With Respect To The Standard Deviation Of U

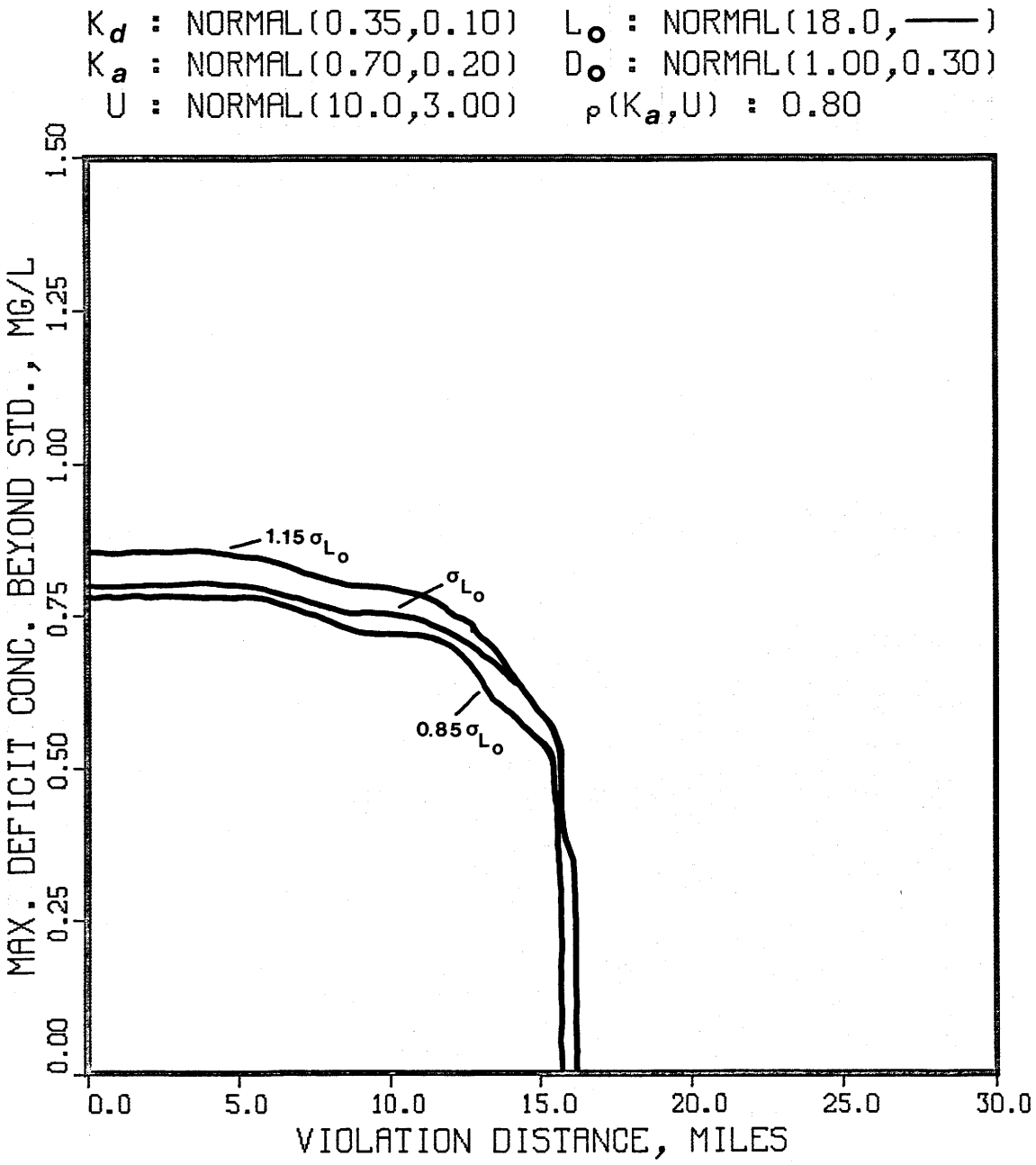

Figure 4.20 Sensitivity Of Six Percent Risk With Respect To The Standard Deviation Of L_o

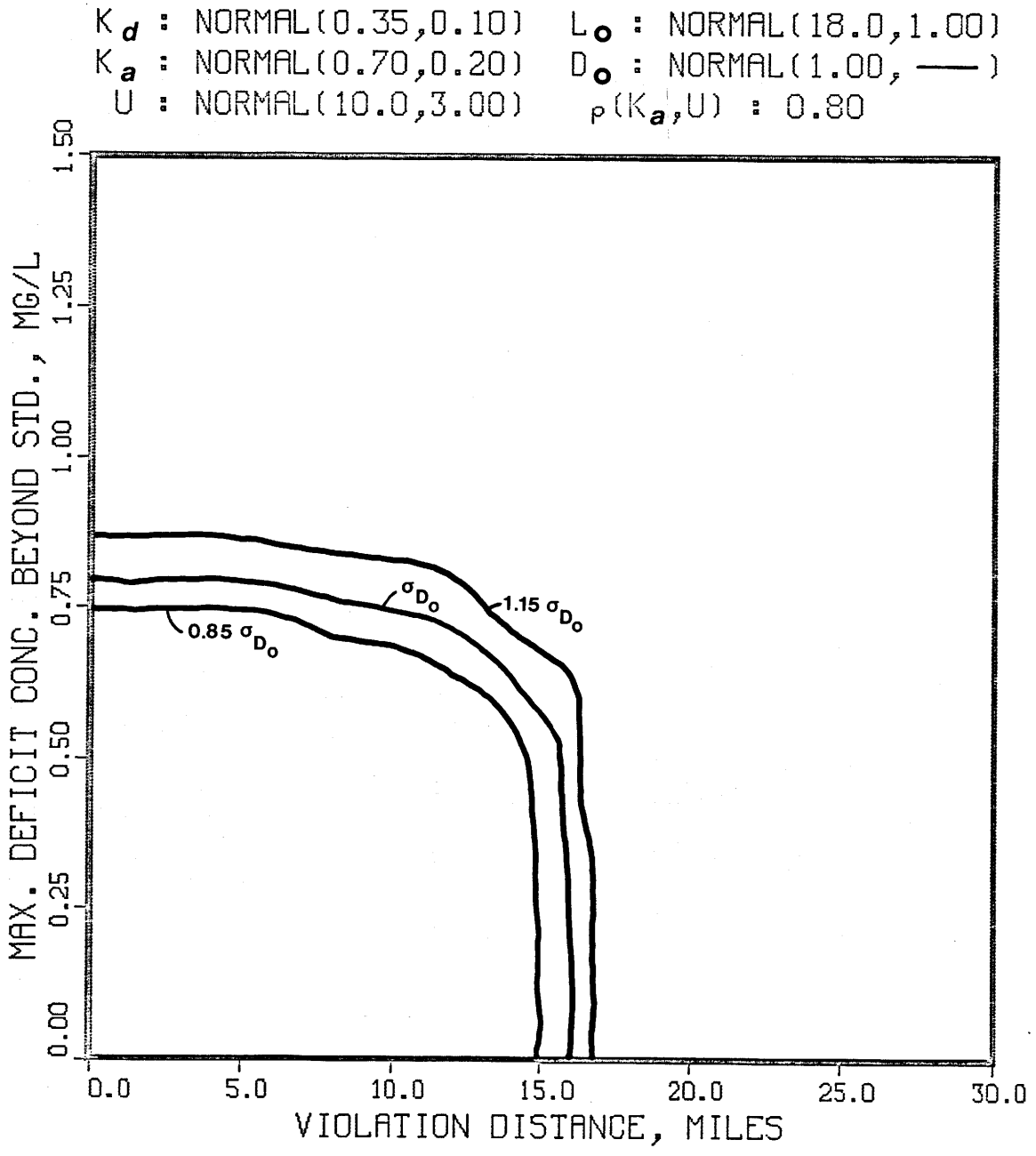

Figure 4.21 Sensitivity Of Six Percent Risk With Respect To The Standard Deviation of D_o

accurately quantify the risk associated with the various violation conditions.

4.7 SUMMARY AND IMPLICATIONS

This paper has presented a methodology for assessing the joint risk associated with maximum and average DO deficits exceeding specified standards and the length of such violations in stream systems receiving waste effluents. Moreover, this method allows this risk to be calculated on the basis of several assumptions for the type of probability distributions assigned to each parameter in the Streeter-Phelps equation. The flexibility provided by this type of model formulation permits a unique analysis of each site under investigation.

The results of this study show that the evaluation of the joint risk is highly sensitive to the type of distribution assumed for each parameter in the water quality model. In addition, a sensitivity analysis revealed that prediction of these risks are greatly impacted by variations in the mean values of each parameter in the model, especially K_d, K_a, and L_o. It is clear from the results of this study that an accurate assessment of the risk associated with various water quality violation conditions is based on the proper evaluation of the statistical properties and type of distribution assumed for each parameter in the model.

In conclusion, water quality regulations have failed to include the inherent stochastic nature of the stream environment under their control. Unrealistic standards have been enacted and remain enforced

which are based on a deterministic evaluation of the stream environment. Implied in the method and results presented in this study is the development of improved water quality regulations incorporating the risks associated with various DO violations. It is believed that the quantification of these risks will aid in the decision making processes employed by water quality management agencies and promote further investigations into the development of more realistic water quality standards incorporating the natural random behavior of aquatic environments.

CHAPTER 5

UNCERTAINTY ANALYSIS OF STREAM DISSOLVED OXYGEN

5.1 INTRODUCTION

Water quality modeling and prediction is an exceedingly enigmatic task. Such complexities are the result of the inherent randomness exhibited throughout the stream environment. Not only are the physical and biological processes not clearly understood, but as shown in Chapter 4, an imposing number of uncertainties are also associated with the various processes occurring within the aquatic environment. Several authors have already attempted to analyze these uncertainties. For example, Loucks and Lynn (1966) investigated the effect of variations in streamflow and waste flow on the probability distribution of DO; Padget and Rao (1979) presented a joint probability distribution for BOD and DO; and Kothandaraman (1969) and Chadderton et al. (1982) have cited the stochastic nature of the model parameters in the Streeter-Phelps equation.

Once the existence of such uncertainties is realized, the prediction of the concentration of DO and critical location X_c (point where the DO concentration is at a minimum) within a given reach of stream is no longer deterministic. Rather, the DO deficit computed by Eq. (1.5) and the critical location, computed by Eq. (1.7), are themselves random variables, each associated with its own probability distribution. However, in most cases, the exact distribution of the DO

deficit and the critical location is not known and is, therefore, frequently assumed.

Knowing the importance of proper water quality prediction in the management of this vital resource, it is the intent of this chapter to present an analysis for determining the appropriate probability distribution associated with the DO concentration and critical location X_c within a given reach of stream using first-order uncertainty analysis. By doing so, the risk of violating a minimum level of DO at any specified location in the stream system can be assessed. In addition, confidence intervals for both the DO deficit and critical location can also be derived from this information.

5.2 FIRST-ORDER ANALYSIS OF UNCERTAINTY

The use of first-order uncertainty analysis is quite popular in all fields of engineering. Owing such popularity to its relative ease in application to a wide array of problems. Detailed analysis and development of first-order uncertainty methods can be found in Benjamin and Cornell (1970) and Cornell (1972). Moreover, Burges and Lettenmaier (1975) have utilized the methods of first-order analysis to investigate the uncertainty in predictions of BOD and DO within the stochastic stream environment.

Essentially, first-order uncertainty analysis provides a methodology for obtaining an estimate for the moments of a single random variable or a function of several random variables. First-order analysis estimates the uncertainty in a deterministic model formulation involving parameters which are not known with certainty. By using

first-order analysis, the combined effect of uncertainty in a model formulation, resulting from the use of uncertain parameters, can be estimated (Tung and Mays, 1980).

First-order uncertainty analysis can be characterized by two major components: (1) single moment (variance) treatment of the random variables and (2) the use of first-order approximation of any functional relationship (e.g., the use of Taylor's series expansion). The first major component implies that the random element of any variable is defined exclusively by its first non-zero moment or simply the variance of the variable itself. Thus, information pertaining to the character of a random variable, Y, is provided solely by its mean ($\overline{Y}$) and variance $(\sigma_y)^2$.

The second component states that only the first-order terms in a Taylor's series expansion will be utilized in the analysis of a functional relationship containing random variables or processes. With exception to the evaluation of the mean (in which second-order terms may be utilized), any attempt to retain terms higher than first-order in the expansion requires more information about the random variables than that provided by their first and second moments (Cornell, 1972).

To present the general methodology of first-order analysis, consider a random variable, Y, which is a function N random variable X_i (multivariate case). Mathematically, Y can be expressed as:

$$Y = g\ (\underline{X}) \qquad (5.1)$$

where $\underline{X} = (X_1, X_2, \ldots, X_N)$, a vector containing N random variables X_i. Through the use of Taylor's series expansion, the random variable Y can be approximated by

$$Y \overset{2}{=} g(\underline{\bar{X}}) + \sum_{i=1}^{N} \left[\frac{\partial g}{\partial X_i} \right]_{\underline{X} = \underline{\bar{X}}} (X_i - \bar{X}_i)$$

$$+ \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left[\frac{\partial^2 g}{\partial X_i \partial X_j} \right]_{\underline{X} = \underline{\bar{X}}} (X_i - \bar{X}_i)(X_j - \bar{X}_j) \tag{5.2}$$

in which $\underline{\bar{X}} = (\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_N)$, a vector containing the mean of N random variables, $\overset{2}{=}$ represents equal in the sense of a second order approximation.

Then, the second-order approximation of the expected value of Y is

$$E[Y] \overset{2}{=} g(\underline{\bar{X}}) + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left[\frac{\partial^2 g}{\partial X_i \partial X_j} \right]_{\underline{X} = \underline{\bar{X}}} \mathrm{Cov}[X_i, X_j] \tag{5.3}$$

in which $\mathrm{Cov}[x_i, x_j]$ is the covariance between random variables x_i and x_j. It should be noted that the second term in the above equation reduces to the sum of the variance if the random variables X_i are independent.

It follows that the first-order approximation of the variance of Y is

$$\sigma_y^2 = \mathrm{Var}[Y] \overset{1}{=} \sum_{i=1}^{N} \sum_{j=1}^{N} \left[\frac{\partial g}{\partial X_i} \right]_{\underline{\bar{X}}} \left[\frac{\partial g}{\partial X_j} \right]_{\underline{\bar{X}}} \mathrm{Cov}[X_i, X_j] \tag{5.4}$$

If the X_i's are independent, Eq. (5.4) reduces to

$$\sigma_y^2 \overset{1}{=} \sum_{i=1}^{N} \left[\frac{\partial g}{\partial X_i} \right]^2_{\underline{X} = \overline{\underline{X}}} \sigma_i^2 \tag{5.5}$$

where $\overset{1}{=}$ means equal in a first-order sense (Benjamin and Cornell, 1970; Burgess and Lettenmaier, 1975) and σ_i^2 is the variance corresponding to random variable X_i.

5.3 UNCERTAINTY ANALYSIS OF THE WATER QUALITY MODEL

In Chapter 4, significant discussion is given to the type of uncertainties encountered in the modeling of the water quality process, i.e., inherent, model, and parameter uncertainties. Knowing the existence of these uncertainties in the aquatic environment, such conditions should be incorporated into the modeling process in order to improve model accountability. To do so, first-order uncertainty analysis is utilized.

Utilizing a second-order Taylor's expansion, Eq. (1.5) can be expressed as

$$D_x^2 \overset{2}{=} D_x(\overline{K}_d, \overline{K}_a, \overline{U}, \overline{L}_o, \overline{D}_o) + P'_{K_d}(K_d - \overline{K}_d) + P'_{K_a}(K_a - \overline{K}_a) + P'_U(U - \overline{U})$$

$$+ P'_{L_o}(L_o - \overline{L}_o) + P'_{D_o}(D_o - \overline{D}_o) + \tfrac{1}{2}P''_{K_d,K_d}(K_d - \overline{K}_d)^2 + \tfrac{1}{2}P''_{K_a,K_a}(K_a - \overline{K}_a)$$

$$+ \tfrac{1}{2}P''_{U,U}(U - \overline{U})^2 + \tfrac{1}{2}P''_{L_o,L_o}(L_o - \overline{L}_o)^2 + \tfrac{1}{2}P''_{D_o,D_o}(D_o - \overline{D}_o)^2$$

$$+ P''_{K_d,K_a}(K_d - \bar{K}_d)(K_a - \bar{K}_a) + P''_{K_d,U}(K_d - \bar{K}_d)(U - \bar{U})$$

$$+ P''_{K_d,L_o}(K_d - \bar{K}_d)(L_o - \bar{L}_o) + P''_{K_d,D}(K_d - \bar{K}_d)(D_o - \bar{D}_o)$$

$$+ P''_{K_a,U}(K_a - \bar{K}_a)(U - \bar{U}) + P''_{K_a,L_o}(K_a - \bar{K}_a)(L_o - \bar{L}_o)$$

$$+ P''_{K_a,D_o}(K_a - \bar{K}_a)(D_o - \bar{D}_o) + P''_{U,L_o}(U - \bar{U})(L_o - \bar{L}_o)$$

$$+ P''_{U,D_o}(U - \bar{U})(D_o - \bar{D}_o) + P''_{L_o,D_o}(L_o - \bar{L}_o)(D_o - \bar{D}_o) \qquad (5.6)$$

where

$P'_X = \partial D_x/\partial X$ and $P''_{XY} = \partial^2 D_x/\partial X \partial Y$ evaluated at the mean of the model parameters $(\bar{K}_d, \bar{K}_a, \bar{U}, \bar{L}_o, \bar{D}_o)$.

It follows that the second-order approximation of the expected value of the DO deficit given by Eq. (1.5) at any location can be expressed as:

$$E[D_x] \overset{2}{\cong} D_x(\bar{K}_d, \bar{K}_a, \bar{U}, \bar{L}_o, \bar{D}_o) + \tfrac{1}{2}\mathrm{Var}[K_d]P''_{K_d,K_d}$$

$$+ \tfrac{1}{2}\mathrm{Var}[K_a]P''_{K_a,K_a} + \tfrac{1}{2}\mathrm{Var}[U]P''_{U,U}$$

$$+ \tfrac{1}{2}\mathrm{Var}[L_o]P''_{L_o,L_o} + \tfrac{1}{2}\mathrm{Var}[D_o]P''_{D_o,D_o}$$

$$+ \text{Cov}[K_a, U] P''_{K_a, U} \tag{5.7}$$

in which Var[] and Cov[] represents the variance and covariance operators, respectively. It should also be noted that provisions for a positive correlation between parameters K_a and U are included in Eq. (5.7). The existence and relevance of such a correlation is discussed in detail in Chapter 4.

By considering all the model parameters in Eq. (1.5) to be independent, the first-order approximation of the variance of the DO deficit can be written as:

$$\sigma^2_{D_x} = \text{Var}[D_x] \stackrel{1}{=} (P'_{K_d})^2 \text{Var}[K_d] + (P'_{K_a})^2 \text{Var}[K_a] + (P'_U)^2 \text{Var}[U]$$

$$+ (P'_{L_o})^2 \text{Var}[L_o] + (P'_{D_o})^2 \text{Var}[D_o] \tag{5.8}$$

In continuing this analysis, a first-order approximation (assuming all model parameters to be independent) for the third (skewness) and fourth (kurtosis) moments of the DO deficit are computed as

$$\gamma_{D_x} = E[(D_x - \bar{D}_x)^3] / \text{Var}[D_x]^{1.5}$$

$$\stackrel{1}{=} \left\{ (P'_{K_d})^3 \gamma_{K_d} \text{Var}[K_d]^{1.5} + (P'_{K_a})^3 \gamma_{K_a} \text{Var}[K_a]^{1.5} \right.$$

$$+ (P'_U)^3 \gamma_U \text{Var}[U]^{1.5} + (P'_{L_o})^3 \gamma_{L_o} \text{Var}[L_o]^{1.5}$$

$$\left. + (P'_{D_o})^3 \gamma_{D_o} \text{Var}[D_o]^{1.5} \right\} / \text{Var}[D_x]^{1.5} \tag{5.9}$$

and

$$K_{D_x} = E[(D_x - \bar{D}_x)^4] \,/\mathrm{Var}[D_x]^2$$

$$\doteq \Big\{(P'_{K_d})^4 K_{K_d} \mathrm{Var}[K_d]^2 + (P'_{K_a})^4 K_{K_a} \mathrm{Var}[K_a]^2$$

$$+ (P'_U)^4 K_U \mathrm{Var}[U]^2 + (P'_{L_o})^4 K_{L_o} \mathrm{Var}[L_o]^2$$

$$+ (P'_{D_o})^4 K_{D_o} \mathrm{Var}[D_o]^2 + 6(P'_{K_d} P'_{K_a})^2 \mathrm{Var}[K_d]\,\mathrm{Var}[K_a]$$

$$+ 6(P'_{K_d} P'_U)^2 \mathrm{Var}[K_d]\,\mathrm{Var}[U] + 6(P'_{K_d} P'_{L_o})^2 \mathrm{Var}[K_d]\,\mathrm{Var}[L_o]$$

$$+ 6(P'_{K_d} P'_{D_o})^2 \mathrm{Var}[K_d]\,\mathrm{Var}[D_o] + 6(P'_{K_a} P'_U)^2 \mathrm{Var}[K_a]\,\mathrm{Var}[U]$$

$$+ 6(P'_{K_a} P'_{L_o})^2 \mathrm{Var}[K_a]\,\mathrm{Var}[L_o] + 6(P'_{K_a} P'_{D_o})^2 \mathrm{Var}[K_a]\,\mathrm{Var}[U]$$

$$+ 6(P'_U P'_{L_o})^2 \mathrm{Var}[U]\,\mathrm{Var}[D_o] + 6(P'_U P'_{D_o})^2 \mathrm{Var}[U]\,\mathrm{Var}[D_o]$$

$$+ 6(P'_{L_o} P'_{D_o})^2 \mathrm{Var}[L_o]\,\mathrm{Var}[D_o]\Big\} \Big/ \mathrm{Var}[D_x]^2 \qquad (5.10)$$

where γ_{D_x}, K_{D_x} and $\gamma_{()}$, $K_{()}$ are the skew and kurtosis of the DO deficit at any given location x and individual model parameters, respectively. Detail expressions of the first P' and second P" partial derivatives of the DO deficit given by Eq (1.5) are outlined in Appendix B.

5.4 PROBABILITY DISTRIBUTION OF THE DISSOLVED OXYGEN DEFICIT

By considering the stream system to be an inherently random environment, the DO deficit (or concentration) is itself a random variable. Thus, in order to evaluate the probability of violating a given water quality standard, knowledge of the probability distribution associated with the instream DO deficit is required. Furthermore, by knowing the distribution, it is then possible to quantify the DO deficit at any location with a given level of confidence. Otherwise, the quantification of the DO deficit at any location in a stream environment under uncertainty is, at best, simply conjecture.

Although significant research has been conducted in the uncertainty analysis of stream dissolved oxygen, most of these studies have been concerned with variations in DO concentrations due to model parameter uncertainty (Kothandaraman and Ewing, 1969; Hornberger, 1979; Chadderton et al., 1982). However, there have been some attempts to derive analytical expressions for the probability distribution associated with the DO deficit. Thayer and Krutchkoff (1967) utilized a stochastic birth and death process to obtain an expression for the probability distribution of DO without considering the uncertainties of the model parameters; Esen and Rathbun (1976) assumed the reaeration and deoxgyenation rate coefficients to be normally distributed and investigated the probability distribution for DO and BOD using a random walk approach; Padgett et al. (1977) developed a joint probability density function for BOD and DO by solving a random differential equation; and Padgett and Rao (1979) utilized a nonparametric

probability density estimator to again obtain an expression for the joint probability density function of BOD and DO. From a practical viewpoint, the main disadvantage to each of the aforementioned methods is that the resulting probability distributions derived for the DO deficit are too complicated. Such sophisticated approaches make it difficult for most engineers to assess the probability of violating a given water quality standard when using the methods above.

Thus, in support of a more tractable methodology, the present study is directed toward the utilization of commonly used probability distributions to describe the random characteristics of the DO deficit computed by the Streeter-Phelps equation, i.e., Eq. (1.5). The candidate probability distributions considered for the DO deficit include the normal, lognormal, gamma, and Weibull distributions. The mean and variance of the DO deficit at any given location are estimated by first-order analysis using Eqs. (5.7) and (5.8). The parameters in each of the candidate probability distribution models can be obtained through the moment-parameter relationships shown in Table 4.2.

Once the first two moments of the DO deficit are estimated and the probability distribution of the model is assumed, the statistical characteristics of DO deficit can be completely defined. This is because the moments of any order of a random variable having a specified probability density function are uniquely related to the parameters in the probability model. Relations of the skew coefficient and kurtosis to the parameters in the candidate probability models are given in Table 5.1 (Hastings and Peacock, 1974; Patel et al., 1976).

TABLE 5.1 THIRD AND FOURTH MOMENTS FOR SEVERAL CONTINUOUS PROBABILITY DISTRIBUTIONS

DISTRIBUTION	SKEWNESS	KURTOSIS	REMARK
Normal	0	3.0	
Log-normal	$(w + 2)\sqrt{w-1}$	$w^4 + 2w^3 + 3w^2 - 3$	$w = 1 + C_v^2$
Gamma[1]	$2/\sqrt{\alpha}$	$3 + 6/\alpha$	
Weibull[1,2]	$\beta^3\ \Gamma(1 + 3/\alpha)$	$\beta^4\ \Gamma(1 + 4/\alpha)$	

[1]Parameters α and β are defined in Table 4.2.

[2]These formulae are for computing the moments about the origin; not for the skewness and kurtosis directly.

Instead of making such a strong assumption about the probability density function of a specific form for the DO deficit, an approach of using Fisher-Cornish asymptotic expansion (Fisher, 1950; Fisher and Cornish, 1968; Kendall and Stuart, 1977) is applied. This method relates the quantile of any standardized distribution to the standard normal quantile and higher order moments. In this case, the quantile of order p for the DO deficit at any location x, can be approximated without making an assumption about its distribution as follows

$$D_x(p) = E[D_x] + \xi_p \sqrt{Var(D_x)} \tag{5.11}$$

in which $D_x(p)$ is the p-th order quantile of standardized DO deficit with skew coefficient γ_{D_x} and kurtosis K_{D_x} and other higher moments of D_x. Because only the first four moments of D_x are approximated through first-order analysis in this study, i.e., Eqs. (5.7) - (5.10), Fisher-Cornish asymptotic expansion for ξ_p can be expressed as

$$\xi_p \approx z_p + \gamma_{D_x} H_2(z_p)/6 + K_{D_x} H_3(z_p)/24 - \gamma^2_{D_x} [(2H_3(z_p) + H_1(z_p)]/36 \tag{5.12}$$

in which z_p is p-th order quantile from standard normal distribution, $H_1(z_p)$, $H_2(z_p)$ and $H_3(z_p)$ are Hermit polynomials which can be computed by (Abramowitz and Stegun, 1970)

$$H_r(z_p) = z_p^r - \frac{r^2}{2\cdot 1!} z_p^{r-2} + \frac{r^4}{2^2\cdot 2!} z_p^{r-4} - \frac{r^6}{2^3\cdot 3!} z_p^{r-6} + \ldots \tag{5.13}$$

5.5 PERFORMANCE EVALUATION OF THE DISTRIBUTIONS FOR DISSOLVED OXYGEN DEFICIT

The idea of applying first-order analysis for estimating the statistical moments of the DO deficit, along with making an assumption of the probability density function for the DO deficit, is straightforward and practical. However, among the various probability distribution models that are commonly used, the ultimate question to be raised is, "which probability distribution model (or models) best describe the random behavior of DO deficit in a stream?"

To evaluate the relative performance of each of the candidate probability distributions considered, three performance criteria are adopted herein: (1) biasness (BIAS), (2) mean absolute error (MAE), and (3) mean square error (MSE). Each of the three criteria are used simultaneously in an attempt to identify the best probability model for describing the random characteristics of DO deficit at a given location.

These criteria are mathematically defined as

(i) Biasness,

$$BIAS = \int_o^1 (\hat{x}_{p,f} - x_p)dp \tag{5.14}$$

(ii) Mean absolute error,

$$MAE = \int_o^1 \left| \hat{x}_{p,f} - x_p \right| dp \tag{5.15}$$

(iii) Mean square error

$$MSE = \int_0^1 (\hat{x}_{p,f} - x_p)^2 dp \qquad (5.16)$$

where x_p and $\hat{x}_{p,f}$ are, respectively, the true value and the estimate of the p-th order quantile determined from the assumed probability model, f. It should be noticed that the true value of the quantile for the DO deficit cannot be determined exactly due to the complexity of Eq. (1.5). As an alternative, Monte Carlo simulation is applied for obtaining and estimate of the 'true' quantile for the DO deficit. The Monte Carlo simulation for this task is described in the following subsection.

5.5.1 Derivation of the 'True' Distribution of DO by Monte Carlo Simulation

Recall, that in this study, the DO deficit at any downstream location x can be computed by Eq. (1.5). However, when applying Eq. (1.5) to a stream environment under uncertainty, the model parameters (K_d, K_a, U, L_o, and D_o) are treated as random variables, each characterized by their own probability distribution. Thus, the DO deficit computed by Eq. (1.5) is itself a random variable characterized by its own distribution. An illustration of the distribution of the DO deficit is given in Figure 5.1.

To determine the probability distribution of the DO deficit at a given location, Monte Carlo simulation techniques are employed, allowing each of the model parameters (K_d, K_a, U, L_o, and D_o) to be

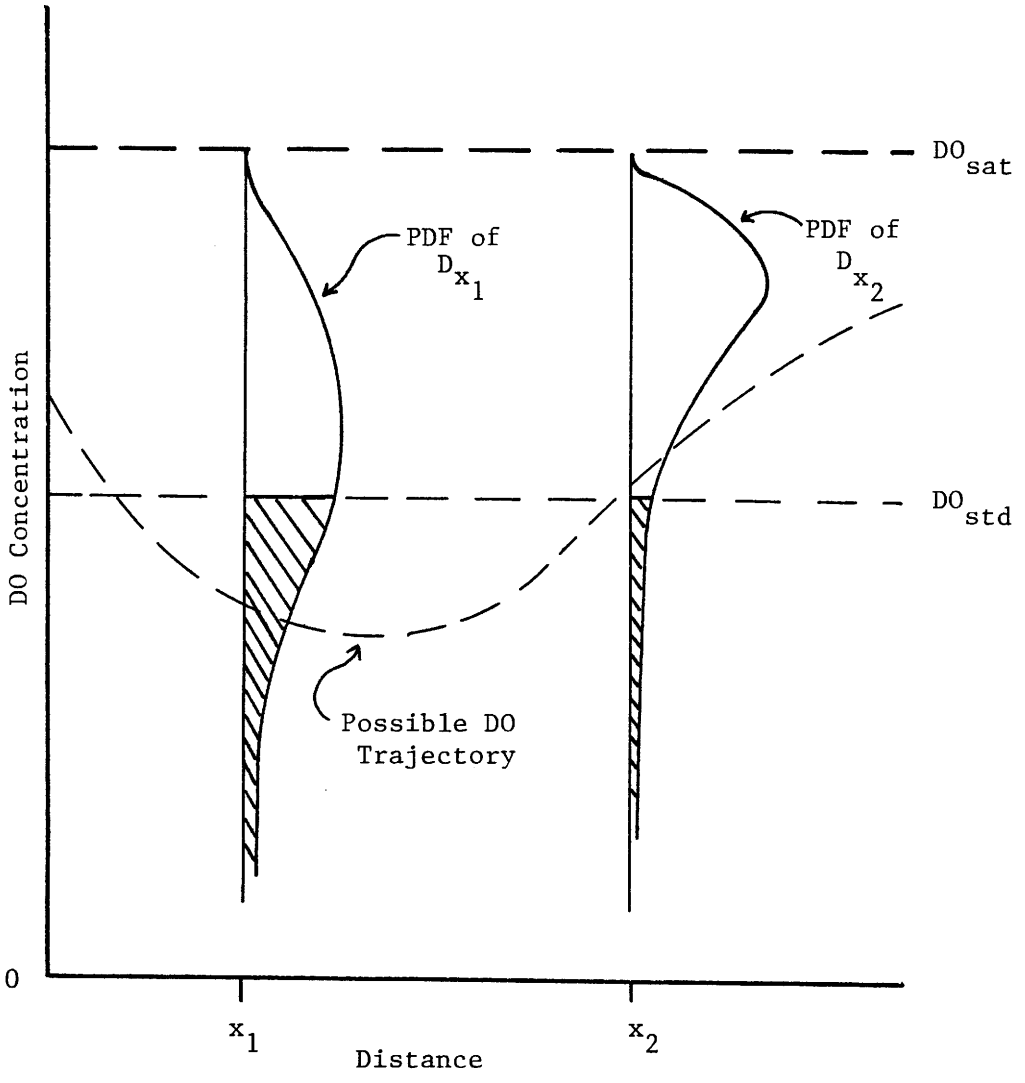

Figure 5.1 Schematic Diagram Of The Probability Density Function For The DO Deficit

assigned one of four distributions: normal, lognormal, gamma, and Weibull. (A detailed discussion of the elements of Monte Carlo simulation has already been presented in Chapter 4.) In addition, the statistical properties of the model parameter used throughout this chapter are listed in Table 5.2. Simulation procedures are performed such that 10 groups of 999 DO deficits are generated using Eq. (1.5) and one of the four distributions mentioned above for each of the model parameters. For example, during the first simulation run, 10 groups of 999 DO deficits (using Eq. 1.5) were generated under an independent and all normal assumption for all the water quality parameters. Then, in successive runs, different distributions are assigned to each of model parameters. It should also be pointed out that provisions for considering a positive correlation ($\rho = 0.8$) between model parameters K_a and U, are included in this simulation exercise. Noting that when such a correlation is considered, a bivariate normal distribution is utilized.

During the simulation runs, each of the 10 groups of 999 DO deficits are ranked in ascending order. Specifically, the minimum value of the DO deficit generated is assigned to position 1 and the maximum value to position 999. Then, quantiles of the DO deficit are computed for several probability levels p by simply locating the value of the deficit in position $(999 + 1)p$. Additionally in order to reduce sampling errors, each of the respective quantiles obtained for the 10 groups are then arithmetically averaged.

Table 5.2 THE STATISTICAL PROPERTIES OF THE MODEL PARAMETERS USED TO INVESTIGATE THE DISTRIBUTION OF THE DISSOLVED OXYGEN DEFICIT AND CRITICAL LOCATION

MODEL PARAMETERS	MEAN	STANDARD DEVIATION	UNITS
K_d	0.35	0.10	$days^{-1}$
K_a	0.70	0.20	$days^{-1}$
U	10.00	3.00	miles/day
L_o	18.00	5.00	mg/l
D_o	1.00	0.30	mg/l

5.5.2 Results and Discussions

In this study, the analysis of the goodness-of-fit for the probability distribution of the DO deficit are conducted within a single reach (described in Table 5.2) at downstream locations of 10 and 20 miles. For the purpose of illustration, a sample result of the DO deficit quantiles under various distributional assumptions and the corresponding BIAS, MAE, and MSE is given in Table 5.3. Results of more extensive experiments are given in Tables 5.4(a)-(c) through 5.6(a)-(c). Examining the biasness presented for each of the cases in Tables 5.4(a)-(b), it is evident that the assumption of a lognormal distribution for the DO deficit is consistently the lowest reported value. In fact, upon review of Tables 5.5(a)-(b) and 5.6(a)-(b) in which the MAE and MSE are tabulated, the lognormal distribution is again consistently the lowest reported value for each combination of distribution and correlation assumed for the model parameters. When combining the information provided by the three fitting criteria, it is determined that among the candidate distributions investigated the lognormal distribution presents the 'best' fit to the simulated values for the DO deficit. Moreover, it one were to select a second best distribution according to Tables 5.4-5.6, the selection of the gamma distribution would be made.

Depending on the probability distribution assumed for the parameters in the water quality model, Eq. (1.5), the relative performance, according to the three criteria, of the normal distribution, Weibull distribution, and the assumption of a Fisher-Cornish asymptotic

TABLE 5.3 EXAMPLE OF THE RESULTS OBTAINED FOR THE DISTRIBUTION OF DISSOLVED OXYGEN DEFICIT (mg/1)

PROB.	NORMAL	LGNML	GAMMA	WEIBULL	FISHER-CORNISH	SIMUL.
.010	1.27	2.22	1.98	1.58	- .40	1.48
.025	1.82	2.49	2.29	2.01	1.07	1.80
.050	2.29	2.74	2.59	2.42	2.09	2.08
.100	2.84	3.06	2.97	2.92	3.03	2.46
.150	3.21	3.30	3.25	3.28	3.54	2.75
.200	3.50	3.51	3.49	3.56	3.89	2.98
.300	3.98	3.87	3.87	4.02	4.36	3.38
.400	4.39	4.21	4.24	4.42	4.69	3.72
.500	4.77	4.55	4.60	4.79	4.95	4.06
.600	5.15	4.92	4.99	5.15	5.20	4.44
.700	5.56	5.35	5.43	5.54	5.45	4.83
.750	5.78	5.60	5.67	5.74	5.59	5.06
.800	6.04	5.89	5.95	5.97	5.76	5.34
.850	6.33	6.26	6.26	6.24	5.97	5.67
.900	6.70	6.75	6.75	6.56	6.27	6.09
.950	7.24	7.55	7.55	7.03	6.81	6.79
.975	7.72	8.32	8.32	7.42	7.41	7.36
.990	8.27	9.31	9.31	7.87	8.31	8.06
BIAS	.5705	.5577	.5648	.5683	.5803	
MAE	.5758	.5577	.5648	.5730	.6564	
MSE	.6013	.5738	.5790	.5999	.7403	

TABLE 5.4(a) BIASNESS FOR THE DO DEFICIT BETWEEN SIMULATION RESULTS AND VARIOUS ASSUMED DISTRIBUTIONS AT A DOWNSTREAM LOCATION OF 10 MILES.

DISTRIBUTIONS ASSUMED FOR MODEL PARAMETERS						BIASNESS (in mg/l)				
K_d	K_a	U	L_o	D_o	$\rho(K_2,U)$	N	LN	G	W	FC
N	N	N	N	N	0.0	.593	.580	.587	.591	.593
N	N	N	N	N	0.8	.561	.547	.557	.558	.561
LN	LN	LN	LN	LN	0.0	.560	.547	.554	.557	.554
LN	LN	LN	LN	LN	0.8	.568	.554	.564	.565	.563
G	G	G	G	G	0.0	.560	.547	.554	.558	.556
W	W	W	W	W	0.0	.571	.558	.565	.568	.580
LN	N	G	W	LN	0.0	.537	.524	.531	.535	.541

NOTE: N-Normal; LN-Lognormal; G-Gamma; W-Weibull; FC-Fisher-Cornish

TABLE 5.4(b) BIASNESS FOR THE DO DEFICIT BETWEEN SIMULATION RESULTS AND VARIOUS ASSUMED DISTRIBUTIONS AT A DOWNSTREAM LOCATION OF 20 MILES.

DISTRIBUTIONS ASSUMED FOR MODEL PARAMETERS						BIASNESS (in mg/l)				
K_d	K_a	U	L_o	D_o	$\rho(K_a,U)$	N	LN	G	W	FC
N	N	N	N	N	0.0	.417	.401	.412	.413	.417
N	N	N	N	N	0.8	.431	.416	.426	.428	.431
LN	LN	LN	LN	LN	0.0	.370	.354	.365	.366	.366
LN	LN	LN	LN	LN	0.8	.398	.383	.394	.395	.394

NOTE: N-Normal; LN-Lognormal; G-Gamma; W-Weibull; FC-Fisher-Cornish

TABLE 5.5(a) MEAN ABSOLUTE ERROR (MAE) FOR THE DO DEFICIT BETWEEN SIMULATION RESULTS AND VARIOUS ASSUMED DISTRIBUTIONS AT A DOWNSTREAM LOCATION OF 10 MILES.

DISTRIBUTIONS ASSUMED FOR MODEL PARAMETERS						MEAN ABSOLUTE ERROR (in mg/l)				
K_d	K_a	U	L_o	D_o	$\rho(K_a,U)$	N	LN	G	W	FC
N	N	N	N	N	0.0	.597	.580	.587	.610	.597
N	N	N	N	N	0.8	.607	.547	.557	.633	.612
LN	LN	LN	LN	LN	0.0	.596	.547	.554	.600	.558
LN	LN	LN	LN	LN	0.8	.624	.554	.564	.628	.582
G	G	G	G	G	0.0	.583	.547	.554	.591	.559
W	W	W	W	W	0.0	.576	.558	.565	.573	.656
LN	N	G	W	LN	0.0	.548	.524	.531	.559	.598

NOTE: N-Normal; LN-Lognormal; G-Gamma; W-Weibull; FC-Fisher-Cornish

TABLE 5.5(b) MEAN ABSOLUTE ERROR (MAE) FOR THE DO DEFICIT BETWEEN SIMULATION RESULTS AND VARIOUS ASSUMED DISTRIBUTIONS AT A DOWNSTREAM LOCATION OF 20 MILES.

DISTRIBUTIONS ASSUMED FOR MODEL PARAMETERS						MEAN ABSOLUTE ERROR (in mg/1)				
K_d	K_a	U	L_o	D_o	$\rho(K_a,U)$	N	LN	G	W	FC
N	N	N	N	N	0.0	.458	.401	.412	.488	.458
N	N	N	N	N	0.8	.462	.416	.426	.487	.460
LN	LN	LN	LN	LN	0.0	.461	.354	.365	.444	.408
LN	LN	LN	LN	LN	0.8	.475	.383	.394	.464	.422

NOTE: N-Normal; LN-Lognormal; G-Gamma; W-Weibull; FC-Fisher-Cornish

TABLE 5.6(a) MEAN SQURAE ERROR (MSE) FOR THE DO DEFICIT BETWEEN SIMULATION RESULTS AND VARIOUS ASSUMED DISTRIBUTIONS AT A DOWNSTREAM LOCATION OF 10 MILES.

DISTRIBUTIONS ASSUMED FOR MODEL PARAMETERS						MEAN SQUARE ERROR (in mg/l)				
K_d	K_a	U	L_o	D_o	$\rho(K_a,U)$	N	LN	G	W	FC
N	N	N	N	N	0.0	.623	.590	.592	.635	.623
N	N	N	N	N	0.8	.644	.562	.567	.669	.654
LN	LN	LN	LN	LN	0.0	.638	.551	.566	.644	.584
LN	LN	LN	LN	LN	0.8	.669	.558	.577	.676	.623
G	G	G	G	G	0.0	.622	.551	.561	.633	.589
W	W	W	W	W	0.0	.601	.574	.579	.600	.740
LN	N	G	W	LN	0.0	.585	.530	.538	.593	.666

NOTE: N-Normal; LN-Lognormal; G-Gamma; W-Weibull; FC-Fisher-Cornish

TABLE 5.6(b) MEAN SQUARE ERROR (MSE) FOR THE DO DEFICIT BETWEEN SIMULATION RESULTS AND VARIOUS ASSUMED DISTRIBUTIONS AT A DOWNSTREAM LOCATION OF 20 MILES.

DISTRIBUTIONS ASSUMED FOR MODEL PARAMETERS						MEAN SQUARE ERROR (in mg/l)				
K_d	K_a	U	L_o	D_o	$\rho(K_a,U)$	N	LN	G	W	FC
N	N	N	N	N	0.0	.489	.434	.422	.513	.489
N	N	N	N	N	0.8	.488	.435	.431	.509	.486
LN	LN	LN	LN	LN	0.0	.502	.358	.381	.486	.435
LN	LN	LN	LN	LN	0.8	.511	.389	.411	.507	.448

NOTE: N-Normal; LN-Lognormal; G-Gamma; W-Weibull; FC-Fisher-Cornish

expansion, vary. However, of the majority of cases investigated, the assumption of a Fisher-Cornish asymptotic expansion for the distribution of the DO deficit performs better or equally as well when compared with the results for the normal and Weibull distributions.

Recall, the Fisher-Cornish asymptotic expansion approximates the quantiles of any standardized random variable depending on statistical moments higher than order two. Hence, the accuracy of approximating the distribution of the DO deficit using the Fisher-Cornish expansion relies heavily on the accurate estimation of the skew coefficient and kurtosis, which are in turn estimated by first-order analysis in this study. Knowing this fact, close examinations are made to compare the values of the skew coefficient and kurtosis of the DO deficit calculated by the first-order analysis with those from the simulation. Discrepancies between the first-order analysis approximations and sample statistics from the simulation were observed. The results of this investigation reveal that such discrepancy becomes more pronounced as the order of moment increases. This indicates that the skew coefficient and kurtosis of the DO deficit estimated by first-order analysis is not quite satisfactory. This is most likely attributed to the nonlinearity involved in the computation of the DO deficit using Eq. (1.5), which makes the use of first-order analysis less desirable for estimating high order moments (Gardner et al., 1981; Hornberger and Spear, 1981).

5.6 UNCERTAINTY ANALYSIS OF THE CRITICAL LOCATION

As a result of the uncertainty involved in the stochastic stream environment, the determination of the critical location, using Eq. (1.7), is itself a random variable, commanding a similar analysis as that performed on the DO deficit. Thus, to estimate the distribution of the critical location under such conditions, first-order analysis is agin employed. To illustrate the concept of the probability distribution associated with the critical, a schematic diagram is provided in Figure 5.2.

Recalling the expression for determining the critical location given by Eq. (1.7), Taylor's series first-order expansion leads to bhe following approximation

$$X_c \overset{1}{\cong} X_c(\bar{K}_d,\bar{K}_a,\bar{U},\bar{L}_o,\bar{D}_o) + F'_{K_d}(K_d - \bar{K}_d) + F'_{K_a}(K_a - \bar{K}_a)$$

$$+ F'_U(U - \bar{U}) + F'_{L_o}(L_o - \bar{L}_o) + F'_{D_o}(D_o - \bar{D}_o) \qquad (5.17)$$

where

$F'_X = \partial X_c/\partial X$ and $F''_{X,Y} = \partial^2 X_c/\partial X \partial Y$ evaluated at the mean values of the model parameters. The analytical expressions for each partial derivative can be found in Appendix C.

It follows that the first-order approximation of the expected critical location X_c, can be written as

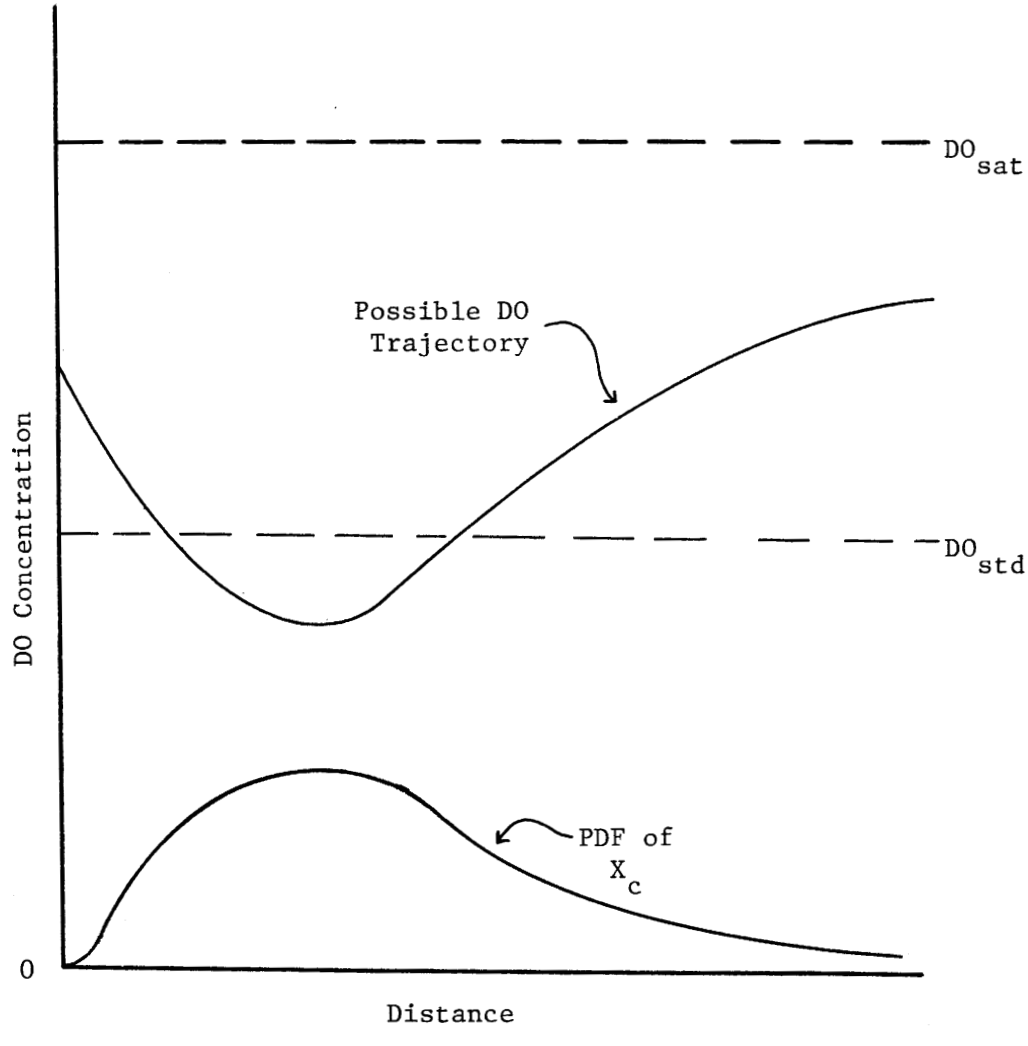

Figure 5.2 Schematic Diagram Of The Probability Density Function For The Critical Location

$$E[X_c] \overset{1}{\cong} X_c(\bar{K}_d, \bar{K}_a, \bar{U}, \bar{L}_o, \bar{D}_o). \tag{5.18}$$

This simply is Eq. (1.5) evaluated at the mean of the model parameter.

The use of first-order analysis can be continued to obtain estimates for the variance, skewness, and kurtosis of X_c as follows:

$$\begin{aligned} \mathrm{Var}[X_c] = \sigma^2_{X_c} &= E[(X_c - \bar{X}_c)^2] \\ &\overset{1}{\cong} F'^2_{K_d} \mathrm{Var}(K_d) + F'^2_{K_a} \mathrm{Var}(K_a) + F'^2_{U} \mathrm{Var}(U) \\ &+ F'^2_{L_o} \mathrm{Var}(L_o) + F'^2_{D_o} \mathrm{Var}(D_o) \end{aligned} \tag{5.19}$$

$$\begin{aligned} \gamma_{X_c} &= E[(X_c - \bar{X}_c)^3]/\mathrm{Var}[X_c]^{1.5} \\ &\overset{1}{\cong} \Big\{ F'^3_{K_d} \gamma_{K_d} [\mathrm{Var}(K_d)]^{1.5} + F'^3_{K_a} \gamma_{K_a} [\mathrm{Var}(K_a)]^{1.5} \\ &+ F'^3_{U} \gamma_{U} [\mathrm{Var}(U)]^{1.5} + F'^3_{L_o} \gamma_{D_o} [\mathrm{Var}(L_o)]^{1.5} \\ &+ F'^3_{D_o} \gamma_{D_o} [\mathrm{Var}(D_o)]^{1.5} \Big\} \Big/ [\mathrm{Var}(X_C)]^{1.5} \end{aligned} \tag{5.20}$$

$$\begin{aligned} K_{X_c} &= E[(X_c - \bar{X}_c)^4]/\mathrm{Var}[X_c]^2 \\ &\overset{1}{\cong} \Big\{ F'^4_{K_d} K_{K_d} [\mathrm{Var}(K_d)]^2 + F'^4_{K_a} K_{K_a} [\mathrm{Var}(K_a)]^2 \\ &+ F'^4_{U} K_{U} [\mathrm{Var}(U)]^2 + F'^4_{L_o} K_{L_o} [\mathrm{Var}(L_o)]^2 \end{aligned}$$

$$+ F'^{4}_{D_o} K_{D_o} [\mathrm{Var}(D_o)]^2 \Big\} \Big/ [\mathrm{Var}(X_c)]^2 \qquad (5.21)$$

where γ_{X_c} and K_{X_c} are the skew coefficient and kurtosis of the critical location, respectively.

5.7 PROBABILITY DISTRIBUTIONS FOR THE CRITICAL LOCATION

Unlike the assessment of the probability distribution for the DO deficit in which there have been several previous studies made, the assessment of an appropriate distribution for describing the random characteristics of the critical location has remained virtually unexplored to date. Throughout the review of the literature available on the stochastic analysis of the stream environment, most of these articles are primarily concerned with DO-BOD interactions, and although the analysis of the critical location in a stochastic environment remains relatively unaccounted for in the literature, this is not to be taken to mean that such information is meaningless or of little significance.

Quite the contrary, the identification of the critical location plays a major role in the regulatory process and monitoring of any stream system to which waste effluents are discharged. Because the critical location is the point at which the DO concentration is at its minimum. This point, from a monitoring viewpoint, has the greatest significance within any reach of the stream system. However, estimating the critical location within a stochastic environment is not an easy task.

Realizing the importance of such considerations, it is the intent of the remaining sections of this chapter to present an analysis for estimating the probability distribution associated with the critical location. Again, the objective of the investigation is to examine the appropriateness of using the more commonly available probability distribution models in describing the random characteristics of the critical location, X_c. Specifically, the assumed distributions of normal, lognormal, gamma, and Weibull are again applied, along with the Fisher-Cornish asymptotic expansion.

5.8 PERFORMANCE EVALUATION OF THE DISTRIBUTION ASSUMED FOR THE CRITICAL LOCATION

Identical procedures as those employed for evaluating the candidate probability distributions for the DO deficit are applied herein, except the function of interest is now that of the critical location, X_c, given by Eq. (1.7). Values of performance criteria, i.e. BIAS, MAE, MSE, under various conditions are given in Tables 5.7-5.9. Examining the results presented in Tables 5.7-5.9, the choice of the distribution for the critical location is not as clearly revealing as that for the DO deficit. For the majority, the gamma distribution appears to be best in describing the randomness of X_c according to the MAE and MSE criteria.

In general, the overall performances of all five distributions considered somewhat less than desirable, especially for Fisher-Cornish asymptotic expansion. These results again place serious doubt on the estimation ability of first-order analysis in conditions where the

TABLE 5.7 BIASNESS FOR THE CRITICAL LOCATION BETWEEN SIMULATION RESULTS AND VARIOUS ASSUMED DISTRIBUTIONS.

DISTRIBUTIONS ASSUMED FOR MODEL PARAMETERS						BIASNESS (in miles)				
K_d	K_a	U	L_o	D_o	$\rho(K_a,U)$	N	LN	G	W	FC
N	N	N	N	N	0.0	-.782	-.848	-.808	-.800	-.782
N	N	N	N	N	0.8	.0342	.010	.039	.041	.028
LN	LN	LN	LN	LN	0.0	-.703	-.768	-.728	-.720	-.726
LN	LN	LN	LN	LN	0.8	.011	-.014	.015	.017	-.059
G	G	G	G	G	0.0	-.795	-.861	-.820	-.813	-.811
W	W	W	W	W	0.0	-.703	-.769	-.728	-721	-670
G	LN	N	G	W	0.0	-.645	-.711	-.670	-.663	-.638

NOTE: N-Normal; LN-Lognormal; G-Gamma; W-Weibull; FC-Fisher-Cornish

TABLE 5.8 MEAN ABSOLUTE ERROR (MAE) FOR THE CRITICAL LOCATION BETWEEN SIMULATION RESULTS AND VARIOUS ASSUMED DISTRIBUTIONS.

DISTRIBUTIONS ASSUMED FOR MODEL PARAMETERS						MEAN ABSOLUTE ERROR (in miles)				
K_d	K_a	U	L_o	D_o	$\rho(K_a,U)$	N	LN	G	W	FC
N	N	N	N	N	0.0	.831	1.085	.889	.834	.831
N	N	N	N	N	0.8	.219	.346	.240	.301	4.269
LN	LN	LN	LN	LN	0.0	.985	.768	.728	.934	.735
LN	LN	LN	LN	LN	0.8	.354	.077	.126	.574	4.798
G	G	G	G	G	0.0	.955	.922	.820	.941	.823
W	W	W	W	W	0.0	.713	.940	.751	.721	1.858
G	LN	N	G	W	0.0	.645	.954	.751	.664	.699

NOTE: N-Normal; LN-Lognormal; G-Gamma; W-Weibull; FC-Fisher-Cornish

TABLE 5.9 MEAN SQUARE ERROR (MSE) FOR THE CRITICAL LOCATION BETWEEN SIMULATION RESULTS AND VARIOUS ASSUMED DISTRIBUTIONS.

DISTRIBUTIONS ASSUMED FOR MODEL PARAMETERS						MEAN SQUARE ERROR (in miles)				
K_d	K_a	U	L_o	D_o	$\rho(K_a,U)$	N	LN	G	W	FC
N	N	N	N	N	0.0	1.697	1.306	1.124	1.890	1.697
N	N	N	N	N	0.8	.364	.571	.444	.462	5.099
LN	LN	LN	LN	LN	0.0	1.672	.892	.857	1.708	1.240
LN	LN	LN	LN	LN	0.8	.464	.113	.167	.739	5.787
G	G	G	G	G	0.0	1.653	1.129	1.004	1.765	1.402
W	W	W	W	W	0.0	1.374	1.012	.821	1.497	2.506
G	LN	N	G	W	0.0	1.098	1.066	.821	1.253	1.211

NOTE: N-Normal; LN-Lognormal; G-Gamma; W-Weibull; FC-Fisher-Cornish

functional relationship of interest is highly nonlinear. It appears that the ability of first-order analysis to accurately estimate higher order moments (such as skewness and kurtosis) of a functional relationship diminishes as the degree of nonlinearity of the function increases.

Before a final decision is made as to the type of distribution to be selected for the critical location among those considered, the results given in Tables 5.10(a) and 5.10(b) should be considered. In these tables, the 90 percent confidence intervals of X_c for each of the assumed distributions are reported, along with the confidence intervals from Monte Carlo simulation and the Fisher-Cornish asymptotic expansion. It should be pointed out that the 90 percent confidence intervals reported for the assumed distributions in Table 5.10(a) are independent of the type of distribution assumed for the model parameters. This is due to the fact that each of the common distributions utilized here can be appropriately characterized by the mean and variance of X_c, which is in turn computed solely by the mean and variance of the model parameters. The mean and variance of the model parameters does not change as the distributions assumed for these parameters are varied.

In addition, separate values are reported for a zero and positive correlation between model parameters K_a and U. Although such considerations are not included in the development of the moments for X_c using first-order analysis, such a correlation can be considered during the simulation portion of these procedures.

TABLE 5.10(a) NINETY PERCENT CONFIDENCE INTERVALS (MILES) FOR THE CRITICAL LOCATION UNDER VARIOUS DISTRIBUTION ASSUMPTIONS

$\rho(K_a,U)$	NORMAL	LOGNORMAL	GAMMA	WEIBULL
0.0	(7.24,29.1)	(9.53,30.6)	(8.80,30.6)	(8.10,28.4)
0.8	(11.5,24.8)	(12.3,25.5)	(12.1,25.5)	(10.9,24.6)

TABLE 5.10(b) NINETY PERCENT CONFIDENCE INTERVALS (MILES) FOR THE CRITICAL LOCATION USING MONTE CARLO SIMULATION AND THE FISHER-CORNISH ASYMPTOTIC EXPANSION

	DISTRIBUTIONS ASSUMED FOR MODEL PARAMETERS					NINETY PERCENT CONFIDENCE INTERVAL	
$\rho(K_a,U)$	K_d	K_a	U	L_o	D_o	Simulation	Fisher-Cornish
0.0	N	N	N	N	N	(8.57,32.9)	(7.24,29.1)
	LN	LN	LN	LN	LN	(9.78,32.3)	(8.10,29.7)
	G	G	G	G	G	(9.24,32.8)	(7.76,29.5)
	W	W	W	W	W	(10.6,31.7)	(6.73,27.4)
	G	LN	N	G	W	(8.55,31.6)	(7.04,28.9)
0.8	N	N	N	N	N	(11.1,25.5)	(13.0,23.3)
	LN	LN	LN	LN	LN	(12.3,25.2)	(15.7,24.6)

Note: N-Normal; LN-Lognormal; G-Gamma; W-Weibull

When actually comparing the numerical values presented in Tables 5.10(a) and 5.10(b), it is obvious that the range of values presented are quite extended. For example, Table 5.10(b) reports the 90 percent confidence interval for X_c, using the simulation procedures, to be between 8.57 and 32.9 miles under all normal and uncorrelated assumptions for the model parameters. It is also interesting to observe that, when the correlation coefficient between K_a and U exists, the resulting 95 percent confidence interval length for X_c is nearly shortened by half.

Finally, the percentage of overlap between the confidence intervals computed under each of the assumed distributions (normal, lognormal, gamma, Weibull, and Fisher-Cornish) and that obtained through simulation procedures are reported in Table 5.11. Again, the assumption of a gamma distribution for the critical location results in the closest characterization of the 'true' confidence intervals obtained through simulation. This provides an additional piece of evidence supporting the use of a gamma distrubtion to model the random behavior of the critical location.

Unfortunately, from a practical viewpoint, the results obtained for the confidence intervals, in Tables 5.10(a) and 5.10(b), provide little, if any, significant information in identifying an exact or narrow range containing the critical location in a stochastic stream setting. The results from this approach are simply too widespread to be of any use in improving the monitoring or sampling process. The wide range of values reported can again be explained by the highly

TABLE 5.11 PERCENTAGE OF OVERLAPPING FOR NINETY PERCENT CONFIDENCE INTERVALS WITH THAT OF SIMULATION UNDER VARIOUS DISTRIBUTIONAL ASSUMPTIONS

DISTRIBUTIONS ASSUMED FOR MODEL PARAMETERS						PERCENTAGE OF OVERLAPPING FOR 90% C.I.				
$\rho(K_a, U)$	K_d	K_a	U	L_o	D_o	N	LN	G	W	FC
	N	N	N	N	N	84.4	86.6	89.6	81.5	84.4
	LN	LN	LN	LN	LN	85.8	92.4	92.4	82.7	88.4
0.0	G	G	G	G	G	94.3	89.4	90.7	81.3	86.0
	W	W	W	W	W	87.7	94.8	94.8	94.4	79.6
	G	LN	N	G	W	89.2	91.4	94.6	86.1	88.3
	N	N	N	N	N	92.4	91.7	93.1	93.8	71.5
0.8	LN	LN	LN	LN	LN	96.9	100.0	100.0	95.3	69.0

nonlinear character associated with Eq. (1.7). However, the importance of identifying the critical location has not been forgotten. In continuing the search, the following chapter is devoted in entirety, to finding a meaningful location for the critical point within a given reach of stream under uncertainty.

CHAPTER 6

DETERMINATION OF THE CRITICAL LOCATIONS IN A STOCHASTIC STREAM ENVIRONMENT

6.1 INTRODUCTION

Simply by its definition, the importance of knowing the location of the critical point in a stream system receiving waste discharge is obvious. In a deterministic stream system, the critical point represents a unique location at which the dissolved oxygen concentration is at a minimum. From a regulatory viewpoint, it is this location which presents the water quality managing agency with the greatest threat to water quality violation (i.e., DO concentrations that are below the minimum standard). No other point within each reach of the stream system possesses such character. Thus, in order to appropriately protect the stream environment from excessive DO depletion, the ability to determine the location of the critical point commands the water quality manager's greatest attention.

Moreover, great savings in terms of water quality monitoring costs can be accomplished if the location of the critical point can be identified or at least established within a narrow range within the stream system. By knowing the general location of the critical point within each reach, monitoring stations could be established in these regions without considering points outside the region which present a lesser threat to violate water quality standards. Consequently, savings

in the costs of laboratory analysis, sampling, and monitoring devices would be realized.

In a deterministic stream system subjected to point-source pollution, finding the critical location within a reach is quite straightforward. By simply inputting the numerical values associated with the stream parameters into Eq. (1.7), the critical location is determined. However, as was shown in Chapter 5, the ability to identify the critical location in a stochastic stream environment is no longer such a trivial computation, and although an appropriate probability distribution is established for the critical location in Chapter 5, the use of confidence interval information was shown to be practically meaningless in effectively identifying the location of the critical point. For this reason, it is the intent of this chapter to present methodologies by which the critical location in a stochastic stream environment can be determined.

6.2 DEFINITIONS OF THE CRITICAL LOCATION IN A STOCHASTIC STREAM ENVIRONMENT

As discussed in Chapters 4 and 5, the stream environment is inherently random by nature, and as such, it should be treated appropriately in the modeling of its components. For instance, having accepted the prevalent stochastic behavior within the stream environment, each of the model parameters in Eq. (1.7) for computing the critical location should be treated as random variables. In doing so, the uncertainties in the model parameters give rise to the random behavior present in the computation of the critical location using Eq.

(1.7). In fact, as was shown in Chapter 5, the uncertainty associated with the critical location under such conditions is quite extended.

In order to provide an effective means of monitoring the effects of waste discharge on the DO profile within any reach, the critical location in a stochastic stream environment must be defined. In this chapter, the critical locations are determined using the following four criteria:

(1) the location determined by Eq. (1.7) using the mean values of water quality parameters, $X_c^{(1)}$;

(2) the location at which the variance of the DO deficit given by Eq. (5.8) is maximum, $X_c^{(2)}$;

(3) the location where the probability of violating a specified DO standard is maximum, $X_c^{(3)}$; and

(4) the location "most likely" to be critical according to the distribution model assumed for the critical location in a particular reach $X_c^{(4)}$.

The significance and rationale of each of these criteria in defining the critical location in a stochastic stream environment are discussed in the following sections.

6.2.1 The Critical Location Determined by Using Mean Valued Water Quality Parameters

Basically, this is a deterministic approach for finding the critical location in which the mean values of the water quality parameters are utilized in Eq. (1.7). However simplistic in idealogy, the utility of such an approach for locating the critical point should not

be initially discounted. This approach can become a competitive tool as compared with the other methods. In fact, such an approach may lead to results which are quite similar to those obtained from more sophisticated techniques. In theory, the critical location, so determined, corresponds approximately the averaged critical location.

6.2.2 Critical Location Associated with the Maximum Variance of DO Deficit

To understand the utility of this second criteria, recall the DO profile presented in Figure 1.2, noting that such a figure exists only when a stream environment is considered deterministically. On the other hand, in a stochastic system, the DO deficit at any point in the stream system is no longer a fixed, unique value. Moreover, the value of a DO deficit at any location is subject to a certain degree of variation.

The location with a maximum variance of the DO deficit is the one associated with the largest uncertainty. By definition, the rationale for considering such a location for the critical point is evident because this point may possess significant potential for violating a minimum specified standard. Although the point of minimum expected DO might be known, this location may not represent a point posing the greatest threat to water quality violation. For instance, consider a point upstream and downstream of the location with minimum expected DO. If the variance of the DO deficit at either of these points is larger than that at the point of minimum DO, these other points may, in fact, pose a greater threat to possible violations of minimum DO standards.

This consideration is obviously more important as the DO profile near the point of minimum DO becomes less acute.

Several authors have already attempted to analyze the variance associated with the DO profile under the assumption of a stochastic stream environment. In review of such articles, conflicting results have been reported. Thayer and Krutchkoff (1967) and Padgett (1978) have cited that the location of maximum DO variance coincides with the point of minimum expected DO. On the other hand, Burger and Lettenmaier (1975) and Esen and Rathbun (1976) have contradicted this earlier research, citing that the point of maximum variance is located at a downstream distance approximately twice that of the location with minimum expected DO.

Although its true location remains unresolved, the importance of knowing the point at which the variance of the DO deficit is maximum is quite clear. This point uniquely represents the location in the stream system where the uncertainty in DO prediction is the largest. Thus in recognizing the threat of water quality violation associated with this point, it has been selected as one of the possible criteria for determining the critical location.

6.2.3 Critical Location Associated with the Maximum Probability of Violating Water Quality Standard

Unlike any other point in the stream system, the location where the probability of violating a minimum DO standard is maximum represents a point posing the greatest threat to water quality transgression. No other point possesses such character. By definition, the importance of

this location is self-evident. It is this location, amongst all others in the stream environment, at which the potential for the destruction of aquatic biota is most vulnerable.

Noting the significance of such a point, the knowledge of the location associated with the maximum probability of violating minimum DO concentrations can play an important role in the overall management of stream water quality. From this discussion, it is quite obvious that such a point should be considered as one of the principal candidates for the critical location in the stochastic stream environment.

6.2.4 The Location Most Likely to be Critical

When acknowledging the uncertainty associated with the stream environment, the computation of the critical location, using Eq. (1.7), no longer reveals a fixed, unique value. Instead, the critical location is subject to probabilistic considerations, characterized by a unique probability distribution. Whether the distribution of the critical location is assumed or known, such information is the basis for this final criteria.

As with any distribution, the value most likely to occur is more commonly known as the mode. Thus, when considering the distribution of the critical location, it is this point amongst all others, that occurs most frequently. Hence, the mode of the distribution (either assumed or determined) for the critical location is selected as the final candidate for the critical point location.

6.3 DETERMINATION OF THE CRITICAL LOCATIONS

With the exception of finding the critical location using the mean values of the water quality parameters, each of the remaining criteria seek to find a critical location associated with the maximum value of their respective functions (i.e. the variance of the DO deficit, the probability of violating specified DO standards, and the ordinate of probability density function of the critical location). Each of these locations could be determined analytically, but this would require the specification of the functions and their derivatives, together with a continuous solution for the points at which the first derivatives are zero. Such procedures are computationally formidable and impractical.

As an alternative to the analytical approach for solving the maximization of these criteria, various search techniques can be employed. Specifically, the Fibonacci search technique is selected to perform the tasks outlined in this study. In order to fully appreciate the utility of this technique, a general description of the Fibonacci search technique is provided in the following section.

6.3.1 The Fibonacci Search Technique

The Fibonacci search is a univariate unconstrained optimization technique. This technique is one of many sequential search methods available (Beveridge and Schechter, 1970; Sivazlian and Stanfel, 1974). Such procedures are quite effective in determining the optimum solution of unimodel functional relationships (i.e., the DO deficit profile and the probability density function of the critical location). However, if

more than one peak of the function exists, sequential search techniques cannot insure that the solution obtained is, in fact, the global optimum.

The basic methodology of any sequential search technique is to decide, after each set of experiments, where the most promising areas of search are located. Then, to continue the search in these regions, each time eliminating an additional undesirable portion. In this manner, the exploration is confined to successively smaller regions, until the final interval of search satisfies the desired convergence limits. This final interval, however small, is yet unexplored but is known to contain the optimum solution. However, by assigning a small value to the desired limits of convergence, the optimum solution can essentially be identified with sufficient accuracy (Beveridge and Schechter, 1970).

In a similar fashion to the general procedures outlined above, the Fibonacci search technique can be described as an interval elimination method. In this method, the location of points for function evaluation are based on the use of positive integers known as "Fibonacci" numbers. As a historical note, this procedure was originally developed by a thirteenth-century Italian, Leonardo de Pisa, who was interested in modeling the monthly growth of a population of rabbits. The sequence of numbers which served as his model became known as the Fibonacci sequence, F_i , i=0,1,2...,, where the recurrence relation is given by

$$F_o = F_1 = 1$$

$$F_{i+1} = F_i + F_{i-1}, \; i \geq 1 \qquad (6.1)$$

Clearly, the numbers begin to grow large very rapidly as the sequence progresses beyond the first few terms. This property will prove to be the greatest advantage in using the Fibonacci search technique (Sivazlian and Stanfel, 1974).

The Fibonacci search is such that it can reduce an initial search interval [a,b] to $1/F_N$ its original size in just N function evaluations. Once the desired level of convergent accuracy is specified, the number of function evaluations, N, is determined. The basic strategy of this technique is to reduce the original search interval to a length of F_{N-1} after the first iteration, F_{N-2} after the second iteration, F_{N-i} after the i-th; and $F_{N-(N-1)}$ after the (N-1)th iteration. As such, this technique will require N-1 iterations, thus, N function evaluations are required (Sivazlian and Stanfel, 1974). An outline of the Fibonacci algorithm for maximizing a functions is as follows (Kuester and Mize, 1973):

(i) Designate the search interval as L_1 between points a_1 and b_1, such that $b_1 > a_1$.

(ii) Specify the desired accuracy, α, and the maximum number of iterations, N, such that

$$\alpha = \frac{1}{F_N}$$

$$F_o = F_1 = 1$$

$$F_{i+1} = F_i + F_{i-1}, \quad 1 \leq i \leq N - 1$$

where F_i is the i-th Fibonacci number.

(iii) Place the first two points, X_1 and X_2 ($X_1 < X_2$) within L_1 at a distance d_1 from each boundary a_1 and b_1

$$d_1 = \frac{F_{N-2}}{F_N} L_1$$

$$X_1 = a_1 + d_1; \quad X_2 = b_1 - d_1$$

(iv) Evaluate the objective function at X_1 and X_2. Designate each as $g(X_1)$ and $g(X_2)$. Then, narrow the interval to be searched as follows:

$a_1 \leq X^* \leq X_2$ for $g(X_1) > g(X_2)$

$X_1 \leq X^* \leq b_1$ for $g(X_1) < g(X_2)$

where X^* is the location of the maximum. The new search interval is given as

$$L_2 = \frac{F_{N-1}}{F_N} L_1 = L_1 - d_1$$

with boundaries a_2 and b_2 where $a_2 = a_1$, $b_2 = X_2$ for $g(X_1) > g(X_2)$ or $a_2 = X_1$, $b_2 = b_1$ for $g(X_1) < g(X_2)$.

(v) Place third point in the new interval L_2, symmetric about the remaining point

$$d_2 = \frac{F_{N-3}}{F_{N-1}} L_2$$

$$X_3 = a_2 + d_2 \quad \text{or} \quad b_2 - d_2$$

(vi) Evaluate the objective function at X_3, $g(X_3)$, compare with function value remaining in the interval and reduce the interval such that

$$L_3 = \frac{F_{N-2}}{F_N} L_1 = L_2 - d_2$$

(vii) Continue the process for N evaluations (N-1 iterations) such that in general

$$d_j = \frac{F_{N-(j+1)}}{F_{N-(j-1)}} L_j \tag{6.2}$$

$$X_{j+1} = a_j + d_j \text{ or } b_j - d_j \tag{6.3}$$

$$L_j = \frac{F_{N-(j-1)}}{F_N} L_1 = L_{j-1} - d_{j-1} \tag{6.4}$$

Once i=N-1 number of iterations have been performed, the final interval $[a_i,b_i]$ is known to contain the optimum solution, X^*, and is of length less than or equal to the desired level of accuracy, α. Hence, the optimum solution is found. A flow chart illustrating the Fibonacci search procedures is presented in Figure 6.1.

6.3.2 Finding the Critical Location Using Mean Valued Water Quality Parameters

Using this approach to determine the critical location, $X_c^{(1)}$, the mean values of the water quality parameters are simply substituted into Eq. (1.7) for computation. Essentially the method is equivalent to the

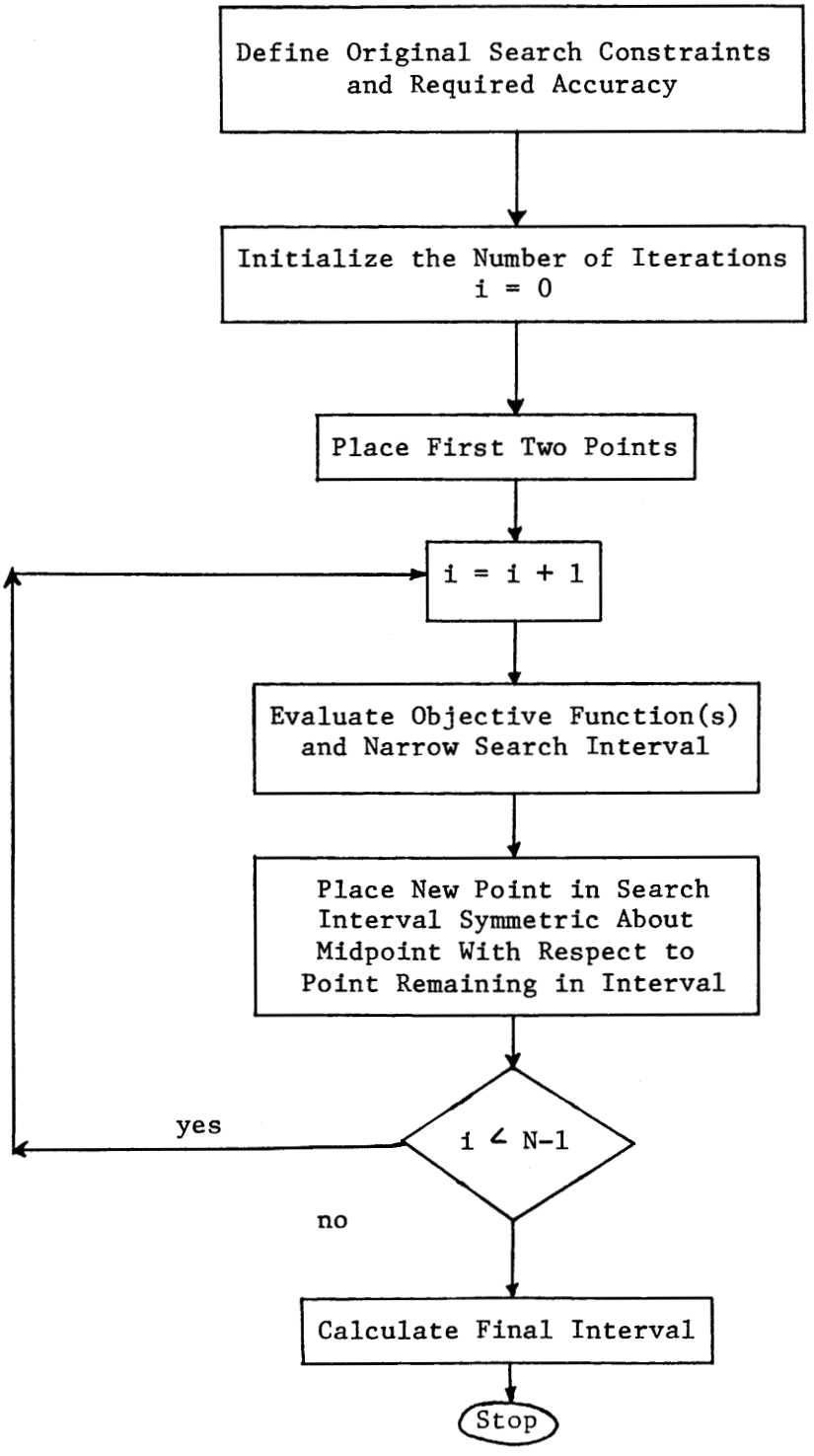

Figure 6.1. Fibonacci Search Algorithm (Kuester and Mize, 1973)

deterministic evaluation of the critical location using the mean values of water quality parameters.

6.3.3 Finding the Critical Location Associated with the Maximum Variance of the DO Deficit

To compute this location, an expression for the variance of the DO deficit as function of distance from the discharge point must be defined. In Chapter 5 such an expression has already been developed using first-order analysis, i.e., Eq. (5.8). For this case, Eq. (5.8) is a univariate function of the downstream location, x, with values for the statistical properties of the stream parameters (K_d,K_a,U,L_o, and D_o) being known. Thus, the essence of this approach is to find a critical location, $X_c^{(2)}$, such that the variance of the DO deficit, given by Eq. (5.8), is maximized. To do so, Eq. (5.8) is designated as the function of interest (i.e., the objective function). Then, applying the techniques of Fibonacci search outlined earlier, the location at which the variance of the DO deficit is maximum is obtained.

6.3.4 Finding the Location Associated with the Maximum Probability of Violating DO Standard

Although the results obtained in Chapter 5 support the use of a lognormal distribution to describe the DO deficit at any location, several probability distributions are again assumed for the purpose of providing both model flexibility and discussion regarding the sensitivity of the calculation of the critical location to the various distribution utilized. Specifically, the DO deficit is assumed to follow one

of four distributions: normal, lognormal, gamma, and Edgeworth's asymptotic expansion.

Evaluating the probability of violating a minimum DO standard at any downstream location x by using normal, lognormal, or gamma probability model is straightforward. In this study, these probabilities are evaluated using subroutines from IMSL.

In addition, Edgeworth's asymptotic expansion is also employed to provide a means for approximating the probability of a known quantile without having to assume or adopt any distribution of a specific form (Abramowitz and Stegun, 1972; Kendall and Stuart, 1977). It, however, requires the knowledge of higher order moments of the random variable under investigation. By knowing the moments of the DO deficit for orders up to four (kurtosis) from Chapter 5, Edgeworth's asymptotic expansion is truncated to give the following approximation:

$$F(w) \approx \Phi(w) - [\gamma_X \Phi^{(2)}(w)/6] + [K'_X \Phi^{(3)}(w)/24] + [\gamma_X^2 \Phi^{(5)}(w)/72] \qquad (6.5)$$

where F(w) is the cumulative probability for the standardized quantile, w; $\Phi(w)$ is the standard normal cumulative probability; γ_X and K'_X are the skewness and coefficient of excess (kurtosis minus 3) of the random variable under investigation, respectively; and $\Phi^{(r)}(w)$ is computed as follows

$$\Phi^{(r)}(w) = \frac{d^r}{dw^r}\left[\Phi(w)\right] = (-1)^r H_r(w)\Phi(w) \qquad (6.6)$$

where $H_r(w)$ is the r-th Hermit polynomial given previously in Eq. (5.13) and $\Phi(w)$ is the normal probability density function given as

$$\Phi(w) = \exp[-w^2/2]/\sqrt{2\pi} \tag{6.7}$$

In the present study the random variable under study is the DO deficit at any downstream location x from the discharge point. The standardized DO, W, can be obtained as

$$W = [D_{std} - E(D_x)]/[Var(D_x)]^{\frac{1}{2}} \tag{6.8}$$

such that $D_{std} = DO_{sat} - DO_{std}$ where DO_{sat} and DO_{std} are the saturated DO concentration and minimum required DO standard, respectively; $E(D_x)$ and $Var(D_x)$ are the expectation and variance of the DO deficit at any downstream location x from the discharge point which can be estimated by Eqs. (5.7) and (5.8), respectively. The probability of violating DO standard at any location x can be found as

$$Pr(D_x \geq D_{std}) = 1 - F(W) \tag{6.9}$$

Based on this criterion the task is to determine the critical location, $X_c^{(3)}$, at which the probability of violating the required DO concentration standard, DO_{std}, is maximum. To do this, appropriate probability distributions for the DO deficit at the downstream location, x, is assumed along with the statistical properties of the stream parameters (K_d,K_a,U,L_o, and D_o). Using this information as the objective function, the critical location, $X_c^{(3)}$, for each of the distributions assumed for the DO deficit can be found using the Fibonacci search procedures.

6.3.5 Finding the Location Most Likely to be Critical

Again, several distributions are assumed for the critical location: normal, lognormal, gamma, and Edgeworth's asymptotic expansion. Although the results of Chapter 5 found that the gamma distribution best described the random behavior of the critical location, such procedures are carried out in an attempt to provide model flexibility and discussion concerning the sensitivity of the computed critical location to the use of different distributions.

As it was pointed out earlier, the most likely point to be critical, $X_c^{(4)}$, is simply the mode of the distribution assumed for the critical location. Thus, to find the mode of each of the distributions, equations from Patel, et al. (1976) and Haan (1977) can be utilized:

(i) Under the assumption of a normal distribution for the critical location,

$$X_c^{(4)} = \mu_{mode} = E(X_c) \tag{6.10}$$

where $E(X_c)$ is the expectation of the critical location obtainable from Eq. (5.7);

(ii) Under the assumption of a lognormal distribution,

$$X_c^{(4)} = \exp(\bar{Y} - \sigma_Y^2) \tag{6.11}$$

such that

$$\bar{Y} = \tfrac{1}{2}\ln[E(X_c)^2/(C_v^2 + 1)] \tag{6.12}$$

$$\sigma_Y^2 = \ln(C_v^2 + 1) \tag{6.13}$$

$$C_v = \sqrt{Var(X_c)}\ /E(X_c) \tag{6.14}$$

where $Var(X_c)$ is the variance of the critical location defined in Eq. (5.8).

(iii) Under the assumption of a gamma distribution,

$$X_c^{(4)} = (\Psi - 1)/\Upsilon \tag{6.15}$$

such that

$$\Psi = E(X_c)^2/Var(X_c) \tag{6.16}$$

$$\Upsilon = E(X_c)/Var(X_c) \tag{6.17}$$

Finally, the mode of using Edgeworth's asymptotic expansion can be found by locating the point at which the ordinate of the density function of the X_c is maximum. This can be done using the Fibonacci search technique with the objective function

$$f(y) \approx \Phi(y) - [\gamma_{X_c} \Phi^{(3)}_{(y)}/6] + [K'_{X_c} \Phi^{(4)}_{(y)}/24] + [\gamma^2_{X_c} \Phi^{(6)}_{(y)}/72] \tag{6.18}$$

where f(y) is the density function for the standardized critical location using Edgeworth's expansion (Abramowitz and Stegun, 1972); y is the standardized X_c defined as

$$Y = [X_c - E(X_c)]/\sqrt{Var(X_c)} \tag{6.19}$$

and X_c is the downstream critical location (miles) under investigation.

6.4 NUMERICAL EXAMPLE AND DISCUSSIONS

In order to compute the critical locations based on each of the four criteria, knowledge of the mean, standard deviation (or variance), skewness, and kurtosis of the stream water parameters (K_d,K_a,U,L_o and D_o) is required. For this example, the mean and standard deviation of the model parameters are assumed to be the same as those shown in Table 5.2. Additionally, fifteen combinations of skewness, kurtosis, and correlation (between K_a and U) are considered and given in Table 6.1. It should be pointed out that during each of the fifteen cases performed in Table 6.1, the mean and standard deviations of the stream parameters remained unchanged. Each of the criteria for determining the critical location in a stochastic stream environment are computed using this example. The result of the numerical computations are displayed in Tables 6.2 through 6.5.

Examining the results obtained for the computation of the critical location using the mean values of the water quality parameters presented in Table 6.2, it is revealed that the calculation of the critical location using this first criteria is independent of the correlation between parameters K_a and U. This is because, in Eq. (1.7), only the mean values of water quality parameters are used in the computation. Correlations between model parameters are not used in Eq. (1.7), thus the calculation of the critical location remains unaffected by such consideration.

In addition to finding the critical location, $X_c^{(i)}$ (i=1,2,3,4) under each of the four criteria, the probability of violating the

TABLE 6.1 COMBINATIONS OF SKEW, KURTOSIS, AND CORRELATION CONSIDERED

Case No	$\rho(K_a,U)$	K_d		K_a		U		L_o		D_o	
		1	2	1	2	1	2	1	2	1	2
1	0.0	0.0	3.0	0.0	3.0	0.0	3.0	0.0	3.0	0.0	3.0
2	0.8	0.0	3.0	0.0	3.0	0.0	3.0	0.0	3.0	0.0	3.0
3	0.0	0.0	2.0	0.0	2.0	0.0	2.0	0.0	2.0	0.0	2.0
4	0.0	0.0	4.0	0.0	4.0	0.0	4.0	0.0	4.0	0.0	4.0
5	0.0	-.5	2.0	-.5	2.0	-.5	2.0	-.5	2.0	-.5	2.0
6	0.0	-.5	3.0	-.5	3.0	-.5	3.0	-.5	3.0	-.5	3.0
7	0.0	-.5	4.0	-.5	4.0	-.5	4.0	-.5	4.0	-.5	4.0
8	0.8	0.0	2.0	0.0	2.0	0.0	2.0	0.0	2.0	0.0	2.0
9	0.8	0.0	4.0	0.0	4.0	0.0	4.0	0.0	4.0	0.0	4.0
10	0.8	0.5	2.0	0.5	2.0	0.5	2.0	0.5	2.0	0.5	2.0
11	0.8	0.5	3.0	0.5	3.0	0.5	3.0	0.5	3.0	0.5	3.0
12	0.8	0.5	4.0	0.5	4.0	0.5	4.0	0.5	4.0	0.5	4.0
13	0.8	-.5	2.0	-.5	2.0	-.5	2.0	-.5	2.0	-.5	2.0
14	0.8	-.5	3.0	-.5	3.0	-.5	3.0	-.5	3.0	-.5	3.0
15	0.8	-.5	4.0	-.5	4.0	-.5	4.0	-.5	4.0	-.5	4.0

1
2= skew coefficient ( )
= kurtosis ( )

TABLE 6.2 CRITICAL LOCATIONS FOUND USING MEAN VALUED WATER QUALITY PARAMETERS

$\rho(K_a,U)$	$X_c^{(1)}$ (miles)	Probability of Violating 4 mg/1 DO Standard			
		Normal	Lognormal	Gamma	Edgeworth[1]
0.0	18.2	0.284	0.242	0.258	0.284-0.301
0.8	18.2	0.317	0.269	0.278	0.302-0.335

[1]These values represent the range of probabilities for all cases in Table 6.1.

TABLE 6.3 CRITICAL LOCATIONS ASSOCIATED WITH MAXIMUM VARIANCE OF DO DEFICIT

$\rho(K_a,U)$	$X_c^{(2)}$ (miles)	Probability of Violating 4 mg/1 DO Standard			
		Normal	Lognormal	Gamma	Edgeworth[1]
0.0	31.9	0.106	0.105	0.112	0.106-0.111
0.8	31.9	0.110	0.107	0.115	0.104-0.115

[1]These values represent the range of probabilities for all cases in Table 6.1.

TABLE 6.4 CRITICAL LOCATIONS ASSOCIATED WITH THE MAXIMUM PROBABILITY OF VIOLATING THE MINIMUM DO STANDARD (4 mg/l)

Case No.	Critical Location, $X_c^{(3)}$ (miles)				[1]Probability of Violating 4 mg/l DO Standard			
	Normal	Lognormal	Gamma	Edgeworth	Normal	Lognormal	Gamma	Edgeworth
1	15.81	15.65	15.71	15.81	0.294	0.250	0.267	0.294
2	15.79	15.65	15.70	15.79	0.328	0.280	0.297	0.328
3	15.81	15.65	15.71	15.81	0.294	0.250	0.267	0.302
4	15.81	15.65	15.71	15.79	0.294	0.250	0.267	0.286
5	15.81	15.65	15.71	15.69	0.294	0.250	0.267	0.312
6	15.81	15.65	15.71	15.68	0.294	0.250	0.267	0.304
7	15.81	15.65	15.71	15.67	0.294	0.250	0.267	0.296
8	15.79	15.65	15.70	15.80	0.328	0.280	0.297	0.335
9	15.79	15.65	15.70	15.78	0.328	0.280	0.297	0.321
10	15.79	15.65	15.70	15.89	0.328	0.280	0.297	0.326
11	15.79	15.65	15.70	15.88	0.328	0.280	0.297	0.319
12	15.79	15.65	15.70	15.87	0.328	0.280	0.297	0.311
13	15.79	15.65	15.70	15.67	0.328	0.280	0.297	0.347
14	15.79	15.65	15.70	15.66	0.328	0.280	0.297	0.340
15	15.69	15.65	15.70	15.66	0.328	0.280	0.297	0.333

[1]Assuming a lognormal distribution for DO deficit used in computing the critical location $X_c^{(3)}$

TABLE 6.5 THE LOCATIONS MOST LIKELY TO BE CRITICAL

Case No.	Critical Location, $X_c^{(4)}$ (miles)				[1]Probability of Violating 4 mg/l DO Standard			
	Normal	Lognormal	Gamma	Edgeworth	Normal	Lognormal	Gamma	Edgeworth
1	18.17	15.05	15.74	18.17	0.294	0.250	0.267	0.293
2	18.17	15.05	15.74	18.17	0.328	0.280	0.297	0.328
3	18.17	15.05	15.74	18.17	0.294	0.250	0.267	0.302
4	18.17	15.05	15.74	18.17	0.294	0.250	0.267	0.286
5	18.17	15.05	15.74	19.11	0.294	0.250	0.267	0.312
6	18.17	15.05	15.74	18.84	0.294	0.250	0.267	0.304
7	18.17	15.05	15.74	18.68	0.294	0.250	0.267	0.296
8	18.17	15.05	15.74	18.17	0.328	0.280	0.297	0.335
9	18.17	15.05	15.74	18.17	0.328	0.280	0.297	0.321
10	18.17	15.05	15.74	17.22	0.328	0.280	0.297	0.326
11	18.17	15.05	15.74	17.51	0.328	0.280	0.297	0.319
12	18.17	15.05	15.74	17.66	0.328	0.280	0.297	0.311
13	18.17	15.05	15.74	19.11	0.328	0.280	0.297	0.347
14	18.17	15.05	15.74	18.84	0.328	0.280	0.297	0.340
15	18.17	15.05	15.74	18.68	0.328	0.280	0.297	0.333

[1]Assuming a gamma distribution for DO deficit used in computing the critical location $X_c^{(4)}$

minimum DO standard at four types of critical locations $X_c^{(i)}$ is also assessed using the assumption of a normal, lognormal, and gamma distribution as well as Edgeworth approximation for the DO deficit. The resulting probabilities of violation for the critical locations found using mean valued water quality parameters are presented in Table 6.2. This information is important in analyzing the risk of potential damaging effects to be suffered by the stream environment under various distribution assumptions for the DO deficit.

It is also observed from Table 6.2 that the probability of violation increases when a positive correlation between K_a and U is considered. To explain these results, recall equations (5.7) and 5.8) for computing the expectation and variance of the DO deficit, respectively. By considering a positive correlation between K_a and U, the expectation and variance for the DO deficit at a given location are increased. Thus, the magnitude of standardized DO, W, is reduced, refer to Eq. (6.8). Hence, the probability of violation, according to each of the distributions, is increased.

It should also be pointed out that probability evaluation using Edgeworth's expansion is dependent on the skewness and kurtosis of the DO deficit, refer to Eq. (6.5), which in turn are dependent on the skewness and kurtosis of each water quality parameter, refer to Eqs. (5.9) and (5.10). Thus, unique values for the probability of violation can be obtained for each of the 15 cases presented in Table 6.1. From Table 6.2, it is revealed that the probability of violation in relatively insensitive to changes in the skewness and kurtosis for the water

quality parameters. More important is the fact that whether or not a positive correlation between model parameters K_a and U is included. Knowing this, the probability of violating the minimum DO standard using Edgeworth's expansion for the distribution of DO deficit are grouped according to those cases which consider correlation between K_a and U and those which do not. The range of values for each case is presented in the final column of Table 6.2 for the critical location found using mean valued water quality parameters. Similar results were obtained for the remaining criteria and are presented in Tables 6.3-6.5.

Interestingly, the results obtained for the critical location associated with the maximum variance of the DO deficit (see Table 6.3) are about twice as large as the critical locations computed using the remaining criteria. These results agree closely with those obtained by Burges and Lettenmaier (1975) and Esen and Rathbun (1976) in which they report the maximum variance to be located at a downstream distance approximately twice that of the location for minimum expected DO. However, as it was shown earlier, these results are in direct conflict with those obtained by Thayer and Krutchkoff (1967) and Padgett (1978).

In addition, the calculation of the critical location for the maximum variance criteria is seemingly unaffected by considering a positive correlation between K_2 and U. However, recall Eq. (5.8), from this it is evident that the consideration of correlation between parameters K_a and U is not included in the development of an equation for computing the variance of the DO deficit. Thus, identical values for the critical location are obtained when a zero or positive correlation between K_a and

U is considered. In analyzing Eq. (5.8) this should always be the result for the first-order uncertainty analysis presented here.

The results from the third criteria, that using the critical location associated with maximum probability of violating the minimum DO standard, are presented in Table 6.4. One can also see, in Table 6.4, the changes in the critical locations computed using the assumption of Edgeworth asymptotic expansion for the DO deficit and the various combinations of skewness, kurtosis and correlation. Although the difference in the values reported for the various cases using Edgeworth expansion seem relatively small, it should be noted that these distances are reported in terms of miles. Hence, small changes, such as two or three tenths, actually represent several hundred, possibly thousands of feet difference between these values. This may, in fact, become quite a significant factor in establishing an adequate monitoring system to control water quality conditions at the critical location, while attempting to simultaneously reduce the cost of the instrumentation and labor required to accomplish these tasks.

As in the result for the other criteria, the probability of violating a minimum DO standard of 4 mg/l is also reported for each of the critical locations, $X_c^{(3)}$, computed and are displayed in Table 6.4. For this third criteria, the probabilities of violation are determined for every critical location computed under each of the distributions assumed for the DO deficit. However, the computation of these probabilities resulted in values which were relatively insensitive to changes in the assumption of the distribution for the DO deficit.

Hence, having reported that a lognormal distribution best fit the random behavior of the DO deficit at any location, only the probabilities of violating the minimum DO standard for the critical locations, $X_c^{(3)}$, found under the assumption of a lognormal distribution for the DO deficit are reported.

Finally, in Table 6.5, the most likely critical locations are displayed, and again, as in the results for the previous criteria, the computation of the critical location, $X_c^{(4)}$, is relatively insensitive to changes in the correlation between K_a and U for the assumption of a normal, lognormal, and gamma distribution for the critical location. Once again these results can be explained by the fact that such correlation is absent in the development of the first-order uncertainty analysis which governs this fourth criteria. Specifically, recall Eqs. (5.18) through (5.21) for computing the first four moments of X_c. Alternatively, note that the computation of the most likely critical location is dependent on knowing the distribution for X_c, which in turn is determined by Eqs. (5.18) through (5.21). From these equations, it is evident that such a correlation is not considered in the first-order uncertainty analysis of X_c, and hence, will have no effect on the outcome of the computation of the most likely point to be critical.

Contrastingly, the results for the critical locations, $X_c^{(4)}$, found using Edgeworth asymptotic expansion for the distribution of the critical location show larger differences for the variety of combinations of skewness and kurtosis selected. This can be seen, for example, in the results for case numbers 1, 2, 10, and 13. Cases 1 and 2 have

exactly the same skew and kurtosis; the only difference is that case 1 considers zero correlation between K_a and U and case 2 considers a positive correlation. The critical locations computed under these conditions (cases 1 and 2) resulted in similar values (for the same reasons cited above). On the contrary, cases 10 and 13 both considered a positive correlation between K_a and U, but changes were made in the skewness and kurtosis for the water quality parameters. It is evident that these changes led to approximately an 11 percent difference between the critical locations represented in cases 10 and 13 when using Edgeworth approximation for the distribution of the critical location.

In following the procedures of this study, the probabilities of violating the minimum DO standard of 4 mg/l are calculated for every critical location, $X_c^{(4)}$, computed under each of the assumed distributions for the critical location. As before, such calculations were relatively insensitive to type of distribution assumed for the critical location. Thus, having cited that a gamma distribution best fit the random behavior of the critical location in Chapter 5, only the probabilities of violation under the assumption of a gamma distribution for the critical location are reported in Table 6.5. The trends displayed in this table are again similar to those in Table 6.4 and those exhibited throughout this investigation.

6.5 CONCLUSIONS

This chapter has attempted to lay a foundation for methodologies to determine the critical locations in a stochastic stream environment. From this study, unless other criteria are developed it would seem that

the methods of determining the critical point by locating the position at which the probability of violating a minimum water quality standard is maximum or the most likely point to be critical would be the most appropriate approaches to use, theoretically. However, due to the large uncertainty involved in determining the distribution of the critical location itself (refer to Chapter 5). It would seem that the method of determining the point most likely to be critical could possibly be quite unreliable by nature. Hence, it is this author's opinion that the method of determining the critical location, $X_c^{(3)}$, associated with the position at which the probability of violating a minimum water quality standard would be the best approach both in theory and reliability. It is this point, $X_c^{(3)}$, which poses the greatest threat to water quality violation, by definition, in the stream environment under uncertainty.

CHAPTER 7

OPTIMAL STOCHASTIC WASTE LOAD ALLOCATION

7.1 INTRODUCTION

As an overview, water quality management can be defined as a tool, used by society, to control the physical, chemical, and biological characteristics of water. Historically, such efforts have been guided toward the goal of controlling the impacts of society on the quality of water. However, it should not be forgotten that water quality in a stream is the result of the activities of society and the inherently random processes of nature itself (Ward and Loftis, 1983). If water quality management is to be implemented in a conscious manner, it must acknowledge both the activities of society and the inherently random nature of the stream environment.

Unfortunately, despite significant research in the area of water quality management to date, many of the research efforts to develop predictive water quality models have been based on a deterministic evaluation of the stream environment. Only during relatively recent times has the random nature of the stream environment been recognized in the waste load allocation (WLA) process.

There have been several articles advocating the concept of variable treatment levels according to the seasonal variation of flow, which since have proven to be cost effective (Yaron, 1979; Bathala et al., 1979; Boner and Furland, 1982). Moreover, there have been some notable works in the development of stochastic WLA models such as

Lohani and Thanh (1979) and Yaron (1979). However, their models are not complete in the sense that they consider only either the streamflow or background pollution as random variables. The limitations of such models is pointed out by Brill et al. (1979). The main reason for such simplifications and assumptions is to preserve the mathematical tractability for solving the problem using well-known linear programming techniques.

More recently, Burn and McBean (1985) have reported the development of a stochastic optimal WLA model using a chance-constrained formulation. In their paper, Burn and McBean state that the principal uncertainties are in the technological coefficients and pollutant loadings. The conclusion of their article notes the potential for including stochastic considerations in the WLA problem.

However, such research has yet to answer all the questions pertaining to effective water quality management in the uncertain environment of the natural stream setting. In fact, to manage the quality of water resources by considering all the inherent processes, both deterministic and stochastic, is a seemingly insurmountable task. On the other hand, the possibility of improvements or expansions of the current research in this field are virtually unbounded. In light of this fact, it is the intent of this chapter to present a refined approach utilizing chance-constrained optimization in conjunction with Monte Carlo simulation in an attempt to incorporate the stochastic nature of the stream environment into the water quality management process. The goal of this research is to improve model performance

beyond such methods already in existence. It is believed that such an endeavor will contribute to current management efforts which are directed toward the protection of valuable water resources from the ever increasing threat of contamination from industrial and/or municipal users.

7.2 GENERALIZED CHANCE-CONSTRAINED FORMULATION

In all fields of science and engineering, the decision-making process is generally dependent on several variables. More often than not at least one of these variables cannot be assessed with certainty. This fact could not be more evident as it is in the case of deciding "how to effectively manage our environment?" In particular, the environment in which decisions are to be made concerning in-stream water quality management are inherently subject to many uncertainties. The stream system itself, through nature, is an animate environment abundant with ever-changing processes, both physical and biological.

If one were to attempt to manage such an environment deterministically, as was done in Chapter 2, this would imply that the compliance of water quality requirements at each control point in the WLA model would be assured with ubiquitous certainty. However, as discussed in Chapters 4 and 5, the existence of the uncertain nature associated with the stream environment cannot be ignored. By acknowledging such uncertainty, it would seem more appropriate and realistic to examine the constraint performance in a probabilistic manner.

Recall the linear programming (LP) model presented in Eqs. (1.9) and (1.10). By imposing a restriction on the constraints such that

their performances will be met with reliability , the original model is transformed into the following chance-constrained formulation:

$$\text{Maximize } \underline{C}^T \underline{X} \tag{7.1}$$

subject to

$$\Pr\left\{ \underline{\underline{A}}\, \underline{X} \leq \underline{b} \right\} \geq \underline{\alpha}' \tag{7.2}$$

$$\underline{X} \geq \underline{0}$$

where $\underline{\alpha}'$ represents an m-dimensional column vector containing the desired levels of reliability for each constraint, $\underline{0} < \underline{\alpha}' < \underline{1}$; $\Pr\{\ \}$ is the probability operator; $\underline{X}$ and $\underline{C}$ are n-dimensional vectors containing the decision variables and their associated unit costs, respectively; $\underline{b}$ is an m-dimensional vector of the maximum allowable units of a specific resource which are available for allocation (or simply call it the right-hand-side, RHS); and $\underline{\underline{A}}$ is an m x n matrix of the technological coefficients (Taha, 1982). For a detailed analysis of chance-constrained problems, the reader should refer to Cooper and Charnes (1963) and Kolbin (1977).

In chance-constrained models, elements in $\underline{\underline{A}}$, $\underline{b}$, and $\underline{C}$ can be considered as random variables. When the objective function coefficients c_j's are random variables it is conventional to replace them with their expected values. Hence, three cases remain: (1) element of the technological coefficient matrix (a_{ij}'s) are random variables; (2) the elements of RHS vector b_i's are random variables; and (3) the

combined random effects of a_{ij} and b_i are considered simultaneously. Without losing generality, the description of chance-constrained formulation will focus on the case in which the elements a_{ij}'s are considered as the only random variables.

It should also be noted that a probabilistic statement of the constraints, like that in Eq. (7.2), is not mathematically operational. Further modification or transformation is required. To do so, it is necessary to develop a deterministic equivalent for Eq. (7.2).

7.3 A DETERMINISTIC EQUIVALENT FOR THE CHANCE-CONSTRAINED FORMULATION

Under the notion of uncertainty, the occurrence of the elements of matrix $\underline{\underline{A}}$ can be described by a probability distribution, not necessarily known, with mean $E[a_{ij}]$ and variance $Var[a_{ij}]$ in which $E[]$ and $Var[]$ are the expectation and variance operators, respectively. Consider now the i-th constraint,

$$Pr\left\{\sum_{j=1}^{n} a_{ij}x_j \leq b_i\right\} \geq \alpha_i' \tag{7.3}$$

We now define a new random variable T_i as

$$T_i = \sum_{j=1}^{n} a_{ij}x_j \tag{7.4}$$

Under the assumption of independency for the random elements a_{ij}'s, the mean and variance of T_i can be expressed as

$$E[T_i] = \sum_{j=1}^{n} E[a_{ij}]x_j \tag{7.5}$$

$$Var[T_i] = \sum_{j=1}^{n} Var[a_{ij}]x_j^2 \tag{7.6}$$

Hence, it follows that

$$Pr\left\{T_i \leq b_i\right\} = Pr\left\{Z_i - \frac{b_i - E[T_i]}{\sqrt{Var[T_i]}}\right\} \geq \alpha_i' \tag{7.7}$$

where Z_i is a standardized random variable with mean zero and unit variance. Therefore,

$$Pr\left\{T_i \leq b_i\right\} = F_Z\left\{\frac{b_i - E[T_i]}{\sqrt{Var[T_i]}}\right\} \tag{7.8}$$

where F_Z represents the cummulative distribution function (CDF) of the standardized random variable, Z. By substituting Eq. (7.8) into Eq. (7.7), the deterministic equivalent of chance constraint Eq. (7.3) can be obtained

$$E[T_i] + F_Z^{-1}(\alpha_i')\sqrt{Var\ [T_i]} \leq b_i \tag{7.9}$$

where $F_Z^{-1}(\alpha_i')$ is the appropriate quantile for the α_i' percentage given by the CDF of Z.

To express more explicitly in terms of decision variables x_j's, Eq. (7.3) can be written as

$$\sum_{j=1}^{n} E[a_{ij}]x_j + F_Z^{-1}(\alpha_i')\sqrt{\text{Var}[a_{ij}]x_j^2} \leq b_i \tag{7.10}$$

As can be seen, the resulting deterministic equivalent of chance constraints, when a_{ij}'s are random, are no longer linear.

7.4 OPTIMAL STOCHASTIC WASTE LOAD ALLOCATION MODEL

The deterministic WLA model presented in Chapter 2, Eqs. (2.2), (2.3), (2.11), and (2.14), is used as the basic model for deriving the optimal stochastic WLA model considered here. Under the assumption of uncertainty within the stream environment, the constraints on water quality given by Eq. (2.3) are expressed probabilistically as

$$\Pr\left\{ a_{oi} + \sum_{j=1}^{n_i} \theta_{ij} L_j + \sum_{j=1}^{n_i} \psi_{ij} D_j \leq DO_i^{sat} - DO_i^{std} \right\} \geq \alpha_i' \tag{7.11}$$

in which

$$a_{oi} = \left[L_o Q_o \prod_{\ell=1}^{n_i-1} b_{\ell,\ell+1} + L_o Q_o \sum_{p=2}^{n_i-1} \left(\prod_{\ell=1}^{n_i-p} b_{\ell,\ell+1} \right) d_{n_i-p+1,n_i-p+1} \left(\prod_{k=n_i-p+1}^{n_i-1} b_{k,k+1}^{a} \right) + D_o Q_o \prod_{k=1}^{n_i-1} b_{k,k+1}^{a} \right] \Bigg/ \left(Q_o + \sum_{m=1}^{n_i} q_m \right) \tag{7.12}$$

Variables in Eqs. (7.11) and (7.12) are defined in Chapter 2. The corresponding deterministic equivalent of Eq. (7.11) is

$$\sum_{j=1}^{n_i} E[\theta_{ij}]L_j + \sum_{j=1}^{n_i} E[\psi_{ij}]D_j + F_Z^{-1}(\alpha_i')\left\{\text{Var}[a_{oi}] + \sum_{j=1}^{n_i}\text{Var}[\theta_{ij}]L_j^2\right.$$

$$\left. + \sum_{j=1}^{n_i} \text{Var}[\psi_{ij}]D_j^2\right\}^{1/2} \leq R_i \qquad (7.13)$$

in which $R_i = DO_i^{sat} - DO_i^{std} - E[a_{oi}]$

In summary, the optimal stochastic WLA model considered herein using chance-constrained formulation consists of an objective function given by Eq. (2.2), subject to constraints Eqs. (2.3), (7.13), and (2.14).

Note that Eq. (7.13) involves a square root of the sum of variances for the technological coefficients multiplied by the square of the unknown decision variable, L_j and D_j. The deterministic equivalent of the chance-constrained formulation is nonlinear. As such, the use of LP techniques for problem solving is prohibited. To solve the optimal stochastic WLA model, it is necessary to assess the statistical properties of the random terms in the chance-constrained formulation of Eq. (7.13) and to develop a methodology for treating the nonlinear terms corresponding to the square of the decision variables. To do this, it will be shown in the following section that the mean and variance of each of the random technological coefficients θ_{ij} and ψ_{ij} in the WLA model can be derived from Monte Carlo simulation. Finally, the nonlinearity of the deterministic equivalents of the chance constraints in WLA model is ignored and the "linearized" optimal stochastic WLA model is solved iteratively.

7.5 ASSESSMENTS OF THE STATISTICAL PROPERTIES OF RANDOM TECHNOLOGICAL COEFFICIENTS IN STOCHASTIC WLA MODEL

To solve the optimal stochastic WLA model, it is necessary to determine values for the statistical properties (i.e., the mean and variance) of the random technological coefficients, θ_{ij} and ψ_{ij}. As evidenced in Eqs. (2.3) through (2.9), the equations which govern the technological coefficients for the WLA problem are functions dependent on several stream parameters (i.e., K_d, K_a, U, L_o and D_o) in each reach of channel, each of which are considered as random variables in this study. Because of the nonlinearity, the use of analytical techniques, such as first-order analysis, to determine the statistical properties of the random technological coefficients would be an extremely formidable task, especially for those water quality constraints corresponding to the control points located in the downstream reaches. Alternatively, by utilizing the theory underlying the "law of large numbers" and Monte Carlo simulation procedures, estimates for the mean and variance of the random technological coefficients can be readily obtained with the aid of a digital computer.

In essence, the law of large numbers states that as the sample size becomes sufficiently large, the probability that the sample mean and variance are close to their respective true population values approaches one. Thus, by using Monte Carlo simulation to generate a sufficiently large sample, the mean and variance of the random technological coefficients can be estimated. Specifically, the procedures are performed in four basic steps:

1. Select (or assume) a distribution and appropriate statistical properties (i.e., mean and variance) for each of the water quality model parameters K_d, K_a, U, L_o and D_o in each reach of channel.
2. According to each of the distributions selected, randomly generate values for the model parameters.
3. Compute the technological coefficients , θ_{ij} and ψ_{ij}, using Eqs. (2.3) through (2.11).
4. Repeat steps 2 and 3 for a sufficiently large number of times. Then simply compute the mean and variance of the simulated θ_{ij} and ψ_{ij}.

7.6 TECHNIQUE FOR SOLVING OPTIMAL STOCHASTIC WLA MODEL

The deterministic WLA models presented in Chapter 2 follow an LP format which can be easily solved by the simplex algorithm. However, the deterministic equivalent transformation of chance-constrained water quality constraints leads to the presence of several nonlinearities (see Eq. (7.13)) which cannot be solved directly by the LP technique. Hence, the problem becomes one of nonlinear optimization which can be solved by various nonlinear programming techniques such as the generalized reduced gradient technique (Lasdon and Warren, 1979). Alternatively, this chapter adopts the procedure to linearize the nonlinear terms of the water quality constraints in the stochastic WLA model and solve the linearized model by the LP technique, iteratively.

Tung (1986) proposed an approach of using the first-order Taylor's expansion to linearize a nonlinear constraint. The

linearization requires an initial assumption of the solution to the optimization problem which is not known. As a result, the linearized problem has to be solved iteratively until the solution converges. Since the linearization process utilized by Tung (1986) is a cumbersome exercise and the resulting linearized model has to then be solved iteratively, it is decided that, in this chapter, the assumed solutions to the stochastic WLA model will be used to calculate the value of the nonlinear terms, and then, the nonlinear terms will be moved to the RHS of the constraints. The resulting "linearized" water quality constraints in the stochastic WLA model can be written as

$$\sum_{j=1}^{n_i} E[\theta_{ij}]L_j + \sum_{j=1}^{n_i} E[\psi_{ij}]D_j \leq R_i - F_Z^{-1}(\alpha_i')$$

$$\left\{ Var[a_{oi}] + \sum_{j=1}^{n_i} Var[\theta_{ij}]\hat{L}_j^2 + \sum_{j=1}^{n_i} Var[\psi_{ij}]\hat{D}_j^2 \right\}^{\frac{1}{2}} \qquad (7.14)$$

where $\hat{L}_j$ and $\hat{D}_j$ are assumed solutions to the optimal stochastic WLA model.

Consequently, the linearized stochastic WLA model can then be solved by the LP technique iteratively, each time comparing the values of the current solutions with those obtained in the previous iteration. Then, updating the assumed solution values, used to compute the right-hand-side, until convergence criteria are met between two successive iterations. To clarify these procedures, the iterative solution approach can be described as follows:

1. Provide an initial estimate of the effluent waste and DO deficit concentrations at each discharge location.
2. Use the estimated $\hat{L}_j$ and $\hat{D}_j$ to compute the RHS of Eq. (7.14).
3. Solve the linearized model by the linear programming technique.
4. Compare the current optimal solutions of effluent waste discharge and DO deficits with estimates from the previous iteration.
5. Stop the iterations and determine the optimal solutions if the difference between solutions from two consecutive iterations are within a specified tolerance. Otherwise, update current solutions and repeat steps 2 through 4.

From this, it should be apparent that the nonlinear character present in the deterministic equivalent of the chance-constrained WLA model is essentially reduced to an iterative, deterministic LP problem. To further illustrate the algorithm, a flow chart depicting the above procedures is shown in Figure 7.1. Of course, alternative stopping rules such as specifying the maximum number of iterations, can be added in order to prevent excessive iteration during the computation procedures.

Prior to the application of these procedures, an assumption for the distribution of the standardized random variable Z must be made in order to determine an appropriate value for the term $F_Z^{-1}(\alpha_i')$ in Eq. (7.14). In effect, this is the same as that in making an assumption

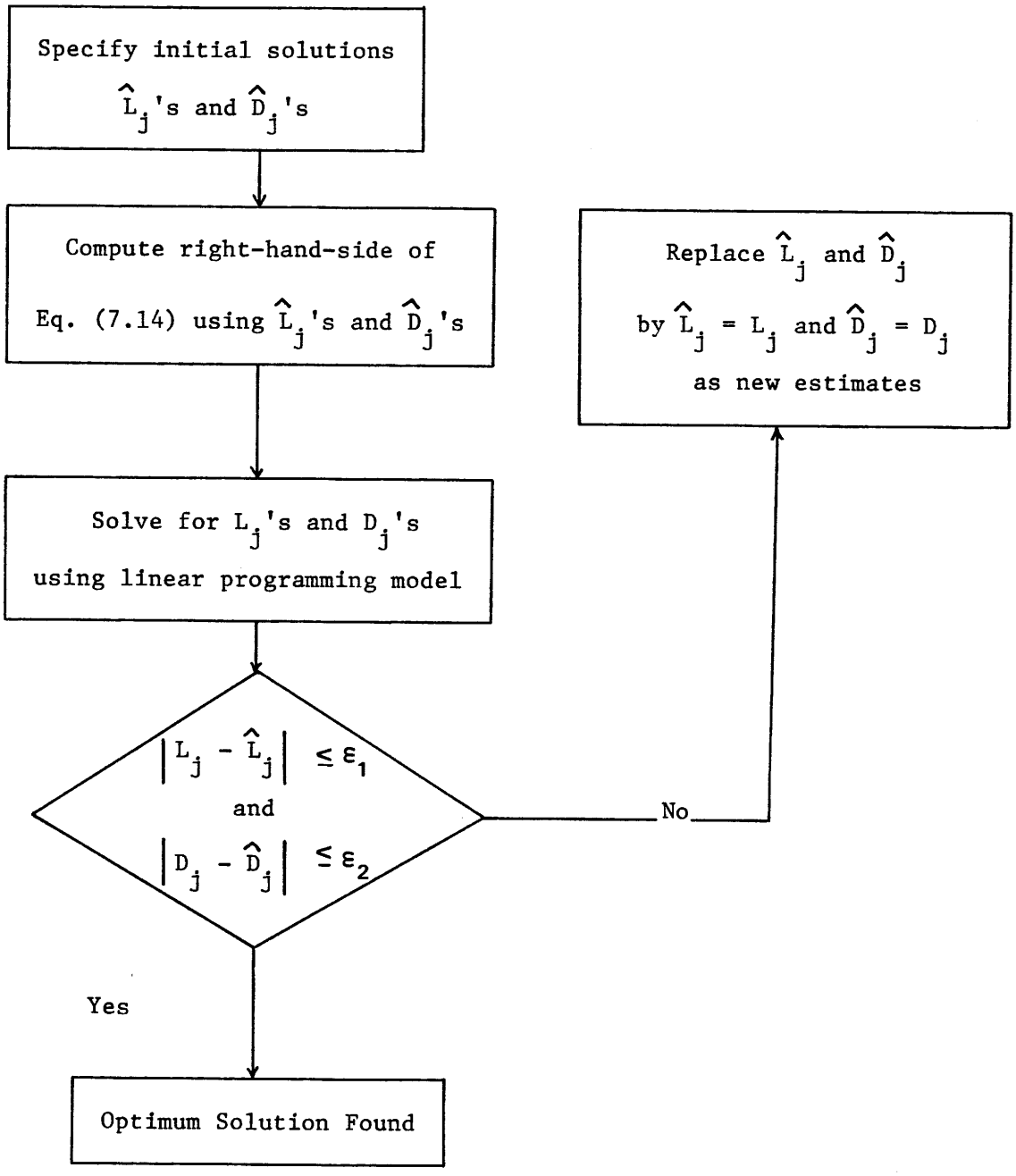

Figure 7.1 Flow Chart for Solving Linearized Stochastic Waste Load Allocation Model.

for the summation of the technological coefficients times the decision variables (see Eqs. (7.7), (7.10), and (7.12)).

Due to the nonlinear nature of the stochastic WLA model, it should also be pointed out that, in general, the optimum solution obtained cannot be assured to be the global optimum. Thus it is suggested that a few runs of these procedures with different initial solutions should be carried out to ensure the model solution converges to the overall optimum. Moreover, it is suggested that a reasonable initial solution for these iterations is to select waste effluent concentrations for each discharger which are comparable to the upper bounds on their respective levels of treatment (refer to Chapter 2). By doing so, the initial solutions for each waste discharge begin at their respective lower limits. Then, if the stochastic WLA solution is infeasible during the first iteration, more than likely a feasible solution to the WLA problem does not exist. Knowing this, time and computational effort can be saved in needless searching for an optimal solution which may not exist.

7.7 SENSITIVITY OF THE STATISTICAL PROPERTIES OF THE TECHNOLOGICAL COEFFICIENTS

In using Monte Carlo simulation, it is known that the mean and variance of the random technological coefficients in the stochastic WLA model depends on the distributions of the water quality parameters. In an attempt to assess the sensitivity of the technological coefficients to various assumptions for the distributions and correlation (between K_a and U) of the stream quality model parameters, an example has been

selected from the information provided in Tables 2.1(a)-(b) and 7.1(a)-(b). Furthermore, to lessen the computational burden and complexity of this analysis, only the information concerning the mean, Tables 2.1(a)-(b), and standard deviation, Table 7.1, of the model parameters for the first two reaches are selected. The mean values of the technological coefficients for these procedures are presented in Table 7.2.

From Table 7.2, it is evident that the mean of the random technological coefficients for the two-reach example are relatively insensitive to changes in both the distributions assumed for the stream model parameters and the correlation between parameters K_a and U. Based on the experimental study, it is observed that changes in the distributions and correlation assumed for the model parameters resulted in differences between the respective technological coefficients for the various assumptions which were generally less than 5 percent, and in almost all cases, these differences were less than 10 percent. Additionally, it should be pointed out that the computation for the technological coefficients using Monte Carlo simulation became stable when the sample size generated reached 1,000. Though not presented here, the results for the standard deviation of the technological coefficients are quite similar.

In conclusion of these results, although the computation of the technological coefficients was found to be relatively insensitive to changes in the assumptions for the distribution and correlation of the model parameters, information concerning the distribution and

TABLE 7.1 STANDARD DEVIATIONS SELECTED FOR THE PHYSICAL STREAM CHARACTERISTICS

(a) For Each Reach

Reach	Deoxygenation Coefficient (K_d)	Reaeration Coefficient (K_a)	Average Stream Velocity (U)
1-6	0.2	0.4	4.0
Units	1/days	1/days	ft^3/sec

(b) Background Characteristics

Upstream Waste Concentration (L_o)	Upstream Flow Rate (Q_o)	Upstream DO Deficit (D_o)
1.0	20.0	0.3
mg/l BOD	ft^3/sec	mg/l

TABLE 7.2 SENSITIVITY ANALYSIS OF THE MEAN TECHNOLOGICAL COEFFICIENTS TO CHANGES IN THE DISTRIBUTIONS AND CORRELATION ASSUMED FOR THE WATER QUALITY PARAMETERS

Assumed distributions For Stream Water Quality Parameters	$\rho(K_a, U)$	θ_{ij}		ψ_{ij}	
Normal	0.8	.000E + 00	.000E + 00	.136E - 02	.000E + 00
		.247E - 03	.000E + 00	.331E - 03	.000E + 00
		.148E - 03	.000E + 00	.726E - 04	.283E + 00
		.951E - 04	.468E - 01	.154E - 04	.555E - 01
Normal	0.0	.000E + 00	.000E + 00	.136E - 02	.000E + 00
		.241E - 03	.000E + 00	.337E - 03	.000E + 00
		.148E - 03	.000E + 00	.727E - 04	.283E + 00
		.950E - 04	.460E - 01	.178E - 04	.578E - 01
Lognormal	0.0	.000E + 00	.000E + 00	.134E - 02	.000E + 00
		.240E - 03	.000E + 00	.334E - 03	.000E + 00
		.148E - 03	.000E + 00	.715E - 04	.280E + 00
		.950E - 04	.459E - 01	.184E - 04	.583E - 01

TABLE 7.2 (continued)

Assumed distributions For Stream Water Quality Parameters	$\rho(K_a, U)$	θ_{ij}		ψ_{ij}	
Gamma	0.0	.000E + 00	.000E + 00	.134E - 02	.000E + 00
		.242E - 03	.000E + 00	.342E - 03	.000E + 00
		.150E - 03	.000E + 00	.680E - 04	.280E + 00
		.952E - 04	.463E - 01	.163E - 04	.564E - 01
Weibull	0.0	.000E + 00	.000E + 00	.137E - 02	.000E + 00
		.252E - 03	.000E + 00	.350E - 03	.000E + 00
		.154E - 03	.000E + 00	.743E - 04	.284E + 00
		.946E - 04	.468E - 01	.176E - 04	.567E - 01

correlation of the model parameters should be included if known or can be justified from the data. To clarify this statement, consider a situation in which one has sufficient evidence to justify the use of a lognormal distribution for K_a. It would seem unreasonable to ignore this information for K_a simply by knowing that such changes will have little effect on the outcome of the mean and variance of the technological coefficients. Instead, if information pertaining to a model parameter is known, one should include this information into the modeling process in order to improve model predictability and justification of one's approach.

7.8 NUMERICAL EXAMPLE AND DISCUSSION OF MODEL PERFORMANCE

The mean and standard deviations for the stream model parameters are shown in Tables 2.1 and 7.1; however, this time the information for all six reaches are used. An illustration of this six-reach example is similar to that given in Figure 2.2.

To assess the statistical properties (i.e., mean and variance) of the technological coefficients for this example, 999 sets of technological coefficients are generated for each of the assumptions of all normal and lognormal distributions for the stream model parameters. From this, the mean and variance of the technological coefficients are computed for each of the assumptions concerning the model parameters. This information was then placed into the LP formulation of the stochastic WLA model presented in Section 7.4. Additionally, assumptions of a normal and lognormal distribution for the random variable

$$a_{oi} + \sum_{j=1}^{n_i} \theta_{ij} L_j + \sum_{j=1}^{n_i} \psi_{ij} D_j$$

in Eq. (7.11) are made, along with the various reliabilities, α_i', for the water quality constraints, ranging from 0.85 to 0.99. The optimal solutions to the stochastic WLA problem under these various assumptions are reported in Tables 7.3 through 7.6.

In following the development of the WLA model presented in Chapter 2, two types of equity between the dischargers are again considered (i.e., equal percent removal and equal effluent concentration). In examining the results presented in Tables 7.3-7.6, the total amount of optimal waste discharge is reduced as the reliability of the water quality constraints is increased. These results can be explained by the fact that as the reliability on water quality is increased, it is equivalent to impose stricter standards or requirements on water quality assurance. Thus, to meet the increased responsibility in terms of water quality reliability, the amount of waste discharge must be reduced in order to reduce the risk of water quality violation at each control point. By continuing to increase the desired reliability for the water quality constraints, at some point these restrictions become too stringent and the feasible solutions to the problem are not obtainable.

Interestingly, at the outset, there was concern pertaining to the ability of the proposed solution methodology to converge. However, once the example for the model was performed, these concerns were, fortunately, shown to be unfounded. In fact, a convergence criteria of

TABLE 7.3 OPTIMAL STOCHASTIC WASTE LOAD ALLOCATION UNDER AN ALL NORMAL ASSUMPTION FOR THE STREAM WATER QUALITY PARAMETERS AND THE EQUITY OF EQUAL PERCENT REMOVAL[1]

$F_Z(z)$	Reliability (α')	No. of Iterations to Converge	Waste Load Allocations for Each Discharger[2]					
			No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
Normal	0.85	4	242.4	6.0	117.7	115.5	265.4	72.5
	0.90	5	233.6	6.0	113.4	109.7	255.8	69.9
	0.95	6	220.9	6.0	107.2	101.2	241.9	66.1
	0.99	"Infeasible"	-	-	-	-	-	-
Lognormal	0.85	4	244.1	6.0	118.5	116.7	267.3	73.1
	0.90	5	232.4	6.0	112.8	108.8	254.4	69.5
	0.95	6	215.0	6.0	104.4	97.3	235.4	64.4
	0.99	"Infeasible"	-	-	-	-	-	-

[1] 5 percent maximum allowable difference in the equity considered.

[2] Measured in terms of mg/l BOD.

TABLE 7.4 OPTIMAL STOCHASTIC WASTE LOAD ALLOCATION UNDER AN ALL NORMAL ASSUMPTION FOR THE STREAM WATER QUALITY PARAMETERS AND THE EQUITY OF EQUAL EFFLUENT CONCENTRATION[1]

$F_Z(z)$	Reliability (α')	No. of Iterations to Converge	Waste Load Allocations for Each Discharger[2]					
			No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
Normal	0.85	4	162.9	6.0	162.9	112.9	162.9	162.9
	0.90	5	157.2	6.0	157.2	107.2	157.2	157.2
	0.95	6	150.0	6.0	134.5	100.0	150.0	150.0
	0.99	"Infeasible"	-	-	-	-	-	-
Lognormal	0.85	4	163.9	6.0	163.9	113.9	163.9	163.9
	0.90	5	156.4	6.0	156.4	106.4	156.4	156.4
	0.95	"Infeasible"	-	-	-	-	-	-
	0.99	"Infeasible"	-	-	-	-	-	-

[1] 50 mg/1 BOD maximum allowable difference in the equity considered.

[2] Measured in terms of mg/1 BOD.

TABLE 7.5 OPTIMAL STOCHASTIC WASTE LOAD ALLOCATION UNDER AN ALL LOGNORMAL ASSUMPTION FOR THE STREAM WATER QUALITY PARAMETERS AND THE EQUITY OF EQUAL PERCENT REMOVAL[1]

$F_Z(z)$	Reliability (α)	No. of Iterations for Convergence	Waste Load Allocations for Each Discharger[2]					
			No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
Normal	0.85	4	248.1	6.0	120.4	119.3	271.6	74.2
	0.90	5	239.5	6.0	116.3	113.6	262.3	71.7
	0.95	5	228.0	6.0	110.7	105.9	249.6	68.2
	0.99	5	209.7	6.0	101.8	93.8	229.6	62.7
Lognormal	0.85	4	249.4	6.0	121.1	120.2	273.1	74.6
	0.90	5	238.2	6.0	115.6	112.7	260.8	71.3
	0.95	6	221.9	6.0	107.7	101.9	242.9	66.4
	0.99	"Infeasible"	-	-	-	-	-	-

[1] 5 percent maximum allowable difference in the equity considered.

[2] Measured in terms of mg/l BOD.

TABLE 7.6 OPTIMAL STOCHASTIC WASTE LOAD ALLOCATION UNDER AN ALL LOGNORMAL ASSUMPTION FOR THE STREAM WATER QUALITY PARAMETERS AND THE EQUITY OF EQUAL EFFLUENT CONCENTRATION[1]

$F_Z(z)$	Reliability (α)	No. of Iterations to Converge	Waste Load Allocations for Each Discharger[2]					
			No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
Normal	0.85	4	166.5	6.0	166.5	116.5	166.5	166.5
	0.90	5	161.0	6.0	161.0	111.0	161.0	161.0
	0.95	6	153.5	6.0	153.5	103.5	153.5	153.5
	0.99	"Infeasible"	-	-	-	-	-	-
Lognormal	0.85	4	167.3	6.0	167.3	117.3	167.3	167.3
	0.90	5	160.2	6.0	160.2	110.2	160.2	160.2
	0.95	4	150.0	6.0	148.4	100.0	150.0	150.0
	0.99	"Infeasible"	-	-	-	-	-	-

[1] 50 mg/l BOD maximum allowable difference in the equity considered.

[2] Measured in terms of mg/l BOD.

0.1 percent between the solutions of successive iterations was adopted, and surprisingly, the number of iterations for each of various runs presented in Tables 7.3 to 7.6 are less than 6.

Finally, it should be noted that there are only slight differences for the optimal solutions when comparisons are made between the assumptions of a normal or lognormal distribution for $F_Z(z)$. However, in each case, the assumption of a lognormal distribution resulted in larger allowable quantities of waste discharge when compared to that of a normal distribution assumption. Furthermore, the stochastic WLA problem turns infeasible for a lower reliability requirement under lognormal consumption of F_Z.

7.9 SUMMARY AND CONCLUSIONS

It is believed that within this chapter the foundation for the development of an effective stochastic optimal WLA model has been presented using chance-constrained formulation and Monte Carlo simulation procedures. The results obtained from the execution of the model presented here, for the complex, multiple discharge example cited, reveal the economic tradeoffs between the reliability specified for the water quality requirements and the total amount of waste discharge. Specifically, increased total waste discharge are concomitant with lower overall treatment costs. On the other hand, increased reliability requirements translates into a reduction in the allowable waste discharge in order to meet the more stringent water quality requirements. However, these factors have been shown to be

inversely related to each other. By increasing the reliability requirements, the total waste discharge must be reduced.

Such trade-offs can have significant implications in the overall WLA decision-making process. Those in charge of managing the stream environment must be cognizant of both the need to ensure water quality protection and the desire to meet this need at a reduced cost. Thus, the decision-maker must decide the level of reliability which is acceptable to protect the aquatic environment with a given level of certainty, while addressing the desire of treatment facilities to hold down costs.

In addition to model development, the results of this study have also revealed the insensitivity of the statistical properties of the technological coefficients, derived from the Streeter-Phelps equation, to changes in the assumption of the distribution for the water quality model parameters, such as the deoxygenation and reaeration coefficients, average stream velocity, and upstream waste conditions. More importantly, the method and ideas presented in this chapter may, in fact, provide beneficial insight into improved water quality management in a more realistic manner.

CHAPTER 8

SUMMARY

8.1 SUMMARY OF STUDY RESULTS AND METHODOLOGIES

In summary, this research was conducted in two parts. The first part was devoted to improving current deterministic approaches to the problem of effective WLA. The second part was directed toward investigations into the stochastic nature of the stream environment and its relationship to the WLA problem. To review each of the ideas presented within this study, a summary of the results and methodologies of each chapter are presented in the following subsections.

8.1.1 Summary of Deterministic Waste Load Allocation Methodologies

The improvements proposed for the deterministic evaluation of the WLA problem are presented in Chapters 2 and 3. Specifically, in Chapter 2, the fixed control point approach, widely used in the past, is shown to be inefficient with respect to the amount of computer storage required to solve the WLA problem and the possibility of water quality violations at or near the critical location. To circumvent the inadequacy of the fixed control point approach, a new technique was developed which utilizes the convex nature of the typical DO sag curve. This new approach solves the WLA model iteratively, each time a single, moving control point is assigned to the critical location within each reach computed from the current solutions of the linear programming formulation. More importantly, this new moving control point approach

possesses the advantages of considerable savings in computer storage requirements and solutions, when generated deterministically, contain no possibility of a water quality violation at any point in the river system under investigation. Thus, the efficiency and model predictability of the moving control point methodology is shown to be superior to that of the fixed control point approach so widely used in the past.

In Chapter 3, a multiobjective approach is applied to the WLA problem in conjunction with the moving control point model developed in Chapter 2. A two-objective WLA problem is formulated based on the goals of maximizing total waste discharge, while minimizing the maximum difference in equity (i.e., equal effluent concentrations or equal percent removal) between the various discharges. By doing so, it is believed that a more realistic approach to the problem of WLA is presented. To solve the multiobjective problem, two classes of methodologies are utilized: (1) generating techniques; and (2) techniques which incorporate prior knowledge of preference between alternatives.

Within the class of generating techniques, the constraint method is selected for solving the two-objective WLA problem formulated. In essence, the constraint method provides the analyst with a means of determining the set of noninferior solutions showing the trade-off between objectives for the multiobjective WLA problem. From this information, the selection of a best compromising alternative is then left to the decision-maker once he and/or she has determined their preference between the objectives.

Alternatively, a methodology which incorporates prior knowledge of preference between the feasible alternatives is also presented. Within this class of multiobjective solution techniques, an exciting and relatively new procedure is adopted to solve the two-objective WLA problem, i.e., fuzzy linear programming. The main thrust of this approach is to appropriately define a membership function which provides the model formulation with information pertaining to the relative preference for each alternative. To do this, two membership functions are utilized: linear and logistic memberships.

Unlike the set of alternatives provided by the constraint method, fuzzy linear programming identifies the optimal alternative with the aid of preference information specified by the membership function. Interestingly, the results obtained from each of the model formulations (i.e., one incorporating a linear membership function and the other logistic membership) revealed the same optimal waste load allocations when applied to the six-reach example from Chadderton et al. (1981). Upon further investigation, such results were shown to always be true. This is due to the fact that the feasible regions by each formulation, i.e., linear or logistic, share the same unique boundary, along which the optimal solution to the two-objective linear programming problem lies. Proof of the similarity between the feasible regions of each formulation was present in Appendix A.

8.1.2 Summary of Stochastic Investigations in the Stream Environment Under Uncertainty

Investigations into the stochastic nature of the stream environment are presented in Chapters 4, 5, 6, and 7. Specifically, in Chapter 4, the joint risk of violating various water quality conditions (i.e, DO deficits beyond the standard and lengths of violation) are investigated based on several assumptions for the population distribution of the stream parameters (K_d, K_a, U, L_o, and D_o) and the correlation between K_a and U. The results of this investigation revealed that the computation of the joint risk is significantly affected by the type of distribution assumed for each stream parameter. Moreover, a sensitivity analysis revealed that the prediction of these risks are most affected by variations in the mean values of each of the model parameters, especially those of K_d, K_a,and L_o. More importantly, it is believed that, in quantifying the risk of violating various water quality regulations, information leading to and promoting more realistic water quality standards incorporating the inherent random behavior of the stream environment can be obtained.

To better understand and model the uncertain nature of stream water quality, Chapter 5 presents a methodology for determining the probability distributions of the DO deficit at any point and critical location within a single reach under investigation. Such an approach is based on Monte Carlo simulation and first-order uncertainty analysis of the statistical properties (i.e., mean, variance, skewness, and kurtosis) for the DO deficit and critical location, found by using Eqs. (1.5) and (1.7). Moreover, using fitting criteria based on the mean

absolute and mean squared errors, the lognormal and gamma distributions are shown to best describe the random behavior of the DO deficit at any point and the critical location within a reach, respectively. From such information, for example, the probability that the DO deficit at a given location will be less than or equal to some specified value can be quantified.

Although a "best" distribution (i.e., the gamma distribution) is identified for describing the random behavior of the critical point, information obtained from 90 percent confidence intervals for this distribution reveal that such knowledge is virtually meaningless if one is to obtain a reasonable estimate of the actual position of the critical location in a stream environment under uncertainty.

Acknowledging the importance from a monitoring and regulatory viewpoint of knowing the position of the critical location, Chapter 6 presents a methodology for estimating various critical locations in a stochastic stream environment based on four criteria: (1) average water quality model parameters; (2) the position of maximum variance in the DO deficit; (3) the location of maximum probability of violating a minimum water quality standard; and (4) the position most likely to be critical (i.e., the mode of the distribution assumed for the critical location). A search technique, called Fibonacci search, is employed to help identify the critical locations based on these criteria. From this investigation, it is determined that the method of estimating the critical location associated with the maximum probability of violation

is the most appropriate approach, both in theory and reliability, of those investigated.

Finally, in Chapter 7, the investigations into the uncertain nature of the stream environment are culminated with the development of a stochastic optimal WLA model using chance-constrained formulation. To determine the required statistical information (i.e., mean and variance) of the technological coefficients associated with the water quality constraints, Monte Carlo simulation procedures are utilized. Additionally, it is found that the mean and variance of the technological coefficients are relatively insensitive to various selections for the distributions of the water quality model parameters (K_d, K_a, U, L_o, and D_o). From all this, it is believed that a refined approach to the problem of WLA under uncertainty is present.

8.1.3 Remarks

It seems appropriate at this point that a note of caution should be given to the reader concerning the limitations of the methodologies presented here. Up to this point, the discussion and conclusion of results has emphasized, for the most part, the positive aspects of this study. However, before implementing the methods of this study, the reader should be very careful to understand the limitations of each approach presented in the previous chapters.

Specifically, in all the chapters, a simplified model relating the interaction between BOD and DO is utilized in which a number of oxygen sources and sinks are excluded, for example, the processes of sedimentation, benthic demand, nitrification, and photosynthesis. In

Chapters 5 and 6, the reader should recognize the significance of approximation techniques based on Taylor's series expansion in which higher order terms have been truncated.

Moreover, it is simply the author's intent at this point to make the reader aware of the potential for discrepancy between the processes and models presented within this study and those which he and/or she might encounter in applying the methods of this study to "real-world" problems. On the other hand, it is believed that the methodologies presented here provide a foundation upon which improved water quality protection incorporating the stochastic nature of the aquatic environment can be built.

8.2 SUGGESTIONS FOR FUTURE RESEARCH

Further extensions of this research might include:

1. The multiobjective approach to WLA can be expanded to include a number of other objectives, for example, a third objective to minimize the DO deficit could be added to improve the accurate reflection of goals in the WLA formulation.
2. The water quality model utilized throughout this study (i.e., the original Streeter-Phelps equation) can be modified to accommodate a variety of DO sources and sinks noted in Section 1.2.2. Such modifications would include additional terms to account for photosynthesis, respiration, sedimentation, etc.

3. In the development of the distribution for the critical location, first-order uncertainty analysis of the mean of the critical location is based solely on the first term of Taylor's series expansion of Eq. (1.7). To improve model accuracy, second-order terms incorporating the variance of the model parameters should be developed.
4. To take advantage of the savings in computer storage realized by the moving control point approach outlined in Chapter 2, such procedures could possibly utilize the stochastic WLA model presented in Chapter 7. However, to do this, an appropriate means of determining the critical location in a stream environment must be developed, possibly from the methodologies presented in Chapter 6.
5. Various economic considerations can be incorporated into the WLA procedures presented here, for example, the costs of operating and constructing treatment plant facilities.
6. Only two forms of equity (i.e., that of equal percent removal and equal effluent concentrations) are considered in the WLA formulation. However, Chadderton et al. (1981) have expressed the notion of approximately twenty forms of equity to be considered in the WLA process, some of which could be at one time or another incorporated into the models of Chapters 2 and 7.
7. The four measures of determining the critical location in a stream environment under uncertainty present in Chapter 6

are not to be considered exclusive. The possibility of more appropriate measures exist.

8. The spatial correlation of the water quality parameters can be investigated and incorporated into the probabilistic analysis in the second half of this study.

8.3 FINAL COMMENTS

Accurate prediction and protection of the quality of this nation's water resources is dependent on a comprehensive understanding of the complex and random behavior exhibited by nature. More importantly, future growth and sustenance of this and other countries is founded upon the ability of society to preserve and protect the quality of our water resources. If future generations are not to be deprived of such resources, measures to protect the quality of water worldwide must be developed and continually improved.

Unfortunately, only in recent times has the inherent stochastic nature of the stream environment been incorporated into the analysis and modeling of its behavior. In realizing the vivacious, enigmatic character of the aquatic environment, such can no longer be ignored. The complexity of this environment must be investigated and understood if the management of the quality of our water resources is to be truly effective.

It is believed that the methodologies presented here, in fact, make meaningful contributions to these goals. Moreover, it is the hope of this author that research of this kind will inspire others to improve methods for protecting the quality of our water resources.

Water, taken for granted by most but essential to all, must be protected from the growing threat of contamination resulting from increased human activity. The old adage, "an ounce of prevention is worth a pound of cure" could not be more important than it is for the idea of preserving the quality of this nation's surface water resources, now and for the future.

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WASTE LOAD ALLOCATION IN STOCHASTIC STREAM ENVIRONMENTS

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FINAL REPORT

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APPENDIX A

FORMAL PROOF THAT THE CONSTRAINTS DEFINED BY THE LINEAR AND LOGISTIC MEMBERSHIP FUNCTIONS ARE IDENTICAL

Recalling Eqs. (3.21) and (3.22), the constraints defined by the linear membership function in the two-objective programming model can be written as

$$-\frac{Z_1}{d_1} + \lambda \leq -\frac{L_1}{d_1} \tag{A.1}$$

$$-\frac{Z_2}{d_2} + \lambda \leq -\frac{L_2}{d_2} \tag{A.2}$$

Subtracting Eq. (A.2) from Eq. (A.1) we obtain

$$-\frac{Z_1}{d_1} + \frac{Z_2}{d_2} = -\frac{L_1}{d_1} + \frac{L_2}{d_2} \tag{A.3}$$

On the other hand, recalling Eqs. (3.28) and (3.29), the constraints defined by the logistic membership function can be written as

$$-\beta_1 Z_1 + \eta \leq \alpha_1 \tag{A.4}$$

$$-\beta_2 Z_2 + \eta \leq \alpha_2 \tag{A.5}$$

Subtracting Eq. (A.5) from Eq. (A.4), we obtain

$$-\beta_1 Z_1 + \beta_2 Z_2 = \alpha_1 - \alpha_2 \tag{A.6}$$

where according to Eqs. (3.17) and (3.18)

$$\beta_1 = \frac{1}{d_1}(C_1 - C_2); \quad \beta_2 = \frac{1}{d_2}(C_1 - C_2)$$

and

$$\alpha_1 = \frac{U_1}{d_1} C_2 - \frac{L_1}{d_1} C_1; \quad \alpha_2 = \frac{U_2}{d_2} C_2 - \frac{L_2}{d_1}$$

such that

$$C_1 = \ln[P_u/(1 - P_u)] \text{ and } C_2 = \ln[P_\ell/(1 - P_\ell)]$$

Next, by substituting this information into Eq. (A.6), the following equation is derived

$$-\frac{1}{d_1}(C_1 - C_2)Z_1 + \frac{1}{d_2}(C_1 - C_2)Z_2 = \frac{U_1}{d_1} C_2 - \frac{L_1}{d_1} C_1 - \frac{U_2}{d_2} C_2 + \frac{L_2}{d_2} C_1 \tag{A.7}$$

By rearranging terms, Eq. (A.7) can be expressed as

$$-\frac{Z_1}{d_1}(C_1 - C_2) + \frac{Z_2}{d_2}(C_1 - C_2) = \frac{L_1C_1 - U_1C_2}{d_1} + \frac{U_2C_2 - L_2C_1}{d_2} \tag{A.8}$$

and since $U_k = L_k + d_k$, it can be shown that Eq. (A.8) is reduced to

$$\frac{Z_1}{d_1} - \frac{Z_2}{d_2} (C_1 - C_2) = \frac{L_1}{d_1} - \frac{L_2}{d_2} (C_1 - C_2) \tag{C.8}$$

which differs from Eq. (A.3) simply by a constant $(C_1 - C_2)$.

Finally, by dividing each term by $(C_1 - C_2)$ we get

$$\frac{Z_1}{d_1} - \frac{Z_2}{d_2} = \frac{L_1}{d_1} - \frac{L_2}{d_2} \tag{A.9}$$

From this, it is evident that Eq. (A.9) derived from the logistic membership is identical to that of Eq. (A.3) obtained from the linear membership function.

APPENDIX B

SOME IMPORTANT PARTIAL DERIVATIVES OF THE DISSOLVED OXYGEN DEFICIT EQUATION

Remember,

$$D_x = \frac{L_o K_d}{K_a - K_d} [\exp(-K_d x/U) - \exp(-K_a x/U)] + D_o \exp(-K_a x/U) \qquad (B.1)$$

It follows,

$$\partial D_x/\partial K_d = L_o K_a (E_1 - E_2)/(K_a - K_d)^2 - L_o K_d E_1 x/[(K_a - K_d)U]$$

$$\partial D_x/\partial K_a = L_o K_d (E_2 - E_1)/(K_a - K_d)^2 + L_o K_d E_1 x/[(K_a - K_d)U] - D_o E_2 x/U$$

$$\partial D_x/\partial U = L_o (K_d^2 E_1 x - K_d K_a x E_2)/(K_a - K_d)U^2] + D_o K_a x E_2/U^2$$

$$\partial D_x/\partial L_o = K_d (E_1 - E_2)/(K_a - K_d)$$

$$\partial D_x/\partial D_o = E_2$$

where

$$E_1 = \exp(-K_d x/U)$$

$$E_2 - \exp(-K_a x/U)$$

and

$$\partial^2 D_x/\partial K_d^2 = 2L_o K_a(E_1 - E_2)/(K_a - K_d)^3 - 2L_o K_a E_1 x/[(K_a - K_d)^2 U]$$

$$- L_o K_d E_1 x^2/[(K_a - K_d)U^2]$$

$$\partial^2 D_x/\partial K_a^2 = 2L_o K_d(E_1 - E_2)/(K_a - K_d)^3 - 2L_o K_d E_2 x/[(K_a - K_d)^2 U]$$

$$- L_o K_d E_2 x^2/[(K_a - K_d)U^2] + D_o E_2 x^2/U^2$$

$$\partial^2 D_x/\partial U^2 = [(2xU - K_d x^2)/(K_a - K_d)U^4]\, L_o K_d^2 E_1$$

$$+ (2U - K_a x)/[(K_a - K_d)U^4]\, L_o K_d K_a E_2 x$$

$$+ [(K_a x - 2U)/U^4]\, D_o K_a E_2 x$$

$$\partial^2 D_x/\partial L_o^2 = 0$$

$$\partial^2 D_x/\partial D_o^2 = 0$$

$$\partial^2 D_x/\partial K_a \partial U = - L_o K_d^2 E_1 x/[(K_a - K_d)^2 U^2]$$

$$+ [K_d K_a xU - K_d x(K_a - K_d)U + K_d K_a x^2(K_a - K_d)]L_o E_2/[(K_a - K_d)^2 U^3]$$

$$- [(K_a x - U)D_o E_2 x]/U^3$$

APPENDIX C

SOME IMPORTANT PARTIAL DERIVATIVES OF THE CRITICAL LOCATION EQUATION

Remember,

$$X_c = [U/(K_a - K_d)] \ln \left\{(K_a/K_d)[1 - (K_a - K_d)D_o/(K_dL_o)]\right\}$$

To begin, let

$$f = K_a/K_d \quad \text{and} \quad C = [1 - (K_a - K_d)D_o/(K_dL_o)]$$

therefore,

$$\partial X_c/\partial K_d = [U/(K_a - K_d)^2] \ln(fC)$$

$$+ U/[(K_a - K_d)fC] \quad -f^2C/K_a + f[K_aD_o/(K_d^2L_o)]$$

$$\partial X_c/\partial K_a = [-U/(K_a - K_d)^2] \ln(fC)$$

$$+ U/[(K_a - K_d)fC] \quad [C/K_d + f[K_aD_o/(K_d^2L_o)]$$

$$\partial X_c/\partial U = [1/(K_a - K_d)] \ln(fC)$$

$$\partial X_c/\partial L_o = U/[(K_a - K_d)fC] \quad - K_a(K_a - K_d)D /(K_dL_o)^2]$$

$$\partial X_c/\partial L_o = U/[(K_a - K_d)fC] \quad - (K_a - K_d)/(K_dL_o)$$

APPENDIX D

LISTING OF COMPUTER PROGRAM FOR

DETERMINISTIC OPTIMAL WASTE LOAD ALLOCATION

USING MOVABLE CONTROL POINT APPROACH

'WLAMCP'

```
      PROGRAM WLAMCP(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7,
     X TAPE8,TAPE99)
      DIMENSION KD(15),U(15),DISLOC(15),BODCOEF(15,15),QT(15),DOD(15)
      DIMENSION XCNTRLP(150),RHS(200),DOCOEF(200,30),Q(15),DOSAT(15)
      DIMENSION KA(15),NCNTRLP(15),NCNT(15),BODSTD(15),DOSTD(15)
      DIMENSION TCHCOEF(220,30),BOD(15),EQCOEF(100,15),LOWBDOD(15)
      DIMENSION PCTREMV(15),XINBOD(15),XC(15),DELTA(15),UPBDOD(15)
     X ,DOD1(15),BOD1(15),NDOPT(15),YCNTRLP(150)
      DIMENSION DELTBOD(15),UPBTRT(15),LOWBTRT(15),IDISCH(15)
      REAL LO,KD,KA,LOWERB(20),UPPERB(20),UPPER(20),LOWBTRT,LOWBDOD
      REAL BASCB(50),BASLB(50),BASUB(50),BOUND,BLOW(100),LJ,
     X     CANDA(50,6),CANDCJ(6),CJ,COLA(200),MEMORY(15000),
     X     UZERO(50),XBZERO(50),YQ(50),Z,CJX(50),VALUE(50)
      INTEGER BNDTYP,COLLEN,COLMAX,FACTOR,IOERR,IOLOG,ITER1,ITER2,
     X        LENMA,LENMI,LENMY,LOOK,M,MAPA(20),MAPI(20),MAXA,MAXM,
     X        MAXN,N,NTYPE2,P,PRINT,TERMIN,UNBDDQ,BASIS(50),CAND(6),
     X        CANDI(50,6),CANDL(6),COLI(200),ROWTYP(200),STATUS(50)
      INTEGER IOIN,NCOLSA
      COMMON/XMPCOM/BIG,SMALL,ZL,ZLC,EPS1,EPS2,EPS3,EPS4,EPS5,EPS6
      REAL BIG,SMALL,ZL,ZLC,EPS1,EPS2,EPS3,EPS4,EPS5,EPS6
C
C
C     PROGRAM 'WLAMCP' WAS WRITTEN BY WADE HATHHORN AND DR. Y.K. TUNG
C     WITH FINAL REVISIONS BEING MADE ON JULY 10, 1986.  THIS PROGRAM
C     IS PART OF WATER QUALITY RESEARCH FUNDED BY THE WYOMING WATER
C     RESEARCH CENTER  LARAMIE, WYOMING  82071.
C
C
C     THIS PROGRAM IS DESIGNED TO SOLVE WATER QUALITY PROBLEMS
C     CONCERNING THE OPTIMAL ALLOCATION OF WASTE DISCHARGE TO
C     SEVERAL USERS OF A STREAM SYSTEM.  THIS PROGRAM IS USED
C     IN CONJUNCTION WITH AN ADDITIONAL PROGRAM CALLED XMP
C     WHICH SOLVES THE LINEAR PROGRAMMING PROBLEM SET UP BY THIS
C     PROGRAM.  THUS, IN ORDER TO EXECUTE THIS PROGRAM, THE USER
C     MUST HAVE A COMPILED VERSION OF 'XMP' AVAILABLE IN THEIR
C     LOCAL COMPUTER LIBRARY.
C     WATER QUALITY CONSTRAINTS TO THIS PROGRAM ARE BASED ON TRANSFER
C     COEFFICIENTS DERIVED FROM THE STREETER PHELPS EQUATION AND
C     WATER QUALITY STANDARDS INPUT BY THE USER.  ADDITIONALLY, EQUITY
C     CONSIDERATIONS BETWEEN THE VARIOUS USERS OF THE STREAM SYSTEM
C     MAKE UP ANOTHER SET OF CONSTRIANTS TO THE LINEAR PROGRAMMING
C     PROBLEM SET UP BY THIS PROGRAM.  UNIQUE TO THIS PROGRAM IS THE
C     FACT THAT EACH OF THE WATER QUALITY CONSTRAINTS ARE BASED UPON
C     MOVALBLE CONTROL POINT LOCATIONS DERIVED BY CALCULATING THE
C     'CRITICAL POINT' WITHIN EACH REACH.  BY REDUCING THE NUMBER OF
C     CONSTRAINTS USING A SINGLE MOVABLE CONTROL POINT LOCATION IN
C     EACH REACH, GREAT SAVINGS ARE ACCOMPLISHED IN BOTH COMPUTER
C     STORAGE AND PROBLEM SIMPLICITY.
C
C     IN ORDER TO EXECUTE THIS PROGRAM, IN ADDITION TO THE AVAILABILITY
C     OF 'XMP', THE USER MUST SET UP A SINGLE DATA FILE TO BE READ BY
C     THIS PROGRAM IN THE FOLLOWING FORMAT:
C
C    CARD        VARIABLES              DESCRIPTION                  FORMAT
C
C      1      LO,DO,QO,EQDIF,       LO=U/S BOD CONC. MG/L          4F8.0,5I8
C                NREACH             DO=U/S DEFICIT CONC. MG/L
C                 IPOLL             QO=U/S FLOW RATE CFS
C                 INDXBOD           EQDIF = NUMERICAL VALUE
C                 IEQUITY                   CORRESPONDING
C                  IPLOT                    TO THE TYPE OF
C                                           EQUITY CONSIDERED
C
C     NOTE : IF YOU SELECT IEQUITY EQUAL TO :
C                 (0) EQDIF REPRESENTS THE ALLOWABLE DIFFERENCE
C                     IN PERCENT REMOVAL BETWEEN DISCHARGERS
C
C                 (1) EQDIF REPRESENTS THE ALLOWALBLE DIFFERENCE
```

```
C                IN EFFLUENT CONCENTRATIONS BETWEEN DISCHARGERS
C                (MG/L)
C
C                              NREACH = NO. OF REACHES
C                              IPOLL=INDICIES FOR TYPE
C                                    OF POLLUTION TO
C                                    BE CONSIDERED
C                                    (0)=BOD ONLY
C                                    (1)=BOD AND DO DEFICITS
C
C                              INDXBOD=INDICIES TO INCLUDE
C                                     BOD CONSTRAINTS.
C                                     (0)=EXCLUDE BOD CONSTR.
C                                     (1)=INCLUDE BOD CONSTR.
C
C                              IEQUITY=INDICIES TO INDICATE
C                                     TYPE OF EQUITY TO BE
C                                     CONSIDERED
C                                     (0)=EQUAL % REMOVAL
C                                     (1)=EQUAL EFFLUENT CONC.
C
C                              IPLOT=INDICIES TO CREATE
C                                   PLOT OF DO PROFILE
C                                   (0) = DO NOT PLOT
C                                   (1) = PLOT ON VERSATEC
C                                         (COMPUTER CENTER)
C                                   (2) = PLOT ON DATMED
C                                         (WATER CENTER)
C                                   (3) = PLOT ON MODGRAPH
C                                         (C.E. BUILDING)
C
C    2          KD          KD=DEOXYGENATION COEFF.          10F8.0
C                           FOR EACH REACH  1/DAY
C
C    3          KA          KA=REAERATION COEFF.             10F8.0
C                             FOR EACH REACH  1/DAY
C
C    4          U            U=STREAM VELOCITY FOR           10F8.0
C                             EACH REACH MILES/DAY
C
C    5          DISLOC       DISLOC=DISCHARGE LOCATION       10F8.0
C                                 FOR EACH REACH MILES
C
C    6          Q             Q=DISCHARGE FLOW RATE          10F8.0
C                              FOR EACH EFFLUENT CFS
C
C    7          BODSTD        BODSTD=BOD STANDARD FOR        10F8.0
C                                   FOR EACH REACH MG/L
C
C    8          DOSAT         DOSAT=DISSOL. OXYGEN SAT.      10F8.0
C                                 IN EACH REACH MG/L
C
C    9          DOSTD         DOSTD=DISSOL. OXYGEN STD.      10F8.0
C                                 IN EACH REACH MG/L
C
C   10          NCNTRLP       NCNTRLP=NO. OF CONTROL PTS.    10I8
C                                    IN EACH REACH
C
C   11          XINBOD        XINBOD=INFLUENT BOD CONC.      10F8.0
C                                    IN MG/L
C
C   12          UPBTRT         UPBTRT=UPPER BOUNDS FOR       10F8.0
C                                    BOD TREATMENT
C                                    (% REMOVAL AS A
C                                     DECIMAL FRACTION)
C
C   13          LOWBTRT        LOWBTRT=LOWER BOUNDS FOR      10F8.0
C                                     BOD TREATMENT
C
```

```
C                                                  (% REMOVAL AS A
C                                                   DECIMAL FRACTION)
C
C    14          UPBDOD              UPBDOD=UPPER BOUNDS FOR     10F8.0
C                                          DO DEFICITS IN
C                                          EACH EFFL. MG/L
C
C    15          LOWBDOD             LOWBDOD=LOWER BOUNDS FOR    10F8.0
C                                           DO DEFICITS IN
C                                           EACH EFFL. MG/L
C
C    16          IDISCH              IDISCH= INDEX FOR DISCHARGE  10I8
C                                          POINTS
C                                          (0) = TRIBUTARY
C                                          (1) = POLL. EFFL.
C
C*************************************************************************
      READ(5,5)LO,DO,QO,EQDIF,NREACH,IPOLL,INDXBOD,IEQUITY,IPLOT
    5 FORMAT(4F8.0,5I8)
      READ(5,7)(KD(I),I=1,NREACH)
      READ(5,7)(KA(I),I=1,NREACH)
      READ(5,7)(U(I),I=1,NREACH)
      READ(5,7)(DISLOC(I),I=1,NREACH)
      READ(5,7)(Q(I),I=1,NREACH)
      READ(5,7)(BODSTD(I),I=1,NREACH)
      READ(5,7)(DOSAT(I),I=1,NREACH)
      READ(5,7)(DOSTD(I),I=1,NREACH)
      READ(5,7)(XINBOD(I),I=1,NREACH)
      READ(5,7)(UPBTRT(I),I=1,NREACH)
      READ(5,7)(LOWBTRT(I),I=1,NREACH)
      READ(5,7)(UPBDOD(I),I=1,NREACH)
      READ(5,7)(LOWBDOD(I),I=1,NREACH)
      READ(5,8) (IDISCH(I),I=1,NREACH)
    7 FORMAT(10F8.0)
    8 FORMAT(10I8)
C
C
C     WRITE HEADING INFORMATION ON OUTPUT
C
      WRITE(6,10)NREACH,LO,DO,QO,INDXBOD,IEQUITY,IPOLL,EQDIF
   10 FORMAT(1H1/3X,'MOVABLE CONTROL POINT INFORMATION :',
     X //3X,'NREACH = ',I6,10X,'LO = ',F6.2,
     X /3X,'DO = ',F6.2,14X,'QO = ',F6.2,
     X /3X,'INDXBOD = ',I6,9X,'IEQUITY = ',I6,
     X /3X,'IPOLL = ',I6,11X,'EQDIF = ',F6.2)
      WRITE(6,11)(KD(I),I=1,NREACH)
   11 FORMAT(/3X,'KD :',/10(2X,F7.2))
      WRITE(6,12)(KA(I),I=1,NREACH)
   12 FORMAT(/3X,'KA :',/10(2X,F7.2))
      WRITE(6,13)(DISLOC(I),I=1,NREACH)
   13 FORMAT(/3X,'DISCHARGE LOCATIONS :'/10(2X,F7.2))
      WRITE(6,14)(Q(I),I=1,NREACH)
   14 FORMAT(/3X,'EFFLUENT DISCHARGE RATES (CFS) :',/10(2X,F7.2))
      WRITE(6,15)(XINBOD(I),I=1,NREACH)
   15 FORMAT(/3X,'PLANT INFLUENT BOD (MG/L) :',/10(2X,F7.2))
      WRITE(6,16)(IDISCH(I),I=1,NREACH)
   16 FORMAT(/3X,'TYPE OF DISCHARGE (TREATMENT PLANT OR TRIBUTARY)',
     X /10(2X,I7)//)
C
C     SET UP AN ARRAY FOR UPPER AND LOWER BOUNDS ON
C     DECISION VARIABLES.
C
      KJ=NREACH
      DO 18 IJ=1,NREACH
      KJ=KJ+1
      UPPERB(IJ)=(1.-LOWBTRT(IJ))*XINBOD(IJ)
      UPPERB(KJ)=UPBDOD(IJ)
```

```
      LOWERB(IJ)=(1-UPBTRT(IJ))*XINBOD(IJ)
      LOWERB(KJ)=LOWBDOD(IJ)
   18 CONTINUE
C
C
      IOIN=5
      IOERR=6
      IOLOG=6
      IOOUT=6
      IODEB=7
C
C     SET UP NUMBER OF CONTROL POINT LOCATIONS IN EACH REACH
C
      DO 20 I=1,NREACH
      NCNTRLP(I)=1
   20 CONTINUE
C
C     INITIIALIZE CONTROL POINT LOCATIONS AT 1/4 DISTANCE BETWEEN
C     DISCHARGE LOCATIONS AND CREATE FALSE DISCHARGE LOCATION TO
C     ALLOW FOR REGULATION OF FINAL DISCHARGE
C
      DO 25 I=1,NREACH
      IF(I .EQ. NREACH) DISLOC(I+1)=DISLOC(I)+(DISLOC(I)-DISLOC(1))
     * /(NREACH-1)
C
      XCNTRLP(I)=0.25*(DISLOC(I+1)-DISLOC(I))+DISLOC(I)
   25 CONTINUE
C
C     CALL SUBROUTINE TO GENERATE TECHNOLOGICAL COEFFICIENT
C
C
      ITERCHK=0
   30 ITERCHK=ITERCHK+1
      IF(ITERCHK .GT. 10)WRITE(6,35)
   35 FORMAT(/1X,'THE NUMBER OF ITERATIONS EXCEEDS 10')
      IF(ITERCHK .GT. 10)GOTO 600
C
      WRITE(6,37)(XCNTRLP(I),I=1,NREACH)
   37 FORMAT(/5X,'XCNTRLP:'/8(2X,F6.2))
C
      CALL TCHCOE1(NREACH,NT,RHS,TCHCOEF,IPOLL,LO,DO,QO,EQDIF,
     1 KD,KA,U,DISLOC,Q,BODSTD,DOSAT,DOSTD,NCNTRLP,XCNTRLP,INDXBOD,
     2 UPPERB,LOWERB,BLOW,NEQROW,IEQUITY,XINBOD,QT,IDISCH,
     3 NCNT,NDISCH)
C
      IPRINT=2
      BNDTYP=4
      NTYPE2=0
      BOUND=0.
      IF(INDXBOD .EQ. 1)MXX=NT+2*NREACH
      IF(INDXBOD .EQ. 0)MXX=NT+NREACH
      MAXM=MXX+NEQROW
      MAXN=2*NREACH+MAXM
      MAXA=MAXM*MAXN
      COLMAX=MAXM
      P=6
      LOOK=2*NREACH
      FACTOR=50
      LENMY=15000
      PRINT=1
C
C     PRINT= (0) ERROR MESSAGES ONLY
C            (1) TERMINATION CONDITION MESSAGE
C            (2) PRINT OBJECTIVE FUNCTION VALUE AFTER
C                EACH BASIS RE-FACTORIZATION
C            (3) LOG INFORMATION AT EVERY ITERATION
C
      NCOLSA=2*NREACH
```

```
      M=MAXM
      CALL XMAPS(BNDTYP,IOERR,LENMA,LENMI,LENMY,MAPA,MAPI,MAXA,MAXM,
     X           MAXN,MEMORY)
C
      IF(INDXBOD .EQ. 0)MNNXX=MXX
      IF(INDXBOD .EQ. 1)MNNXX=MXX-NREACH
      IXXI=MNNXX+NEQROW
C
C     SET ROW TYPES
C
C     2=TWO SIDED CONSTRAINT
C     1=LESS THAN; EQUAL TO
C     0=EQUATION
C    -1=GREATER THAN; EQUAL TO
C    -2=MEANS FREE ROW
C
      DO 40 IR=1,MAXM
      IF(IR .GT. MNNXX .AND. IR .LE. IXXI)ROWTYP(IR)=2
      IF(IR .GT. MNNXX .AND. IR .LE. IXXI)GOTO 40
      ROWTYP(IR)=1
   40 CONTINUE
      DO 50 JC=1,NCOLSA
      IF(JC .GT. NREACH .AND. IPOLL .EQ. 0)CJX(JC)=0.0
      IF(JC .GT. NREACH .AND. IPOLL .EQ. 0)GOTO 50
      CJX(JC)=1.0
   50 CONTINUE
C
      N=0
      DO 130 JC=1,NCOLSA
      CJ=CJX(JC)
      UJ=UPPERB(JC)
      LJ=LOWERB(JC)
      COLLEN=MAXM
      DO 120 IR=1,COLLEN
      COLI(IR)=IR
      COLA(IR)=TCHCOEF(IR,JC)
  120 CONTINUE
      CALL XADDUB(BNDTYP,IOERR,JC,LENMA,LENMY,LJ,MAPA,MEMORY,UJ)
      CALL XADDAJ(CJ,COLA,COLI,COLLEN,COLMAX,IOERR,JC,
     X            LENMA,LENMY,MAPA,MEMORY,N)
  130 CONTINUE
      DO 140 JC=1,N
  140 STATUS(JC)=0
C
      Z=0.0
      CALL XSLACK(RHS,BASCB,BASIS,BASLB,BASUB,BLOW,BNDTYP,BOUND,
     X COLA,COLI,COLMAX,IOERR,LENMA,LENMI,LENMY,
     X M,MAPA,MAPI,MAXM,MAXN,MEMORY,N,ROWTYP,STATUS,
     X UZERO,XBZERO,Z)
C
C
      CALL XPRIML(RHS,BASCB,BASIS,BASLB,BASUB,BNDTYP,BOUND,
     X CAND,CANDA,CANDCJ,CANDI,CANDL,COLA,COLI,COLMAX,
     X FACTOR,IOERR,IOLOG,ITER1,ITER2,LENMA,LENMI,LENMY,LOOK,
     X M,MAPA,MAPI,MAXM,MAXN,MEMORY,N,NTYPE2,P,PRINT,
     X STATUS,TERMIN,UNBDDQ,UZERO,XBZERO,YQ,Z)
C
C
      WRITE(IOLOG,325) TERMIN
  325 FORMAT(18H TERMINATION CODE=,I4)
      IF(IPRINT .EQ. 0 .OR. IPRINT .EQ. 1)GOTO 326
C
C
      CALL XPRINT(BASIS,BNDTYP,BOUND,IOERR,IOLOG,
     X LENMA,LENMY,M,MAPA,MAXM,MAXN,MEMORY,N,NTYPE2,
     X STATUS,XBZERO,Z)
C
C
```

```
C     WRITE(6,200) (STATUS(J),J=1,N)
C 200 FORMAT(//5X,"STATUS :"/8(I5,2X))
  326 DO 330 J=1,NCOLSA
      VALUE(J)=0.
  330 CONTINUE
      DO 370 J=1,NREACH
      IF(STATUS(J)) 340,350,360
  340 BOD(J)=UPPERB(J)
      GO TO 370
  350 BOD(J)=LOWERB(J)
      GO TO 370
  360 IX=STATUS(J)
      BOD(J)=XBZERO(IX)
  370 CONTINUE
      JDOD=0
      NP1=NREACH+1
      NN=NREACH*2
      DO 410 J=NP1,NN
      JDOD=JDOD+1
      IF(STATUS(J))380,390,400
  380 DOD(JDOD)=UPPERB(J)
      GOTO 410
  390 DOD(JDOD)=LOWERB(J)
      GOTO 410
  400 IX=STATUS(J)
      DOD(JDOD)=XBZERO(IX)
  410 CONTINUE
C
C     CALL SUBROUTINE TO CALCULATE CRITICAL LOCATIONS
C
      CALL CRITIC(QT,DOD,BOD,XC,DO,NREACH,KD,KA,LO,U,QO,Q,DISLOC)
C
      ICHECK=0
      DO 411 IXX=1,NREACH
      DELTA(IXX)=ABS(XCNTRLP(IXX)-XC(IXX))
      IF(DELTA(IXX) .GT. 0.01)ICHECK=ICHECK+1
  411 CONTINUE
      IF(ICHECK .EQ. 0)GOTO 418
C
C      UPDATE CONTROL POINT LOCATIONS
C
      DO 412 IXXX=1,NREACH
      IF(XC(IXXX) .GE. DISLOC(IXXX+1))XCNTRLP(IXXX)=DISLOC(IXXX+1)
      IF(XC(IXXX) .LT. DISLOC(IXXX+1))XCNTRLP(IXXX)=XC(IXXX)
      IF(XCNTRLP(IXXX) .EQ. DISLOC(IXXX))NCNTRLP(IXXX)=0
      IF(XCNTRLP(IXXX) .EQ. DISLOC(IXXX+1))NCNTRLP(IXXX)=0
  412 CONTINUE
      IF(ITERCHK .EQ. 1)GOTO 415
C
C     PROVIDE CHECK ON THE BOD ASSIGNED DURING THE ITERATIONS
C
      JCHCK=0
      DO 414 IXXXX=1,NREACH
      DELTBOD(IXXXX)=ABS(BOD1(IXXXX)-BOD(IXXXX))/BOD(IXXXX)
      IF(DELTBOD(IXXXX) .GT. 0.05)JCHCK=JCHCK+1
  414 CONTINUE
      IF(JCHCK .EQ. 0)GOTO 418
  415 DO 416 I=1,NREACH
      BOD1(I)=BOD(I)
      DOD1(I)=DOD(I)
  416 CONTINUE
      GOTO 30
  418 DO 419 LX=1,NREACH
      PCTREMV(LX)=(XINBOD(LX)-BOD(LX))/XINBOD(LX)*100.0
  419 CONTINUE
      WRITE(IOOUT,420)(I,I=1,NREACH)
  420 FORMAT(/9X,6(6X,'REACH',I1))
      WRITE(IOOUT,430)(BOD(J),J=1,NREACH)
```

```
  430 FORMAT(/7X,'BOD',10(3X,E9.4))
      WRITE(IOOUT,440)(DOD(J),J=1,NREACH)
  440 FORMAT(/7X,'DOD',10(3X,E9.4))
      WRITE(IOOUT,450)(PCTREMV(J),J=1,NREACH)
  450 FORMAT(/2X,'%REMOVAL',5X,8(F5.2,7X))
      IF(IEQUITY .EQ. 0)WRITE(IOOUT,460)
      IF(IEQUITY .EQ. 1)WRITE(IOOUT,480)
  460 FORMAT(//2X,'THE EQUITY CONSIDERED IS EQUAL % REMOVAL')
  480 FORMAT(//2X,'THE EQUITY CONSIDERED IS EQUAL EFFLUENT CONC.')
C
CC    CREATE DO PROFILE
C
      NCP=20
      DO 500 I123=1,NREACH
      NDOPT(I123)=NCP
  500 CONTINUE
      CALL DATGEN(NDOPT,YCNTRLP,DISLOC,NREACH)
C     WRITE(6,550)(NDOPT(I1),I1=1,NREACH)
C 550 FORMAT(/'NDOPT:'/10(2X,I3))
      CALL DOCOMP(NREACH,LO,DO,QO,KD,KA,U,DISLOC,Q,DOSTD,DOSAT,
     X NDOPT,YCNTRLP,IDISCH,BOD,DOD,NPOINT)
C
CC    CALL SUBROUTINE TO CREATE PLOT OF DO PROFILE
CC    (OPTIONAL; SEE COMMENTS AT TOP OF PROGRAM)
C
      IF(IPLOT .EQ. 1)CALL PLOTTER(NPOINT,NREACH,DISLOC,DOSTD,DOSAT,
     X NCP,IPLOT)
  600 STOP
      END
C
CC    ***********************************************************************
C     SUBROUTINE TCHCOE1 IS WHERE THE TECHNOLOGICAL COEFFICIENTS FOR
C     THE LINEAR PROGRAMMING PROBLEM ARE DEFINED
CC    ***********************************************************************
C
      SUBROUTINE TCHCOE1(NREACH,NT,RHS,TCHCOEF,IPOLL,LO,DO,QO,EQDIF,
     1 KD,KA,U,DISLOC,Q,BODSTD,DOSAT,DOSTD,NCNTRLP,XCNTRLP,INDXBOD,
     2 UPPERB,LOWERB,BLOW,NEQROW,IEQUITY,XINBOD,QT,IDISCH,
     3 NCNT,NDISCH)
      DIMENSION KD(15),U(15),DISLOC(15),BODCOEF(15,15),QT(15),IDISCH(15)
      DIMENSION XCNTRLP(150),RHS(200),DOCOEF(200,30),Q(15),DOSAT(15)
      DIMENSION KA(15),NCNTRLP(15),NCNT(15),BODSTD(15),DOSTD(15)
      DIMENSION TCHCOEF(220,30),UPPERB(20),LOWERB(20),EQCOEF(100,15)
      REAL LO,KD,KA,BLOW(100),XINBOD(15)
C
      CQT=QO
C
CC    CALCULATE TOTAL FLOWS IN EACH REACH AND DETERMINE TOTAL
CC    NUMBER OF CONTROL POINTS.
C
      NDISCH=0
      NT=0
      DO 50 J=1,NREACH
      CQT=CQT+Q(J)
      QT(J)=CQT
      NT=NT+NCNTRLP(J)
      NCNT(J)=NT
      NDISCH=NDISCH+IDISCH(J)
   50 CONTINUE
C
CC    INITIALIZE ARRAYS
CC
C
      DO 85 I=1,NREACH
      DO 80 J=1,NREACH
      BODCOEF(I,J)=0.0
   80 CONTINUE
   85 CONTINUE
```

```
      NEQROW=(NDISCH**2 - NDISCH)/2
      DO 87 I=1,NEQROW
      DO 86 J=1,NREACH
      EQCOEF(I,J)=0.0
   86 CONTINUE
   87 CONTINUE
      NRDOD=NT+NREACH
      NCDOD=2*NREACH
      DO 95 J=1,NCDOD
      DO 90 I=1,NRDOD
      DOCOEF(I,J)=0.0
   90 CONTINUE
   95 CONTINUE
      NROWTOL=NRDOD+NREACH+NEQROW
      DO 97 I=1,NCDOD
      DO 96 J=1,NROWTOL
      TCHCOEF(J,I)=0.0
   96 CONTINUE
   97 CONTINUE
      IF(INDXBOD .EQ. 0)GOTO 410
C
C     DETERMINE TECHNOLOGICAL COEFFICIENTS FOR BOD
C
      DO 300 J=1,NREACH
      TFBOD=1.0
      DO 200 I=J,NREACH
      IF(I .EQ. J)GOTO 150
      TFBOD=TFBOD*EXP(-KD(I-1)*(DISLOC(I)-DISLOC(I-1))/U(I-1))
  150 BODCOEF(I,J)=TFBOD*Q(J)/QT(I)
  200 CONTINUE
  300 CONTINUE
C     DO 320 I=1,NREACH
C     WRITE(7,310)(BODCOEF(I,J),J=1,NREACH)
C 310 FORMAT(5(1X,E10.3))
C 320 CONTINUE
C
C     DETERMINE RHS FOR BOD
C
      TFBOD=1.0
      IROW=NT+NREACH+NEQROW
      DO 400 I=1,NREACH
      IROW=IROW+1
      IF(I .EQ. 1)GOTO 350
      TFBOD=TFBOD*EXP(-KD(I-1)*(DISLOC(I)-DISLOC(I-1))/U(I-1))
  350 RHS(IROW)=BODSTD(I)-QO*LO*TFBOD/QT(I)
  400 CONTINUE
C
C     CALCULATE TECHNOLOGICAL COEFFICIENTS FOR DO (BASED ON BOD)
C
  410 NROW=1
      IM1=0
      DO 700 I=1,NREACH
      IM1=I-1
      IF(I .NE. 1)NROW=NCNT(IM1)+IM1+1
      IRCNT=IM1
      TERM1=1.0
      TERM2=0.0
      DOCOEF(NROW,I)=0.0
      DO 600 J=I,NREACH
      IF(NCNTRLP(J) .EQ. 0) IRCNT=IRCNT+1
      IF(NCNTRLP(J) .EQ. 0)GOTO 550
      DO 500 K=1,NCNTRLP(J)
      IRCNT=IRCNT+1
      X=XCNTRLP(IRCNT)-DISLOC(J)
      PART1=Q(I)/QT(J)*TERM1*KD(J)/(KA(J)-KD(J))*
     1 (EXP(-KD(J)*X/U(J))-EXP(-KA(J)*X/U(J)))
      PART2=TERM2*Q(I)/QT(J)*EXP(-KA(J)*X/U(J))
      NROW=NROW+1
```

```
      DOCOEF(NROW,I)=PART1+PART2
  500 CONTINUE
  550 IF(J .EQ. NREACH)GOTO 700
      XR=DISLOC(J+1)-DISLOC(J)
      FPART1=TERM1*KD(J)/(KA(J)-KD(J))*
     1 (EXP(-KD(J)*XR/U(J))-EXP(-KA(J)*XR/U(J)))
      FPART2=TERM2*EXP(-KA(J)*XR/U(J))
      NROW=NROW+1
      DOCOEF(NROW,I)=(FPART1+FPART2)*Q(I)/QT(J+1)
      TERM1=TERM1*EXP(-KD(J)*XR/U(J))
      TERM2=FPART1+FPART2
  600 CONTINUE
  700 CONTINUE
C     NR=NT+NREACH
C     DO 705 IR=1,NR
C     WRITE(7,702)(DOCOEF(IR,JC),JC=1,NREACH)
C 702 FORMAT(5(2X,E10.3))
C 705 CONTINUE
C
C     CALCULATE TECHNOLOGICAL COEFFICIENTS FOR DO (BASED ON DO)
C
      MM1=0
      KROW=1
      DO 1000 M=1,NREACH
      KCNTR=0
      IF(M .EQ. 1)GOTO 720
      MM1=M-1
      DO 710 J=1,MM1
      KCNTR=KCNTR+NCNTRLP(J)
  710 CONTINUE
  720 NCOL=M+NREACH
      TFDO=1.0
      DO 900 N=M,NREACH
      IF(N .EQ. M)GOTO 750
      TFDO=TFDO*EXP(-KA(N-1)*(DISLOC(N)-DISLOC(N-1))/U(N))
  750 DOCOEF(KROW,NCOL)=TFDO*Q(M)/QT(N)
      IF(NCNTRLP(N) .EQ. 0)GOTO 850
      DO 800 L=1,NCNTRLP(N)
      KCNTR=KCNTR+1
      KROW=KROW+1
      TFDOX=TFDO*EXP(-KA(N)*(XCNTRLP(KCNTR)-DISLOC(N))/U(N))
      DOCOEF(KROW,NCOL)=TFDOX*Q(M)/QT(N)
  800 CONTINUE
  850 KROW=KROW+1
  900 CONTINUE
      KROW=NCNT(M)+M+1
 1000 CONTINUE
C
C
C     DETERMINE EQUITY COEFFICIENTS
C
      ICC=0
      LLL=1
      MNN=0
      NRR=NREACH-1
      DO 1030 MNN=1,NRR
      LLL=LLL+1
      IF(IDISCH(MNN) .EQ. 0) GO TO 1030
      DO 1020 JNN=LLL,NREACH
      IF(IDISCH(JNN) .EQ. 0) GO TO 1020
      ICC=ICC+1
      II=ICC+NREACH+NT
      IF(IEQUITY .EQ. 0)EQCOEF(ICC,MNN)=1/XINBOD(MNN)
      IF(IEQUITY .EQ. 0)EQCOEF(ICC,JNN)=-1/XINBOD(JNN)
      IF(IEQUITY .EQ. 1)EQCOEF(ICC,MNN)=1.0
      IF(IEQUITY .EQ. 1)EQCOEF(ICC,JNN)=-1.0
C
C     CREATE RHS FOR EQUITY
```

```
C
      RHS(II)=EQDIF
C
C     CREATE BLOW FOR LOWER BOUNDS ON EQUITY
C     THESE VALUES ARE USED IN TWO-SIDED CONSTRAINTS
C     DEFINED IN THE MAIN PROGRAM
C
      BLOW(II)=-EQDIF
 1020 CONTINUE
 1030 CONTINUE
C
C     DETERMINE RHS FOR DO
C
      LROW=0
      ILN=1
      DO 1200 I=1,NREACH
      ILN=LROW+1
      LROW=LROW+NCNTRLP(I)+1
      DO 1100 J=ILN,LROW
      RHS(J)=DOSAT(I)-DOSTD(I)-DOCOEF(J,1)*QO*LO/QT(1)-
     1 DOCOEF(J,NREACH+1)*QO*DO/QT(1)
 1100 CONTINUE
 1200 CONTINUE
C
C     COMBINE TECHNOLOGICAL COEFFICIENTS INTO A SINGLE
C     ARRAY -- TCHCOEF
C
      MNROW=NT+NREACH
      MNCOL=2*NREACH
      DO 1400 I=1,MNCOL
      DO 1300 J=1,MNROW
      TCHCOEF(J,I)=DOCOEF(J,I)
 1300 CONTINUE
 1400 CONTINUE
      MXROW=NT+NREACH+1
      MBOT=NT+NREACH+NEQROW
      DO 1475 MI=1,NREACH
      MCNTR=0
      DO 1450 NI=MXROW,MBOT
      MCNTR=MCNTR+1
      TCHCOEF(NI,MI)=EQCOEF(MCNTR,MI)
 1450 CONTINUE
 1475 CONTINUE
      JROW=MBOT+1
      JBOTTOM=MBOT+NREACH
      DO 1600 K=1,NREACH
      JCNTR=0
      DO 1500 L=JROW,JBOTTOM
      JCNTR=JCNTR+1
      TCHCOEF(L,K)=BODCOEF(JCNTR,K)
 1500 CONTINUE
 1600 CONTINUE
C
C
C    WRITE OUT TECHNOLOGICAL COEFF.
C
      ITCOL=2*NREACH
C     DO 1700 I=1,NROWTOL
C     WRITE(7,1)
C   1 FORMAT(/1X,'HELLO I AM HERE')
C     WRITE(7,20)(TCHCOEF(I,J),J=1,ITCOL)
C  20 FORMAT (10(2X,E10.3),/)
C1700 CONTINUE
C     DO 1800 I=1,JBOTTOM
C     WRITE(7,30)RHS(I)
C  30 FORMAT(E10.3)
C1800 CONTINUE
      RETURN
```

```
      END
C
C
C
C     ********************************************************
C     SUBROUTINE CRITIC IS TO CALCULATE CRITICAL POINT LOCATIONS
C     ********************************************************
      SUBROUTINE CRITIC(QT,DOD,BOD,XC,DO,NREACH,KD,KA,
     * LO,U,QO,Q,DISLOC)
      DIMENSION QT(15),Q(15),DOD(15),BOD(15),XC(15),KD(15),KA(15)
      DIMENSION U(15),DISLOC(15),XINITD(15),QT1(15),TOPD(15)
      REAL LO,KA,KD,QO,DO,TOPBOD(15),XINIBOD(15)
      QT1(1)=QO
      DO 20 I=1,NREACH
      QT1(I+1)=QT(I)
   20 CONTINUE
      DO 100 I=1,NREACH
      IF(I .EQ. 1)XINITD(I)=DO
      IF(I .EQ. 1)XINIBOD(I)=LO
      IF(I .EQ. 1)GOTO 50
      XINITD(I)=KD(I-1)*TOPBOD(I-1)/(KA(I-1)-KD(I-1))*(EXP(-KD(I-1)*
     1 (DISLOC(I)-DISLOC(I-1))/U(I-1))-EXP(-KA(I-1)*(DISLOC(I)-
     2 DISLOC(I-1))/U(I-1)))+TOPD(I-1)*EXP(-KA(I-1)*
     3 (DISLOC(I)-DISLOC(I-1))/U(I-1))
      XINIBOD(I)=TOPBOD(I-1)*EXP(-KD(I-1)*(DISLOC(I)-DISLOC(I-1))
     1 /U(I-1))
   50 TOPD(I)=(QT1(I)*XINITD(I)+Q(I)*DOD(I))/QT1(I+1)
      TOPBOD(I)=(QT1(I)*XINIBOD(I)+Q(I)*BOD(I))/QT1(I+1)
C
C     CHECK FOR NEGATIVE AND INDEFINITE CRITICAL LOCATIONS
C     THEN CALCULATE XC
C
      CHKCAL=(KA(I)-KD(I))*TOPD(I)/(KD(I)*TOPBOD(I))
      IF(CHKCAL .GE. 1.0)XC(I)=-1.0
      IF(CHKCAL .GE. 1.0)GOTO 90
      XC(I)=DISLOC(I)+U(I)/(KA(I)-KD(I))*ALOG(KA(I)/KD(I)*(1-CHKCAL))
   90 IF(XC(I) .LT. DISLOC(I))XC(I)=DISLOC(I)
  100 CONTINUE
      WRITE(6,110)(XC(I),I=1,NREACH)
  110 FORMAT(/1X,'CRITICAL POINTS:'/1X,7(F6.2,3X))
      RETURN
      END
C
C
C     ********************************************************************
C     SUBROUTINE DOCOMP IS WHERE THE DO PROFILE IS COMPUTED
C     ********************************************************************
C
      SUBROUTINE DOCOMP(NREACH,LO,DO,QO,KD,KA,U,DISLOC,Q,DOSTD,DOSAT,
     X NDOPT,XCNTRLP,IDISCH,BOD,DOD,NPOINT)
      DIMENSION NDOPT(15),KD(15),KA(15),U(15),DISLOC(15),DOSTD(15),
     X DOSAT(15),XCNTRLP(150),IDISCH(15),BOD(15),DOD(15),Q(15),
     X NCNT1(15),QT(15),DOCOEF2(200,30),DOCONC(150),YCNTL(150)
      REAL LO,KD,KA
      CQT=QO
C
C     CALCULATE TOTAL FLOWS IN EACH REACH AND DETERMINE TOTAL
C     NUMBER OF CONTROL POINTS.
C
      NDISCH=0
      NT=0
      DO 50 J=1,NREACH
      CQT=CQT+Q(J)
      QT(J)=CQT
      NT=NT+NDOPT(J)
      NCNT1(J)=NT
      NDISCH=NDISCH+IDISCH(J)
   50 CONTINUE
```

```
C
C
C     INITIALIZE ARRAY
C
      NRDOD=NT+NREACH
      NCDOD=2*NREACH
      DO 95 J=1,NCDOD
      DO 90 I=1,NRDOD
      DOCOEF2(I,J)=0.0
   90 CONTINUE
   95 CONTINUE
C
C     CALCULATE TECHNOLOGICAL COEFFICIENTS FOR DO (BASED ON BOD)
C
      NROW=1
      IM1=0
      DO 700 I=1,NREACH
      IRCNT=0
      IF(I .EQ. 1)GOTO 460
      IM1=I-1
      NROW=NCNT1(IM1)+IM1+1
      DO 450 L=1,IM1
      IRCNT=IRCNT+NDOPT(L)
  450 CONTINUE
  460 TERM1=1.0
      TERM2=0.0
      DOCOEF2(NROW,I)=0.0
      DO 600 J=I,NREACH
      DO 500 K=1,NDOPT(J)
      IRCNT=IRCNT+1
      X=XCNTRLP(IRCNT)-DISLOC(J)
      PART1=Q(I)/QT(J)*TERM1*KD(J)/(KA(J)-KD(J))*
     1 (EXP(-KD(J)*X/U(J))-EXP(-KA(J)*X/U(J)))
      PART2=TERM2*Q(I)/QT(J)*EXP(-KA(J)*X/U(J))
      NROW=NROW+1
      DOCOEF2(NROW,I)=PART1+PART2
  500 CONTINUE
      IF(J .EQ. NREACH)GOTO 700
      XR=DISLOC(J+1)-DISLOC(J)
      FPART1=TERM1*KD(J)/(KA(J)-KD(J))*
     1 (EXP(-KD(J)*XR/U(J))-EXP(-KA(J)*XR/U(J)))
      FPART2=TERM2*EXP(-KA(J)*XR/U(J))
      NROW=NROW+1
      DOCOEF2(NROW,I)=(FPART1+FPART2)*Q(I)/QT(J+1)
      TERM1=TERM1*EXP(-KD(J)*XR/U(J))
      TERM2=FPART1+FPART2
  600 CONTINUE
  700 CONTINUE
C     NR=NT+NREACH
C     DO 705 IR=1,NR
C     WRITE(7,702)(DOCOEF2IR,JC),JC=1,NREACH)
C 702 FORMAT(5(2X,E10.3))
C 705 CONTINUE
C
C     CALCULATE TECHNOLOGICAL COEFFICIENTS FOR DO (BASED ON DO)
C
      MM1=0
      KROW=1
      DO 1000 M=1,NREACH
      KCNTR=0
      IF(M .EQ. 1)GOTO 720
      MM1=M-1
      DO 710 J=1,MM1
      KCNTR=KCNTR+NDOPT(J)
  710 CONTINUE
  720 NCOL=M+NREACH
      TFDO=1.0
      DO 900 N=M,NREACH
```

```
      IF(N .EQ. M)GOTO 750
      TFDO=TFDO*EXP(-KA(N-1)*(DISLOC(N)-DISLOC(N-1))/U(N))
  750 DOCOEF2(KROW,NCOL)=TFDO*Q(M)/QT(N)
      DO 800 L=1,NDOPT(N)
      KCNTR=KCNTR+1
      KROW=KROW+1
      TFDOX=TFDO*EXP(-KA(N)*(XCNTRLP(KCNTR)-DISLOC(N))/U(N))
      DOCOEF2(KROW,NCOL)=TFDOX*Q(M)/QT(N)
  800 CONTINUE
      KROW=KROW+1
  900 CONTINUE
      KROW=NCNT1(M)+M+1
 1000 CONTINUE
C     DO 1050 IROW=1,KROW
C     WRITE(7,1025)(DOCOEF2(IROW,JC),JC=1,NCDOD)
C1025 FORMAT(/'DOCOEF2:'/10(2X,E10.3))
C1050 CONTINUE
C
C
      I=0
      NPOINT=NT+NREACH
      DO 1230 IX=1,NREACH
      DO 1200 IYY=1,NDOPT(IX)+1
      I=I+1
      DOCONC(I)=0.0
      TOLDEF=DOCOEF2(I,1)*QO*LO/QT(1)+DOCOEF2(I,NREACH+1)*QO*DO/QT(1)
      DO 1100 J=1,NREACH
      TOLDEF=TOLDEF+BOD(J)*DOCOEF2(I,J)+DOD(J)*DOCOEF2(I,J+NREACH)
 1100 CONTINUE
      DOCONC(I)=DOSAT(IX)-TOLDEF
 1200 CONTINUE
 1230 CONTINUE
C
C     PRINT OUT DO PROFILE
C
      WRITE(6,1245)
 1245 FORMAT(//20X,'DO CONCENTRARION PROFILE'/)
      WRITE(6,1250)(DOCONC(IP),IP=1,NPOINT)
 1250 FORMAT(/11(2X,F5.2))
      WRITE(6,1270)
 1270 FORMAT(//20X,'CONTROL POINT LOCATIONS'/)
      KOUNT=1
      DO 1280 ID=1,NREACH
      WRITE(6,1275)DISLOC(ID),(XCNTRLP(IC),IC=KOUNT,NCNT1(ID))
 1275 FORMAT(11(1X,F6.2))
      KOUNT=NCNT1(ID)+1
 1280 CONTINUE
C
C     WRITE DO PROFILE TO TAPE FOR USE IN POTTING ROUTINE
C
      WRITE(8,1300)(DOCONC(IP8),IP8=1,NPOINT)
 1300 FORMAT(11(1X,F6.2))
      KOUNT8=1
      DO 1350 ID8=1,NREACH
      WRITE(8,1300)DISLOC(ID8),(XCNTRLP(IC8),IC8=KOUNT8,NCNT1(ID8))
      KOUNT8=NCNT1(ID8)+1
 1350 CONTINUE
      RETURN
      END
C     ********************************************************
C     SUBROUTINE DATGEN IS TO GENERATE CONTROL POINT LOCATIONS
C     ********************************************************
      SUBROUTINE DATGEN(NDOPT,XCNTRLP,DISLOC,NREACH)
      DIMENSION NDOPT(15),XCNTRLP(150),DISLOC(15)
      MCNTR=0
      DO 100 I=1,NREACH
      IF(I .EQ. NREACH) DISLOC(I+1)=DISLOC(I)+(DISLOC(I)-DISLOC(1))
     1 /(NREACH-1)
```

```
      DO 50 J=1,NDOPT(I)
      MCNTR=MCNTR+1
      XCNTRLP(MCNTR)=(DISLOC(I+1)-DISLOC(I))/(NDOPT(I)+1)*J
     1 +DISLOC(I)
   50 CONTINUE
  100 CONTINUE
      RETURN
      END
C
C
C     ******************************************************
C     SUBROUTINE PLOTTER IS WHERE PLOT OF DO PROFILE IS CREATED
C     ******************************************************
C
      SUBROUTINE PLOTTER(NPOINT,NREACH,DISLOC,DOSTD,DOSAT,NCP,IPLOT)
      DIMENSION YDAT(150),XDAT(150),DISLOC(15),DOSTD(15),NCNT1(15),
     X XLINE(2),YLINE(2),DOSAT(15)
C
C     NOTE: (1) YDAT: DO CONCENTRATION VALUES
C           (2) XDAT: DOWNSTREAM LOCATIONS
C
      REWIND 8
      READ(8,100)(YDAT(IP8),IP8=1,NPOINT)
  100 FORMAT(11(1X,F6.2))
      DO 200 I=1,NREACH
      IPOS=(I-1)*NCP+I
      IPOSP1=IPOS+1
      IEND=I*(NCP+1)
      READ(8,100)XDAT(IPOS),(XDAT(IC8),IC8=IPOSP1,IEND)
  200 CONTINUE
C
      WRITE(6,100)(YDAT(I),I=1,NPOINT)
      WRITE(6,100)(XDAT(I),I=1,NPOINT)
C
      NP1=NREACH+1
C
      START=DOSTD(1)-1.0
      STEPX=DISLOC(NP1)/NREACH
C
C     TO PLOT ON WATER RESEARCH TERMINALS
C
      IF(IPLOT .EQ. 2)CALL DATMED(0)
C
C     TO PLOT ON C.E. TERMINALS
C
      IF(IPLOT .EQ. 3)CALL TEKALL(4010,960,0,1,0)
C
C     TO CREATE HARDCOPY OF PLOT
C
      IF(IPLOT .EQ. 1)CALL VRSTEC(0,0,0)
C
C     BEGIN CALLS TO PLOTTING ROUTINES
C
      CALL PAGE(8.50,11.0)
      CALL BANGLE(-90)
C     CALL NOBRDR
      CALL PHYSOR(1.5,9.0)
      CALL AREA2D(8.0,6.0)
      CALL FRAME
      CALL YINTAX
      CALL XINTAX
      CALL YNAME('DO CONCENTRATION, MG/L$',100)
      CALL XNAME('DOWNSTREAM LOCATION, MILES$',100)
      CALL YTICKS(4)
      CALL XTICKS(5)
C     CALL HEADIN('DO CONCENTRATION PROFILE$',100,1.5,1.0)
      CALL GRAF(0.0,STEPX,DISLOC(NP1),START,1.0,DOSAT(1))
      CALL CURVE(XDAT,YDAT,NPOINT,0)
```

```
C
C     PLOT DO STANDARD (LINE)
C
      XLINE(1)=0.0
      XLINE(2)=DISLOC(NP1)
      YLINE(1)=DOSTD(1)
      YLINE(2)=DOSTD(1)
      CALL DOT
      CALL CURVE(XLINE,YLINE,2,0)
      CALL ENDPL(0)
      CALL DONEPL
      RETURN
      END
```

APPENDIX E

LISTING OF COMPUTER PROGRAM FOR DETERMINISTIC MULTIOBJECTIVE WASTE LOAD ALLOCATION USING CONSTRAINT METHOD AND THE GENERATION OF THE NONINFERIOR SOLUTION SET 'WLAMOBJ'

```
      PROGRAM WLAMOBJ(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7,
     X TAPE99)
C
C  THIS PROGRAM WAS WRITTEN BY WADE HATHHORN AND DR. Y.K. TUNG WITH
C  FINAL REVISIONS BEING MADE ON JUNE 19, 1986. THE ESSENCE OF THIS
C  PROGRAM IS THE DEVELOPMENT OF AN INDEIFFERENCE CURVE DEPICTING THE
C  THE TRADEOFF BETWEEN MAXIMIZING WASTE LOAD ALLOCATION AND MINIMIZING
C  THE DIFFERENCE IN EQUITY BETWEEN THE VARIOUS USERS OF THE STREAM
C  SYSTEM.  THE DEVELOLPMENT OF SUCH A CURVE PROVIDES A USEFUL TECHNIQUE
C  IN THE ANALYSIS OF MULTIOBJECTIVE WATER QUALITY PROBLEMS.
C  IN ORDER TO ACCOMPLISH THESE TASKS, THIS PROGRAM UTILIZES BOTH
C  LINEAR PROGRAMMING TECHNIQUES AND THE CONSTRAINT METHOD FOR THE
C  DEVELOPMENT OF THE INDIFFERENCE CURVE DESCRIBED ABOVE.
C
C     IN ORDER TO SOLVE THE LINEAR PROGRAMMING PROBLEM FORMULATED
C     BY THIS PROGRAM, SEVERAL CALLS ARE MADE TO A CANNED PROGRAM
C     CALLED 'XMP'.  THUS TO EXECUTE THIS PROGRAM, THE USER MUST
C     HAVE A COMPILED VERSION OF 'XMP' AVAILABLE IN THEIR LOCAL
C     LIBRARY.
C
C  TO EXECUTE THIS PROGRAM, ONE INPUT FILE MUST BE MADE WHICH
C  CONTAINS THE FOLLOWING INFORMATION AND FORMAT:
C
C
C
C   CARD       VARIABLES            DESCRIPTION                    FORMAT
C
C     1     LO,DO,QO,STEPDIF,    LO=U/S BOD CONC. MG/L          5F8.0,5I8
C             LASTDIF            DO=U/S DEFICIT CONC. MG/L
C              NREACH            QO=U/S FLOW RATE CFS
C               IPOLL            STEPDIF=INCREMENT (STEP SIZE)
C              INDXBOD                  TO CONTROL THE MAX.
C              IEQUITY                  DIFFERENCE IN EQUITY
C               IPLOT                   SELECTED (SEE IEQUITY
C                                       BELOW)
C
C                                LASTDIF=LAST MAX. DIFFERENCE IN
C                                       EQUITY TO BE CONSIDERED
C
C                                NREACH= NO. OF REACHES
C                                IPOLL=INDICIES FOR TYPE
C                                      OF POLLUTION TO
C                                      BE CONSIDERED
C                                      (0)=BOD ONLY
C                                      (1)=BOD AND DO DEFICITS
C
C                                INDXBOD=INDICIES TO INCLUDE
C                                       BOD CONSTRAINTS.
C                                       (0)=EXCLUDE BOD CONSTR.
C                                       (1)=INCLUDE BOD CONSTR.
C
C                                IEQUITY=INDICIES TO INDICATE
C                                       TYPE OF EQUITY TO BE
C                                       CONSIDERED
C                                       (0)=EQUAL % REMOVAL
C                                       (1)=EQUAL EFFLUENT CONC.
C
C                                IPLOT = INDICIES TO GENERATE
C                                       PLOT OF INDIFFERENCE
C                                       CURVE
C                                       (0) = DO NOT PLOT
C                                       (1) = PLOT TO VERSATEC
C                                             (COMPUTER CENTER)
C                                       (2) = PLOT TO DATMED
C                                             (WATER CENTER)
C                                       (3) = PLOT TO MODGRAPH
C                                             (C.E. BUILDING)
C
```

```
C     2            KD              KD=DEOXYGENATION COEFF.          10F8.0
C                                  FOR EACH REACH  1/DAY
C
C     3            KA              KA=REAERATION COEFF.             10F8.0
C                                    FOR EACH REACH  1/DAY
C
C     4            U                U=STREAM VELOCITY FOR           10F8.0
C                                    EACH REACH MILES/DAY
C
C     5          DISLOC             DISLOC=DISCHARGE LOCATION       10F8.0
C                                         FOR EACH REACH MILES
C
C     6            Q                Q=DISCHARGE FLOW RATE           10F8.0
C                                    FOR EACH EFFLUENT CFS
C
C     7          BODSTD             BODSTD=BOD STANDARD FOR         10F8.0
C                                         FOR EACH REACH MG/L
C
C     8           DOSAT             DOSAT=DISSOL. OXYGEN SAT.       10F8.0
C                                         IN EACH REACH MG/L
C
C     9           DOSTD             DOSTD=DISSOL. OXYGEN STD.       10F8.0
C                                         IN EACH REACH MG/L
C
C    10          XINBOD             XINBOD=INFLUENT BOD CONC.       10F8.0
C                                         IN MG/L
C
C    11          UPBTRT             UPBTRT=UPPER BOUNDS FOR         10F8.0
C                                         BOD TREATMENT
C                                         (% REMOVAL AS A
C                                          DECIMAL FRACTION)
C
C    12          LOWBTRT            LOWBTRT=LOWER BOUNDS FOR        10F8.0
C                                         BOD TREATMENT
C                                         (% REMOVAL AS A
C                                          DECIMAL FRACTION)
C
C    13          UPBDOD             UPBDOD=UPPER BOUNDS FOR         10F8.0
C                                         DO DEFICITS IN
C                                         EACH EFFL. MG/L
C
C    14          LOWBDOD            LOWBDOD=LOWER BOUNDS FOR        10F8.0
C                                         DO DEFICITS IN
C                                         EACH EFFL. MG/L
C
C    15          IDISCH             IDISCH = INDEX FOR DISCHARGE    10I8
C                                          POINTS
C                                          (0) = TRIBUTARY
C                                          (1) = PLANT EFFL.
C
C
C*************************************************************************
      DIMENSION KD(15),U(15),DISLOC(15),BODCOEF(15,15),QT(15),DOD(15)
      DIMENSION XCNTRLP(50),RHS(200),DOCOEF(200,30),Q(15),DOSAT(15)
      DIMENSION KA(15),NCNT(15),BODSTD(15),DOSTD(15),NCNTRLP(15),
     X UPBTRT(15),LOWBTRT(15),UPBDOD(15),LOWBDOD(15),XC(15),
     X DELTBOD(15),DOD1(15),BOD1(15),DELTA(15),IDISCH(15)
      DIMENSION TCHCOEF(220,30),BOD(15),EQCOEF(100,15)
      DIMENSION PCTREMV(15),XINBOD(15),ZDIFF(20),ZTOTAL(20)
      REAL LO,KD,KA,LOWERB(20),UPPERB(20),UPPER(20),LJ,
     X LOWBDOD,LOWBTRT,LASTDIF
      REAL BASCB(80),BASLB(80),BASUB(80),BOUND,BLOW(1),
     X     CANDA(80,8),CANDCJ(8),CJ,COLA(200),MEMORY(20000),
     X     UZERO(80),XBZERO(80),YQ(80),Z,CJX(80),VALUE(80)
      INTEGER BNDTYP,COLLEN,COLMAX,FACTOR,IOERR,IOLOG,ITER1,ITER2,
     X        LENMA,LENMI,LENMY,LOOK,M,MAPA(20),MAPI(20),MAXA,MAXM,
```

```
     X        MAXN,N,NTYPE2,P,PRINT,TERMIN,UNBDDQ,BASIS(80),CAND(8),
     X        CANDI(80,8),CANDL(8),COLI(200),ROWTYP(200),STATUS(80)
      INTEGER IOIN,NCOLSA,ZITER
      COMMON/XMPCOM/BIG,SMALL,ZL,ZLC,EPS1,EPS2,EPS3,EPS4,EPS5,EPS6
      REAL BIG,SMALL,ZL,ZLC,EPS1,EPS2,EPS3,EPS4,EPS5,EPS6
C
C
      IOIN=5
      IOERR=6
      IOLOG=6
      IOOUT=6
      IODEB=7
C
C
C     READ IN PROGRAM INFORMATION (DATA)
C
      CALL READIN(KD,KA,U,DISLOC,Q,BODSTD,DOSAT,DOSTD,
     * XINBOD,UPBTRT,UPBDOD,LOWBTRT,LOWBDOD,LO,DO,QO,NREACH,
     * IPOLL,INDXBOD,IEQUITY,IDISCH,IPLOT,STEPDIF,LASTDIF)
C
C
C     SET UP AN ARRAY FOR UPPER AND LOWER BOUNDS ON
C     DECISION VARIABLES.
C
      KJ=NREACH
      DO 10 IJ=1,NREACH
      KJ=KJ+1
      UPPERB(IJ)=(1-LOWBTRT(IJ))*XINBOD(IJ)
      UPPERB(KJ)=UPBDOD(IJ)
      LOWERB(1J)=(1-UPBTRT(IJ))*XINBOD(IJ)
      LOWERB(KJ)=LOWBDOD(IJ)
   10 CONTINUE
C
      ZITER=0
      DIFMAX=0.0
C
C     SET UP NUMBER OF CONTROL POINT LOCATIONS IN EACH REACH
C
   12 DO 15 IK=1,NREACH
      NCNTRLP(IK)=1
   15 CONTINUE
C
C     PROVIDE COUNTER TO INCREMENT THE ALLOWED MAX. EQUITY DIFFERENCE
C
      ZITER=ZITER+1
      DIFMAX=DIFMAX+STEPDIF
      IF(DIFMAX .GT. LASTDIF) GOTO 510
C
C     INITIIALIZE CONTROL POINT LOCATIONS AT 1/4 DISTANCE BETWEEN
C     DISCHARGE LOCATIONS AND CREATE FALSE DISCHARGE LOCATION TO
C     ALLOW FOR REGULATION OF FINAL DISCHARGE
C
      DO 25 I=1,NREACH
      IF(I .EQ. NREACH) DISLOC(I+1)=DISLOC(I)+(DISLOC(I)-DISLOC(1))
     * /(NREACH-1)
      XCNTRLP(I)=0.25*(DISLOC(I+1)-DISLOC(I))+DISLOC(I)
   25 CONTINUE
C
C     PROVIDE CHECK ON THE NUMBER OF ITERATONS
C
      ITERCHK=0
   30 ITERCHK=ITERCHK+1
      IF(ITERCHK .GT. 10)WRITE(6,32)
   32 FORMAT(/1X,'THE NUMBER OF ITERATIONS EXCEEDS 10')
      IF(ITERCHK .GT. 10)GOTO 600
C
C     SET IOBJ= MAXIMIZE BOD AND/OR DOD LOADING
C
```

```
      IPRINT=2
      BNDTYP=4
      NTYPE2=0
      BOUND=0.
      IF(INDXBOD .EQ. 1)MXX=NT+2*NREACH
      IF(INDXBOD .EQ. 0)MXX=NT+NREACH
      MAXM=MXX+NEQROW
      MAXN=2*NREACH+MAXM
      MAXA=MAXM*MAXN
      COLMAX=MAXM
      P=8
      LOOK=2*NREACH
      FACTOR=50
      LENMY=20000
      PRINT=1
C
CC    PRINT= (0) ERROR MESSAGES ONLY
CC           (1) TERMINATION CONDITION MESSAGE
CC           (2) PRINT OBJECTIVE FUNCTION VALUE AFTER
CC               EACH BASIS RE-FACTORIZATION
CC           (3) LOG INFORMATION AT EVERY ITERATION
C
      NCOLSA=2*NREACH
      M=MAXM
C
CC    CALL SUBROUTINE TCHCOE1 TO GENERATE TECHNOLOGICAL COEFFICIENTS
C
      CALL TCHCOE1(NREACH,NT,RHS,TCHCOEF,IPOLL,LO,DO,QO,DIFMAX,
     1 KD,KA,U,DISLOC,Q,BODSTD,DOSAT,DOSTD,NCNTRLP,XCNTRLP,INDXBOD,
     2 UPPERB,LOWERB,NEQROW,IEQUITY,XINBOD,IDISCH,
     3 QT,NCNT,NDISCH,NCOLSA)
C
CC    SET ROW TYPES
CC
CC    2=TWO SIDED CONSTRAINT
CC    1=LESS THAN; EQUAL TO
CC    0=EQUATION
CC   -1=GREATER THAN; EQUAL TO
CC   -2=MEANS FREE ROW
C
      DO 40 IR=1,MAXM
      ROWTYP(IR)=1
   40 CONTINUE
C
CC    SET OBJECTIVE FUNCTION COEFFICIENTS FOR MAX SUM OF LOADING
C
      DO 50 JC=1,NCOLSA
      CJX(JC)=1.0
      IF(IPOLL .EQ. 0 .AND. JC .GT. NREACH)CJX(JC)=0.0
   50 CONTINUE
C
CC    THE NEXT WRITE STATEMENT IS USED FOR DEBUGGING PURPOSES
CC
CC    WRITE(7,51) (CJX(JC),JC=1,NCOLSA)
C  51 FORMAT('CJX:'/10(2X,E10.3))
CC
CC
      CALL XMAPS(BNDTYP,IOERR,LENMA,LENMI,LENMY,MAPA,MAPI,MAXA,MAXM,
     X           MAXN,MEMORY)
C
      N=0
      DO 130 JC=1,NCOLSA
      CJ=CJX(JC)
      UJ=UPPERB(JC)
      LJ=LOWERB(JC)
      COLLEN=MAXM
      DO 120 IR=1,COLLEN
      COLI(IR)=IR
```

```
      COLA(IR)=TCHCOEF(IR,JC)
  120 CONTINUE
      CALL XADDUB(BNDTYP,IOERR,JC,LENMA,LENMY,LJ,MAPA,MEMORY,UJ)
      CALL XADDAJ(CJ,COLA,COLI,COLLEN,COLMAX,IOERR,JC,
     X            LENMA,LENMY,MAPA,MEMORY,N)
  130 CONTINUE
      DO 140 JC=1,N
  140 STATUS(JC)=0
C
      Z=0.0
      CALL XSLACK(RHS,BASCB,BASIS,BASLB,BASUB,BLOW,BNDTYP,BOUND,
     X COLA,COLI,COLMAX,IOERR,LENMA,LENMI,LENMY,
     X M,MAPA,MAPI,MAXM,MAXN,MEMORY,N,ROWTYP,STATUS,
     X UZERO,XBZERO,Z)
C
C
      CALL XPRIML(RHS,BASCB,BASIS,BASLB,BASUB,BNDTYP,BOUND,
     X CAND,CANDA,CANDCJ,CANDI,CANDL,COLA,COLI,COLMAX,
     X FACTOR,IOERR,IOLOG,ITER1,ITER2,LENMA,LENMI,LENMY,LOOK,
     X M,MAPA,MAPI,MAXM,MAXN,MEMORY,N,NTYPE2,P,PRINT,
     X STATUS,TERMIN,UNBDDQ,UZERO,XBZERO,YQ,Z)
C
C
      WRITE(IOLOG,325) TERMIN
  325 FORMAT(18H TERMINATION CODE=,I4)
      IF(IPRINT .EQ. 0 .OR. IPRINT .EQ. 1)GOTO 326
C
C
      CALL XPRINT(BASIS,BNDTYP,BOUND,IOERR,IOLOG,
     X LENMA,LENMY,M,MAPA,MAXM,MAXN,MEMORY,N,NTYPE2,
     X STATUS,XBZERO,Z)
C
C
C     WRITE(6,200) (STATUS(J),J=1,N)
C 200 FORMAT(//5X,"STATUS :"/8(I5,2X))
  326 DO 330 J=1,NCOLSA
      VALUE(J)=0.
  330 CONTINUE
      DO 370 J=1,NREACH
      IF(STATUS(J)) 340,350,360
  340 BOD(J)=UPPERB(J)
      GO TO 370
  350 BOD(J)=LOWERB(J)
      GO TO 370
  360 IX=STATUS(J)
      BOD(J)=XBZERO(IX)
  370 CONTINUE
      JDOD=0
      NP1=NREACH+1
      DO 410 J=NP1,NCOLSA
      JDOD=JDOD+1
      IF(STATUS(J))380,390,400
  380 DOD(JDOD)=UPPERB(J)
      GOTO 410
  390 DOD(JDOD)=LOWERB(J)
      GOTO 410
  400 IX=STATUS(J)
      DOD(JDOD)=XBZERO(IX)
  410 CONTINUE
C
C     CALL SUBROUTINE TO CALCULATE CRITICAL LOCATIONS
C
      CALL CRITIC(QT,DOD,BOD,XC,DO,NREACH,KD,KA,LO,U,QO,Q,DISLOC)
C
      ICHECK=0
      DO 412 IXX=1,NREACH
      DELTA(IXX)=ABS(XCNTRLP(IXX)-XC(IXX))
      IF(DELTA(IXX) .GT. 0.01)ICHECK=ICHECK+1
```

```
412 CONTINUE
    IF(ICHECK .EQ. 0)GOTO 415
    DO 413 IXXX=1,NREACH
    IF(XC(IXXX) .GE. DISLOC(IXXX+1))XCNTRLP(IXXX)=DISLOC(IXXX+1)
    IF(XC(IXXX) .LT. DISLOC(IXXX+1))XCNTRLP(IXXX)=XC(IXXX)
    IF(XCNTRLP(IXXX) .EQ. DISLOC(IXXX))NCNTRLP(IXXX)=0
    IF(XCNTRLP(IXXX) .EQ. DISLOC(IXXX+1))NCNTRLP(IXXX)=0
413 CONTINUE
415 IF(ITERCHK .EQ. 1)GOTO 417
    JCHCK=0
    DO 416 IXXXX=1,NREACH
    DELTBOD(IXXXX)=ABS(BOD1(IXXXX)-BOD(IXXXX))/BOD(IXXXX)
    IF(DELTBOD(IXXXX) .GT. 0.05)JCHCK=JCHCK+1
416 CONTINUE
    IF(JCHCK .EQ. 0)GOTO 419
417 DO 418 I=1,NREACH
    BOD1(I)=BOD(I)
    DOD1(I)=DOD(I)
418 CONTINUE
    GOTO 30
419 DO 420 LX=1,NREACH
    PCTREMV(LX)=(XINBOD(LX)-BOD(LX))/XINBOD(LX)*100.0
420 CONTINUE
    WRITE(IOOUT,425)(I,I=1,NREACH)
425 FORMAT(/9X,6(6X,'REACH',I1))
    WRITE(IOOUT,430)(BOD(J),J=1,NREACH)
430 FORMAT(/7X,'BOD',10(3X,E9.4))
    WRITE(IOOUT,440)(DOD(J),J=1,NREACH)
440 FORMAT(/7X,'DOD',10(3X,E9.4))
    WRITE(IOOUT,450)(PCTREMV(J),J=1,NREACH)
450 FORMAT(/2X,'%REMOVAL',5X,8(F5.2,7X))
    WRITE(IOOUT,455)DIFMAX
455 FORMAT(/2X,'MAX. DIFFERENCE IN EQUITY =',E9.4)
    WRITE(IOOUT,457)Z
457 FORMAT(/1X,'TOTAL BOD AND/OR DOD =',E10.4)
    IF(IEQUITY .EQ. 0)WRITE(IOOUT,460)
    IF(IEQUITY .EQ. 1)WRITE(IOOUT,480)
460 FORMAT(//2X,'THE EQUITY CONSIDERED IS EQUAL % REMOVAL')
480 FORMAT(//2X,'THE EQUITY CONSIDERED IS EQUAL EFFLUENT CONC.')
    ZDIFF(ZITER)=DIFMAX
    ZTOTAL(ZITER)=Z
    GOTO 12
510 WRITE(6,520)
520 FORMAT(/5X,'MAX. DIFF.',10X,'TOTAL BOD/DO')
    KMM=ZITER-1
    DO 550 IMM=1,KMM
530 WRITE(6,540)ZDIFF(IMM),ZTOTAL(IMM)
540 FORMAT(/3X,F10.4,11X,F10.4)
550 CONTINUE
    IF(IPLOT .GT. 0)CALL PLOT(KMM,ZDIFF,ZTOTAL,IPLOT,IEQUITY,
   X STEPDIF,ZITER)
600 STOP
    END
C
C
C
C
C     ************************************************************
C     SUBROUTINE TCHCOEF IS WHERE THE TECHNOLOGICAL COEFFICIENTS
C     FOR BOTH THE WATER QUALITY CONSTRAINTS AND EQUITY CONSID-
C     ERATIONS ARE DEVELOPED.
C     ************************************************************
C
      SUBROUTINE TCHCOE1(NREACH,NT,RHS,TCHCOEF,IPOLL,LO,DO,QO,DIFMAX,
     1 KD,KA,U,DISLOC,Q,BODSTD,DOSAT,DOSTD,NCNTRLP,XCNTRLP,INDXBOD,
     2 UPPERB,LOWERB,NEQROW,IEQUITY,XINBOD,IDISCH,
     3 QT,NCNT,NDISCH,NCOLSA)
      DIMENSION KD(15),U(15),DISLOC(15),BODCOEF(15,15),QT(15)
```

```
      DIMENSION XCNTRLP(50),RHS(200),DOCOEF(200,30),Q(15),DOSAT(15)
      DIMENSION KA(15),NCNTRLP(15),NCNT(15),BODSTD(15),DOSTD(15)
      DIMENSION TCHCOEF(220,30),UPPERB(20),LOWERB(20),EQCOEF(100,15)
      DIMENSION UPBTRT(10),LOWBTRT(10),UPBDOD(10),LOWBDOD(10),
     X IDISCH(15)
      REAL LO,KD,KA,XINBOD(15),LOWBTRT,LOWBDOD,LOWERB
C
C     CALCULATE TOTAL FLOWS IN EACH REACH AND DETERMINE TOTAL
C     NUMBER OF CONTROL POINTS.
C
      NDISCH=0
      CQT=QO
      NT=0
      DO 50 J=1,NREACH
      CQT=CQT+Q(J)
      QT(J)=CQT
      NT=NT+NCNTRLP(J)
      NCNT(J)=NT
      NDISCH=NDISCH+IDISCH(J)
   50 CONTINUE
      NEQROW=(NDISCH**2 - NDISCH)
C
C
C     INITIALIZE ARRAYS
C
C
      DO 85 I=1,NREACH
      DO 80 J=1,NCOLSA
      BODCOEF(I,J)=0.0
   80 CONTINUE
   85 CONTINUE
      DO 87 I=1,NEQROW
      DO 86 J=1,NREACH
      EQCOEF(I,J)=0.0
   86 CONTINUE
   87 CONTINUE
      NCR=NT+NREACH
      DO 95 J=1,NCOLSA
      DO 90 I=1,NCR
      DOCOEF(I,J)=0.0
   90 CONTINUE
   95 CONTINUE
      ICR=NT+NREACH+NEQROW
      DO 97 J=1,NCOLSA
      DO 96 I=1,ICR
      TCHCOEF(I,J)=0.0
   96 CONTINUE
   97 CONTINUE
      IF(INDXBOD .EQ. 0)GOTO 420
C
C     DETERMINE TECHNOLOGICAL COEFFICIENTS FOR BOD
C
      DO 300 J=1,NREACH
      TFBOD=1.0
      DO 200 I=J,NREACH
      IF(I .EQ. J)GOTO 150
      TFBOD=TFBOD*EXP(-KD(I-1)*(DISLOC(I)-DISLOC(I-1))/U(I-1))
  150 BODCOEF(I,J)=TFBOD*Q(J)/QT(I)
  200 CONTINUE
  300 CONTINUE
C
C     DETERMINE RHS FOR BOD
C
      TFBOD=1.0
      IROW=NT+NREACH+NEQROW
      DO 400 I=1,NREACH
      IROW=IROW+1
      IF(I .EQ. 1)GOTO 350
```

```
      TFBOD=TFBOD*EXP(-KD(I-1)*(DISLOC(I)-DISLOC(I-1))/U(I-1))
  350 RHS(IROW)=BODSTD(I)-QO*LO*TFBOD/QT(I)
  400 CONTINUE
C
C     CALCULATE TECHNOLOGICAL COEFFICIENTS FOR DO (BASED ON BOD)
C
  420 NROW=1
      IM1=0
      DO 700 I=1,NREACH
      IM1=I-1
      IF(I .NE. 1)NROW=NCNT(IM1)+IM1+1
      IRCNT=IM1
      TERM1=1.0
      TERM2=0.0
      DOCOEF(NROW,I)=0.0
      DO 600 J=I,NREACH
      IF(NCNTRLP(J) .EQ. 0)IRCNT=IRCNT+1
      IF(NCNTRLP(J) .EQ. 0)GOTO 550
      DO 500 K=1,NCNTRLP(J)
      IRCNT=IRCNT+1
      X=XCNTRLP(IRCNT)-DISLOC(J)
      PART1=Q(I)/QT(J)*TERM1*KD(J)/(KA(J)-KD(J))*
     1 (EXP(-KD(J)*X/U(J))-EXP(-KA(J)*X/U(J)))
      PART2=TERM2*Q(I)/QT(J)*EXP(-KA(J)*X/U(J))
      NROW=NROW+1
      DOCOEF(NROW,I)=PART1+PART2
  500 CONTINUE
  550 IF(J .EQ. NREACH)GOTO 700
      XR=DISLOC(J+1)-DISLOC(J)
      FPART1=TERM1*KD(J)/(KA(J)-KD(J))*
     1 (EXP(-KD(J)*XR/U(J))-EXP(-KA(J)*XR/U(J)))
      FPART2=TERM2*EXP(-KA(J)*XR/U(J))
      NROW=NROW+1
      DOCOEF(NROW,I)=(FPART1+FPART2)*Q(I)/QT(J+1)
      TERM1=TERM1*EXP(-KD(J)*XR/U(J))
      TERM2=FPART1+FPART2
  600 CONTINUE
  700 CONTINUE
C
C     CALCULATE TECHNOLOGICAL COEFFICIENTS FOR DO (BASED ON DO DEFICITS)
C
      MM1=0
      KROW=1
      DO 1000 M=1,NREACH
      KCNTR=0
      IF(M .EQ. 1)GOTO 720
      MM1=M-1
      DO 710 J=1,MM1
      KCNTR=KCNTR+NCNTRLP(J)
  710 CONTINUE
  720 NCOL=M+NREACH
      TFDO=1.0
      DO 900 N=M,NREACH
      IF(N .EQ. M)GOTO 750
      TFDO=TFDO*EXP(-KA(N-1)*(DISLOC(N)-DISLOC(N-1))/U(N))
  750 DOCOEF(KROW,NCOL)=TFDO*Q(M)/QT(N)
      DO 800 L=1,NCNTRLP(N)
      KCNTR=KCNTR+1
      KROW=KROW+1
      TFDOX=TFDO*EXP(-KA(N)*(XCNTRLP(KCNTR)-DISLOC(N))/U(N))
      DOCOEF(KROW,NCOL)=TFDOX*Q(M)/QT(N)
  800 CONTINUE
      KROW=KROW+1
  900 CONTINUE
      KROW=NCNT(M)+M+1
 1000 CONTINUE
C
C
```

```
C     CREATE RHS FOR EQUITY
C
      NX=NT+NREACH+1
      NNXX=NT+NREACH+NEQROW
      DO 1010 II=NX,NNXX
      RHS(II)=DIFMAX
 1010 CONTINUE
C
C
C     DETERMINE EQUITY COEFFICIENTS
C
      ICC=0
      IICC=0
      LLL=1
      MNN=0
      NRR=NREACH-1
      DO 1030 MNN=1,NRR
      LLL=LLL+1
      IF(IDISCH(MNN) .EQ. 0)GOTO 1030
      DO 1020 JNN=LLL,NREACH
      IF(IDISCH(JNN) .EQ. 0)GOTO 1020
      ICC=ICC+1
      IF(IEQUITY .EQ. 0)EQCOEF(ICC,MNN)=1/XINBOD(MNN)
      IF(IEQUITY .EQ. 0)EQCOEF(ICC,JNN)=-1/XINBOD(JNN)
      IF(IEQUITY .EQ. 1)EQCOEF(ICC,MNN)=1.0
      IF(IEQUITY .EQ. 1)EQCOEF(ICC,JNN)=-1.0
      IICC=ICC+NEQROW/2
      IF(IEQUITY .EQ. 0)EQCOEF(IICC,MNN)=-1/XINBOD(MNN)
      IF(IEQUITY .EQ. 0)EQCOEF(IICC,JNN)=1/XINBOD(JNN)
      IF(IEQUITY .EQ. 1)EQCOEF(IICC,MNN)=-1.0
      IF(IEQUITY .EQ. 1)EQCOEF(IICC,JNN)=1.0
 1020 CONTINUE
 1030 CONTINUE
C     DETERMINE RHS FOR DO
C
      LROW=0
      ILN=1
      DO 1200 I=1,NREACH
      ILN=LROW+1
      LROW=LROW+NCNTRLP(I)+1
      DO 1100 J=ILN,LROW
      RHS(J)=DOSAT(I)-DOSTD(I)-DOCOEF(J,1)*QO/QT(1)*LO-
     1 DOCOEF(J,NREACH+1)*QO*DO/QT(1)
 1100 CONTINUE
 1200 CONTINUE
C
C     COMBINE TECHNOLOGICAL COEFFICIENTS INTO A SINGLE
C     ARRAY -- TCHCOEF
C
      MNROW=NT+NREACH
      DO 1400 I=1,NCOLSA
      DO 1300 J=1,MNROW
      TCHCOEF(J,I)=DOCOEF(J,I)
 1300 CONTINUE
 1400 CONTINUE
      MXROW=NT+NREACH+1
      MBOT=NT+NREACH+NEQROW
      DO 1475 MI=1,NREACH
      MCNTR=0
      DO 1450 NI=MXROW,MBOT
      MCNTR=MCNTR+1
      TCHCOEF(NI,MI)=EQCOEF(MCNTR,MI)
 1450 CONTINUE
 1475 CONTINUE
C
      IF(INDXBOD .EQ. 0)GOTO 1700
C
C     ADD CONSTRAINTS FOR BOD IF REQUESTED
```

```
C
      JBOTP1=MBOT+1
      JBOTTOM=MBOT+NREACH
      DO 1600 K=1,NREACH
      JCNTR=0
      DO 1500 L=JBOTP1,JBOTTOM
      JCNTR=JCNTR+1
      TCHCOEF(L,K)=BODCOEF(JCNTR,K)
 1500 CONTINUE
 1600 CONTINUE
 1700 CONTINUE
C
C     THE NEXT WRITE STATEMENT IS USED FOR DEBUGGING PURPOSES
C
C     DO 1710 I=1,MBOT
C     WRITE(7,1705) (TCHCOEF(I,J),J=1,LASTCOL),RHS(I)
C1705 FORMAT(/10(2X,E10.3))
C1710 CONTINUE
      RETURN
      END
C
C     ***************************************************
C     SUBROUTINE READIN IS WHERE PROGRAM INFORMATION IS
C     READ IN.
C     ***************************************************
      SUBROUTINE READIN(KD,KA,U,DISLOC,Q,BODSTD,DOSAT,DOSTD,
     * XINBOD,UPBTRT,UPBDOD,LOWBTRT,LOWBDOD,LO,DO,QO,NREACH,
     * IPOLL,INDXBOD,IEQUITY,IDISCH,IPLOT,STEPDIF,LASTDIF)
      REAL LO,KD,KA,LOWBTRT,LOWBDOD,LASTDIF
      DIMENSION KD(15),KA(15),U(15),DISLOC(15),Q(15),BODSTD(15)
      DIMENSION DOSAT(15),DOSTD(15),XINBOD(15)
      DIMENSION UPBTRT(10),UPBDOD(10),LOWBTRT(10),LOWBDOD(10)
      DIMENSION XCNTRLP(50),IDISCH(15)
      READ(5,5)LO,DO,QO,STEPDIF,LASTDIF,NREACH,IPOLL,INDXBOD,
     X IEQUITY,IPLOT
    5 FORMAT(5F8.0,5I8)
      READ(5,7)(KD(I),I=1,NREACH)
      READ(5,7)(KA(I),I=1,NREACH)
      READ(5,7)(U(I),I=1,NREACH)
      READ(5,7)(DISLOC(I),I=1,NREACH)
      READ(5,7)(Q(I),I=1,NREACH)
      READ(5,7)(BODSTD(I),I=1,NREACH)
      READ(5,7)(DOSAT(I),I=1,NREACH)
      READ(5,7)(DOSTD(I),I=1,NREACH)
      READ(5,7)(XINBOD(I),I=1,NREACH)
      READ(5,7)(UPBTRT(I),I=1,NREACH)
      READ(5,7)(LOWBTRT(I),I=1,NREACH)
      READ(5,7)(UPBDOD(I),I=1,NREACH)
      READ(5,7)(LOWBDOD(I),I=1,NREACH)
      READ(5,8)(IDISCH(I),I=1,NREACH)
    7 FORMAT(10 F8.0)
    8 FORMAT(10I8)
C
C     WRITE HEADING INFORMATION ON OUTPUT
C
      WRITE(6,10)NREACH,LO,DO,QO,STEPDIF,LASTDIF,INDXBOD,IEQUITY,IPOLL
   10 FORMAT(/3X,'MULTI-OBJECTIVE OPTIMIZATION BY CONSTRAINT METHOD :',
     X //3X,'NREACH = ',I6,10X,'LO = ',F6.2,
     X /3X,'DO = ',F6.2,14X,'QO = ',F6.2,
     X /3X,'STEPDIF = ',F6.2,9X,'LASTDIF = ',F6.2,
     X /3X,'INDXBOD = ',I6,9X,'IEQUITY = ',I6,
     X /3X,'IPOLL = ',I6)
      WRITE(6,11)(KD(I),I=1,NREACH)
   11 FORMAT(/3X,'KD :',/10(2X,F7.2))
      WRITE(6,12)(KA(I),I=1,NREACH)
   12 FORMAT(/3X,'KA :',/10(2X,F7.2))
      WRITE(6,13)(DISLOC(I),I=1,NREACH)
   13 FORMAT(/3X,'DISCHARGE LOCATIONS :'/10(2X,F7.2))
```

```
      WRITE(6,14)(Q(I),I=1,NREACH)
   14 FORMAT(/3X,'EFFLUENT DISCHARGE RATES (CFS) :',/10(2X,F7.2))
      WRITE(6,15)(XINBOD(I),I=1,NREACH)
   15 FORMAT(/3X,'PLANT INFLUENT BOD (MG/L) :',/10(2X,F7.2))
      WRITE(6,16)(IDISCH(I),I=1,NREACH)
   16 FORMAT(/3X,'TYPE OF DISCHARGE (TREATMENT PLANT OR TRIBUTARY)',
     X /10(2X,I7)//)
      RETURN
      END
C
C     ********************************************************
C     SUBROUTINE CRITIC IS TO CALCULATE CRITICAL POINT LOCATIONS
C     ********************************************************
      SUBROUTINE CRITIC(QT,DOD,BOD,XC,DO,NREACH,KD,KA,
     * LO,U,QO,Q,DISLOC)
      DIMENSION QT(15),Q(15),DOD(15),BOD(15),XC(15),KD(15),KA(15)
      DIMENSION U(15),DISLOC(15),XINITD(15),QT1(15),TOPD(15)
      REAL LO,KA,KD,QO,DO,TOPBOD(15),XINIBOD(15)
      QT1(1)=QO
      DO 20 I=1,NREACH
      QT1(I+1)=QT(I)
   20 CONTINUE
      DO 100 I=1,NREACH
      IF(I .EQ. 1)XINITD(I)=DO
      IF(I .EQ. 1)XINIBOD(I)=LO
      IF(I .EQ. 1)GOTO 50
      XINITD(I)=KD(I-1)*TOPBOD(I-1)/(KA(I-1)-KD(I-1))*(EXP(-KD(I-1)*
     1 (DISLOC(I)-DISLOC(I-1))/U(I-1))-EXP(-KA(I-1)*(DISLOC(I)-
     2 DISLOC(I-1))/U(I-1)))+TOPD(I-1)*EXP(-KA(I-1)*
     3 (DISLOC(I)-DISLOC(I-1))/U(I-1))
      XINIBOD(I)=TOPBOD(I-1)*EXP(-KD(I-1)*(DISLOC(I)-DISLOC(I-1))
     1 /U(I-1))
   50 TOPD(I)=(QT1(I)*XINITD(I)+Q(I)*DOD(I))/QT1(I+1)
      TOPBOD(I)=(QT1(I)*XINIBOD(I)+Q(I)*BOD(I))/QT1(I+1)
C
C     CHECK FOR NEGATIVE AND INDEFINITE CRITICAL LOCATIONS
C     THEN CALCULATE XC
C
      CHKCAL=(KA(I)-KD(I))*TOPD(I)/(KD(I)*TOPBOD(I))
      IF(CHKCAL .GE. 1.0)XC(I)=-1.0
      IF(CHKCAL .GE. 1.0)GOTO 90
      XC(I)=DISLOC(I)+U(I)/(KA(I)-KD(I))*ALOG(KA(I)/KD(I)*(1-CHKCAL))
   90 IF(XC(I) .LT. DISLOC(I))XC(I)=DISLOC(I)
  100 CONTINUE
      WRITE(6,110)(XC(I),I=1,NREACH)
  110 FORMAT(/1X,7(F6.2,3X))
      RETURN
      END
C
C*************************************************************
C     SUBROUTINE PLOT IS TO GENERATE PLOT OF INDIFFERENCE CURVE
C*************************************************************
C
C
      SUBROUTINE PLOT(KMM,ZDIFF,ZTOTAL,IPLOT,IEQUITY,STEPDIF,ZITER)
      DIMENSION ZDIFF(20),ZTOTAL(20)
      INTEGER ZITER
C
C
C     NOTE:  (1) ZDIFF@D MAXIMUM DIFFERENCE IN EQUITY
C            (2) ZTOTAL@D TOTAL BOD AND/OR DO DEFICITS
C
C
C     TO PLOT ON VERSATEC
C
      IF(IPLOT .EQ. 1)CALL VRSTEC(0,0,0)
C
C     TO PLOT ON DATMED
```

```
C
      IF(IPLOT .EQ. 2)CALL DATMED(0)
C
CC    TO PLOT ON MODGRAPH
C
      IF(IPLOT .EQ. 3)CALL TEKALL(4010,960,0,1,0)
C
CC    SET UP X AXIS VALUES
C
      INVN=5
      STARTX=INT(ZTOTAL(1)/100.0)*100.0
      INTX=INT((INT(ZTOTAL(KMM)/100.0) - INT(ZTOTAL(1)/100.0))/INVN)
      STEPX=(INTX+1)*100.0
      FINALX=STEPX*INVN + STARTX
C
CC    SET UP Y AXIS VALUES
C
      STARTY=0.0
      IF(IEQUITY .EQ. 0)STEPY=2.0*STEPDIF*100.0
      IF(IEQUITY .EQ. 1)STEPY=2.0*STEPDIF
      FINALY=STEPY*ZITER/2. + STARTY
C
CC    BEGIN CALLS TO PLOTTING ROUTINE
C
      CALL PAGE(8.20,11.0)
      CALL BANGLE(-90)
C     CALL NOBRDR
      CALL PHYSOR(1.5,9.0)
      CALL AREA2D(8.0,6.0)
      CALL FRAME
      CALL XINTAX
      CALL YINTAX
      IF(IEQUITY .EQ. 0)CALL YNAME('MAX. DIFF. IN EQUITY, %$',100)
      IF(IEQUITY .EQ. 1)CALL YNAME('MAX. DIFF. IN EQUITY, MG/L$',100)
      CALL XNAME('TOTAL BOD AND/OR DO DEFICITS, MG/L$',100)
      CALL YTICKS(2)
      CALL XTICKS(5)
C
CC    CHANGE VALUES IN ARRAY ZDIFF FROM DECIMAL FRACTION TO PERCENT
C
      DO 500 I=1,KMM
      IF(IEQUITY .EQ. 0)ZDIFF(I)=ZDIFF(I)*100.0
  500 CONTINUE
      CALL GRAF(STARTX,STEPX,FINALX,STARTY,STEPY,FINALY)
      CALL CURVE(ZTOTAL,ZDIFF,KMM,0)
      CALL ENDPL(0)
      CALL DONEPL
      RETURN
      END
```

APPENDIX F

LISTING OF COMPUTER PROGRAM FOR

DETERMINISTIC MULTIOBJECTIVE WASTE LOAD ALLOCATION

USING FUZZY LINEAR PROGRAMMING

'WLAFUZZ'

```
      PROGRAM WLAFUZZ(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7,
     X TAPE8,TAPE99)
      DIMENSION KD(15),U(15),DISLOC(15),BODCOEF(15,15),QT(15),DOD(15)
      DIMENSION XCNTRLP(150),RHS(200),DOCOEF(200,30),Q(15),DOSAT(15)
      DIMENSION KA(15),NCNTRLP(15),NCNT(15),BODSTD(15),DOSTD(15)
      DIMENSION TCHCOEF(200,30),BOD(15),EQCOEF(100,15)
      DIMENSION PCTREMV(15),XINBOD(15),UPBTRT(15),LOWBTRT(20),
     X DELTA(30),DELTBOD(30),XC(15),BOD1(15),DOD1(15),
     X UPBDOD(15),LOWBDOD(15),IDISCH(15),NDOPT(15),YCNTRLP(150)
      REAL LO,KD,KA,LOWERB(20),UPPERB(20),LJ,LOWBTRT,PU,PL,LOWBDOD
      REAL BASCB(80),BASLB(80),BASUB(80),BOUND,BLOW(1),
     X     CANDA(80,8),CANDCJ(8),CJ,COLA(200),MEMORY(30000),
     X     UZERO(80),XBZERO(80),YQ(80),Z,CJX(80),VALUE(80)
      INTEGER BNDTYP,COLLEN,COLMAX,FACTOR,IOERR,IOLOG,ITER1,ITER2,
     X        LENMA,LENMI,LENMY,LOOK,M,MAPA(20),MAPI(20),MAXA,MAXM,
     X        MAXN,N,NTYPE2,P,PRINT,TERMIN,UNBDDQ,BASIS(80),CAND(8),
     X        CANDI(80,8),CANDL(8),COLI(200),ROWTYP(200),STATUS(80)
      INTEGER IOIN,NCOLSA
      COMMON/XMPCOM/BIG,SMALL,ZL,ZLC,EPS1,EPS2,EPS3,EPS4,EPS5,EPS6
      REAL BIG,SMALL,ZL,ZLC,EPS1,EPS2,EPS3,EPS4,EPS5,EPS6
C
CC
CC    WLAFUZZ IS AN OPTIMAL WASTE LOAD ALLOCATION PROGRAM DESIGNED
CC    TO SOLVE A MULTIOBJECTIVE WATER QUALITY PROBLEM CONSIDERING
CC    THE MAXIMIZATION OF WASTE DISCHARGE (MEASURED BY BOD) AND
CC    MINIMIZATION OF THE DIFFERENCE IN EQUITY (MEASURED BY THE LEVEL
CC    OF TREATMENT) BETWEEN SEVERAL USERS OF A STREAM SYSTEM.
CC    IN ADDITION, THIS PROGRAM INCORPORTATES THE USE OF MOVABLE
CC    CONTROL POINT LOCATIONS IN AN ATTEMPT TO REDUCE THE NUMBER
CC    OF CONSTRAINTS IN THE PROBLEM FORMULATION.  FUZZY PROGRAMMING
CC    TECHNIQUES ARE UTILIZED TO SOLVE THE MULTIOBJECTIVE PROBLEMS
CC    IN CONJUNCTION WITH WATER QUALITY CONSTRAINTS BASED ON THE
CC    STREETER-PHELPS EQUATION.  THE USER OF THIS PROGRAM MAY MAKE
CC    SEVERAL CHOICES FOR THE TYPE OF EQUITY, POLLUTION, AND MEMBERSHIP
CC    FUNCTION TO BE USED IN THE SOLUTION OF EACH PROBLEM.  IN ORDER TO
CC    APPROPRIATELY SELECT THE SPECIFIC PROBLEM TO BE SOLVED, SEE THE
CC    COMMENTS PRESENTED BELOW FOR THE REQUIRED INPUT DATA.
CC    THIS PROGRAM MAKES SEVERAL SUBROUTINE CALLS TO A "CANNED" PROGRAM
CC    CALLED "XMP".  THE ESSENCE OF THIS PROGRAM IS TO SET UP THE
CC    MULTIOBJECTIVE PROBLEM IN A LINEAR PROGRAMMING FORMAT, THEN
CC    SOLVE THIS PROBLEM UTILIZING THE SUBROUTINES AVAILABLE IN XMP.
CC    IN ORDER TO EXECUTE THIS PROGRAM, THE USER MUST HAVE A COMPILED
CC    VERSION OF XMP IN THEIR CURRENT LIBRARY.
CC
CC    THIS PROGRAM WAS WRITTEN BY WADE HATHHORN AND DR. Y.K. TUNG
CC    IN ANSWER TO RESEARCH INTERESTS PROVIDED BY THE WYOMING
CC    WATER RESEARCH CENTER  LARAMIE, WYOMING  82071.  FINAL REVISIONS
CC    WERE MADE ON JULY 2, 1986.
CC
CC    IN ORDER TO EXECUTE THIS PROGRAM, THE USER MUST PROVIDE INPUT
CC    INFORMATION UNDER THE FOLLOWING FORMAT:
CC
CC
CC  CARD       VARIABLES            DESCRIPTION                    FORMAT
CC
CC    1     LO,DO,QO,NREACH,     LO=U/S BOD CONC. MG/L          3F8.0,6I8
CC              INDXBOD           DO=U/S DEFICIT CONC. MG/L
CC              IEQUITY           QO=U/S FLOW RATE CFS
CC              IMEMBER           NREACH= NO. OF REACHES
CC               IPOLL
CC                                INDXBOD=INDICIES TO INCLUDE
CC                                       BOD CONSTRAINTS.
CC                                       (0)=EXCLUDE BOD CONSTR.
CC                                       (1)=INCLUDE BOD CONSTR.
CC
CC                                IEQUITY=INDICIES TO INDICATE
CC                                       TYPE OF EQUITY TO BE
C                                        CONSIDERED
```

```
C                                (0)=EQUAL % REMOVAL
C                                (1)=EQUAL EFFLUENT CONC.
C
C                           IMEMBER=INDICIES TO INDICATE
C                                  TYPE OF MEMBERSHIP
C                                  FUNCTION TO BE CONSIDERED
C                                  (0)=LINEAR
C                                  (1)=LOGISTIC
C
C
C                           IPOLL=INDICIES FOR TYPE
C                                OF POLLUTION TO
C                                BE CONSIDERED
C                                (0)=BOD ONLY
C                                (1)=BOD AND DO DEFICITS
C
C                           IPLOT = INDICIES TO GENERATE
C                                  PLOT OF DO PROFILE
C                                  (0) = DO NOT PLOT
C                                  (1) = PLOT ON VERSATEC
C                                        (COMPUTER CENTER)
C                                  (2) = PLOT ON DATMED
C                                        (WATER CENTER)
C                                  (3) = PLOT ON MODGRAPH
C                                        (C.E. BUILDING)
C
C    2          KD          KD=DEOXYGENATION COEFF.          10F8.0
C                           FOR EACH REACH  1/DAY
C
C    3          KA          KA=REAERATION COEFF.             10F8.0
C                             FOR EACH REACH  1/DAY
C
C    4          U             U=STREAM VELOCITY FOR          10F8.0
C                             EACH REACH MILES/DAY
C
C    5        DISLOC          DISLOC=DISCHARGE LOCATION      10F8.0
C                                  FOR EACH REACH MILES
C
C    6          Q             Q=DISCHARGE FLOW RATE          10F8.0
C                             FOR EACH EFFLUENT CFS
C
C    7        BODSTD          BODSTD=BOD STANDARD FOR        10F8.0
C                                  FOR EACH REACH MG/L
C
C    8         DOSAT          DOSAT=DISSOL. OXYGEN SAT.      10F8.0
C                                IN EACH REACH MG/L
C
C    9         DOSTD          DOSTD=DISSOL. OXYGEN STD.      10F8.0
C                                IN EACH REACH MG/L
C
C
C   10         XINBOD         XINBOD=INFLUENT BOD CONC.      10F8.0
C                                  IN MG/L
C
C   11         UPBTRT          UPBTRT=UPPER BOUNDS FOR       10F8.0
C                                  BOD TREATMENT
C                                  (% REMOVAL AS A
C                                   DECIMAL FRACTION)
C
C   12         LOWBTRT         LOWBTRT=LOWER BOUNDS FOR      10F8.0
C                                  BOD TREATMENT
C                                  (% REMOVAL AS A
C                                   DECIMAL FRACTION)
C
C   13         UPBDOD          UPBDOD=UPPER BOUNDS FOR       10F8.0
C                                  DO DEFICITS IN
C                                  EACH EFFL. MG/L
C
```

```
C    14          LOWBDOD              LOWBDOD=LOWER BOUNDS FOR    10F8.0
C                                            DO DEFICITS IN
C                                            EACH EFFL. MG/L
C
C    15          IDISCH               IDISCH = INDICIES FOR      10I8
C                                            TYPE OF DISCHARGE
C                                            POINT
C                                            (0) = TRIBUTARY
C                                            (1) = PLANT EFFL.
C
C   NOTE:  IF YOU INPUT IMEMBER=1 (LOGISTIC MEMBERSHIP FUNCTION), YOU
C          MUST INCLUDE AN ADDITIONAL CARD CONTAINING THE UPPER AND
C          LOWER BOUNDS ON THE PROBABILITY TO BE CONSIDERD BY USING
C          A LOGISITC MEMBERSHIP FUNCTION (SEE CARD 16)
C
C    16          PU,PL                PU=UPPER BOUUND OF          2F8.0
C                                         LOGISTIC PROB.
C
C                                     PL=LOWER BOUND OF
C                                         LOGISTIC PROB.
C**************************************************************************
C
C
C     READ IN REQUIRED INPUT DATA
C
      READ(5,5)LO,DO,QO,NREACH,INDXBOD,IEQUITY,IMEMBER,IPOLL,IPLOT
    5 FORMAT(3F8.0,6I8)
      READ(5,7)(KD(I),I=1,NREACH)
      READ(5,7)(KA(I),I=1,NREACH)
      READ(5,7)(U(I),I=1,NREACH)
      READ(5,7)(DISLOC(I),I=1,NREACH)
      READ(5,7)(Q(I),I=1,NREACH)
      READ(5,7)(BODSTD(I),I=1,NREACH)
      READ(5,7)(DOSAT(I),I=1,NREACH)
      READ(5,7)(DOSTD(I),I=1,NREACH)
      READ(5,7)(XINBOD(I),I=1,NREACH)
      READ(5,7)(UPBTRT(I),I=1,NREACH)
      READ(5,7)(LOWBTRT(I),I=1,NREACH)
      READ(5,7)(UPBDOD(I),I=1,NREACH)
      READ(5,7)(LOWBDOD(I),I=1,NREACH)
      READ(5,8)(IDISCH(I),I=1,NREACH)
      IF(IMEMBER .EQ. 1)READ(5,7)PU,PL
    7 FORMAT(10F8.0)
    8 FORMAT(10I8)
C
C     WRITE HEADING INFORMATION ON OUTPUT
C
      WRITE(6,10)NREACH,LO,DO,QO,INDXBOD,IEQUITY,IMEMBER,IPOLL
   10 FORMAT(1H1/3X,'FUZZY PROGRAMMING INFORMATION :',
     X //3X,'NREACH = ',I6,10X,'LO = ',F6.2,
     X /3X,'DO = ',F6.2,14X,'QO = ',F6.2,
     X /3X,'INDXBOD = ',I6,9X,'IEQUITY = ',I6,
     X /3X,'IMEMBER = ',I6,9X,'IPOLL = ',I6)
      WRITE(6,11)(KD(I),I=1,NREACH)
   11 FORMAT(/3X,'KD :',/10(2X,F7.2))
      WRITE(6,12)(KA(I),I=1,NREACH)
   12 FORMAT(/3X,'KA :',/10(2X,F7.2))
      WRITE(6,13)(DISLOC(I),I=1,NREACH)
   13 FORMAT(/3X,'DISCHARGE LOCATIONS :'/10(2X,F7.2))
      WRITE(6,14)(Q(I),I=1,NREACH)
   14 FORMAT(/3X,'EFFLUENT DISCHARGE RATES (CFS) :',/10(2X,F7.2))
      WRITE(6,15)(XINBOD(I),I=1,NREACH)
   15 FORMAT(/3X,'PLANT INFLUENT BOD (MG/L) :',/10(2X,F7.2))
      WRITE(6,16)(IDISCH(I),I=1,NREACH)
   16 FORMAT(/3X,'TYPE OF DISCHARGE (TREATMENT PLANT OR TRIBUTARY)',
     X /10(2X,I7)//)
      IF(IMEMBER .EQ. 1) WRITE(6,17) PL,PU
```

```
   17 FORMAT(/3X,'PL = ',F6.4,3X,'PU = ',F6.4)
C
CCC
CC     SET UP AN ARRAY FOR UPPER AND LOWER BOUNDS ON
CC     DECISION VARIABLES.  THIS STEP IS REQUIRED
CC     FOR THE OPERATION OF XMP.
C
       KJ=NREACH
       DO 23 IJ=1,NREACH
       KJ=KJ+1
       UPPERB(IJ)=(1.-LOWBTRT(IJ))*XINBOD(IJ)
       UPPERB(KJ)=UPBDOD(IJ)
       LOWERB(IJ)=(1-UPBTRT(IJ))*XINBOD(IJ)
       LOWERB(KJ)=LOWBDOD(IJ)
   23 CONTINUE
       IPI=2*NREACH+1
       UPPERB(IPI)=2000.0
       LOWERB(IPI)=0.0
CC
CC
       IOIN=5
       IOERR=6
       IOLOG=6
       IOOUT=6
       IODEB=7
       BNDTYP=4
       NTYPE2=0
       IPRINT=2
       BOUND=0.
       IOBJ=0
       LOOK=6
       P=8
       FACTOR=50
       LENMY=30000
       PRINT=1
C
CC     PRINT= (0) ERROR MESSAGES ONLY
CC            (1) TERMINATION CONDITION MESSAGE
CC            (2) PRINT OBJECTIVE FUNCTION VALUE AFTER
CC                EACH BASIS RE-FACTORIZATION
CC            (3) LOG INFORMATION AT EVERY ITERATION
CC
CC
CC
CC     IOBJ= INDICATOR TO CONTROL PROBLEM ITERATIONS
CC          (1) = MAX. BOD AND/OR DO DEFICITS
CC          (2) = MIN. MAX. DIFFERENCE IN EQUITY
CC          (3) = MULTIOBJECTIVE PROBLEM
CC
CC
CC
C
   25 IOBJ=IOBJ+1
       WRITE(6,26)IOBJ
   26 FORMAT(1H1/5X,'THE CURRENT OBJECTIVE IS ',I4,/)
C
CC     INITIALIZE THE NUMBER OF CONTROL POINTS IN EACH REACH
C
       DO 27 I=1,NREACH
       NCNTRLP(I)=1
   27 CONTINUE
C
CCC
CCC
CC     INITIIALIZE CONTROL POINT LOCATIONS AT 1/4 DISTANCE BETWEEN
CC     DISCHARGE LOCATIONS AND CREATE FALSE DISCHARGE LOCATION TO
CC     ALLOW FOR REGULATION OF FINAL DISCHARGE
C
       DO 30 I=1,NREACH
```

```
      IF(I .EQ. NREACH) DISLOC(I+1)=DISLOC(I)+(DISLOC(I)-DISLOC(1))
     * /(NREACH-1)
      XCNTRLP(I)=0.25*(DISLOC(I+1)-DISLOC(I))+DISLOC(I)
   30 CONTINUE
C
CCC
CCC   CALL SUBROUTINE TO GENERATE TECHNOLOGICAL COEFFICIENT
CCC
CCC
CCC   PROVIDE COUNTER TO CHECK NUMBER OF ITERATIONS
C
      ITERCHK=0
   35 ITERCHK=ITERCHK+1
      IF(ITERCHK .GT. 10)WRITE(6,37)
   37 FORMAT(/1X,'THE NUMBER OF ITERATIONS EXCEEDS 10')
      IF(ITERCHK .GT. 10)GOTO 600
C
CC
      CALL TCHCOE1(NREACH,NT,RHS,TCHCOEF,LO,DO,QO,IDISCH,IOBJ,
     1 KD,KA,U,DISLOC,Q,BODSTD,DOSAT,DOSTD,NCNTRLP,XCNTRLP,INDXBOD,
     2 UPPERB,LOWERB,NEQROW,IEQUITY,XINBOD,QT,ITERCHK)
C
CCC
CCC   DEFINITIONS FROM XMP:  MAXM=MAXIMUM NUMBER OF CONSTRAINTS
CCC                          MAXA=MAXIMUM NUMBER OF NON-ZEROS
CCC                          MAXN=MAXIMUM NUMBER OF VARIABLES
CCC                          COLMAX=MAXIMUM NUMBER OF NON-ZEROS
CCC                                IN ANY MATRIX COLUMN
C
      IF(INDXBOD .EQ. 1)MXX=NT+2*NREACH
      IF(INDXBOD .EQ. 0)MXX=NT+NREACH
      IF(IOBJ .LE. 2)MAXM=MXX+NEQROW
      IF(IOBJ .EQ. 3)MAXM=MXX+NEQROW+2
      IF(IOBJ .LE. 2)MAXN=2*NREACH+MAXM+1
      IF(IOBJ .EQ. 3 .AND. IMEMBER .EQ. 0)MAXN=2*NREACH+MAXM+2
      IF(IOBJ .EQ. 3 .AND. IMEMBER .EQ. 1)MAXN=2*NREACH+MAXM+3
      MAXA=MAXM*MAXN
      COLMAX=MAXM
      IF(IOBJ .LE. 2)NCOLSA=2*NREACH+1
      IF(IOBJ .EQ. 3 .AND. IMEMBER .EQ. 0)NCOLSA=2*NREACH+2
      IF(IOBJ .EQ. 3 .AND. IMEMBER .EQ. 1)NCOLSA=2*NREACH+3
      M=MAXM
      CALL XMAPS(BNDTYP,IOERR,LENMA,LENMI,LENMY,MAPA,MAPI,MAXA,MAXM,
     X           MAXN,MEMORY)
C
CC    SET THE TYPE OF CONSTRAINTS TO BE CONSIDERED
C           2=TWO-SIDED CONSTRAINT
C           1=LESS THAN; EQUAL TO
C           0=EQUATION
C          -1=GREATER THAN; EQUAL TO
C          -2=FREE ROW (FUNCTIONAL; SEE XMP)
C
      MAXMM1=MAXM-1
      DO 40 IR=1,MAXM
      ROWTYP(IR)=1
   40 CONTINUE
      IF(IOBJ.EQ.3) GO TO 55
C
CC    SET UP OBJECTIVE FUNCTION COEFFICIENTS FOR EACH OBJECTIVE
C
      DO 50 JC=1,NCOLSA
      CJX(JC)=0.0
      IF(IOBJ .EQ. 2)GOTO 49
      IF(JC. LE. NREACH )CJX(JC)=1.0
      IF(JC .GT. NREACH .AND. JC .LT. NCOLSA .AND. IPOLL .EQ. 0)
     * CJX(JC)=0.0
      IF(JC .GT. NREACH .AND. JC .LT. NCOLSA .AND. IPOLL .EQ. 1)
     * CJX(JC)=1.0
```

```
      IF(JC .EQ. NCOLSA .AND. IOBJ .EQ. 1)CJX(JC)=0.0
   49 IF(JC .EQ. NCOLSA .AND. IOBJ .EQ. 2)CJX(JC)=-1.0
   50 CONTINUE
      GO TO 60
   55 DO 57 JC=1,NCOLSA
      CJX(JC)=0.0
      IF(IMEMBER .EQ. 0)CJX(NCOLSA)=1.0
      IF(IMEMBER .EQ. 1)CJX(NCOLSA-1)=1.0
      IF(IMEMBER .EQ. 1)CJX(NCOLSA)=-1.0
   57 CONTINUE
C
   60 N=0
C
C     ADD NUMBER OF CONSTRAINTS EQUAL TO THE NUMBER OF OBJECTIVES
C     FOR THE MULTIOBJECTIVE PROBLEM
C
      IF(IOBJ .EQ. 3)CALL ADDSTUF(TCHCOEF,IMEMBER,RANGBOD,RANGDIF,
     * Z11,Z12,Z21,Z22,PL,PU,MAXM,NREACH,RHS,UPPERB,LOWERB,IPOLL)
C
C
C           PRINT OUT A-MATRIX AND RHS FOR DEBUGGING PURPOSES
C
C     IF(ITERCHK .NE. 1) GO TO 90
C     WRITE(7,64)
C  64 FORMAT(1H1/,3X,'C(J) :')
C     WRITE(7,65) (CJX(JC),JC=1,NCOLSA)
C  65 FORMAT(/10(2X,E10.3))
C     WRITE(7,66)
C  66 FORMAT(/3X,'A(I,J) :')
C     DO 68 I=1,MAXM
C     WRITE(7,67)(TCHCOEF(I,J),J=1,NCOLSA)
C  67 FORMAT (/10(2X,E10.3))
C  68 CONTINUE
C     WRITE(7,75)
C  75 FORMAT(/3X,'RHS :')
C     DO 80 I=1,MAXM
C     WRITE(7,78)RHS(I)
C  78 FORMAT(E10.3)
C  80 CONTINUE
C  90 CONTINUE
C
C
C     SET UP OBJECTIVE ROW AND TECHNOLOGICAL COEFFICIENTS FOR USE
C     IN XMP
C
      DO 130 JC=1,NCOLSA
      CJ=CJX(JC)
      UJ=UPPERB(JC)
      LJ=LOWERB(JC)
      COLLEN=MAXM
      DO 120 IR=1,COLLEN
      COLI(IR)=IR
      COLA(IR)=TCHCOEF(IR,JC)
  120 CONTINUE
      CALL XADDUB(BNDTYP,IOERR,JC,LENMA,LENMY,LJ,MAPA,MEMORY,UJ)
      CALL XADDAJ(CJ,COLA,COLI,COLLEN,COLMAX,IOERR,JC,
     X           LENMA,LENMY,MAPA,MEMORY,N)
  130 CONTINUE
      DO 140 JC=1,N
  140 STATUS(JC)=0
C
      Z=0.0
      CALL XSLACK(RHS,BASCB,BASIS,BASLB,BASUB,BLOW,BNDTYP,BOUND,
     X COLA,COLI,COLMAX,IOERR,LENMA,LENMI,LENMY,
     X M,MAPA,MAPI,MAXM,MAXN,MEMORY,N,ROWTYP,STATUS,
     X UZERO,XBZERO,Z)
C
C
```

```
      CALL XPRIML(RHS,BASCB,BASIS,BASLB,BASUB,BNDTYP,BOUND,
     X CAND,CANDA,CANDCJ,CANDI,CANDL,COLA,COLI,COLMAX,
     X FACTOR,IOERR,IOLOG,ITER1,ITER2,LENMA,LENMI,LENMY,LOOK,
     X M,MAPA,MAPI,MAXM,MAXN,MEMORY,N,NTYPE2,P,PRINT,
     X STATUS,TERMIN,UNBDDQ,UZERO,XBZERO,YQ,Z)
C
C
      WRITE(IOLOG,325) TERMIN
  325 FORMAT(18H TERMINATION CODE=,I4)
      IF(IPRINT .EQ. 0 .OR. IPRINT .EQ. 1)GOTO 326
C
C
      CALL XPRINT(BASIS,BNDTYP,BOUND,IOERR,IOLOG,
     X LENMA,LENMY,M,MAPA,MAXM,MAXN,MEMORY,N,NTYPE2,
     X STATUS,XBZERO,Z)
C
C
C     WRITE(6,200) (STATUS(J),J=1,N)
C 200 FORMAT(//5X,"STATUS :"/8(I5,2X))
  326 DO 330 J=1,NCOLSA
      VALUE(J)=0.
  330 CONTINUE
      TOLBOD=0.
      DO 375 J=1,NREACH
      IF(STATUS(J)) 340,350,360
  340 BOD(J)=UPPERB(J)
      GO TO 370
  350 BOD(J)=LOWERB(J)
      GO TO 370
  360 IX=STATUS(J)
      BOD(J)=XBZERO(IX)
  370 CONTINUE
      TOLBOD=TOLBOD+BOD(J)
  375 CONTINUE
      JDOD=0
      NNN=2*NREACH
      NP1=NREACH+1
      DO 410 J=NP1,NCOLSA
      JDOD=JDOD+1
      IF(STATUS(J))380,390,400
  380 DOD(JDOD)=UPPERB(J)
      GOTO 405
  390 DOD(JDOD)=LOWERB(J)
      GOTO 405
  400 IX=STATUS(J)
      DOD(JDOD)=XBZERO(IX)
  405 CONTINUE
      IF(IPOLL .EQ. 1 .AND. J .LE. NNN)TOLBOD=TOLBOD+DOD(JDOD)
  410 CONTINUE
C
C     CALL SUBROUTINE TO CALCULATE CRITICAL LOCATIONS
C
C     THE USE OF CRITICAL POINT LOCATIONS DEFINED BY THE STREETER-
C     PHELPS EQUATION ARE ESSENTIAL IN THE USE OF MOVABLE CONTROL
C     POINT LOCATIONS.  WATER QUALITY CONSTRAINTS ARE BASED ON THE
C     LOCATION OF CONTROL POINTS WITHIN THE STREAM SYSTEM.  AT EACH
C     CONTROL POINT LOCATION, WATER QUALITY STANDARDS MUST BE MET.
C     BY AN ITERATIVE PROCEEDURE, THE WATER QUALITY PROBLEM CAN BE
C     SOLVED UNTIL BOTH THE CRITICAL LOCATION AND CONTROL POINTS
C     COINCIDE.
C
C
      CALL CRITIC(QT,DOD,BOD,XC,DO,NREACH,KD,KA,LO,U,QO,Q,DISLOC)
C
C
C     PROVIDE CHECK BETWEEN CONTROL POINT AND CRITICAL POINT LOCATIONS
C
      ICHECK=0
```

```
      DO 412 IXX=1,NREACH
      DELTA(IXX)=ABS(XCNTRLP(IXX)-XC(IXX))
      IF(DELTA(IXX) .GT. 0.01)ICHECK=ICHECK+1
  412 CONTINUE
      IF(ICHECK .EQ. 0)GOTO 419
C
C     IF CONTROL POINT AND CRITICAL POINT LOCATIONS DO NOT COINCIDE,
C     REDEFINE CONTROL POINT LOCATIONS
C
      DO 413 IXXX=1,NREACH
      IF(XC(IXXX) .GE. DISLOC(IXXX+1))XCNTRLP(IXXX)=DISLOC(IXXX+1)
      IF(XC(IXXX) .LT. DISLOC(IXXX+1))XCNTRLP(IXXX)=XC(IXXX)
      IF(XCNTRLP(IXXX) .EQ. DISLOC(IXXX))NCNTRLP(IXXX)=0
      IF(XCNTRLP(IXXX) .EQ. DISLOC(IXXX+1))NCNTRLP(IXXX)=0
  413 CONTINUE
      IF(ITERCHK .EQ. 1)GOTO 415
C
C     PROVIDE CHECK ON THE BOD ASSIGNED DURING THE ITERATIONS
C
      JCHCK=0
      DO 414 IXXXX=1,NREACH
      DELTBOD(IXXXX)=ABS(BOD1(IXXXX)-BOD(IXXXX))/BOD(IXXXX)
      IF(DELTBOD(IXXXX) .GT. 0.05)JCHCK=JCHCK+1
  414 CONTINUE
      IF(JCHCK .EQ. 0)GOTO 419
  415 DO 416 I=1,NREACH
      BOD1(I)=BOD(I)
      DOD1(I)=DOD(I)
  416 CONTINUE
      GOTO 35
  419 DO 420 LX=1,NREACH
      PCTREMV(LX)=(XINBOD(LX)-BOD(LX))/XINBOD(LX)*100.0
  420 CONTINUE
      WRITE(IOOUT,425)(I,I=1,NREACH)
  425 FORMAT(/9X,6(6X,'REACH',I1))
      WRITE(IOOUT,430)(BOD(J),J=1,NREACH)
  430 FORMAT(/7X,'BOD',10(3X,E9.4))
      WRITE(IOOUT,440)(DOD(J),J=1,NREACH)
  440 FORMAT(/7X,'DOD',10(3X,E9.4))
      WRITE(IOOUT,450)(PCTREMV(J),J=1,NREACH)
  450 FORMAT(/2X,'%REMOVAL',5X,8(F5.2,7X))
      IF(IOBJ .LE. 2)WRITE(IOOUT,455)DOD(JDOD)
      IF(IOBJ .EQ. 3 .AND. IMEMBER .EQ. 1)WRITE(IOOUT,455)DOD(JDOD-2)
      IF(IOBJ .EQ. 3 .AND. IMEMBER .EQ. 0)WRITE(IOOUT,455)DOD(JDOD-1)
  455 FORMAT(/2X,'MAX. DIFFERENCE IN EQUITY =',E9.4)
      WRITE(IOOUT,457)TOLBOD
  457 FORMAT(/1X,'TOTAL BOD AND/OR DOD =',E10.4)
      IF(IEQUITY .EQ. 0)WRITE(IOOUT,460)
      IF(IEQUITY .EQ. 1)WRITE(IOOUT,480)
  460 FORMAT(//2X,'THE EQUITY CONSIDERED IS EQUAL % REMOVAL')
  480 FORMAT(//2X,'THE EQUITY CONSIDERED IS EQUAL EFFLUENT CONC.')
      IF(IOBJ .EQ. 1) GO TO 490
      IF(IOBJ .EQ. 2) GO TO 500
      IF(IOBJ .EQ. 3) GO TO 510
  490 Z11=Z
      Z21=DOD(JDOD)
      WRITE(6,495)Z11,Z21
  495 FORMAT(/1X,'Z11= ',F10.4,5X,'Z21= ',F10.4)
C
C     CREATE DO PROFILE
C
      NCP=20
      DO 496 I123=1,NREACH
      NDOPT(I123)=NCP
  496 CONTINUE
      CALL DATGEN(NDOPT,YCNTRLP,DISLOC,NREACH)
C     WRITE(6,497)(NDOPT(I1),I1=1,NREACH)
C 497 FORMAT(/'NDOPT:'/10(2X,I3))
```

```
      CALL DOCOMP(NREACH,LO,DO,QO,KD,KA,U,DISLOC,Q,DOSTD,DOSAT,
     X NDOPT,YCNTRLP,IDISCH,BOD,DOD,NPOINT)
C
C     CALL SUBROUTINE TO CREATE PLOT OF DO PROFILE
C     (OPTIONAL; SEE COMMENTS AT TOP OF PROGRAM)
C
      IF(IPLOT .GE. 1)CALL PLOTTER(NPOINT,NREACH,DISLOC,DOSTD,DOSAT,
     X NCP,IPLOT)
      GO TO 25
  500 Z12=TOLBOD
      Z22=Z
      RANGBOD=Z11-Z12
      RANGDIF=-(Z22-Z21)
      WRITE(6,505)RANGBOD,RANGDIF
  505 FORMAT(/1X,'RANGBOD= ',F10.4,5X,'RANGDIF= ',F10.4,/)
C
C     CREATE DO PROFILE
C
      NCP=20
      DO 507 I123=1,NREACH
      NDOPT(I123)=NCP
  507 CONTINUE
      CALL DATGEN(NDOPT,YCNTRLP,DISLOC,NREACH)
C     WRITE(6,508)(NDOPT(I1),I1=1,NREACH)
C 508 FORMAT(/'NDOPT:'/10(2X,I3))
      CALL DOCOMP(NREACH,LO,DO,QO,KD,KA,U,DISLOC,Q,DOSTD,DOSAT,
     X NDOPT,YCNTRLP,IDISCH,BOD,DOD,NPOINT)
C
C     CALL SUBROUTINE TO CREATE PLOT OF DO PROFILE
C     (OPTIONAL; SEE COMMENTS AT TOP OF PROGRAM)
C
      IF(IPLOT .GE. 1)CALL PLOTTER(NPOINT,NREACH,DISLOC,DOSTD,DOSAT,
     X NCP,IPLOT)
      GO TO 25
  510 CONTINUE
      IF(IMEMBER .EQ. 1) GO TO 520
      WRITE(6,515) Z
  515 FORMAT(//3X,'LINEAR FUZZY MEMBERSHIP FUNCTION WITH A MINIMUM VALUE
     1 OF ',3X,F6.3)
      GO TO 530
  520 EXPZ=EXP(Z)
      VALMEM=EXPZ/(1.+EXPZ)
      WRITE(6,525) VALMEM
  525 FORMAT(//3X,'LOGISTIC FUZZY MEMBERSHIP FUNCTION WITH A MINIMUM VAL
     1UE OF ',3X,F6.3)
  530 CONTINUE
C
C     CREATE DO PROFILE
C
      NCP=20
      DO 540 I123=1,NREACH
      NDOPT(I123)=NCP
  540 CONTINUE
      CALL DATGEN(NDOPT,YCNTRLP,DISLOC,NREACH)
C     WRITE(6,550)(NDOPT(I1),I1=1,NREACH)
C 550 FORMAT(/'NDOPT:'/10(2X,I3))
      CALL DOCOMP(NREACH,LO,DO,QO,KD,KA,U,DISLOC,Q,DOSTD,DOSAT,
     X NDOPT,YCNTRLP,IDISCH,BOD,DOD,NPOINT)
C
C     CALL SUBROUTINE TO CREATE PLOT OF DO PROFILE
C     (OPTIONAL; SEE COMMENTS AT TOP OF PROGRAM)
C
      IF(IPLOT .GE. 1)CALL PLOTTER(NPOINT,NREACH,DISLOC,DOSTD,DOSAT,
     X NCP,IPLOT)
  600 STOP
      END
C
C     ***************************************************************
```

```
C     SUBROUTINE TCHCOE1 IS DESIGNED TO GENERATE THE TECHNOLOGICAL
C     COEFFICIENTS AND RIGHT HAND SIDE FOR EACH OF THE CONSTRAINTS
C     CONTROLLING WATER QUALITY AND EQUITY
C     ************************************************************
C
      SUBROUTINE TCHCOE1(NREACH,NT,RHS,TCHCOEF,LO,DO,QO,IDISCH,IOBJ,
     1 KD,KA,U,DISLOC,Q,BODSTD,DOSAT,DOSTD,NCNTRLP,XCNTRLP,INDXBOD,
     2 UPPERB,LOWERB,NEQROW,IEQUITY,XINBOD,QT,ITERCHK)
      DIMENSION KD(15),U(15),DISLOC(15),BODCOEF(15,15),QT(15)
      DIMENSION XCNTRLP(150),RHS(200),DOCOEF(200,30),Q(15),DOSAT(15)
      DIMENSION KA(15),NCNTRLP(15),NCNT(15),BODSTD(15),DOSTD(15)
      DIMENSION TCHCOEF(200,30),UPPERB(20),LOWERB(20),EQCOEF(100,15)
      DIMENSION UPBTRT(10),LOWBTRT(10),UPBDOD(10),LOWBDOD(10)
      DIMENSION IDISCH(15)
      REAL LO,KD,KA,XINBOD(15),LOWBTRT,LOWBDOD,LOWERB
C
C
C
C     CALCULATE TOTAL FLOWS IN EACH REACH AND DETERMINE TOTAL
C     NUMBER OF CONTROL POINTS.
C
      CQT=QO
      NT=0
      NDISCH=0
      DO 50 J=1,NREACH
      CQT=CQT+Q(J)
      QT(J)=CQT
      NT=NT+NCNTRLP(J)
      NCNT(J)=NT
      NDISCH=NDISCH+IDISCH(J)
   50 CONTINUE
C
C     SET THE NUMBER OF EQUITY ROWS IN THE MATRIX CONTAINING
C     THE TECHNOLOGICAL COEFFICIENTS
C
      NEQROW=(NDISCH**2 - NDISCH)
C
C     INITIALIZE ARRAYS
C
      DO 85 I=1,NREACH
      DO 80 J=1,NREACH
      BODCOEF(I,J)=0.0
   80 CONTINUE
   85 CONTINUE
      LASTCOL=2*NREACH+1
      DO 87 I=1,NEQROW
      DO 86 J=1,LASTCOL
      EQCOEF(I,J)=0.0
   86 CONTINUE
   87 CONTINUE
      NRDOD=NT+NREACH
      NCDOD=2*NREACH+3
      DO 95 I=1,NRDOD
      DO 90 J=1,NCDOD
      DOCOEF(I,J)=0.0
   90 CONTINUE
   95 CONTINUE
      NROWTOL=NRDOD+NREACH+NEQROW
      DO 97 I=1,NROWTOL
      DO 96 J=1,NCDOD
      TCHCOEF(I,J)=0.0
   96 CONTINUE
   97 CONTINUE
C
      IF(INDXBOD .EQ. 0)GOTO 410
C
C     DETERMINE TECHNOLOGICAL COEFFICIENTS FOR BOD
C
```

```
C     WRITE(7,110)
C 110 FORMAT(/3X,'BODCOEF(I,J) :')
      DO 300 J=1,NREACH
      TFBOD=1.0
      DO 200 I=J,NREACH
      IF(I .EQ. J)GOTO 150
      TFBOD=TFBOD*EXP(-KD(I-1)*(DISLOC(I)-DISLOC(I-1))/U(I-1))
  150 BODCOEF(I,J)=TFBOD*Q(J)/QT(I)
  200 CONTINUE
C     WRITE(7,250) (BODCOEF(I,K),K=1,NREACH)
C 250 FORMAT(/10(2X,E10.3))
  300 CONTINUE
C
C     DETERMINE RHS FOR BOD
C
      TFBOD=1.0
      IROW=NT+NREACH+NEQROW
      DO 400 I=1,NREACH
      IROW=IROW+1
      IF(I .EQ. 1)GOTO 350
      TFBOD=TFBOD*EXP(-KD(I-1)*(DISLOC(I)-DISLOC(I-1))/U(I-1))
  350 RHS(IROW)=BODSTD(I)-QO*LO*TFBOD/QT(I)
  400 CONTINUE
C
C     CALCULATE TECHNOLOGICAL COEFFICIENTS FOR DO (BASED ON BOD)
C
  410 NROW=1
      IM1=0
      DO 700 I=1,NREACH
      IM1=I-1
      IF(I .NE. 1)NROW=NCNT(IM1)+IM1+1
      IRCNT=IM1
      TERM1=1.0
      TERM2=0.0
      DOCOEF(NROW,I)=0.0
      DO 600 J=I,NREACH
      IF(NCNTRLP(J) .EQ. 0)IRCNT=IRCNT+1
      IF(NCNTRLP(J) .EQ. 0)GOTO 550
      DO 500 K=1,NCNTRLP(J)
      IRCNT=IRCNT+1
C
C     CREATE TECHNOLOGICAL COEFFICIENTS FOR CONTROL POINT LOCATAIONS
C
      X=XCNTRLP(IRCNT)-DISLOC(J)
      PART1=Q(I)/QT(J)*TERM1*KD(J)/(KA(J)-KD(J))*
     1 (EXP(-KD(J)*X/U(J))-EXP(-KA(J)*X/U(J)))
      PART2=TERM2*Q(I)/QT(J)*EXP(-KA(J)*X/U(J))
      NROW=NROW+1
      DOCOEF(NROW,I)=PART1+PART2
  500 CONTINUE
  550 IF(J .EQ. NREACH)GOTO 700
C
C     CREATE TECHNOLOGICAL COEFFICIENTS FOR DISCHARGE LOCATIONS
C
      XR=DISLOC(J+1)-DISLOC(J)
      FPART1=TERM1*KD(J)/(KA(J)-KD(J))*
     1 (EXP(-KD(J)*XR/U(J))-EXP(-KA(J)*XR/U(J)))
      FPART2=TERM2*EXP(-KA(J)*XR/U(J))
      NROW=NROW+1
      DOCOEF(NROW,I)=(FPART1+FPART2)*Q(I)/QT(J+1)
      TERM1=TERM1*EXP(-KD(J)*XR/U(J))
      TERM2=FPART1+FPART2
  600 CONTINUE
  700 CONTINUE
C     WRITE(7,703)
C 703 FORMAT(/3X,'DOCOEF(I,J) :')
C     DO 706 I=1,NROW
C     WRITE(7,705) (DOCOEF(I,J),J=1,NREACH)
```

```
C 705 FORMAT(/10(2X,E10.3))
C 706 CONTINUE
C
C     CALCULATE TECHNOLOGICAL COEFFICIENTS FOR DO (BASED ON DO DEFICITS)
C
      MM1=0
      KROW=1
      DO 1000 M=1,NREACH
      KCNTR=0
      IF(M .EQ. 1)GOTO 720
      MM1=M-1
      DO 710 J=1,MM1
      KCNTR=KCNTR+NCNTRLP(J)
  710 CONTINUE
  720 NCOL=M+NREACH
      TFDO=1.0
      DO 900 N=M,NREACH
      IF(N .EQ. M)GOTO 750
      TFDO=TFDO*EXP(-KA(N-1)*(DISLOC(N)-DISLOC(N-1))/U(N))
  750 DOCOEF(KROW,NCOL)=TFDO*Q(M)/QT(N)
      DO 800 L=1,NCNTRLP(N)
      KCNTR=KCNTR+1
      KROW=KROW+1
      TFDOX=TFDO*EXP(-KA(N)*(XCNTRLP(KCNTR)-DISLOC(N))/U(N))
      DOCOEF(KROW,NCOL)=TFDOX*Q(M)/QT(N)
  800 CONTINUE
      KROW=KROW+1
  900 CONTINUE
      KROW=NCNT(M)+M+1
 1000 CONTINUE
C     DO 1002 I=1,KROW
C     WRITE(7,1001) (DOCOEF(I,J),J=1,NCOL)
C1001 FORMAT(/10(2X,E10.3))
C1002 CONTINUE
C
C
C     DETERMINE EQUITY COEFFICIENTS
C
      ICC=0
      IICC=0
      LLL=1
      MNN=0
      NRR=NREACH-1
      DO 1030 MNN=1,NRR
      LLL=LLL+1
      IF(IDISCH(MNN) .EQ. 0)GOTO 1030
      DO 1020 JNN=LLL,NREACH
      IF(IDISCH(JNN) .EQ. 0)GOTO 1020
      ICC=ICC+1
      IF(IEQUITY .EQ. 0)EQCOEF(ICC,MNN)=1/XINBOD(MNN)
      IF(IEQUITY .EQ. 0)EQCOEF(ICC,JNN)=-1/XINBOD(JNN)
      IF(IEQUITY .EQ. 1)EQCOEF(ICC,MNN)=1.0
      IF(IEQUITY .EQ. 1)EQCOEF(ICC,JNN)=-1.0
      EQCOEF(ICC,LASTCOL)=-1.0
      IICC=ICC+NEQROW/2
      IF(IEQUITY .EQ. 0)EQCOEF(IICC,MNN)=-1/XINBOD(MNN)
      IF(IEQUITY .EQ. 0)EQCOEF(IICC,JNN)=1/XINBOD(JNN)
      IF(IEQUITY .EQ. 1)EQCOEF(IICC,MNN)=-1.0
      IF(IEQUITY .EQ. 1)EQCOEF(IICC,JNN)=1.0
      EQCOEF(IICC,LASTCOL)=-1.0
 1020 CONTINUE
 1030 CONTINUE
C     WRITE(7,1033)
C1033 FORMAT(/3X,'EQCOEF(I,J) :')
C     DO 1040 I=1,IICC
C     WRITE(7,1035)(EQCOEF(I,J),J=1,LASTCOL)
C1035 FORMAT(/10(2X,E10.3))
C1040 CONTINUE
```

```
C
C     DETERMINE RHS FOR DO
C
      LROW=0
      ILN=1
      DO 1200 I=1,NREACH
      ILN=LROW+1
      LROW=LROW+NCNTRLP(I)+1
      DO 1100 J=ILN,LROW
      RHS(J)=DOSAT(I)-DOSTD(I)-DOCOEF(J,1)*QO/QT(1)*LO-
     1 DOCOEF(J,NREACH+1)*QO*IO/QT(1)
 1100 CONTINUE
 1200 CONTINUE
C
C     CREATE RHS FOR EQUITY
C
      NX=NT+NREACH+1
      NNXX=NT+NREACH+NEQROW
      DO 1250 II=NX,NNXX
      RHS(II)=0.0
 1250 CONTINUE
C
C
C     COMBINE TECHNOLOGICAL COEFFICIENTS INTO A SINGLE
C     ARRAY -- TCHCOEF
C
      MNROW=NT+NREACH
      MNCOL=2*NREACH
      DO 1400 I=1,MNROW
      DO 1300 J=1,MNCOL
      TCHCOEF(I,J)=DOCOEF(I,J)
 1300 CONTINUE
 1400 CONTINUE
      MXROW=NT+NREACH+1
      MBOT=NT+NREACH+NEQROW
      DO 1475 MI=1,LASTCOL
      MCNTR=0
      DO 1450 NI=MXROW,MBOT
      MCNTR=MCNTR+1
      TCHCOEF(NI,MI)=EQCOEF(MCNTR,MI)
 1450 CONTINUE
 1475 CONTINUE
      IF(INDXBOD .EQ. 0)GOTO 1650
      JROW=MBOT+1
      JBOTTOM=MBOT+NREACH
      DO 1600 K=1,NREACH
      JCNTR=0
      DO 1500 L=JROW,JBOTTOM
      JCNTR=JCNTR+1
      TCHCOEF(L,K)=BODCOEF(JCNTR,K)
 1500 CONTINUE
 1600 CONTINUE
C
 1650 CONTINUE
C
C
C    WRITE OUT TECHNOLOGICAL COEFF.
C     USED AS A DEBUGGING DEVICE
C
C     ITCOL=2*NREACH+3
C     DO 1700 I=1,NROWTOL
C     WRITE(7,1)
C   1 FORMAT(/1X,'HELLO I AM HERE')
C     WRITE(7,1690)(TCHCOEF(I,J),J=1,ITCOL)
C1690 FORMAT (10(2X,E10.3),/)
C1700 CONTINUE
C     DO 1800 I=1,JBOTTOM
C     WRITE(7,1750)RHS(I)
```

```
C1750 FORMAT(E10.3)
C1800 CONTINUE
      RETURN
      END
C
C
C     ********************************************************************
C     SUBROUTINE CRITIC IS DESIGNED TO GENERATE CRITICAL POINT LOCATIONS
C     ********************************************************************
C
      SUBROUTINE CRITIC(QT,DOD,BOD,XC,DO,NREACH,KD,KA,
     X LO,U,QO,Q,DISLOC)
      DIMENSION QT(15),Q(15),DOD(15),BOD(15),XC(15),KD(15),KA(15)
      DIMENSION U(15),DISLOC(15),XINITD(15),QT1(15),TOPD(15)
      REAL LO,KA,KD,QO,DO,TOPBOD(15),XINIBOD(15)
      QT1(1)=QO
      DO 20 I=1,NREACH
      QT1(I+1)=QT(I)
   20 CONTINUE
      DO 100 I=1,NREACH
      IF(I .EQ. 1)XINITD(I)=DO
      IF(I .EQ. 1)XINIBOD(I)=LO
      IF(I .EQ. 1)GOTO 50
      XINITD(I)=KD(I-1)*TOPBOD(I-1)/(KA(I-1)-KD(I-1))*(EXP(-KD(I-1)*
     1 (DISLOC(I)-DISLOC(I-1))/U(I-1))-EXP(-KA(I-1)*(DISLOC(I)-
     2 DISLOC(I-1))/U(I-1)))+TOPD(I-1)*EXP(-KA(I-1)*
     3 (DISLOC(I)-DISLOC(I-1))/U(I-1))
      XINIBOD(I)=TOPBOD(I-1)*EXP(-KD(I-1)*(DISLOC(I)-DISLOC(I-1))
     1 /U(I-1))
   50 TOPD(I)=(QT1(I)*XINITD(I)+Q(I)*DOD(I))/QT1(I+1)
      TOPBOD(I)=(QT1(I)*XINIBOD(I)+Q(I)*BOD(I))/QT1(I+1)
C
C     CHECK FOR NEGATIVE AND INDEFINITE CRITICAL LOCATIONS
C     THEN CALCULATE XC
C
      CHKCAL=(KA(I)-KD(I))*TOPD(I)/(KD(I)*TOPBOD(I))
      IF(CHKCAL .GE. 1.0)XC(I)=-1.0
      IF(CHKCAL .GE. 1.0)GOTO 90
      XC(I)=DISLOC(I)+U(I)/(KA(I)-KD(I))*ALOG(KA(I)/KD(I)*(1-CHKCAL))
   90 IF(XC(I) .LT. DISLOC(I))XC(I)=DISLOC(I)
  100 CONTINUE
      WRITE(6,110)(XC(I),I=1,NREACH)
  110 FORMAT(//3X,'XC :'/3X,7(F6.2,3X))
      RETURN
      END
C
C     ************************************************************
C     SUBROUTINE ADDSTUF IS TO ADD ROWS TO THE MATRIX CONTAINING
C     THE TECHNOLOGICAL COEFFICIENTS AND RHS IN ORDER TO PERFORM
C     PERFORM FUZZY PROGRAMMING TECHNIQUES.
C     ************************************************************
      SUBROUTINE ADDSTUF(TCHCOEF,IMEMBER,RANGBOD,RANGDIF,Z11,Z12,
     * Z21,Z22,PL,PU,MAXM,NREACH,RHS,UPPERB,LOWERB,IPOLL)
      DIMENSION TCHCOEF(200,30),RHS(200),UPPERB(20),LOWERB(20)
      REAL PL,PU,LOWERB,PPLL,PPUU
C
C
      MAXMM1=MAXM-1
      JCOL=2*NREACH+2
       JCOLM1=JCOL-1
       JCOLP1=JCOL+1
      IF(IPOLL .EQ. 0)NN=NREACH
      IF(IPOLL .EQ. 1)NN=2*NREACH
      IF(IMEMBER .EQ. 1)GOTO 500
C
C     ADD TECHNOLOGICAL COEFFICIENTS AND RHS TO ACCOUNT FOR LINEAR
C     MEMBERSHIP FUNCTION
C
```

```
      DO 100 J=1,NN
      TCHCOEF(MAXMM1,J)=-1.0/RANGBOD
  100 CONTINUE
      TCHCOEF(MAXMM1,JCOL)=1.0
C
C      CONSTRAINT FOR MAX. DIFFERENCE IN EQUITY
C
      TCHCOEF(MAXM,JCOLM1)=1.0/RANGDIF
      TCHCOEF(MAXM,JCOL)=1.0
C     DO 120 IR=MAXMM1,MAXM
C     WRITE(7,110) (TCHCOEF(IR,JC),JC=1,JCOL)
C 110 FORMAT(/10(2X,E10.3))
C 120 CONTINUE
      RHS(MAXMM1)=-Z12/RANGBOD
      RHS(MAXM)=Z21/RANGDIF
      GOTO 900
C
C     ADD TECHNOLOGICAL COEFFIIENTS AND RHS TO ACCOUNT FOR LOGISTIC
C     MEMBERSHIP FUNCTION
C
  500 PPLL=PL/(1.-PL)
      PPUU=PU/(1.-PU)
      ALPHA1=Z11/RANGBOD*ALOG(PPLL)-Z12/RANGBOD*ALOG(PPUU)
      ALPHA2=Z22/RANGDIF*ALOG(PPLL)-Z21/RANGDIF*ALOG(PPUU)
      BETA1=1./RANGBOD*(ALOG(PPUU)-ALOG(PPLL))
      BETA2=1./RANGDIF*(ALOG(PPUU)-ALOG(PPLL))
C     WRITE(6,599) ALPHA1,BETA1,ALPHA2,BETA2
C 599 FORMAT(/5X,'ALPHA1 =',1X,E11.4,'  BETA1 =',1X,E11.4/
C    1 5X,'ALPHA2 = ',E11.4,'  BETA2 = ',E11.4)
      DO 600 J=1,NN
      TCHCOEF(MAXMM1,J)=-BETA1
  600 CONTINUE
      TCHCOEF(MAXMM1,JCOL)=+1.0
      TCHCOEF(MAXMM1,JCOLP1)=-1.0
      TCHCOEF(MAXM,JCOLM1)=BETA2
      TCHCOEF(MAXM,JCOL)=1.0
      TCHCOEF(MAXM,JCOLP1)=-1.0
      RHS(MAXMM1)=ALPHA1
      RHS(MAXM)=-ALPHA2
  900 CONTINUE
C
C     ADD BOUNDS ON ADDITIONAL DECISION VARIABLE
C
      IF(IMEMBER .EQ. 0)LOWERB(JCOL)=0.0
      IF(IMEMBER .EQ. 0)UPPERB(JCOL)=1.0
      IF(IMEMBER .EQ.1)LOWERB(JCOL)=0.0
      IF(IMEMBER .EQ. 1)UPPERB(JCOL)=100000.
      IF(IMEMBER .EQ. 1)LOWERB(JCOLP1)=0.0
      IF(IMEMBER .EQ. 1)UPPERB(JCOLP1)=100000.0
      RETURN
      END
C
C     ****************************************************************
C     SUBROUTINE DOCOMP IS WHERE THE DO PROFILE IS COMPUTED
C     ****************************************************************
C
      SUBROUTINE DOCOMP(NREACH,LO,DO,QO,KD,KA,U,DISLOC,Q,DOSTD,DOSAT,
     X NDOPT,XCNTRLP,IDISCH,BOD,DOD,NPOINT)
      DIMENSION NDOPT(15),KD(15),KA(15),U(15),DISLOC(15),DOSTD(15),
     X DOSAT(15),XCNTRLP(150),IDISCH(15),BOD(15),DOD(15),Q(15),
     X NCNT1(15),QT(15),DOCOEF2(200,30),DOCONC(150),YCNTL(150)
      REAL LO,KD,KA
      CQT=QO
C
C     CALCULATE TOTAL FLOWS IN EACH REACH AND DETERMINE TOTAL
C     NUMBER OF CONTROL POINTS.
C
      NDISCH=0
```

```
      NT=0
      DO 50 J=1,NREACH
      CQT=CQT+Q(J)
      QT(J)=CQT
      NT=NT+NDOPT(J)
      NCNT1(J)=NT
      NDISCH=NDISCH+IDISCH(J)
   50 CONTINUE
C
C
C     INITIALIZE ARRAY
C
      NRDOD=NT+NREACH
      NCDOD=2*NREACH
      DO 95 J=1,NCDOD
      DO 90 I=1,NRDOD
      DOCOEF2(I,J)=0.0
   90 CONTINUE
   95 CONTINUE
C
C     CALCULATE TECHNOLOGICAL COEFFICIENTS FOR DO (BASED ON BOD)
C
      NROW=1
      IM1=0
      DO 700 I=1,NREACH
      IRCNT=0
      IF(I .EQ. 1)GOTO 460
      IM1=I-1
      NROW=NCNT1(IM1)+IM1+1
      DO 450 L=1,IM1
      IRCNT=IRCNT+NDOPT(L)
  450 CONTINUE
  460 TERM1=1.0
      TERM2=0.0
      DOCOEF2(NROW,I)=0.0
      DO 600 J=I,NREACH
      DO 500 K=1,NDOPT(J)
      IRCNT=IRCNT+1
      X=XCNTRLP(IRCNT)-DISLOC(J)
      PART1=Q(I)/QT(J)*TERM1*KD(J)/(KA(J)-KD(J))*
     1 (EXP(-KD(J)*X/U(J))-EXP(-KA(J)*X/U(J)))
      PART2=TERM2*Q(I)/QT(J)*EXP(-KA(J)*X/U(J))
      NROW=NROW+1
      DOCOEF2(NROW,I)=PART1+PART2
  500 CONTINUE
      IF(J .EQ. NREACH)GOTO 700
      XR=DISLOC(J+1)-DISLOC(J)
      FPART1=TERM1*KD(J)/(KA(J)-KD(J))*
     1 (EXP(-KD(J)*XR/U(J))-EXP(-KA(J)*XR/U(J)))
      FPART2=TERM2*EXP(-KA(J)*XR/U(J))
      NROW=NROW+1
      DOCOEF2(NROW,I)=(FPART1+FPART2)*Q(I)/QT(J+1)
      TERM1=TERM1*EXP(-KD(J)*XR/U(J))
      TERM2=FPART1+FPART2
  600 CONTINUE
  700 CONTINUE
C     NR=NT+NREACH
C     DO 705 IR=1,NR
C     WRITE(7,702)(DOCOEF2IR,JC),JC=1,NREACH)
C 702 FORMAT(5(2X,E10.3))
C 705 CONTINUE
C
C     CALCULATE TECHNOLOGICAL COEFFICIENTS FOR DO (BASED ON DO)
C
      MM1=0
      KROW=1
      DO 1000 M=1,NREACH
      KCNTR=0
```

```
      IF(M .EQ. 1)GOTO 720
      MM1=M-1
      DO 710 J=1,MM1
      KCNTR=KCNTR+NDOPT(J)
  710 CONTINUE
  720 NCOL=M+NREACH
      TFDO=1.0
      DO 900 N=M,NREACH
      IF(N .EQ. M)GOTO 750
      TFDO=TFDO*EXP(-KA(N-1)*(DISLOC(N)-DISLOC(N-1))/U(N))
  750 DOCOEF2(KROW,NCOL)=TFDO*Q(M)/QT(N)
      DO 800 L=1,NDOPT(N)
      KCNTR=KCNTR+1
      KROW=KROW+1
      TFDOX=TFDO*EXP(-KA(N)*(XCNTRLP(KCNTR)-DISLOC(N))/U(N))
      DOCOEF2(KROW,NCOL)=TFDOX*Q(M)/QT(N)
  800 CONTINUE
      KROW=KROW+1
  900 CONTINUE
      KROW=NCNT1(M)+M+1
 1000 CONTINUE
C     DO 1050 IROW=1,KROW
C     WRITE(7,1025)(DOCOEF2(IROW,JC),JC=1,NCDOD)
C1025 FORMAT(/'DOCOEF2:'/10(2X,E10.3))
C1050 CONTINUE
C
C
      I=0
      NPOINT=NT+NREACH
      DO 1230 IX=1,NREACH
      DO 1200 IYY=1,NDOPT(IX)+1
      I=I+1
      DOCONC(I)=0.0
      TOLDEF=DOCOEF2(I,1)*QO*LO/QT(1)+DOCOEF2(I,NREACH+1)*QO*DO/QT(1)
      DO 1100 J=1,NREACH
      TOLDEF=TOLDEF+BOD(J)*DOCOEF2(I,J)+DOD(J)*DOCOEF2(I,J+NREACH)
 1100 CONTINUE
      DOCONC(I)=DOSAT(IX)-TOLDEF
 1200 CONTINUE
 1230 CONTINUE
C
C     PRINT OUT DO PROFILE
C
      WRITE(6,1245)
 1245 FORMAT(//20X,'DO CONCENTRARION PROFILE'/)
      WRITE(6,1250)(DOCONC(IP),IP=1,NPOINT)
 1250 FORMAT(/11(2X,F5.2))
      WRITE(6,1270)
 1270 FORMAT(//20X,'CONTROL POINT LOCATIONS'/)
      KOUNT=1
      DO 1280 ID=1,NREACH
      WRITE(6,1275)DISLOC(ID),(XCNTRLP(IC),IC=KOUNT,NCNT1(ID))
 1275 FORMAT(11(1X,F6.2))
      KOUNT=NCNT1(ID)+1
 1280 CONTINUE
C
C     WRITE DO PROFILE TO TAPE FOR USE IN POTTING ROUTINE
C
      WRITE(8,1300)(DOCONC(IP8),IP8=1,NPOINT)
 1300 FORMAT(11(1X,F6.2))
      KOUNT8=1
      DO 1350 ID8=1,NREACH
      WRITE(8,1300)DISLOC(ID8),(XCNTRLP(IC8),IC8=KOUNT8,NCNT1(ID8))
      KOUNT8=NCNT1(ID8)+1
 1350 CONTINUE
      REWIND 8
      RETURN
      END
```

```
C
C
C     ********************************************************
C     SUBROUTINE DATGEN IS TO GENERATE CONTROL POINT LOCATIONS
C     ********************************************************
      SUBROUTINE DATGEN(NDOPT,XCNTRLP,DISLOC,NREACH)
      DIMENSION NDOPT(15),XCNTRLP(150),DISLOC(15)
      MCNTR=0
      DO 100 I=1,NREACH
      IF(I .EQ. NREACH) DISLOC(I+1)=DISLOC(I)+(DISLOC(I)-DISLOC(1))
     1 /(NREACH-1)
      DO 50 J=1,NDOPT(I)
      MCNTR=MCNTR+1
      XCNTRLP(MCNTR)=(DISLOC(I+1)-DISLOC(I))/(NDOPT(I)+1)*J
     1 +DISLOC(I)
   50 CONTINUE
  100 CONTINUE
      RETURN
      END
C
C
C     ********************************************************
C     SUBROUTINE PLOTTER IS WHERE PLOT OF DO PROFILE IS CREATED
C     ********************************************************
C
      SUBROUTINE PLOTTER(NPOINT,NREACH,DISLOC,DOSTD,DOSAT,NCP,IPLOT)
      DIMENSION YDAT(150),XDAT(150),DISLOC(15),DOSTD(15),NCNT1(15),
     X XLINE(2),YLINE(2),DOSAT(15)
C
C     NOTE: (1) YDAT: DO CONCENTRATION VALUES
C           (2) XDAT: DOWNSTREAM LOCATIONS
C
      READ(8,100)(YDAT(IP8),IP8=1,NPOINT)
  100 FORMAT(11(1X,F6.2))
      DO 200 I=1,NREACH
      IPOS=(I-1)*NCP+I
      IPOSP1=IPOS+1
      IEND=I*(NCP+1)
      READ(8,100)XDAT(IPOS),(XDAT(IC8),IC8=IPOSP1,IEND)
  200 CONTINUE
C
C     WRITE(6,100)(YDAT(I),I=1,NPOINT)
C     WRITE(6,100)(XDAT(I),I=1,NPOINT)
C
      NP1=NREACH+1
C
      START=DOSTD(1)-1.0
      STEPX=DISLOC(NP1)/NREACH
C
C     TO PLOT ON WATER RESEARCH TERMINAL
C
      IF(IPLOT .EQ. 2)CALL DATMED(0)
C
C     TO PLOT ON TERMINALS IN C.E. BUILDING
C
      IF(IPLOT .EQ. 3)CALL TEKALL(4010,960,0,1,0)
C
C     TO CREATE HARDCOPY OF PLOT AT COMPUTER CENTER
C
      IF(IPLOT .EQ. 1)CALL VRSTEC(0,0,0)
C
C     BEGIN CALLS TO PLOTTING ROUTINES
C
      CALL PAGE(8.50,11.0)
      CALL BANGLE(-90)
C     CALL NOBRDR
      CALL PHYSOR(1.5,9.0)
      CALL AREA2D(8.0,6.0)
```

```
      CALL FRAME
      CALL YINTAX
      CALL XINTAX
      CALL YNAME('DO CONCENTRATION, MG/L$',100)
      CALL XNAME('DOWNSTREAM LOCATION, MILES$',100)
      CALL YTICKS(4)
      CALL XTICKS(5)
C     CALL HEADIN('DO CONCENTRATION PROFILE$',100,1.5,1.0)
      CALL GRAF(0.0,STEPX,DISLOC(NP1),START,1.0,DOSAT(1))
      CALL CURVE(XDAT,YDAT,NPOINT,0)
C
C     PLOT DO STANDARD (LINE)
C
      XLINE(1)=0.0
      XLINE(2)=DISLOC(NP1)
      YLINE(1)=DOSTD(1)
      YLINE(2)=DOSTD(1)
      CALL DOT
      CALL CURVE(XLINE,YLINE,2,0)
      CALL ENDPL(0)
      CALL DONEPL
      REWIND 8
      RETURN
      END
```

APPENDIX G

LISTING OF COMPUTER PROGRAM FOR ASSESSING THE RISK OF VIOLATING VARIOUS DISSOLVED OXYGEN WATER QUALITY CONDITIONS

'RISKDO'

```
      PROGRAM RISKDO(TAPE4,TAPE5,TAPE6,TAPE7,TAPE8,TAPE9)
C*****************************************************************
C           THIS PROGRAM IS CALLED "MAINDO" MEAINING MAIN DISSOLVED OXYGEN
C     PROGRAM.  IT WAS WRITTEN BY DR. Y.K. TUNG AND WADE HATHHORN WITH
C     THE FINAL VERSION COMPLETED NOVEMBER 11, 1985.  AS AN OVERVIEW,
C     THE PROGRAM GENERATES INFORMATION ABOUT THE RISK OF OBTAINING
C     A GIVEN DEFICIT CONCENTRATION OF DISSOLVED OXYGEN BEYOND A SPECIFIED
C     STANDARD IN COMBINATION WITH A GIVEN VIOLATION DISTANCE.  THE BASIS
C     FOR THE DEVELOPMENT OF THIS PROGRAM IS THE UTILIZATION OF THE
C     STREETER-PHELPS EQUATION AND SIMULATION TECHNIQUES IN CONJUNCTION
C     WITH NUMERICAL APPROXIMATION AND STATISTICAL THEORY.  SEVERAL
C     TECHNIQUES ARE UTILIZED WITHIN THIS PROGRAM AND ARE GENERALLY
C     ASSIGNED WITNIN CORRESPONDING SUBROUTINES, FOR EXAMPLE, SIMULATION
C     IS COMPLETED BY THE SUBROUTINE SIMULDO AND NEWTON-RAPHSON'S
C     APPROXIMATION IS DONE WITHIN SUBROUTINE NWTONDO.  FOR MORE DETAIL
C     ABOUT EACH SUBROUTINE, SEE THE COMMENTS PRIOR TO THE LISTING OF
C     EACH ROUTINE WITHIN THIS PROGRAM.
C
C     *****************************************************************
C
C     TO EXECUTE THIS PROGRAM, TWO INPUT FILES MUST BE PROVIDED:
C
C         1. TAPE4
C         2. TAPE5
C
C     TAPE4 CONSISTS OF 4 MAJOR RECORDS:
C
C     RECORD          VARIABLES                   FORM_FORMAT
C
C       1             NDEF,NDIST                   2I8
C       2              DEFCBS                     10F8.0
C       3              VIOLDIS                    10F8.0
C       4             LSET,DOSTD,DOS,             I8,3F8.0,D12.1
C                     CORR,DSEED
C
C
C                        DSECRIPTION
C
C     NDEF = THE NUMBER OF DEFICIT CONC. BEYOND SPECIFIED STANDARD
C     NDIST = THE NUMBER OF VIOLATION DISTANCES TO BE CONSIDERED
C     DEFCBS = ANY NUMBER OF CARDS ENTERED CONTAINING THE GIVEN DEFICIT
C              CONCENTRATIONS DESIRED.  THE NUMBER OF DEFICITS ENTERED
C              IN MG/L MUST BE EQUAL TO (NDEF).
C     VIOLDIS = ANY NUMBER OF CARDS ENTERED CONTANING THE GIVEN VIOLATION
C               DISTANCES DESIRED.  THE NUMBER OF DISTANCES ENTERED IN
C               MILES MUST BE EQUAL TO (NDIST).
C     LSET = THE NUMBER OF PAIRS TO BE GENERATED
C     DOSTD = DISSOLVED OXYGEN AT THE SPECIFIED STANDARD IN MG/L
C     DOS = DISSOLVED OXYGEN AT SATURATION IN MG/L (GENERALLY 10.0)
C     CORR = CORRELATION COEFFICIENT BETWEEN K2 AND U
C     DSEED = SEED VALUE ENTERED FOR RANDOM NUMBER GERNERATOR
C
C     AN EXAMPLE OF TAPE4 WOULD BE AS FOLLOWS:
C
C       9       9
C     0.1     0.2     0.3     0.4     0.5     0.6     0.7     0.8     0.9
C     2.0     4.0     6.0     8.0    10.0    12.0    14.0    16.0    18.0
C     999     6.0    10.0     0.8  765432.1D0
C
C
C     TAPE5 CONSISTS OF 5 MAJOR RECORDS:
C
C    RECORD      PARAMETER       VARIABLES              FORMAT
C
C      1             K1          MODEL,MEAN,             I8,4F8.0
C                                XSTD,XA,XC
C      2             K2               "                    "
C      3             U                "                    "
```

```
C      4           LO              "                   "
C      5           DO              "                   "
C
C
C                          DESCRIPTION
C
C     THE INPUT IN TAPE5 DESCRIBES THE TYPE AND CHARACTERISTICS OF THE
C     STATISTICAL DISTRIBUTIONS ASSIGNED FOR EACH PARAMETER:
C
C          K1 =  DEOXYGENATION COEFFICIENT
C          K2 =  REAERATION COEFFICIENT
C           U =  STREAM VELOCITY, AVE. (MILES/DAY)
C          LO =  BOD LOADING IN STREAM (MG/L)
C          DO =  INITIAL DISSOLVED OXYGEN DEFICIT (MG/L)
C
C     MODEL = VALUE DESIGNATING THE ASSUMED DISTRIBUTIONS FOR EACH
C             PARAMETER.  VALUES FROM 1 TO 5 MAY BE ENTERED:
C
C                     1 = NORMAL
C                     2 = LOG-NORMAL
C                     3 = GAMA
C                     4 = WIEBULL
C                     5 = BETA
C                     6 = TRIANGULAR (NOT OPERATIONAL AT THIS TIME)
C
C
C     MEAN = MEAN VALUE OF THE CORRESPONDING PARAMETER
C     XSTD = STD. DEVIATION OF THE CORRESPONDING PARAMETER
C     XA = LOWER BOUND WHEN BETA DISTRIBUTION IS ASSUMED; OTHERS USE 0.0
C     XC = UPPER BOUND WHEN BETA DISTRIBUTION IS ASSUMED; OTHERS USE 0.0
C
C     AN EXAMPLE OF TAPE5 WOULD BE AS FOLLOWS:
C
C       1    0.35     0.3     0.0     0.0
C       2    0.70     0.2     0.0     0.0
C       3    10.0     3.0     0.0     0.0
C       4    15.0     3.0     0.0     0.0
C       5     1.0     0.2     0.2     1.5
C
C     ********************************************************************
C
C     ********************************************************************
C
C     THERE ARE SEVERAL OUTPUTS FROM THIS PROGRAM:
C          1. TAPE6
C          2. TAPE7
C          3. TAPE8
C          4. TAPE9
C
C     TAPE6 CONTAINS AN ECHO PRINT OF THE INPUT TAPE5 ALONG WITH A TABULAR
C     PRINTOUT OF THE GENERATED RISK VALUES ASSOCIATED WITH EACH PAIR OF
C     DEFICIT CONCENTRATIONS AND VIOLATION DISTANCES.  ALSO PRINTED OUT
C     WITH TAPE6 IS THE NUMBER AND RISK OF NO VIOLATION AND THE CORRELATIION
C     BETWEEN THE SIMULATION DEFICITS AND VIOLATION DISTANCES.
C
C
C     TAPE7 CONTAINS INFORMATION ABOUT EACH PAIR OF GENERATED VIOLATION
C     DISTANCE AND DEFICIT.  IT SHOULD BE NOTED THAT SINCE THE NUMBER
C     OF GENERATED PAIRS IS USUALLY LARGE FOR SIMULATION, STATEMENTS TO
C     WRITE TO TAPE7 SHOULD BE COMMENTED OUT (C) IN ORDER TO REDUCE
C     THE AMOUNT OF TIME NEEDED TO EXECUTE THIS PROGRAM.  TAPE7 CAN BE
C     GENERATED IF EXTREME INTEREST IS NEEDED FOR EACH ITERATION AND
C     GENERATION OF SIMULATIONS PAIRS.  CURRENTLY STATEMENTS TO WRITE TO
C     TAPE7 ARE COMMENTED OUT WITH A (C).
C
C
C     TAPE8 CONTAINS A LIST OF THE GERNERATED PAIRS OF VIOLATION DISTANCE
C     AND DEFICIT CONCENTRATION, USING MAX. DEFICIT CONCENTRATION,
```

```
C     WITH THE ASSOCIATED RISK.  TAPE8 IS USED
C     WITH A GRAPHICS PROGRAM TO PLOT CONTOURS OF THE PROBABILITY OF
C     VIOLATION UTILIZING THESE GENERATED PAIRS.
C
C     TAPE9 CONTAINS A LIST OF THE GERNERATED PAIRS OF VIOLATION DISTANCE
C     AND DEFICIT CONCENTRATION, USING AVERAGE DEFICIT CONCENTRATION,
C     WITH THE ASSOCIATED RISK.  TAPE8 IS USED
C     WITH A GRAPHICS PROGRAM TO PLOT CONTOURS OF THE PROBABILITY OF
C     VIOLATION UTILIZING THESE GENERATED PAIRS.
C*************************************************************************
C
      DOUBLE PRECISION DSEED
      DIMENSION DEFCBS(30),VIOLDIS(30),SIMDMBS(999),SIMXD(999)
      DIMENSION PRTMATM(30,30),SIMADBS(999),PRTMATA(30,30)
      READ(4,10)NDEF,NDIST
   10 FORMAT(2I8)
      READ(4,20)(DEFCBS(J),J=1,NDEF)
      READ(4,20)(VIOLDIS(I),I=1,NDIST)
   20 FORMAT(10F8.0)
      READ(4,30)LSET,DOSTD,DOS,CORR,DSEED
   30 FORMAT(I8,3F8.0,D12.1)
      CALL SIMULDO(LSET,DOSTD,DOS,CORR,DSEED,SIMDMBS,SIMXD,SIMADBS,INDX)
C     WRITE(6,40)
C  40 FORMAT(///1X,"DEFCBS (MG/L)",5X,"VIOLDIS (MILES)",
C    X 7X,"RISK",5X,"PROB. NO TOLER. VIOL.")
      DO 300 I=1,NDIST
      DO 200 J=1,NDEF
      COUNTM=0.
      RCOUNTM=0.0
      COUNTA=0.
      RCOUNTA=0.
      DO 100 ISET=1,LSET
      IF(SIMDMBS(ISET).GE.DEFCBS(J).AND.SIMXD(ISET).GE.
     X VIOLDIS(I)) COUNTM=COUNTM+1.
      IF(SIMDMBS(ISET).LT.DEFCBS(J).AND.SIMXD(ISET).LT.
     X VIOLDIS(I)) RCOUNTM=RCOUNTM+1.0
      IF(SIMADBS(ISET).GE.DEFCBS(J).AND.SIMXD(ISET).GE.
     X VIOLDIS(I)) COUNTA=COUNTA+1.
      IF(SIMADBS(ISET).LT.DEFCBS(J).AND.SIMXD(ISET).LT.
     X VIOLDIS(I)) RCOUNTA=RCOUNTA+1.0
  100 CONTINUE
      RISKM=COUNTM/(LSET+1)
      RSKNOTM=RCOUNTM/(LSET+1)
      RISKA=COUNTA/(LSET+1.)
      RSKNOTA=RCOUNTA/(LSET+1.)
C     WRITE(6,110)VIOLDIS(I),DEFCBS(J),RISKM,RSKNOTM
C 110 FORMAT(3X,F5.2,14X,F7.2,10X,F6.4,10X,F6.4)
      WRITE(8,120)VIOLDIS(I),DEFCBS(J),RISKM
  120 FORMAT(2F8.2,F8.4)
      WRITE(9,120) VIOLDIS(I),DEFCBS(J),RISKA
      PRTMATM(I,J)=RISKM
      PRTMATA(I,J)=RISKA
  200 CONTINUE
  300 CONTINUE
      WRITE(6,302)
  302 FORMAT(////30X,"VIOLATION DISTANCE",
     X" VS. MAXIMUM DEFICIT CONCENTRATION",
     X" BEYOND STANDARD")
      WRITE(6,305)(VIOLDIS(I),I=1,NDIST)
  305 FORMAT(//9X,15(3X,F5.2))
      DO 340 J=1,NDEF
      WRITE(6,320)DEFCBS(J),(PRTMATM(I,J),I=1,NDIST)
  320 FORMAT(/5X,F4.2,15(2X,F6.3))
  340 CONTINUE
      WRITE(6,350)INDX
  350 FORMAT(//1X,"THE NO. OF NO VIOLATIONS= ",I3)
      RNON=INDX/(LSET+1.0)
      WRITE(6,370)RNON
```

```
  370 FORMAT(/1X,"THE PROBABILITY OF NO VIOLATION=",
     X 2X,F6.4)
      CALL CORRCAL(SIMDMBS,SIMXD,CORXY,LSET)
      WRITE(6,380)CORXY
  380 FORMAT(/1X,"THE CORRELATION OF VIOLATION",
     X" DISTANCE AND MAX. CONCENTRATION BEYOND STD.=",F10.8)
      WRITE(6,385)DOS,DOSTD
  385 FORMAT(/1X,"THE SPECIFIED D.O. AT SATURATION IS ",
     X F6.3,"  MG/L AND THE INPUT D.O. AT THE STANDARD IS ",
     X F6.3,"  MG/L")
      WRITE(6,400)
  400 FORMAT(1H1,////30X,"VIOLATION DISTANCE",
     X" VS. AVERAGE DEFICIT CONCENTRATION",
     X" BEYOND STANDARD")
      WRITE(6,305)(VIOLDIS(I),I=1,NDIST)
      DO 410 J=1,NDEF
      WRITE(6,320)DEFCBS(J),(PRTMATA(I,J),I=1,NDIST)
  410 CONTINUE
      CALL CORRCAL(SIMADBS,SIMXD,CORXY,LSET)
      WRITE(6,420)CORXY
  420 FORMAT(/1X,"THE CORRELATION OF VIOLATION",
     X" DISTANCE AND AVG. CONCENTRATION BEYOND STD.=",F10.8)
      STOP
      END
C
C     ******************************************************************
C
C     SUBROUTINE SIMULDO IS THE SIMULATION PORTION OF THE PROGRAM WHERE
C     AN (LSET) NUMBER OF PAIRS OF VIOLATION DISTANCE AND DEFICIT
C     CONCENTRATION ARE GENERATED ALONG WITH THE ASSOCIATED RISK FOR EACH
C     PAIR.
C
C     ******************************************************************
C
      SUBROUTINE SIMULDO(LSET,DOSTD,DOS,CORR,DSEED,
     * SIMDMBS,SIMXD,SIMADBS,INDX)
      DOUBLE PRECISION DSEED
      INTEGER MODEL
      DIMENSION R(1),WK(3),PARAMTR(5),SIMDMBS(999),SIMXD(999)
      DIMENSION SIGMA(3),RVEC(1,2),WKVEC(1),SIMADBS(999)
      INDX=0
      RCOUNT=0.0
      ISET=0
   10 ISET=ISET+1
      IF(ISET.GT.LSET)GOTO 150
      DO 100 NP=1,5
      READ(5,20) MODEL,XMEAN,XSTD,XA,XC
   20 FORMAT(I8,4F8.0)
      IF(ISET.GT. 1) GO TO 28
      IF(NP.EQ.1)WRITE(6,22)MODEL,XMEAN,XSTD,XA,XC
      IF(NP.EQ.2)WRITE(6,23)MODEL,XMEAN,XSTD,XA,XC
      IF(NP.EQ.3)WRITE(6,24)MODEL,XMEAN,XSTD,XA,XC
      IF(NP.EQ.4)WRITE(6,25)MODEL,XMEAN,XSTD,XA,XC
      IF(NP.EQ.5)WRITE(6,26)MODEL,XMEAN,XSTD,XA,XC
   22 FORMAT(/3X,"K1",5X,I8,4(3X,E12.5))
   23 FORMAT(/3X,"K2",5X,I8,4(3X,E12.5))
   24 FORMAT(/3X,"U",6X,I8,4(3X,E12.5))
   25 FORMAT(/3X,"LO",5X,I8,4(3X,E12.5))
   26 FORMAT(/3X,"DO",5X,I8,4(3X,E12.5))
   28 GO TO(30,40,50,60,70,80),MODEL
C
C      NORMAL DISTRIBUTION
C
   30 IF(NP.EQ.2 .AND. CORR.NE.0.0)GOTO 35
      IF(NP.EQ.3 .AND. CORR.NE.0.0)GOTO 85
      CALL GGNML(DSEED,1,R)
      PARAMTR(NP)=XMEAN+XSTD*R(1)
      GO TO 90
```

```
C
C       STORE THE MEAN AND STD. DEV. OF K2 WHEN CORRELATION BETWEEN K2 AND
C        VELOCITY IS NONZERO.
C
   35 STDK2=XSTD
      XMEANK2=XMEAN
      GOTO 90
C
C       LOG-NORMAL DISTRIBUTION
C
C
C        COMPUTE THE MEAN (XM) AND STD. DEV. (S) OF LOG-TRANSFORMED VARIABLES.
C
   40 CV=XSTD/XMEAN
      S2=ALOG(CV*CV+1.)
      S=SQRT(S2)
      A=XMEAN*XMEAN/(CV*CV+1.)
      XM=ALOG(A)/2.
      IF(NP.EQ.2 .AND. CORR.NE.0.0)GOTO 45
      IF(NP.EQ.3 .AND. CORR.NE.0.0)GOTO 85
      CALL GGNLG(DSEED,1,XM,S,R)
      PARAMTR(NP)=R(1)
      GO TO 90
C
C       STORE THE MEAN AND STD. DEV. OF LOG-TRANSFORMED K2 WHEN CORRELATION
C        BETWEEN K2 AND VELOCITY IS NONZERO.
C
   45 YSTDK2=S
      YMEANK2=XM
      CVK2=CV
      GOTO 90
C
C      GAMMA DISTRIBUTION
C
   50 XVAR=XSTD*XSTD
      B=XVAR/XMEAN
      A=XMEAN/B
      WK(1)=0.
      CALL GGAMR(DSEED,A,1,WK,R)
      PARAMTR(NP)=R(1)*B
      GO TO 90
C
C      WEIBULL DISTRIBUTION
C
   60 CALL SEARCH2(XMEAN,XSTD,AA,BB)
      CALL GGWIB(DSEED,AA,1,R)
      PARAMTR(NP)=BB*R(1)
      GO TO 90
C
C      BETA DISTRIBUTION
C
   70 YMEAN=(XMEAN-XA)/(XC-XA)
      YSTD=XSTD/(XC-XA)
      YVAR=YSTD*YSTD
      YMM1=1.-YMEAN
      A=(YMEAN/YSTD)**2*(1.-YMEAN)-YMEAN
      B=YMEAN*YMM1*YMM1/YVAR-YMM1
      CALL GGBTR(DSEED,A,B,1,R)
      PARAMTR(NP)=XA+(XC-XA)*R(1)
      GO TO 90
C
C      TRIANGULAR DISTRIBUTION
C
   80 BA=XB-XA
      CB=XC-XB
      CA=XC-XA
      FB=BA/CA
      YAB=2./(CA*BA)
```

```
      YBC=2./(CA*CB)
      CALL GGUBS(DSEED,1,R)
      IF(R(1).GT.FB) GO TO 82
      XX=2.*R(1)/YAB
      PARAMTR(NP)=XA+SQRT(XX)
      GO TO 90
   82 XK=2.*XB*XC-XB*XB+2.*(R(1)-FB)/YBC
      XX=XC*XC-XK
      PARAMTR(NP)=XC-SQRT(XX)
      GOTO 90
   85 IF(MODEL.EQ.1) GO TO 87
C        COMPUTE THE CORRELATION COEFFICIENT FOR LOG-TRANSFORMED K2 AND
C         VELOCITY FROM THE CORRELATION CCOEFFICIENT BETWEEN K2 AND VELOCITY
C        OF ORIGINAL SCALE.
C
      CVU=CV
      YSTDU=S
      YMEANU=XM
      F1=ALOG(1.+CORR*CVK2*CVU)
      F2=ALOG(1.+CVK2*CVK2)
      F3=ALOG(1.+CVU*CVU)
      F2=SQRT(F2)
      F3=SQRT(F3)
      YCORR=F1/(F2*F3)
      SIGMA(2)=YCORR*YSTDK2*YSTDU
      SIGMA(1)=YSTDK2*YSTDK2
      SIGMA(3)=YSTDU*YSTDU
      GO TO 88
   87 STDU=XSTD
      XMEANU=XMEAN
      SIGMA(2)=CORR*STDK2*STDU
      SIGMA(1)=STDK2*STDK2
      SIGMA(3)=STDU*STDU
   88 NR=1
      KPAR=2
      IR=1
      WKVEC(1)=0.0
      CALL GGNSM(DSEED,NR,KPAR,SIGMA,IR,RVEC,WKVEC,IER)
      IF(MODEL.EQ.1) GO TO 89
C
C        FIND BIVARIATE NORMAL RANDOM DEVIATES OF LOG-TRANSFORMED K2 AND
C        VELOCITY.  THEN RETRANSFORM THEM BACK TO THE ORIGINAL SCALE.
C
      YK2=RVEC(1,1)+YMEANK2
      YU=RVEC(1,2)+YMEANU
      PARAMTR(NP-1)=EXP(YK2)
      PARAMTR(NP)=EXP(YU)
      GO TO 90
C
C        OBTAIN BIAVARIATE NORMAL RANDOM DEVIATES FOR K2 AND VELOCITY.
C
   89 PARAMTR(NP-1)=RVEC(1,1)+XMEANK2
      PARAMTR(NP)=RVEC(1,2)+XMEANU
   90 CONTINUE
  100 CONTINUE
C
C       CHECK TO EXCLUDE PARAMETER SET WITH NONPOSITIVE VALUES
C
      NEG=0
      DO 110 I=1,5
      IF(PARAMTR(I).LT.0.)NEG=NEG+1
  110 CONTINUE
      IF(NEG.GT.0)ISET=ISET-1
      IF(NEG.GT.0)GOTO 140
C
C        CHECK TO EXCLUDE PARAMETER SET THAT WILL LEAD TO NEGATIVE
C        OR INDEFINITE D.O. DEFICITS AND/OR CRITICAL LOCATIONS
C
```

```
      PARCHK1=PARAMTR(2)-PARAMTR(1)
      PARCHK2=PARAMTR(2)/PARAMTR(1)
      ANY=(PARAMTR(4)/PARAMTR(5))+1.0
      IF(PARCHK1.EQ.0.0)ISET=ISET-1
      IF(PARCHK1.EQ.0.0)GOTO 140
      IF(PARCHK2.GT.ANY)ISET=ISET-1
      IF(PARCHK2.GT.ANY)GOTO 140
      XD=0.0
      DMBS=0.0
      INDIC=0
      CALL NWTONDO(PARAMTR,DOSTD,DOS,DMBS,XD,AVGDCBS,INDX,INDIC)
      IF(INDIC.GT.0)ISET=ISET-1
      IF(INDIC.GT.0)GOTO 140
      SIMDMBS(ISET)=DMBS
      SIMXD(ISET)=XD
      SIMADBS(ISET)=AVGDCBS
C     WRITE(7,130) ISET,SIMXD(ISET),SIMDMBS(ISET),SIMADBS(ISET)
C 130 FORMAT(1X,I4,3X,F5.1,2(3X,F6.3))
  140 REWIND 5
      GOTO 10
  150 CONTINUE
      WRITE(6,160) CORR
  160 FORMAT(/3X,"CORRELATION COEFF. BETWEEN K2 AND U IS",2X,F6.3)
      RETURN
      END
C*************************************************************************
C
C       SEARCH FOR PARAMETERS IN WEIBULL DISTRIBUTION USING METHOD OF MOMENTS.
C
C*************************************************************************
      SUBROUTINE SEARCH2(XMEAN,XSTD,ALPHA,BETA)
      ALPHA=5.0
      ITMAX=20
      XRIGHT=(XSTD/XMEAN)**2+1.
      TOLER=0.01*XRIGHT
      IT=0
   10 IT=IT+1
      IF(IT.GT.ITMAX) GO TO 20
      A1=1./ALPHA
      A2=2.*A1
      A2P1=A2+1.
      A1P1=A1+1.
      T2=GAMMA(A2P1)
      T3=GAMMA(A1P1)
      XLEFT=T2/(T3*T3)
      DIF=XLEFT-XRIGHT
      DIF=ABS(DIF)
      IF(DIF.LE.TOLER) GO TO 20
      R=XLEFT/XRIGHT
      ALPHA=ALPHA*SQRT(R)
      GO TO 10
   20 CONTINUE
      BETA=XMEAN/T3
C     WRITE(6,25) IT,ALPHA,DIF
C  25 FORMAT(2X,"SEARCH 2",5X,"IT=",I3,3X,"ALPHA=",E12.6,
C     X 2X,"DIF=",E12.6)
      IF(ALPHA.LE.1.) WRITE(6,26)
   26 FORMAT(2X,"*** WARNING : ALPHA IS LESS THAN 1. ***")
      RETURN
      END
C
C     ***************************************************************
C
C     SUBROUTINE NWTONDO IS WHERE NEWTON-RAPHSON'S APPROXIMATION IS USED
C     TO DETERMINE THE LENGTH OF VIOLATION.  IN ADDITION, THE ASSOCIATED
C     MAXIMUM DEFICIT OF DISSOLVED OXYGEN BEYOND THE SPECIFIED STANDARD
C     IS CALCULATED USING THE STREETER-PHELPS EQUATION.
C
```

```
C     ************************************************************************
C
      SUBROUTINE NWTONDO(PARAMTR,DOSTD,DOS,DMBS,XD,AVGDCBS,INDX,INDIC)
C
C
      DIMENSION PARAMTR(5)
      REAL K1,K2,LO,K,U
      MAXITER=20
      K1=PARAMTR(1)
      K2=PARAMTR(2)
      U=PARAMTR(3)
      LO=PARAMTR(4)
      DO=PARAMTR(5)
      C=K1/(K2-K1)
      K=1.0-(DO/(LO*C))
C      XC = CRITICAL LOCATION
      XC=(ALOG(K2*K/K1))*(U/(K2-K1))
C        CRITICAL LOCATION CAN NOT BE NONPOSITIVE
      IF(XC.LE.0.0)INDIC=INDIC+1
      IF(XC.LE.0.0)GOTO 500
C        DMAX = MAX. DEFICIT CONCENTRATION
      DMAX=(EXP(-K1*XC/U))*(LO*K1/K2)
C
C      MAXIMUM DO DEFICIT CAN NOT EXCEED SATURATION DO CONCENTRATION
C
      IF(DMAX.GE.DOS) DMAX=DOS
      IF(DMAX.GE.DOS) CALL NEWTON(K1,K2,U,LO,DO,DOS,DMAX,XC,YL,YR)
C     WRITE(7,45)DO,LO,K1,K2,U,DOS,XC,DMAX
C  45 FORMAT(//1X,'DO=',F5.2,3X,'LO=',F5.2,3X,
C    *'K1=',F6.3,3X,'K2=',F6.3,3X,'U=',F5.2,3X,'DOS=',F5.2,
C    */3X,'XC=',F7.3,' MILES',5X,'DMAX=',F7.3)
C        DSTD = DO DEFICIT TO STANDARD
C        DMBS = MAX. DEFICIT BEYAND STANDARD
      DSTD=DOS-DOSTD
      DMBS=DMAX-DSTD
      IF(DMBS.LE.0.)GOTO 480
      XL1=0.3*XC
      XR1=1.8*XC
      ICTR=0
      JCTR=0
      DO 100 ICTR=1,MAXITER
      A1=EXP(-K1*XL1/U)
      A2=EXP(-K2*XL1/U)
      DX=((DO-C*LO)*A2)+(C*LO*A1)-DSTD
      DDX=(((C*LO-DO)*K2/U)*A2)-(C*LO*K1*A1/U)
      XL2=XL1-(DX/DDX)
      IF(XL2.LT.0.0)GOTO 80
      XLCHCK=ABS(XL2-XL1)
      DLCHCK=ABS(DX)
      IF(XLCHCK.LE.0.001)GOTO 150
      IF(ICTR.GE.MAXITER)CALL LHELP(MAXITER)
      XL1=XL2
      GOTO 100
   80 XL1=XL1*0.5
  100 CONTINUE
  150 DO 200 JCTR=1,MAXITER
      A1=EXP(-K1*XR1/U)
      A2=EXP(-K2*XR1/U)
      DX=((DO-LO*C)*A2)+(C*LO*A1)-DSTD
      DDX=(((C*LO-DO)*K2/U)*A2)-(C*LO*K1*A1/U)
      XR2=XR1-(DX/DDX)
      IF(XR2.LE.XC)GOTO 180
      XRCHCK=ABS(XR2-XR1)
      DRCHCK=ABS(DX)
      IF(XRCHCK.LE.0.001)GOTO 250
      IF(JCTR.GE.MAXITER)CALL RHELP(MAXITER)
      XR1=XR2
      GOTO 200
```

```
  180 XR1=XR1*1.5
  200 CONTINUE
  250 CONTINUE
      XR=XR2
      XL=XL2
      IF(DMAX.GE.DOS) GO TO 260
C
C      COMPUTE AVERAGE DEFICIT CONCENTRATION BEYOND STANDARD WITHIN
C      THE REACH OF VIOLATION.
C
      XD=XR-XL
      T1=XL/U
      T2=XR/U
      ELK1=EXP(-K1*T1)
      ERK1=EXP(-K1*T2)
      ELK2=EXP(-K2*T1)
      ERK2=EXP(-K2*T2)
      E1ME2K1=ELK1-ERK1
      E1ME2K2=ELK2-ERK2
      AVGDEFC=(C*LO*U*(E1ME2K1/K1-E1ME2K2/K2)+DO*U*E1ME2K2/K2)/XD
      AVGDCBS=AVGDEFC-DSTD
      GO TO 264
  260 XD=XR-XL
      TXL=XL/U
      TYL=YL/U
      TXR=XR/U
      TYR=YR/U
      ELXK1=EXP(-K1*TXL)
      ELYK1=EXP(-K1*TYL)
      ELXK2=EXP(-K2*TXL)
      ELYK2=EXP(-K2*TYL)
      ELXMYK1=ELXK1-ELYK1
      ELXMYK2=ELXK2-ELYK2
      TERM1=C*LO*U*(ELXMYK1/K1-ELXMYK2/K2)+DO*U*ELXMYK2/K2
      TERM2=DOS*(YR-YL)
      ERXK1=EXP(-K1*TXR)
      ERYK1=EXP(-K1*TYR)
      ERXK2=EXP(-K2*TXR)
      ERYK2=EXP(-K2*TYR)
      ERYMXK1=ERYK1-ERXK1
      ERYMXK2=ERYK2-ERXK2
      TERM3=C*LO*U*(ERYMXK1/K1-ERYMXK2/K2)+DO*U*ERYMXK2/K2
      AVGDEFC=(TERM1+TERM2+TERM3)/XD
      AVGDCBS=AVGDEFC-DSTD
  264 CONTINUE
C     WRITE(7,265)DOSTD,DMBS
C 265 FORMAT(/1X,'THE MAXIMUM DEFICIT BEYOND STD. OF ',
C    *F5.2,' MG/L IS ',F5.2,' MG/L.')
C     WRITE(7,275)XD
C 275 FORMAT(/1X,'THE LENGTH OF VIOL. IS ',F8.2,' MILES.')
C     WRITE(7,285)T1
C 285 FORMAT(/1X,'THE VIOL. BEGINS AT TIME ',F5.2,' DAYS.')
C     WRITE(7,295)T2
C 295 FORMAT(/1X,'THE VIOL. ENDS AT TIME ',F5.2,' DAYS.')
C     WRITE(7,300) AVGDCBS
C 300 FORMAT(/1X,"AVERAGE DEFICIT CONCENTRATION BEYOND"/
C    X 1X,"STANDARD WITHIN REACH OF VIOLATION IS ",F5.2," MG/L")
C     WRITE(7,305)ICTR
C 305 FORMAT(/1X,'THE NO. OF ITERATIONS TO FIND XL IS ',I3)
C     WRITE(7,315)JCTR
C 315 FORMAT(/1X,'THE NO. OF ITERATIONS TO FIND XR IS ',I3)
      XLCHCK=XLCHCK*5280.0
      XRCHCK=XRCHCK*5280.0
C     WRITE(7,325)XLCHCK,XRCHCK
C 325 FORMAT(/5X,'THE VALUES OF XLCHCK=',F8.2,' AND XRCHCK=',
C    *F8.2,' FEET.')
C     WRITE(7,335)DLCHCK,DRCHCK
C 335 FORMAT(/5X,'THE VALUES OF DLCHCK=',F6.3,
```

```
C    *' AND DRCHCK=',F6.3)
     GOTO 500
  480 INDX=INDX+1
     DMBS=0.
     AVGDCBS=0.
C     WRITE(7,485)DOSTD,DMBS
C 485 FORMAT(/1X,'THE MAXIMUM DEFICIT BEYOND STD. OF ',
C    *F5.2,' MG/L IS ',F5.2,' MG/L.')
C     WRITE(7,490)XD
C 490 FORMAT(/1X,'THE LENGTH OF VIOL. IS ',F8.2,' MILES.')
  500 RETURN
     END
C
C
     SUBROUTINE LHELP(MAXITER)
C     WRITE(7,525)MAXITER
C 525 FORMAT(//5X,'***ITERATIONS TO IDENTIFY XL EXCEEDS ',I5)
     RETURN
     END
C
C
     SUBROUTINE RHELP(MAXITER)
C     WRITE(7,535)MAXITER
C 535 FORMAT(//5X,'***ITERATIONS TO IDENTIFY XR EXCEEDS ',I5)
     RETURN
     END
C
C     ***********************************************************************
C
C     SUBROUTINE CORRCAL IS WHERE THE CORRELATION COEFFICIENT BETWEEN
C     THE SIMULATED PAIRS OF TWO RANDOM SERIES.  NON-ZERO VIOLATION
C     DISTANCES AND THE CORRESPONDING MAX. DEFICIT AND AVERAGE DEFICIT
C     BEYOND STANDARD ARE CONSIDERED IN THE CALCULATION.
C
C     ***********************************************************************
C
     SUBROUTINE CORRCAL(X,Y,CORXY,LSET)
     DIMENSION X(999),Y(999)
     SUMX=0.0
     SUMY=0.0
     SUMXSQ=0.0
     SUMYSQ=0.0
     XSQR=0.0
     YSQR=0.0
     XY=0.0
     SUMXY=0.0
     NZERO=0
     DO 100 I=1,LSET
     IF(Y(I).EQ.0.) GO TO 90
     XSQR=X(I)*X(I)
     YSQR=Y(I)*Y(I)
     XY=X(I)*Y(I)
     SUMX=SUMX+X(I)
     SUMY=SUMY+Y(I)
     SUMXSQ=SUMXSQ+XSQR
     SUMYSQ=SUMYSQ+YSQR
     SUMXY=SUMXY+XY
     GO TO 100
   90 NZERO=NZERO+1
  100 CONTINUE
     NONZERO=LSET-NZERO
     COVXY=SUMXY-(SUMX*SUMY/NONZERO)
     STDX=(SUMXSQ-(SUMX*SUMX/NONZERO))**0.5
     STDY=(SUMYSQ-(SUMY*SUMY/NONZERO))**0.5
     CORXY=COVXY/(STDX*STDY)
     RETURN
     END
C     ***********************************************************************
```

```
C     SUBROUTINE NEWTON IS WHERE NEWTON-RAPHSON'S APPROXIMATION IS USED
C     TO DETERMINE THE LENGTH OF ZERO DO CONCENTRATION USING STREET-PHELP EQ.
C     ************************************************************************
C
      SUBROUTINE NEWTON(K1,K2,U,LO,DO,DOS,DMAX,XC,YL,YR)
C
C
      REAL K1,K2,LO,K,U
      IF(DMAX.EQ.DOS) GO TO 260
      MAXITER=20
      C=K1/(K2-K1)
      YL1=0.8*XC
      YR1=1.2*XC
      ICTR=0
      JCTR=0
      DO 100 ICTR=1,MAXITER
      A1=EXP(-K1*YL1/U)
      A2=EXP(-K2*YL1/U)
      DX=((DO-C*LO)*A2)+(C*LO*A1)-DOS
      DDX=(((C*LO-DO)*K2/U)*A2)-(C*LO*K1*A1/U)
      YL2=YL1-(DX/DDX)
      IF(YL2.LT.0.0)GOTO 80
      YLCHCK=ABS(YL2-YL1)
      DLCHCK=ABS(DX)
      IF(YLCHCK.LE.0.001)GOTO 150
      IF(ICTR.GE.MAXITER)CALL LHELP(MAXITER)
      YL1=YL2
      GOTO 100
   80 YL1=YL1*0.9
  100 CONTINUE
  150 DO 200 JCTR=1,MAXITER
      A1=EXP(-K1*YR1/U)
      A2=EXP(-K2*YR1/U)
      DX=((DO-LO*C)*A2)+(C*LO*A1)-DOS
      DDX=(((C*LO-DO)*K2/U)*A2)-(C*LO*K1*A1/U)
      YR2=YR1-(DX/DDX)
      IF(YR2.LE.XC)GOTO 180
      YRCHCK=ABS(YR2-YR1)
      DRCHCK=ABS(DX)
      IF(YRCHCK.LE.0.001)GOTO 250
      IF(JCTR.GE.MAXITER)CALL RHELP(MAXITER)
      YR1=YR2
      GOTO 200
  180 YR1=YR1*1.1
  200 CONTINUE
  250 YL=YL2
      YR=YR2
      GO TO 300
  260 YL=XC
      YR=XC
  300 CONTINUE
      RETURN
      END
```

APPENDIX H

LISTING OF COMPUTER PROGRAM FOR

ESTIMATING THE PROBABILITY DISTRIBUTION

OF THE DISSOLVED OXYGEN DEFICIT AT ANY LOCATION

WITHIN A SINGLE REACH

'DISTRDX'

```
      PROGRAM DISTRDX(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7)
C ***************************************************************
C THE MAIN OBJECTIVE OF THIS PROGRAM IS TO DETERMINE THE DISTRIBUTION
C OF THE STREETER-PHELPS DISSOLVED OXYGEN EQUATION AT A GIVEN
C LOCATION, X.
C
C SEVERAL DISTRIBUTIONS ARE ASSUMED FOR THE CRITICAL LOCATION,
C DX: (1)NORMAL; (2)LOG-NORMAL; (3)GAMMA; (4)WEIBUL.  IN ADDITION,
C FISHER-CORNISH TECHNIQUE IS EMPLOYED TO ALLOW FOR A FLEXIBLE FIT
C TO THE DISTRIBUTION OF DX.  THE BEST FIT IS THEN DETERMINED BY
C THE METHODS OF MEAN ABSOLUTE ERROR (MAE) AND MEAN STANDARD ERROR (MSE).
C EACH OF THE TECNIQUES UTILIZED IN THIS PROGRAM ARE GENERALLY
C OUTLINED IN THEIR RESPECTIVE SUBROUTINES.
C FOR MORE DETAILS ABOUT EACH SUBROUTINE, SEE THE COMMENTS PRIOR
C TO THE LISTING OF EACH ROUTINE IN THE PROGRAM BELOW.
C
C ***************************************************************
C
C TO EXECUTE THIS PROGRAM, ONE INPUT FILE MUST BE PROVIDED. THIS
C INPUT FILE CONSISTS OF FIVE (5) MAJOR RECORDS:
C
C       RECORD            VARIABLES              FORMAT
C
C       1              K1,K2,U,LO,DO,X           10F8.0
C
C       2            STDK1,STDK2,STDU            10F8.0
C                    STDLO,STDDO,CORRK2U
C
C       3                 IDIST(I)               10I8
C
C       4            LSET,NPROB,NGRP,DSEED       3I8,D12.1
C
C       5              PROB(I),I=1,NPROB         10F8.0
C
C
C
C                         DESCRIPTION
C
C   K1 = MEAN VALUE OF THE DEOXYGENATION COEFFICIENT, 1/DAYS
C   K2 = MEAN VALUE OF THE REAERATION COEFFICIENT, 1/DAYS
C    U = MEAN VALUE OF THE STREAM VELOCITY, MILES/DAY
C   LO = MEAN VALUE OF THE INITIAL IN STREAM BOD CONCENTRATION, MG/L
C   DO = MEAN VALUE OF THE INITIAL IN STREAM DO DEFICIT CONC., MG/L
C
C   X = THE DOWNSTREAM LOCATION (IN MILES) AT WHICH THE PROFILE
C       OF THE DISTRIBUTION OF THE DEFICIT IS DETERMINED
C
C   STDK1
C   STDK2
C   STDU  = STANDARD DEVIATIONS OF K1,K2,U,LO,DO
C   STDLO
C   STDDO
C
C   CORRK2U = CORRELATION COEFFICIENT BETWEEN K2 AND U
C
C  IDIST(I),I=1,5    FIVE ELEMENT ARRAY CONTAINING AN INTEGER
C                    CODE VALUE TO INDICATE THE TYPE OF DISTRIBUTION
C                    TO BE ASSUMED FOR K1,K2,U,LO,DO, RESPECTIVELY:
C
C                                IDIST(I)              DISTR. TYPE
C
C                                    1                  NORMAL
C                                    2                  LOG-NORMAL
C                                    3                  GAMMA
C                                    4                  WEIBUL
C
C   LSET = NUMBER OF VILOATION PAIRS (DO DEFICITS AND LENGTH OF VILOATION)
C          TO BE GENERATED BY THE SIMULATION ROUTINE SIMULDX
```

```
C
C   NPROB = NUMBER OF PROBABILITY QUANTILES TO BE USED TO DETERMINE THE
C                   PROFILE OF THE DISTRIBUTION OF DX
C
C   NGRP = NUMBER OF GROUPS OF DATA SETS (EACH LSET IN LENGTH)
C          TO BE GENERATED
C
C   DSEED = SEED INPUT VALUE FOR THE RANDOM NUMBER GENERATOR
C
C   PROB(I),I=1,NPROB          NPROB ELEMENT ARRAY CONTIANING THE QUANTILES
C                              USED IN DETERMINING THE PROFILE OF THE
C                              DISTRIBUTION OF DX
C
C   AN EXAMPLE OF A COMPLETE INPUT FOLLOWS:
C
C
C    0.35    0.70    10.0    18.0     1.0    10.0
C     0.1    0.20    3.00    5.00    0.30     0.0
C       1       1       1       1       1
C     999       9       2  764857.0D1
C    0.10   0.20    0.30    0.40    0.50    0.60    0.80    0.90    0.99
C
C*********************************************************************
C
      DIMENSION PROB(30),WK(3),PARAMTR(5),SIMDX(1000),PARMEAN(5)
      DIMENSION SKEW(5),PARSTD(5),SIGMA(3),RVEC(1,2),AVDX(30)
      DIMENSION WKVEC(1),X1(30),X2(30),X3(30),X4(30),X5(30)
      INTEGER MODEL,IDIST(5)
      DOUBLE PRECISION DSEED
      REAL K1,K2,U,LO,DO,KURTK1,KURTK2,KURTU,KURTLO,KURTDO,KURT(5)
      REAL KURTDX
C
      READ(5,10)K1,K2,U,LO,DO,X
      READ(5,10)STDK1,STDK2,STDU,STDLO,STDDO,CORRK2U
      READ(5,9)(IDIST(I),I=1,5)
      READ(5,11)LSET,NPROB,NGRP,DSEED
      READ(5,10)(PROB(I),I=1,NPROB)
    9 FORMAT(10I8)
   10 FORMAT(10F8.0)
   11 FORMAT(3I8,D12.1)
C
C     WRITE OUTPUT HEADING
C
      WRITE(6,15)
   15 FORMAT(/3X,'OUTPUT FROM THE EXECUTION OF DISTRDX FOLLOWS: '/)
      VARK1=STDK1*STDK1
      VARK2=STDK2*STDK2
      VARU=STDU*STDU
      VARLO=STDLO*STDLO
      VARDO=STDDO*STDDO
      COVK2U=CORRK2U*STDK2*STDU
C
      CALL M3M4PAR(K1,K2,U,LO,DO,STDK1,STDK2,STDU,STDLO,STDDO,
     1 IDIST,SKEW,KURT,PARMEAN,PARSTD)
C
      SKEWK1=SKEW(1)
      SKEWK2=SKEW(2)
      SKEWU=SKEW(3)
      SKEWLO=SKEW(4)
      SKEWDO=SKEW(5)
C
      KURTK1=KURT(1)
      KURTK2=KURT(2)
      KURTU=KURT(3)
      KURTLO=KURT(4)
      KURTDO=KURT(5)
C
      CALL SIMULDX(LSET,CORRK2U,DSEED,PARMEAN,PARSTD,IDIST,X,NGRP,
```

```
     X NPROB,PROB,SKEW,KURT,AVDX)
C
      CALL EXPECDX(X,EDX,VARK1,VARK2,VARU,VARLO,VARDO,
     X K1,K2,U,LO,DO,COVK2U)
C
      CALL VARDX(X,VDX,VARK1,VARK2,VARU,VARLO,VARDO,K1,K2,U,LO,DO,
     X COVK2U)
C
      CALL M3M4DX(X,VDX,K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,
     1 VARDO,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,KURTK1,KURTK2,
     2 KURTU,KURTLO,KURTDO,SKEWDX,KURTDX)
C
      STDDX=SQRT(VDX)
      CVDX=STDDX/EDX
C
      WRITE(6,18)
   18 FORMAT(//3X,"APPROX. STATISTICS FROM THE FIRST-ORDER ANALYSIS")
      WRITE(6,20) EDX,STDDX,VDX,SKEWDX,KURTDX
   20 FORMAT(6X,"MEAN OF DX = ",F8.3," MG/L"/6X,"STD  OF DX = ",
     X F8.3," MG/L"/6X,"VAR  OF DX = ",F8.3/
     X 6X,"SKEW OF DX = ",F8.3/6X,"KURT OF DX = ",F8.3)
      DO 120 I=1,NPROB
C
C     NORMAL DISTRIBUTION
C
      P=PROB(I)
      CALL MDNRIS(P,Z1,IER)
      X1(I)=Z1*STDDX+EDX
C
C     LOG-NORMAL DISTRIBUTION
C
      YBAR=0.5*ALOG(EDX*EDX/(CVDX*CVDX+1.))
      YVAR=ALOG(CVDX*CVDX+1.)
      X2LOG=Z1*SQRT(YVAR)+YBAR
      X2(I)=EXP(X2LOG)
C
C     GAMMA DISTRIBUTION
C
      DXTHETA=1./(CVDX*STDDX)
      DXALPHA=EDX*DXTHETA
      IT=0
      GP1=X2(I)*DXTHETA
  104 IT=IT+1
      IF(IT.GT.40)GOTO 106
      CALL MDGAM(GP1,DXALPHA,PG,IER)
      DIF=PG-PROB(I)
      TOLER=PROB(I)/100.
      IF(ABS(DIF).LE.TOLER)GOTO 106
      R=PROB(I)/PG
      IF(PROB(I).LE.0.5)GP2=GP1*R**0.25
      IF(PROB(I).GT.0.5)GP2=GP1*R**1.25
      GP1=(GP1+GP2)/2.
      GOTO 104
  106 X3(I)=GP1/DXTHETA
C
C     WEIBULL DISTRIBUTION
C
      CALL SEARCH2(EDX,CVDX,ADX,BDX,IT)
      ADXINV=1./ADX
      ALP=-ALOG(1.-P)
      X4(I)=BDX*ALP**ADXINV
C
C     USE OF FISHER-CORNISH ROUTINE
C
      CALL FISCORN(Z1,EDX,STDDX,SKEWDX,KURTDX,X5FC)
      X5(I)=X5FC
  120 CONTINUE
C
```

```
      CALL ERROR1(NPROB,PROB,X1,X2,X3,X4,X5,AVDX)
      STOP
      END
C
C     ****************************************************************
C     FISHER-CORNISH ASYMPTOTIC EXPANSION USING BOTH SKEWNNESS AND
C     KURTOSIS
C     ****************************************************************
C
      SUBROUTINE FISCORN(ZP,MEAN,STD,SKEW,KURT,WP)
      REAL MEAN,KURT,ZP,WP,XP
C
C     COMPUTE THE HERMITE POLYNOMIALS
C
      CALL HERMIT(ZP,H0,H1,H2,H3,H4,H5,H6,H7)
C
      G1=SKEW
      G2=KURT-3.
C
      XP=ZP+G1*H2/6. + G2*H3/24. -G1*G1*(2.*H3+H1)/36.
C     *   - G1*G2*(H4+H2)/24. + G1**3*(12.*H4+19.*H2)/324.
C     *   - G2*G2*(3.*H5+6.*H3+2.*H1)/384.
C     *   + G1**2*G2*(14.*H5+37.*H3+8.*H1)/288.
C     *   - G1**4*(252.*H5+832.*H3+227.*H1)/7776.
C     *   + G1*G2**2*(12.*H6+51.*H4+37.*H2)/576.
C     *   - G1**3*G2*(80.*H6+397.*H4+295.*H2)/1296.
C     *   + G1**5*(960.*H6+5463.*H4+4640.*H2)/29160.
C     *   + G2**3*(9.*H7+58.*H5+86.*H3+12.*H1)/3072.
C     *   - G1**2*G2**2*(594.*H7+4281.*H5+6446.*H3+1126.*H1)/13824.
C     *   + G1**4*G2*(5148.*H7+41104.*H5+65759.*H3+11740.*H1)/62208.
C     *   - G1**6*(154440.*H7+1355556.*H5+2373074.*H3+459445.*H1)/4199040.
C
      WP=MEAN+STD*XP
C
      RETURN
      END
C
C
C     ****************************************************************
C     SUBROUTINE M3M4PAR IS TO CALCULATE THE 3RD AND 4TH MOMENTS
C     OF THE MODEL PARAMETERS
C     ****************************************************************
C
      SUBROUTINE M3M4PAR(K1,K2,U,LO,DO,STDK1,STDK2,STDU,STDLO,STDDO,
     1 IDIST,SKEW,KURT,PARMEAN,PARSTD)
      REAL K1,K2,LO,KURT(5)
      INTEGER IDIST(5)
      DIMENSION SKEW(5),PARMEAN(5),PARSTD(5)
      PARMEAN(1)=K1
      PARMEAN(2)=K2
      PARMEAN(3)=U
      PARMEAN(4)=LO
      PARMEAN(5)=DO
      PARSTD(1)=STDK1
      PARSTD(2)=STDK2
      PARSTD(3)=STDU
      PARSTD(4)=STDLO
      PARSTD(5)=STDDO
      DO 100 I=1,5
      XMEAN=PARMEAN(I)
      XSTD=PARSTD(I)
      XVAR=XSTD*XSTD
      GOTO(10,20,30,40),IDIST(I)
C
C     NORMAL DISTRIBUTION
C
   10 SKEW(I)=0.0
      KURT(I)=3.0
```

```
      GOTO 100
C
C     LOG-NORMAL DISTRIBUTION
C
   20 DXV=XSTD/XMEAN
      OMEGA=1.+DXV**2
      OM1=OMEGA-1.
      SKEW(I)=(OMEGA+2.)*SQRT(OM1)
      KURT(I)=OMEGA**4+2.*OMEGA**3+3.*OMEGA**2-3.
      GOTO 100
C
C      GAMMA DISTRIBUTION
C
   30 ALPHA=(XMEAN/XSTD)**2
      SKEW(I)=2./SQRT(ALPHA)
      KURT(I)=6./ALPHA+3.
      GOTO 100
C
C     WEIBULL DISTRIBUTION
C
   40 DXV=XSTD/XMEAN
      CALL SEARCH2(XMEAN,DXV,ALPHA,BETA,IT)
      AINV=1./ALPHA
      XMU20=XVAR+XMEAN**2
      A3=1.+3.*AINV
      A4=1.+4.*AINV
      XMU30=BETA**3*GAMMA(A3)
      XMU3=XMU30-3.*XMEAN*XMU20+2.*XMEAN**3
      SKEW(I)=XMU3/(XSTD**3)
      XMU40=BETA**4*GAMMA(A4)
      XMU4=XMU40-4.*XMEAN*XMU30+6.*XMEAN**2*XMU20-3.*XMEAN**4
      KURT(I)=XMU4/(XSTD**4)
  100 CONTINUE
      RETURN
      END
C     ******************************************************************
C     SUBROUTINE HERMIT IS WHERE SIX ORDERS OF THE HERMIT FUNCTION
C     ARE  CALCULATED.
C     ******************************************************************
C
      SUBROUTINE HERMIT(X,H0,H1,H2,H3,H4,H5,H6,H7)
C
      H0=1.0
      H1=X
      H2=(X**2)-1.0
      H3=(X**3)-(3.*X)
      H4=(X**4)-6.*(X**2)+3.0
      H5=(X**5)-10.*(X**3)+(15.*X)
      H6=(X**6)-15.*(X**4)+45.*(X**2)-15.0
      H7=(X**7)-21.*(X**5)+105.*X**3-105.*X
      RETURN
      END
C
C
C     ******************************************************************
C     SUBROUTINE EXPECDX IS WHERE THE EXPECTAION OF THE DEFICIT
C     FUNCTION, DX; IS CALCULATED.
C     ******************************************************************
C
      SUBROUTINE EXPECDX(X,EDX,VARK1,VARK2,VARU,VARLO,VARDO,
     1 K1,K2,U,LO,DO,COVK2U)
C
      REAL K1,K2,LO
C
      CALL CONST2(X,K1,K2,U,LO,DO,CK1P,CK2P,BK2P,CUP,BUP,CK2UP,BK2UP)
C
      E1=EXP(-K1*X/U)
      E2=EXP(-K2*X/U)
```

```
      DBAR=K1*LO/(K2-K1)*(E1-E2) + DO*E2
C
C     COMPUTE A FIRST ORDER APPROXIMATION OF THE EXPECTATION OF DX
C     UTILIZING TAYLOR'S SERIES EXPANSION OF DX (ASSUMING PARAMETERS
C     K2 AND U ARE DEPENDENT)
C
      EDX=DBAR+ 0.5*LO*CK1P*VARK1 + 0.5*(LO*CK2P+BK2P)*VARK2
     1 + 0.5*(LO*CUP+BUP)*VARU + (LO*CK2UP-BK2UP)*COVK2U
C
      RETURN
      END
C
C
C     ****************************************************************
C     SUBROUTINE VARDX IS WHERE THE VARIANCE OF THE DEFICIT FUNCTION,
C     DX, IS CALCULATED.
C     ****************************************************************
C
      SUBROUTINE VARDX(X,VDX,VARK1,VARK2,VARU,VARLO,VARDO,
     X K1,K2,U,LO,DO,COVK2U)
C
      REAL K1,K2,LO
      CALL CONST1(X,K1,K2,U,LO,DO,CK1,CK2,BK2,CU,BU,CLO,CDO)
C
C     COMPUTE A FIRST ORDER APPROXIMATION OF THE VARIANCE OF DX
C     UTILIZING TAYLOR'S SERIES EXPANSION OF DX
C
      VDX=(CK1**2)*(LO**2)*VARK1 + (LO*CK2-BK2)**2*VARK2
     1 + (LO*CU+BU)**2*VARU + (CDO**2)*VARDO
     2 + (CLO**2)*VARLO + 2.*(LO*CK2-BK2)*(LO*CU+BU)*COVK2U
      RETURN
      END
C
C
C     ****************************************************************
C     SUBROUTINE CONST1 IS WHERE SEVERAL CONSTANTS ARE CALCULATED WHICH ARE
C     ASSOCAITED WITH THE PARTIAL DERIVATIVES OF THE DEFICIT FUNCTION,
C     DX, WITH RESPECT TO EACH PARAMETER K1,K2,U,LO,DO.
C     ****************************************************************
C
      SUBROUTINE CONST1(X,K1,K2,U,LO,DO,CK1,CK2,BK2,CU,BU,CLO,CDO)
C
      REAL K1,K2,LO
      E1=EXP(-K1*X/U)
      E2=EXP(-K2*X/U)
      CK1=K2/(K2-K1)**2 * (E1-E2) - K1/(K2-K1)*(X/U)*E1
      CK2= -K1/(K2-K1)**2 *(E1-E2) + K1/(K2-K1)*(X/U)*E2
      BK2=DO*(X/U)*E2
      CU=K1**2/(K2-K1)*(X/U**2)*E1 - K1*K2/(K2-K1)*(X/U**2)*E2
      BU=DO*K2*X/(U**2)*E2
      CDO=E2
      CLO=K1/(K2-K1)*(E1-E2)
      RETURN
      END
C
C
C     ****************************************************************
C     SUBROUTINE CONST2 IS WHERE SEVERAL CONSTANTS ARE CALCULATED
C     WHICH ARE ASSOCIATED WITH THE SECOND PARTIAL DERIVATIVES OF THE
C     DEFICIT FUNCTION, DX, WITH RESPECT TO EACH OF THE VARIABLES K1,
C     K2,U,LO,DO.
C     ****************************************************************
C
      SUBROUTINE CONST2(X,K1,K2,U,LO,DO,CK1P,CK2P,BK2P,CUP,BUP,
     1 CK2UP,BK2UP)
      REAL K1,K2,LO
      E1=EXP(-K1*X/U)
      E2=EXP(-K2*X/U)
```

```
C
      CK1P=2*K2/((K2-K1)**3)*(E1-E2) - 2*K2/((K2-K1)**2)*(X/U)*E1
     1 - K1/(K2-K1)*((X/U)**2)*E1
C
      CK2P=2*K1/((K2-K1)**3)*(E1-E2) - 2*K1/((K2-K1)**2)*(X/U)*E2
     1 - K1/(K2-K1)*((X/U)**2)*E2
C
      BK2P=DO*((X/U)**2)*E2
C
      CUP=(2*(K1**2)/(K2-K1)*X/(U**3) - K1**3/(K2-K1)*(X**2/U**4))*E1
     1 +(2*K1*K2/(K2-K1)*(X/U**3) - K1*(K2**2)/(K2-K1)*(X**2/U**4))*E2
C
      BUP=(K2*X - 2*U)*K2*X*DO*E2/U**4
C
      CK2UP=K1**2/((K2-K1)**2)*(-X/U**2)*E1
     1 + (K1*K2/((K2-K1)**2)*(X/U**2) - K1/(K2-K1)*(X/U**2)
     2 + K1*K2/(K2-K1)*(X**2/U**3))*E2
C
      BK2UP=(K2*X - U)*DO*X*E2/U**3
      RETURN
      END
C
CC
CC    ********************************************************************
C     SUBROUTINE M3M4DX IS WHERE THE 3 RD AND 4 TH MOMENTS (SKEW AND
C     KURTOSIS) OF THE DEFICIT FUNCTION, DX, IS CALCULATED.
CC    ********************************************************************
C
      SUBROUTINE M3M4DX(X,VDX,K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,
     1 VARDO,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,KURTK1,KURTK2,
     2 KURTU,KURTLO,KURTDO,SKEWDX,KURTDX)
C
      REAL K1,K2,LO,KURTDX,KURTK1,KURTK2,KURTU,KURTLO,KURTDO
      CALL CONST1(X,K1,K2,U,LO,DO,CK1,CK2,BK2,CU,BU,CLO,CDO)
C
C     COMPUTE A FIRST ORDER APPROXIMATION OF THE SKEW OF DX UTILIZING
C     TAYLOR'S SERIES EXPANSION
C
      SKEWDX=((LO*CK1)**3*SKEWK1*(VARK1**1.5) + (LO*CK2-BK2)**3
     1 *SKEWK2*(VARK2**1.5) + (LO*CU+BU)**3*SKEWU*(VARU**1.5)
     2 + CDO**3*SKEWDO*(VARDO**1.5) + CLO**3*SKEWLO*(VARLO**1.5))
     3 /VDX**1.5
C
C     COMPUTE A FIRST ORDER APPROXIMATION OF THE KURTOSIS OF DX
C     UTILIZING TAYLOR'S SERIES EXPANSION
C
      PART1=(LO*CK1)**4*KURTK1*(VARK1**2) + (LO*CK2-BK2)**4
     1 *KURTK2*(VARK2**2) + (LO*CU+BU)**4*KURTU*(VARU**2)
     2 + CDO**4*KURTDO*(VARDO**2) + CLO**4*KURTLO*(VARLO**2)
     3 + 6*(LO*CK1)**2*(LO*CK2-BK2)**2*VARK1*VARK2
     4 + 6*(LO*CK1)**2*(LO*CU+BU)**2*VARK1*VARU
     5 + 6*(LO*CK1)**2*CDO**2*VARK1*VARDO
     6 + 6*(LO*CK2-BK2)**2*CDO**2*VARK2*VARDO
     7 + 6*(LO*CK2-BK2)**2*(LO*CU+BU)**2*VARK2*VARU
     8 + 6*(LO*CU+BU)**2*CDO**2*VARU*VARDO
     9 + 6*(LO*CK1)**2*CLO**2*VARK1*VARLO
C
      PART2=6*(LO*CK2-BK2)**2*CLO**2*VARK2*VARLO
     1 + 6*(LO*CU+BU)**2*CLO**2*VARU*VARLO
     2 +6*(CDO**2)*(CLO**2)*VARDO*VARLO
C
      KURTDX=(PART1+PART2)/VDX**2
      RETURN
      END
C
CC
CC    ********************************************************************
C
```

```
C     SUBROUTINE SIMULDX IS THE SIMULATION PORTION OF THE PROGRAM WHERE
C     AN (LSET) NUMBER OF DO DEFICIT CONCENTRATIONS ARE GENERATED
C
C     ***********************************************************************
C
      SUBROUTINE SIMULDX(LSET,CORR,DSEED,PARMEAN,PARSTD,IDIST,X,
     1 NGRP,NPROB,PROB,PARSKEW,PARKURT,AVDX)
      DOUBLE PRECISION DSEED
      INTEGER IDIST(5)
      REAL K1,K2,U,LO,DO
      DIMENSION R(1),WK(3),PARAMTR(5),SIMDX(1000),PARMEAN(5)
      DIMENSION PARSTD(5),DX(10,30),AVDX(30),PROB(30),AWIB(5),BWIB(5)
      DIMENSION SIGMA(3),RVEC(1,2),WKVEC(1),AGAM(5),BGAM(5)
      DIMENSION PARSKEW(5),PARKURT(5),SIMDX1(1000),SLN(5),XMLN(5)
      DIMENSION DXMEAN(10),DXSTD(10),DXSKEW(10),DXKURT(10)
      INDX=0
      RCOUNT=0.0
      DO 200 IGRP=1,NGRP
      ISET=0
   10 ISET=ISET+1
      IF(ISET.GT.LSET)GOTO 150
      DO 100 NP=1,5
      MODEL=IDIST(NP)
      XMEAN=PARMEAN(NP)
      XSTD=PARSTD(NP)
      XSKEW=PARSKEW(NP)
      XKURT=PARKURT(NP)
      IF(IGRP.GT.1) GO TO 28
      IF(ISET.GT. 1) GO TO 28
      IF(NP.EQ.1)WRITE(6,22)MODEL,XMEAN,XSTD,XSKEW,XKURT
      IF(NP.EQ.2)WRITE(6,23)MODEL,XMEAN,XSTD,XSKEW,XKURT
      IF(NP.EQ.3)WRITE(6,24)MODEL,XMEAN,XSTD,XSKEW,XKURT
      IF(NP.EQ.4)WRITE(6,25)MODEL,XMEAN,XSTD,XSKEW,XKURT
      IF(NP.EQ.5)WRITE(6,26)MODEL,XMEAN,XSTD,XSKEW,XKURT
   22 FORMAT(/3X,"K1",5X,I8,4(3X,E12.5))
   23 FORMAT(/3X,"K2",5X,I8,4(3X,E12.5))
   24 FORMAT(/3X,"U",6X,I8,4(3X,E12.5))
   25 FORMAT(/3X,"LO",5X,I8,4(3X,E12.5))
   26 FORMAT(/3X,"DO",5X,I8,4(3X,E12.5))
   28 GO TO(30,40,50,60),MODEL
C
C      NORMAL DISTRIBUTION
C
   30 IF(NP.EQ.2 .AND. CORR.NE.0.0)GOTO 35
      IF(NP.EQ.3 .AND. CORR.NE.0.0)GOTO 85
      CALL GGNML(DSEED,1,R)
      PARAMTR(NP)=XMEAN+XSTD*R(1)
      GO TO 90
C
C       STORE THE MEAN AND STD. DEV. OF K2 WHEN CORRELATION BETWEEN K2 AND
C        VELOCITY IS NONZERO.
C
   35 STDK2=XSTD
      XMEANK2=XMEAN
      GOTO 90
C
C       LOG-NORMAL DISTRIBUTION
C
C
C        COMPUTE THE MEAN (XM) AND STD. DEV. (S) OF LOG-TRANSFORMED VARIABLES.
C
   40 IF(ISET.NE.1 .OR. IGRP.NE.1) GO TO 43
      CV=XSTD/XMEAN
      S2=ALOG(CV*CV+1.)
      SLN(NP)=SQRT(S2)
      A=XMEAN*XMEAN/(CV*CV+1.)
      XMLN(NP)=ALOG(A)/2.
   43 IF(NP.EQ.2 .AND. CORR.NE.0.0)GOTO 45
```

```
      IF(NP.EQ.3 .AND. CORR.NE.0.0)GOTO 85
C     CALL GGNLG(DSEED,1,XM,S,R)
C     PARAMTR(NP)=R(1)
      CALL GGNML(DSEED,1,R)
      Y=XMLN(NP)+R(1)*SLN(NP)
      PARAMTR(NP)=EXP(Y)
      GO TO 90
C
C       STORE THE MEAN AND STD. DEV. OF LOG-TRANSFORMED K2 WHEN CORRELATION
C        BETWEEN K2 AND VELOCITY IS NONZERO.
C
   45 YSTDK2=SLN(NP)
      YMEANK2=XMLN(NP)
      CVK2=CV
      GOTO 90
C
C      GAMMA DISTRIBUTION
C
   50 IF(ISET.NE.1 .OR. IGRP.NE.1) GO TO 55
      XVAR=XSTD*XSTD
      BGAM(NP)=XVAR/XMEAN
      AGAM(NP)=XMEAN/BGAM(NP)
   55 A=AGAM(NP)
      B=BGAM(NP)
      WK(1)=0.
      CALL GGAMR(DSEED,A,1,WK,R)
      PARAMTR(NP)=R(1)*BGAM(NP)
      GO TO 90
C
C      WEIBULL DISTRIBUTION
C
   60 IF(ISET.NE.1 .OR. IGRP.NE.1) GO TO 70
      CV=XSTD/XMEAN
      CALL SEARCH2(XMEAN,CV,AA,BB,IT)
      AWIB(NP)=AA
      BWIB(NP)=BB
   70 AA=AWIB(NP)
      BB=BWIB(NP)
      CALL GGWIB(DSEED,AA,1,R)
      PARAMTR(NP)=BB*R(1)
      GO TO 90
   85 IF(MODEL.EQ.1) GO TO 87
C        COMPUTE THE CORRELATION COEFFICIENT FOR LOG-TRANSFORMED K2 AND
C         VELOCITY FROM THE CORRELATION CCOEFFICIENT BETWEEN K2 AND VELOCITY
C        OF ORIGINAL SCALE.
C
      CVU=CV
      YSTDU=SLN(NP)
      YMEANU=XMLN(NP)
      F1=ALOG(1.+CORR*CVK2*CVU)
      F2=ALOG(1.+CVK2*CVK2)
      F3=ALOG(1.+CVU*CVU)
      F2=SQRT(F2)
      F3=SQRT(F3)
      YCORR=F1/(F2*F3)
      SIGMA(2)=YCORR*YSTDK2*YSTDU
      SIGMA(1)=YSTDK2*YSTDK2
      SIGMA(3)=YSTDU*YSTDU
      GO TO 88
   87 STDU=XSTD
      XMEANU=XMEAN
      SIGMA(2)=CORR*STDK2*STDU
      SIGMA(1)=STDK2*STDK2
      SIGMA(3)=STDU*STDU
   88 NR=1
      KPAR=2
      IR=1
      WKVEC(1)=0.0
```

```
      CALL GGNSM(DSEED,NR,KPAR,SIGMA,IR,RVEC,WKVEC,IER)
      IF(MODEL.EQ.1) GO TO 89
C
C        FIND BIVARIATE NORMAL RANDOM DEVIATES OF LOG-TRANSFORMED K2 AND
C        VELOCITY.  THEN RETRANSFORM THEM BACK TO THE ORIGINAL SCALE.
C
      YK2=RVEC(1,1)+YMEANK2
      YU=RVEC(1,2)+YMEANU
      PARAMTR(NP-1)=EXP(YK2)
      PARAMTR(NP)=EXP(YU)
      GO TO 90
C
C        OBTAIN BIAVARIATE NORMAL RANDOM DEVIATES FOR K2 AND VELOCITY.
C
   89 PARAMTR(NP-1)=RVEC(1,1)+XMEANK2
      PARAMTR(NP)=RVEC(1,2)+XMEANU
   90 CONTINUE
  100 CONTINUE
C
C       CHECK TO EDXLUDE PARAMETER SET WITH NONPOSITIVE VALUES
C
      NEG=0
      DO 110 I=1,5
      IF(PARAMTR(I).LT.0.)NEG=NEG+1
  110 CONTINUE
      IF(NEG.GT.0)ISET=ISET-1
      IF(NEG.GT.0)GOTO 140
      K1=PARAMTR(1)
      K2=PARAMTR(2)
      U=PARAMTR(3)
      LO=PARAMTR(4)
      DO=PARAMTR(5)
      A1=EXP(-K1*X/U)
      A2=EXP(-K2*X/U)
      C=K1/(K2-K1)
      SIMDX(ISET)=((DO-C*LO)*A2)+(C*LO*A1)
C     WRITE(7,130) ISET,SIMDX(ISET)
C 130 FORMAT(1X,I4,3X,F5.1)
  140 GOTO 10
  150 CONTINUE
C
      CALL STAT(LSET,IGRP,SIMDX,DXMEAN,DXSTD,DXSKEW,DXKURT)
C
      CALL RANK(SIMDX,LSET)
      DO 160 I=1,LSET
      N=LSET-I+1
      SIMDX1(N)=SIMDX(I)
  160 CONTINUE
      DO 170 IP=1,NPROB
      IPNT=PROB(IP)*(LSET+1)
      DX(IGRP,IP)=SIMDX1(IPNT)
  170 CONTINUE
  200 CONTINUE
      DO 220 IP=1,NPROB
      SUMDX=0.
      DO 210 IG=1,NGRP
      SUMDX=SUMDX+DX(IG,IP)
  210 CONTINUE
      AVDX(IP)=SUMDX/NGRP
  220 CONTINUE
      WRITE(6,230) CORR
  230 FORMAT(/3X,"CORRELATION COEFF. BETWEEN K2 AND U IS",2X,F6.3)
      WRITE(6,235) NGRP
  235 FORMAT(/3X,"NO. OF GROUPS OF RANDOM NUMBER GENERATED IS",2X,I5)
      WRITE(6,240) X
  240 FORMAT(//3X,"LOCATION EXAMINED IS ",F7.2," MILES DOWNSTREAM FROM
     XTHE DISCHARGE POINT")
      YMN=0.
```

```
      YSTD=0.
      YSKW=0.
      YKUT=0.
      DO 250 IG=1,NGRP
      YMN=YMN+DXMEAN(IG)
      YSTD=YSTD+DXSTD(IG)
      YSKW=YSKW+DXSKEW(IG)
      YKUT=YKUT+DXKURT(IG)
  250 CONTINUE
      AVGMN=YMN/NGRP
      AVGSTD=YSTD/NGRP
      AVGSKW=YSKW/NGRP
      AVGKUT=YKUT/NGRP
      WRITE(6,260) AVGMN,AVGSTD,AVGSKW,AVGKUT
  260 FORMAT(//3X,"SAMPLE STATISTICS FROM SIMULATED OXYGEN DEFICIT :"/
     1 6X,"MEAN =",F8.3/6X,"STD  =",F8.3/6X,"SKEW =",F8.3/
     2 6X,"KURT =",F8.3)
      RETURN
      END
C
C ****************************************************************
C SUBROUTINE ERROR1 IS WHERE THE MEAN ABSOLUTE AND STANDARD
C ERRORS ARE CALCULATED.
C ****************************************************************
C
      SUBROUTINE ERROR1(NPROB,PROB,X1,X2,X3,X4,X5,SIMUL)
      DIMENSION PROB(30),QUANTL(30,5),SIMUL(30),ERROR(30,5),
     C DELTAP(30),ABSERR(30,5),SQRERR(30,5),E1(5),E2(5),E3(5),
     C AVGERR1(30,5),AVGERR2(30,5),AVGERR3(30,5),
     C X1(30),X2(30),X3(30),X4(30),X5(30)
      DO 10 I=1,NPROB
      QUANTL(I,1)=X1(I)
      QUANTL(I,2)=X2(I)
      QUANTL(I,3)=X3(I)
      QUANTL(I,4)=X4(I)
      QUANTL(I,5)=X5(I)
   10 CONTINUE
      DO 30 I=1,NPROB
      DO 25 J=1,5
      ERROR(I,J)=QUANTL(I,J)-SIMUL(I)
      ABSERR(I,J)=ABS(ERROR(I,J))
      SQRERR(I,J)=ERROR(I,J)**2
   25 CONTINUE
   30 CONTINUE
      DO 35 I=2,NPROB
      DELTAP(I)=PROB(I)-PROB(I-1)
   35 CONTINUE
      NPROBP1=NPROB+1
      DELTAP(1)=PROB(1)
      DELTAP(NPROBP1)=1.-PROB(NPROB)
      DO 50 J=1,5
      DO 40 I=2,NPROB
      AVGERR1(I,J)=(ERROR(I,J)+ERROR(I-1,J))/2.
      AVGERR2(I,J)=(ABSERR(I,J)+ABSERR(I-1,J))/2.
      AVGERR3(I,J)=(SQRERR(I,J)+SQRERR(I-1,J))/2.
   40 CONTINUE
      AVGERR1(1,J)=ERROR(1,J)/2.
      AVGERR2(1,J)=ABSERR(1,J)/2.
      AVGERR3(1,J)=SQRERR(1,J)/2.
      AVGERR1(NPROBP1,J)=ERROR(NPROB,J)/2.
      AVGERR2(NPROBP1,J)=ABSERR(NPROB,J)/2.
      AVGERR3(NPROBP1,J)=SQRERR(NPROB,J)/2.
   50 CONTINUE
      DO 70 J=1,5
      E1(J)=0.
      E2(J)=0.
      E3(J)=0.
      DO 60 I=1,NPROBP1
```

```
      E1(J)=E1(J)+DELTAP(I)*AVGERR1(I,J)
      E2(J)=E2(J)+DELTAP(I)*AVGERR2(I,J)
      E3(J)=E3(J)+DELTAP(I)*AVGERR3(I,J)
   60 CONTINUE
   70 CONTINUE
      DO 72 J=1,5
      EE3=E3(J)
      E3(J)=SQRT(EE3)
   72 CONTINUE
      WRITE (6,75)
   75 FORMAT(1H1//5X,"PROB.",3X,"NORMAL",6X,"LGML",5X,"GAMMA",
     X 3X,"WEIBULL",7X,"EW",6X,"SIMUL")
      DO 80 I=1,NPROB
   80 WRITE(6,85) PROB(I),(QUANTL(I,J),J=1,5),SIMUL(I),
     X (ERROR(I,J),J=1,5)
   85 FORMAT (5X,F4.3,6(3X,F7.4)/9X,5(3X,F7.4))
      WRITE (6,90) (E1(J),J=1,5)
   90 FORMAT (/3X,"BIAS :",5(3X,F7.4))
      WRITE (6,95) (E2(J),J=1,5)
   95 FORMAT (/4X,"MAE :",5(3X,F7.4))
      WRITE (6,100) (E3(J),J=1,5)
  100 FORMAT (/4X,"MSE :",5(3X,F7.4))
      RETURN
      END
C
C
C     ************************************************
C     SUBROUTINE SEARCH2 IS WHERE THE PARAMETERS OF THE
C     WEIBULL DISTRIBUTION ARE IDENTIFIED USING THE METHOD
C     OF MOMENTS
C     ************************************************
C
      SUBROUTINE SEARCH2(EDX,DXV,ALPHA,BETA,IT)
      ALPHA=5.0
      ITMAX=20
      XRIGHT=1.+DXV*DXV
      TOLER=0.01*XRIGHT
      IT=0
   10 IT=IT+1
      IF(IT.GT.ITMAX) GO TO 20
      A1=1./ALPHA
      A2=2.*A1
      A2P1=A2+1.
      A1P1=A1+1.
      T2=GAMMA(A2P1)
      T1=GAMMA(A1P1)
      XLEFT=T2/(T1*T1)
      DIF=XLEFT-XRIGHT
      DIF=ABS(DIF)
      IF(DIF.LE.TOLER) GO TO 20
      R=XLEFT/XRIGHT
      ALPHA=ALPHA*SQRT(R)
      GO TO 10
   20 CONTINUE
      BETA=EDX/T1
C     WRITE(7,25) IT,ALPHA,DIF,BETA
C  25 FORMAT(2X,"SEARCH 2",5X,"IT=",I3,3X,"ALPHA=",E12.6,
C    X 2X,"DIF=",E12.6,2X,"BETA=",E12.6)
      IF(ALPHA.LE.1.) WRITE(7,26)
   26 FORMAT(2X,"*** WARNING : ALPHA IS LESS THAN 1. ***")
      RETURN
      END
C
C
C     ************************************************
C     SUBROUTINE RANK IS USED TO RANK ARRAY ELEMENTS IN
C     NUMERICAL ORDER BEGINNING WITH THE LARGEST VALUES
C     ************************************************
```

```
C
      SUBROUTINE RANK(Q,N)
      DIMENSION Q(N)
      NA=N-1
      DO 20 J=1,NA
      M=J
      MA=J+1
      DO 10 I=MA,N
      IF(Q(I).GT.Q(M)) M=I
   10 CONTINUE
      TEMP=Q(J)
      Q(J)=Q(M)
      Q(M)=TEMP
   20 CONTINUE
      RETURN
      END
C
C
C
C     ****************************************************
C     SUBROUTINE STAT IS WHERE SIMPLE STATISTICS ARE COMPUTED
C     ****************************************************
C
      SUBROUTINE STAT(N,IG,SAMPLE,SMPLMN,SMPLSTD,SMPLSKW,SMPLKUT)
      DIMENSION SAMPLE(N),SMPLMN(10),SMPLSTD(10),SMPLSKW(10)
      DIMENSION SMPLKUT(10)
      SUM=0.
      DO 10 I=1,N
      SUM=SUM+SAMPLE(I)
   10 CONTINUE
      AVG=SUM/N
      SUM2=0.
      SUM3=0.
      SUM4=0.
      DO 20 I=1,N
      DEV=SAMPLE(I)-AVG
      SUM2=SUM2+DEV**2
      SUM3=SUM3+DEV**3
      SUM4=SUM4+DEV**4
   20 CONTINUE
      SMPLMN(IG)=AVG
      SMPLVAR=SUM2/N
      SMPLSTD(IG)=SQRT(SMPLVAR)
      SMPLSKW(IG)=SUM3/N/SMPLSTD(IG)**3
      SMPLKUT(IG)=SUM4/N/SMPLSTD(IG)**4
      RETURN
      END
```

APPENDIX I

LISTING OF COMPUTER PROGRAM FOR ESTIMATING THE POSITION OF THE CRITICAL LOCATION IN A STREAM ENVIRONMENT UNDER UNCERTAINTY USING FIBONACCI SEARCH AND MONTE CARLO SIMULATION

'FIBDX'

```
      PROGRAM FIBDX(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C ***************************************************************
C     THIS PROGRAM WAS WRITTEN BY DR. Y.K. TUNG AND WADE HATHHORN
C     WITH THE FINAL VERSION COMPLETED ON JULY 25, 1986.
C      THE ESSENCE OF THIS PROGRAM IS TO LOCATE THE
C     CRITICAL DISTANCE, XC, WHERE THE D.O. LEVELS ARE AT A MINIMUM
C     IN AN ENVIRONMENT CONTAINING UNCERTAINTY BASED ON
C      FOUR SEPARATE CASES:
C
C         1.  XC ASSOCIATED WITH AVERAGE STREAM PARAMETERS
C
C         2.  XC ASSOCIATED WITH THE MAXIMUM VARIANCE OF VIOLATION
C
C         3.  XC ASSOCATIED WITH THE MAXIMUM PROBABILITY OF VIOLATING
C             THE SPECIFIED STANDARD
C
C         4.  XC ASSOCIATED WITH THE MOST LIKELY CRITICAL LOCATION
C
C
C     THE DEVELOPMENT OF THIS PROGRAM IS BASED ON USING
C     THE CRITICAL DISTANCE, XC, CALCULATED FROM THE
C     'STREETER-PHELPS' EQUATION AND FIBONACCI SEARCH TECHNIQUES IN
C     CONJUNCTION WITTH STATISTICAL APPLICATIONS.  SEVERAL TECHNIQUES
C     ARE UTILIZED WITHIN THIS PROGRAM AND ARE GENERALLY ASSIGNED
C     WITHIN CORRESPONDING SUBROUTINES.  FOR EXAMPLE, FIBONACCI SEARCH
C     IS COMPLETED BY THE SUBROUTINE FIBSRCH.  THERE ARE SEVERAL
C     SUBROUTINES WITHIN THIS PROGRAM; FOR MORE DETAIL ABOUT EACH
C     SUBROUTINE, SEE THE COMMENTS PRIOR TO THE LISTING OF EACH ROUTINE
C     WITHIN THIS LISTING.
C
C ***************************************************************************
C
C     TO EXECUTE THIS PROGRAM, ONE INPUT FILE MUST BE PROVIDED.  THIS
C     INPUT FILE CONSISTS OF FIVE (5) MAJOR RECORDS:
C
C          RECORD            VARIALBES                FORMAT
C
C            1             A,B,ACCFT,DOSTD             8F10.0
C                               DOSAT
C
C            2             K1,K2,U,LO,DO               8F10.0
C
C            3             STDK1,STDK2,STDU            8F10.0
C                          STDLO,STDDO,CORK2U
C
C            4             SKEWK1,SKEWK2,SKEWU         8F10.0
C                          SKEWLO,SKEWDO
C
C            5             KURTK1,KURTK2,KURTU         8F10.0
C                          KURTLO,KURTDO
C
C
C                          DESCRIPTION
C
C     A = THE LOWER BOUND OF THE DEFICIT FUNCTION TO BE SEARCHED IN MILES
C     B = THE UPPER BOUND OF THE DEFICIT FUNCTION TO BE SEARCHED IN MILES
C     ACCFT = DESIRED ACCURACY OF THE FIBONACCI SEARCH IN FEET
C     DOSTD = DISSOLVED OXYGEN AT THE SPECIFIED STANDARD IN MG/L
C     DOSAT = DISSOLVED OXYGEN AT SATURATION IN MG/L (GENERALLY 10.0)
C     K1 = MEAN VALUE FOR THE REAERATION COEFFICIENT (STREAM PARAMETER)
C     K2 = MEAN VALUE FOR THE DEOXYGENATION COEFFICIENT (STREAM PARAMETER)
C     U = MEAN VALUE FOR THE STREAM VELOCITY IN MILES/DAY
C     LO = MEAN VALUE FOR THE B.O.D. LOADING IN STREAM IN MG/L
C     DO = MEAN VALUE FOR THE INITIAL D.O. DEFICIT IN STREAM IN MG/L
C
C     STDK1
C     STDK2
C     STDU   = STANDARD DEVIATIONS OF K1,K2,U,LO,DO
```

```
C     STDLO
C     STDDO
C
C     CORK2U = CORRELATION COEFFICIENT OF K2 AND U
C
C     SKEWK1
C     SKEWK2
C     SKEWU  = SKEWNESS OF K1,K2,U,LO,DO
C     SKEWLO
C     SKEWDO
C
C     KURTK1
C     KURTK2
C     KURTU  = KURTOSIS OF K1,K2,U,LO,DO
C     KURTLO
C     KURTDO
C
C ***************************************************************************
C
      DIMENSION F(500)
      REAL K1,K2,LO,KC,KURTDX,KURTXC,KURTK1,KURTK2,KURTU
      REAL KURTLO,KURTDO
      READ(5,1000)A,B,ACCFT,DOSTD,DOSAT
      READ (5,1000) K1,K2,U,LO,DO
      READ (5,1000) STDK1,STDK2,STDU,STDLO,STDDO,CORK2U
      READ(5,1000)SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO
      READ(5,1000)KURTK1,KURTK2,KURTU,KURTLO,KURTDO
 1000 FORMAT(10F8.0)
C
C     WRITE HEADING TO IDENTIFY OUTPUT
C
      WRITE(6,1002)
 1002 FORMAT(/2X,'**** PROGRAM OUTPUT FROM "FIBDX" ****')
C
C
C     CALCULATE THE VARIANCE OF EACH PARAMETER IN THE STREETER-PHELPS
C     EQUATION
C

      VARK1=STDK1*STDK1
      VARK2=STDK2*STDK2
      VARU=STDU*STDU
      VARLO=STDLO*STDLO
      VARDO=STDDO*STDDO
C
C     CALCULATE COVARIANCE BETWEEN K2 AND U
C
      COVK2U=CORK2U*STDK2*STDU
      ALPHA=ACCFT/5280.
      DO 2000 MODEL=1,4
      IF(MODEL.EQ.1)WRITE(6,1004)
      IF(MODEL.EQ.2)WRITE(6,1005)
      IF(MODEL.EQ.3)WRITE(6,1006)DOSTD
      IF(MODEL.EQ.4)WRITE(6,1007)
C
 1004 FORMAT(1H1//5X,'LOCATION XC ASSOCIATED WITH THE AVERAGE',
     1' STREAM PARAMETER VALUES')
 1005 FORMAT(1H1///5X,'FIBONACCI SEARCH FOR THE LOCATION ASSOCIATED',
     1' WITH THE MAXIMUM VARIANCE OF DEFICIT')
 1006 FORMAT(1H1///5X,'FIBONACCI SEARCH FOR THE LOCATION ASSOCIATED',
     1' WITH THE MAXIMUM PROBABILITY OF VIOLATING THE D.O. STD',
     2' OF',F5.2,' MG/L')
 1007 FORMAT(1H1///5X,'FIBONACCI SEARCH ASSOCIATED WITH THE',
     1' LOCATION MOST LIKELY TO BE CRITICAL')
C
      IF(MODEL.EQ.1)CALL MODEL1(K1,K2,U,LO,DO,VARK1,VARK2,VARU,
     1 VARLO,VARDO,COVK2U,MODEL,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,
     2 KURTK1,KURTK2,KURTU,KURTLO,KURTDO,DOSTD,DOSAT,CORK2U)
```

```
C
      IF(MODEL.EQ.2)CALL MODEL2(K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,
     1 VARDO,COVK2U,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,KURTK1,KURTK2,
     2 KURTU,KURTLO,KURTDO,A,B,DOSTD,DOSAT,MODEL,ALPHA,CORK2U)
C
      IF(MODEL.EQ.3)CALL MODEL3(K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,
     1 VARDO,COVK2U,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,KURTK1,KURTK2,
     2 KURTU,KURTLO,KURTDO,A,B,DOSTD,DOSAT,MODEL,ALPHA,CORK2U)
C
      IF(MODEL.EQ.4)CALL MODEL4(K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,
     1 VARDO,COVK2U,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,KURTK1,KURTK2,
     2 KURTU,KURTLO,KURTDO,A,B,DOSTD,DOSAT,MODEL,ALPHA,CORK2U)
 2000 CONTINUE
      STOP
      END
C
C
C
C     ****************************************************************
C     SUBROUTINE FUNC IS WHERE A FUNCTION ASSOCIATED WITH EACH OF
C     THE FOUR CASES IS DERIVED FOR THE CALCULATION OF XC
C     ****************************************************************
C
      SUBROUTINE FUNC(X,Y,MODEL,K1,K2,U,LO,DO,VARK1,
     *VARK2,VARU,VARLO,VARDO,COVK2U,SKEWK1,SKEWK2,SKEWU,SKEWLO,
     *SKEWDO,KURTK1,KURTK2,KURTU,KURTLO,KURTDO,XC,DOSTD,DOSAT,IDIST)
      REAL K1,K2,LO,KC,KURTDX,KURTXC,KURTK1,KURTK2,KURTU
      REAL KURTLO,KURTDO
C
C
      GOTO(110,120,130,140),MODEL
C
C     XC CALCULATED FROM AVERAGE STREAM PARAMETER VALUES
C
  110 XC=ALOG((K2/K1)*(1-(((K2-K1)*DO)/(K1*LO))))*(U/(K2-K1))
      WRITE(6,112)XC
  112 FORMAT(//1X,'THE CORRESPONDING VALUE OF XC = ',F8.3,' MILES')
      GOTO 9999
C
C      CALCULATION FOR VAR(DX)
C
  120 CALL VARDX(X,Y,VARK1,VARK2,VARU,VARLO,VARDO,K1,K2,U,LO,DO,
     X COVK2U)
      GOTO 9999
C
C     CALCULATION FOR MAX. PROB. OF VIOLATION
C
  130 CALL EXPECDX(X,EDX,VARK1,VARK2,VARU,VARLO,VARDO,
     1 K1,K2,U,LO,DO,COVK2U)
C
      CALL VARDX(X,VDX,VARK1,VARK2,VARU,VARLO,VARDO,K1,K2,U,LO,DO,
     X COVK2U)
C
      STDY=SQRT(VDX)
      CVDX=STDY/EDX
      DSTD=DOSAT-DOSTD
      ZI=(DSTD-EDX)/(VDX**0.5)
      GOTO(132,133,134,135),IDIST
C
  132 CALL M3M4DX(X,VDX,K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,
     1 VARDO,COVK2U,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,KURTK1,KURTK2,
     2 KURTU,KURTLO,KURTDO,SKEWDX,KURTDX)
C
      CALL EWPROB(ZI,KURTDX,SKEWDX,Y)
      GOTO 139
C
C     NORMAL
C
```

```
  133 CALL MDNOR(ZI,Y)
      Y=1.-Y
      GOTO 139
C
C     LOG-NORMAL
C
  134 DSTDLOG=ALOG(DSTD)
      YBAR=0.5*ALOG(EDX*EDX/(CVDX*CVDX+1.))
      YVAR=ALOG(CVDX*CVDX+1.)
      ZI=(DSTDLOG-YBAR)/SQRT(YVAR)
      CALL MDNOR(ZI,Y)
      Y=1.-Y
      GOTO 139
C
C     GAMMA
C
  135 ALPHA=1./(CVDX*CVDX)
      THETA=1./(CVDX*STDY)
      XP=DSTD*THETA
      CALL MDGAM(XP,ALPHA,Y)
      Y=1.-Y
  139 GOTO 9999
C
C     CALCULATION FOR MOST LIKELY CRITICAL LOCATION
C
  140 CALL M1M2XC(K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,VARDO,
     1 FC,KC,SK1,SK2,SU,SLO,SDO,EXC,VARXC)
C
      YC=(X-EXC)/(VARXC**0.5)
C
      CALL HERMIT(YC,H0,H1,H2,H3,H4,H5,H6)
C
      CALL M3M4XC(SK1,SK2,SU,SLO,SDO,VARK1,VARK2,VARU,VARLO,
     1 VARDO,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,KURTK1,KURTK2,KURTU,
     2 KURTLO,KURTDO,VARXC,SKEWXC,KURTXC)
C
      ZYC=0.39894*EXP(-0.5*(YC**2))
C
      Y=ZYC*(1.+((SKEWXC*H3/6.)+((KURTXC-3.)*H4/24.)+((SKEWXC**2)
     1 *H6/72.)))
C
 9999 RETURN
      END
C
C
C     ****************************************************************
C     SUBROUTINE EXPECDX IS WHERE THE EXPECTAION OF THE DEFICIT
C     FUNCTION, DX, IS CALCULATED.
C     ****************************************************************
C
      SUBROUTINE EXPECDX(X,EDX,VARK1,VARK2,VARU,VARLO,VARDO,
     1 K1,K2,U,LO,DO,COVK2U)
C
      REAL K1,K2,LO
C
      CALL CONST2(X,K1,K2,U,LO,DO,CK1P,CK2P,BK2P,CUP,BUP,CK2UP,BK2UP)
C
      E1=EXP(-K1*X/U)
      E2=EXP(-K2*X/U)
      DBAR= K1*LO/(K2-K1)*(E1-E2) + DO*E2
C
C     COMPUTE A FIRST ORDER APPROXIMATION OF THE EXPECTATION OF DX
C     UTILIZING TAYLOR'S SERIES EXPANSION OF DX (ASSUMING PARAMETERS
C     K2 AND U ARE DEPENDENT)
C
      EDX=DBAR+ 0.5*LO*CK1P*VARK1 + 0.5*(LO*CK2P+BK2P)*VARK2
     1 + 0.5*(LO*CUP+BUP)*VARU + (LO*CK2UP-BK2UP)*COVK2U
C
```

```
      RETURN
      END
C
C
C     ****************************************************************
C     SUBROUTINE VARDX IS WHERE THE VARIANCE OF THE DEFICIT FUNCTION,
C     DX, IS CALCULATED.
C     ****************************************************************
C
      SUBROUTINE VARDX(X,Y,VARK1,VARK2,VARU,VARLO,VARDO,K1,K2,U,LO,DO,
     X COVK2U)
C
      REAL K1,K2,LO
      CALL CONST1(X,K1,K2,U,LO,DO,CK1,CK2,BK2,CU,BU,CLO,CDO)
C
C     COMPUTE A FIRST ORDER APPROXIMATION OF THE VARIANCE OF DX
C     UTILIZING TAYLOR'S SERIES EXPANSION OF DX
C
      Y=(CK1**2)*(LO**2)*VARK1 + (LO*CK2-BK2)**2*VARK2
     1 + (LO*CU+BU)**2*VARU + (CDO**2)*VARDO + (CLO**2)*VARLO
     2 + 2.*(LO*CK2-BK2)*(LO*CU+BU)*COVK2U
      RETURN
      END
C
C
C
C     ****************************************************************
C     SUBROUTINE M3M4DX IS WHERE THE 3 RD AND 4 TH MOMENTS (SKEW AND
C     KURTOSIS) OF THE DEFICIT FUNCTION, DX, IS CALCULATED.
C     ****************************************************************
C
      SUBROUTINE M3M4DX(X,VDX,K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,
     1 VARDO,COVK2U,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,KURTK1,KURTK2,
     2 KURTU,KURTLO,KURTDO,SKEWDX,KURTDX)
C
      REAL K1,K2,LO,KC,KURTDX,KURTXC,KURTK1,KURTK2,KURTU,KURTLO,KURTDO
C
      CALL CONST1(X,K1,K2,U,LO,DO,CK1,CK2,BK2,CU,BU,CLO,CDO)
C
C     COMPUTE A FIRST ORDER APPROXIMATION OF THE SKEW OF DX UTILIZING
C     TAYLOR'S SERIES EXPANSION
C
      SKEWDX=((LO*CK1)**3*SKEWK1*(VARK1**1.5) + (LO*CK2-BK2)**3
     1 *SKEWK2*(VARK2**1.5) + (LO*CU+BU)**3*SKEWU*(VARU**1.5)
     2 + CDO**3*SKEWDO*(VARDO**1.5) + CLO**3*SKEWLO*(VARLO**1.5))
     3 /VDX**1.5
C
C     COMPUTE A FIRST ORDER APPROXIMATION OF THE KURTOSIS OF DX
C     UTILIZING TAYLOR'S SERIES EXPANSION
C
      PART1=(LO*CK1)**4*KURTK1*(VARK1**2) + (LO*CK2-BK2)**4
     1 *KURTK2*(VARK2**2) + (LO*CU+BU)**4*KURTU*(VARU**2)
     2 + CDO**4*KURTDO*(VARDO**2) + CLO**4*KURTLO*(VARLO**2)
     3 + 6*(LO*CK1)**2*(LO*CK2-BK2)**2*VARK1*VARK2
     4 + 6*(LO*CK1)**2*(LO*CU+BU)**2*VARK1*VARU
     5 + 6*(LO*CK1)**2*CDO**2*VARK1*VARDO
     6 + 6*(LO*CK2-BK2)**2*CDO**2*VARK2*VARDO
     7 + 6*(LO*CK2-BK2)**2*(LO*CU+BU)**2*VARK2*VARU
     8 + 6*(LO*CU+BU)**2*CDO**2*VARU*VARDO
     9 + 6*(LO*CK1)**2*CLO**2*VARK1*VARLO
C
      PART2=6*(LO*CK2-BK2)**2*CLO**2*VARK2*VARLO
     1 + 6*(LO*CU+BU)**2*CLO**2*VARU*VARLO
     2 +6*(CDO**2)*(CLO**2)*VARDO*VARLO
C
      KURTDX=(PART1+PART2)/VDX**2
      RETURN
      END
```

```
C
C     ****************************************************************
C     SUBROUTINE CONST1 IS WHERE SEVERAL CONSTANTS ARE CALCULATED WHICH ARE
C     ASSOCAITED WITH THE PARTIAL DERIVATIVES OF THE DEFICIT FUNCTION,
C     DX, WITH RESPECT TO EACH PARAMETER K1,K2,U,LO,DO.
C     ****************************************************************
C
      SUBROUTINE CONST1(X,K1,K2,U,LO,DO,CK1,CK2,BK2,CU,BU,CLO,CDO)
C
      REAL K1,K2,LO
      E1=EXP(-K1*X/U)
      E2=EXP(-K2*X/U)
      CK1=K2/(K2-K1)**2 * (E1-E2) - K1/(K2-K1)*(X/U)*E1
      CK2= -K1/(K2-K1)**2 *(E1-E2) + K1/(K2-K1)*(X/U)*E2
      BK2=DO*(X/U)*E2
      CU=K1**2/(K2-K1)*(X/U**2)*E1 - K1*K2/(K2-K1)*(X/U**2)*E2
      BU=DO*K2*X/(U**2)*E2
      CDO=E2
      CLO=K1/(K2-K1)*(E1-E2)
      RETURN
      END
C
C
C     ****************************************************************
C     SUBROUTINE CONST2 IS WHERE SEVERAL CONSTANTS ARE CALCULATED
C     WHICH ARE ASSOCIATED WITH THE SECOND PARTIAL DERIVATIVES OF THE
C     DEFICIT FUNCTION, DX, WITH RESPECT TO EACH OF THE VARIABLES K1,
C     K2,U,LO,DO.
C     ****************************************************************
C
      SUBROUTINE CONST2(X,K1,K2,U,LO,DO,CK1P,CK2P,BK2P,CUP,BUP,
     1 CK2UP,BK2UP)
      REAL K1,K2,LO
      E1=EXP(-K1*X/U)
      E2=EXP(-K2*X/U)
C
      CK1P=2*K2/((K2-K1)**3)*(E1-E2) - 2*K2/((K2-K1)**2)*(X/U)*E1
     1 - K1/(K2-K1)*((X/U)**2)*E1
C
      CK2P=2*K1/((K2-K1)**3)*(E1-E2) - 2*K1/((K2-K1)**2)*(X/U)*E2
     1 - K1/(K2-K1)*((X/U)**2)*E2
C
      BK2P=DO*((X/U)**2)*E2
C
      CUP=(2*(K1**2)/(K2-K1)*X/(U**3) - K1**3/(K2-K1)*(X**2/U**4))*E1
     1 +(2*K1*K2/(K2-K1)*(X/U**3) - K1*(K2**2)/(K2-K1)*(X**2/U**4))*E2
C
      BUP=(K2*X - 2*U)*K2*X*DO*E2/U**4
C
      CK2UP=K1**2/((K2-K1)**2)*(-X/U**2)*E1
     1 + (K1*K2/((K2-K1)**2)*(X/U**2) - K1/(K2-K1)*(X/U**2)
     2 + K1*K2/(K2-K1)*(X**2/U**3))*E2
C
      BK2UP=(K2*X - U)*DO*X*E2/U**3
      RETURN
      END
C
C
C     ****************************************************************
C     SUBROUTINE HERMIT IS WHERE SIX ORDERS OF THE HERMIT FUNCTION
C     ARE  CALCULATED.
C     ****************************************************************
C
      SUBROUTINE HERMIT(X,H0,H1,H2,H3,H4,H5,H6)
C
      H0=1.0
      H1=X
      H2=(X**2)-1.0
```

```
      H3=(X**3)-(3*X)
      H4=(X**4)-(6*(X**2))+3.0
      H5=(X**5)-(10*(X**3))+(15*X)
      H6=(X**6)-(15*(X**4))+(45+(X**2))-15.0
      RETURN
      END
C
C
C     ****************************************************************
C     SUBROUTINE CONSTXC IS WHERE SEVERAL CONSTANTS ARE CALCULATED
C     WHICH ARE ASSOCIATED WITH THE PARTIAL DERIVATIVES OF THE CRITICAL
C     LOCATION FUNCTION, XC, WITH RESPECT TO EACH OF THE VARIABLES K1,
C     K2,U,LO,DO.  THESE ARE USED TO DEVELOP A FIRST ORDER APPROXIMATION
C     OF XC IN CONJUNCTION WITH TAYLOR'S SERIES EXPANSION.
C     ****************************************************************
C
      SUBROUTINE CONSTXC(K1,K2,U,LO,DO,SK1,SK2,SU,SLO,SDO,FC,KC)
C
      REAL K1,K2,LO,KC
      KC=1.- (K2-K1)*DO/(K1*LO)
      FC=K2/K1
C
      SK1=U/((K2-K1)**2)*ALOG(FC*KC) + U/((K2-K1)*FC*KC)
     1 *(-K2/(K1**2)*KC + FC*K2*DO/((K1**2)*LO))
C
      SK2= -U/((K2-K1)**2)*ALOG(FC*KC) + U/((K2-K1)*FC*KC)
     1 *(1./K1*KC - FC*DO/(K1*LO))
C
      SLO=U/((K2-K1)*FC*KC)*K2*(K1-K2)*DO/((K1*LO)**2)
      SU=1/(K2-K1)*ALOG(FC*KC)
      SDO=U/((K2-K1)*FC*KC)*(K1-K2)/(K1*LO)
      RETURN
      END
C
C
C     ****************************************************************
C     SUBROUTINE M1M2XC IS WHERE THE 1 ST AND 2 ND MOMENTS (EXPECTATION
C     AND VARIANCE) OF THE CRITICAL LOCATION FUNCTION, XC, IS CALCULATED.
C     ****************************************************************
C
      SUBROUTINE M1M2XC(K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,VARDO
     1 ,FC,KC,SK1,SK2,SU,SLO,SDO,EXC,VARXC)
C
      REAL K1,K2,LO,KC
      CALL CONSTXC(K1,K2,U,LO,DO,SK1,SK2,SU,SLO,SDO,FC,KC)
C
C     COMPUTE A FIRST ORDER APPROXIMATION OF THE EXPECTATION OF XC
C     UTILIZING TAYLOR'S SERIES EXPANSION
C
      EXC=(U/(K2-K1))*ALOG(FC*KC)
C
C     COMPUTE A FIRST ORDER APPROXIMATION OF THE VARIANCE OF XC
C     UTILIZING TAYLOR'S SERIES EXPANSION
C
      VARXC=(SK1**2)*VARK1 + (SK2**2)*VARK2 + (SU**2)*VARU
     1 + (SLO**2)*VARLO + (SDO**2)*VARDO
      RETURN
      END
C
C
C     ****************************************************************
C     SUBROUTINE M3M4XC IS WHERE THE 3RD AND 4TH MOMENTS (SKEW AND
C     KURTOSIS) OF THE CRITICAL LOCATION FUNCTION, XC, ARE COMPUTED
C     ****************************************************************
C
      SUBROUTINE M3M4XC(SK1,SK2,SU,SLO,SDO,VARK1,VARK2,VARU,VARLO,
     1 VARDO,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,KURTK1,KURTK2,KURTU,
     2 KURTLO,KURTDO,VARXC,SKEWXC,KURTXC)
```

```
C
      REAL KURTK1,KURTK2,KURTU,KURTLO,KURTDO,KURTXC
C
C     DETERMINE THE SKEW OF XC UTILIZING A FIRST ORDER APPROXIMATION
C     FROM TAYLOR'S SERIES EXPANSION
C
      SKEWXC = (SK1**3*SKEWK1*(VARK1**1.5) +
     1 SK2**3*SKEWK2*(VARK2**1.5) + SU**3*SKEWU*(VARU**1.5) +
     2 SLO**3*SKEWLO*(VARDO**1.5) + SDO**3*SKEWDO*(VARDO**1.5))
     3 /(VARXC**1.5)
C
C     DETERMINE THE KURTOSIS OF XC UTILIZING A FIRST ORDER APPROXIMATION
C     FROM TAYLOR'S SERIES EXPANSION
C
      PART1 = SK1**4*KURTK1*(VARK1**2) + SK2**4*KURTK2*(VARK2**2)
     1 + SU**4*KURTU*(VARU**2) + SLO**4*KURTLO*(VARLO**2)
     2 + SDO**4*KURTDO*(VARDO**2)
C
      PART2 = 6*(SK1**2)*(SK2**2)*VARK1*VARK2
     1 + 6*(SK1**2)*(SU**2)*VARK1*VARU
     2 + 6*(SK1**2)*(SLO**2)*VARK1*VARLO
     3 + 6*(SK1**2)*(SDO**2)*VARK1*VARDO
     4 + 6*(SK2**2)*(SU**2)*VARK2*VARU
     5 + 6*(SK2**2)*(SLO**2)*VARK2*VARLO
     6 + 6*(SK2**2)*(SDO**2)*VARK2*VARDO
     7 + 6*(SU**2)*(SLO**2)*VARU*VARLO
     8 + 6*(SU**2)*(SDO**2)*VARU*VARDO
     9 + 6*(SLO**2)*(SDO**2)*VARLO*VARDO
C
      KURTXC = (PART1 + PART2)/(VARXC**2)
      RETURN
      END
C
C     ***************************************************************
C     SUBROUTINE FIBSRCH IS WHERE THE FIBONACCI SEARCH TECHNIQUE IS
C     CARRIED OUT.
C     ***************************************************************
C
      SUBROUTINE FIBSRCH(K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,VARDO,
     1 COVK2U,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,KURTK1,KURTK2,KURTU,
     2 KURTLO,KURTDO,XC,DOSTD,DOSAT,A,B,ALPHA,YMAX,AA,BB,IFN,F,MODEL,
     3 IFPRT,IDIST)
C
      DIMENSION F(500)
      REAL K1,K2,LO,KURTDX,KURTXC,KC,KURTK1,KURTK2,KURTU,KURTLO,KURTDO
C
      BB=B
      AA=A
      UNCIV=B-A
      FNPR=UNCIV/ALPHA
      F(1)=1.0
      F(2)=1.0
      DO 5 I=3,500
      F(I)=F(I-1)+F(I-2)
      IF(FNPR-F(I))6,6,5
    5 CONTINUE
    6 IFN=I
      IFPRT=I
   15 P1=AA+(F(IFN-1)/F(IFN))*UNCIV
      P2=AA+(F(IFN-2)/F(IFN))*UNCIV
C
      CALL FUNC(P1,Y1,MODEL,K1,K2,U,LO,DO,VARK1,VARK2,
     X VARU,VARLO,VARDO,COVK2U,SKEWK1,SKEWK2,SKEWU,SKEWLO,
     X SKEWDO,KURTK1,KURTK2,KURTU,KURTLO,KURTDO,XC,DOSTD,DOSAT,IDIST)
C
      CALL FUNC(P2,Y2,MODEL,K1,K2,U,LO,DO,VARK1,VARK2,
     X VARU,VARLO,VARDO,COVK2U,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,
     X KURTK1,KURTK2,KURTU,KURTLO,KURTDO,XC,DOSTD,DOSAT,IDIST)
```

```
C
      IF(Y1-Y2)10,11,12
   12 AA=P2
      YMAX=Y1
      GOTO 13
   10 BB=P1
      YMAX=Y2
      GOTO 13
   11 BB=P1
      AA=P2
      YMAX=Y2
   13 UNCIV=BB-AA
      IFN=IFN-1
      IF(IFN-3)20,15,15
   20 RETURN
      END
C
C
C     ****************************************************************
C     SUBROUTINE MODEL1 IS WHERE OUTPUT IS GENERATED BASED UPON AVERAGE
C     STREAM VALUE PARAMETERS.  WITHIN THIS OUTPUT, THERE IS THE
C     EXPECTATION, VARIANCE, SKEW, AND KURTOSIS OF THE DEFICIT FUNCTION,
C     DX, AND THE PROBABILITY OF VIOLATION.
C     ****************************************************************
C
      SUBROUTINE MODEL1(K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,
     1 VARDO,COVK2U,MODEL,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,KURTK1,
     2 KURTK2,KURTU,KURTLO,KURTDO,DOSTD,DOSAT,CORK2U)
C
      REAL K1,K2,LO,KC,KURTK1,KURTK2,KURTU,KURTLO,KURTDO,KURTDX
      IDIST=0
C
      CALL FUNC(X,Y,MODEL,K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,VARDO,
     1 COVK2U,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,KURTK1,KURTK2,
     2 KURTU,KURTLO,KURTDO,XC,DOSTD,DOSAT,IDIST)
C
      CALL EXPECDX(XC,EDX,VARK1,VARK2,VARU,VARLO,VARDO,
     1 K1,K2,U,LO,DO,COVK2U)
C
      CALL VARDX(XC,VDX,VARK1,VARK2,VARU,VARLO,VARDO,K1,K2,U,LO,DO,
     X COVK2U)
      STDY=VDX**0.5
C
      CALL M3M4DX(XC,VDX,K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,
     1 VARDO,COVK2U,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,KURTK1,KURTK2,
     2 KURTU,KURTLO,KURTDO,SKEWDX,KURTDX)
C
      DSTD=DOSAT-DOSTD
      CALL PRBVIOL(DSTD,EDX,STDY,SKEWDX,KURTDX,
     1 PROBNML,PROBLGN,PROBGAM,PROBEW)
C
      WRITE(6,1015)EDX,VDX,STDY
 1015 FORMAT(//1X,'THE EXPECTATION OF DEFICIT FOR THE FINAL DISTANCE',
     1' IS',F8.4,/1X,'AND THE VARIANCE OF DEFICIT IS',F10.4,
     2 /1X,'WITH A CORRESPONDING STD. DEV. OF ',F10.4)
C
      WRITE(6,1017)PROBNML,PROBLGN,PROBGAM,PROBEW
 1017 FORMAT(//1X,'PROBABILITY OF VIOLATION = ',F8.5,' (NORMAL)',
     1 /26X,'= ',F8.5,' (LOG-NORMAL)',/26X,'= ',F8.5,' (GAMMA)',
     2 /26X,'= ',F8.5,' (EDGEWORTH)')
C
      WRITE(6,1020)K1,SKEWK1,KURTK1,K2,SKEWK2,KURTK2,U,SKEWU,KURTU,
     1 LO,SKEWLO,KURTLO,DO,SKEWDO,KURTDO,CORK2U
 1020 FORMAT(/1X,'AND AVERAGE STREAM PARAMETER VALUES OF K1 = ',F8.3,
     1 9X,'SKEW K1 = ',F10.5,9X,'KURT K1 = ',F10.5,/40X,'K2 = ',F8.3,
     2 9X,'SKEW K2 = ',F10.5,9X,'KURT K2 = ',F10.5,/40X,' U = ',
     3 F8.3,9X,'SKEW U  = ',F10.5,9X,'KURT U  = ',F10.5,/40X,
     4 'LO = ',F8.3,9X,'SKEW LO = ',F10.5,9X,'KURT LO = ',F10.5,
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     5 /40X,'DO = ',F8.3,9X,'SKEW DO = ',F10.5,9X,'KURT DO = ',
     6 F10.5,/3X,'CORRELATION COEFFICENT BETWEEN K2 AND U IS',
     7 F7.3,///)
C
      RETURN
      END
C
C
C     ****************************************************************
C     SUBROUTINE MODEL2 IS WHERE THE CRITICAL LOCATION IS IDENTIFIED
C     USING FIBONACCI SEARCH BASED UPON THE MAXIMUM VARIANCE OF
C     VIOLATION.  THE OUTPUT CONTAINS THE EXPECTATION, VARIANCE,
C     SKEW, AND KURTOSIS OF THE DEFICIT FUNCTION, DX, AND THE PROBABILITY
C     OF VIOLATION.
C     ****************************************************************
C
      SUBROUTINE MODEL2(K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,
     1 VARDO,COVK2U,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,KURTK1,
     2 KURTK2,KURTU,KURTLO,KURTDO,A,B,DOSTD,DOSAT,MODEL,ALPHA,CORK2U)
C
      REAL K1,K2,LO,KURTXC,KURTDX,KURTK1,KURTK2,KURTU,KURTLO
      REAL KURTDO,KC
      DIMENSION F(500)
C
      CALL FIBSRCH(K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,VARDO,
     1 COVK2U,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,KURTK1,KURTK2,
     2 KURTU,KURTLO,KURTDO,XC,DOSTD,DOSAT,A,B,ALPHA,YMAX,AA,BB,
     3 IFN,F,MODEL,IFPRT,IDIST)
C
      WRITE(6,1008)A,B,ALPHA,IFPRT,IFPRT,F(IFPRT)
 1008 FORMAT(//////5X,'THE DISTANCE INTERVAL TO BE SEARCHED EXTENDS',
     1' FROM ', F10.5,' TO ',F10.5,' MILES',//5X,'THE DESIRED',
     2' INTERVAL OF UNCERTAINTY IS ',E13.5,' MILES'//5X,I3,
     3 ' FIBONACCI NUMBERS WERE GENERATED IN ORDER TO ACHIEVE',
     4 ' THIS FINAL INTERVAL OF UNCERTAINTY',//5X,' THE ',I3,
     5'TH FIBONACCI NUMBER IS ', E13.5,///)
C
      WRITE(6,1010)AA,BB,YMAX
 1010 FORMAT(5X,'THE FINAL INTERVAL OF UNCERTAINTY EXTENDS FROM ',
     1 E13.5,' TO ',E13.5,' MILES'//5X,'THE MAXIMUM VALUE OF THE',
     2' FUNCTION OBTAINED FOR THIS INTERVAL IS ',E13.5)
C
      X=(AA+BB)/2.
C
      CALL EXPECDX(X,EDX,VARK1,VARK2,VARU,VARLO,VARDO,
     1 K1,K2,U,LO,DO,COVK2U)
C
      CALL VARDX(X,VDX,VARK1,VARK2,VARU,VARLO,VARDO,K1,K2,U,LO,DO,
     X COVK2U)
      STDY=VDX**0.5
C
      CALL M3M4DX(X,VDX,K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,
     1 VARDO,COVK2U,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,KURTK1,KURTK2,
     2 KURTU,KURTLO,KURTDO,SKEWDX,KURTDX)
C
      DSTD=DOSAT-DOSTD
      CALL PRBVIOL(DSTD,EDX,STDY,SKEWDX,KURTDX,
     1 PROBNML,PROBLGN,PROBGAM,PROBEW)
C
      WRITE(6,1015)EDX,VDX,STDY
 1015 FORMAT(//1X,'THE EXPECTATION OF DEFICIT FOR THE FINAL DISTANCE',
     1' IS',F8.4,/1X,'AND THE VARIANCE OF DEFICIT IS',F10.4,
     2 /1X,'WITH A CORRESPONDING STD. DEV. OF ',F10.4)
C
      WRITE(6,1017)PROBNML,PROBLGN,PROBGAM,PROBEW
 1017 FORMAT(//1X,'PROBABILITY OF VIOLATION = ',F8.5,' (NORMAL)',
     1 /26X,'= ',F8.5,' (LOG-NORMAL)',/26X,'= ',F8.5,' (GAMMA)',
     2 /26X,'= ',F8.5,' (EDGEWORTH)')
```

```
C
      WRITE(6,1020)K1,SKEWK1,KURTK1,K2,SKEWK2,KURTK2,U,SKEWU,KURTU,
     1 LO,SKEWLO,KURTLO,DO,SKEWDO,KURTDO,CORK2U
 1020 FORMAT(/1X,'AND AVERAGE STREAM PARAMETER VALUES OF K1 = ',F8.3,
     1 10X,'SKEW K1 =',F10.5,10X,'KURT K1 = ',F10.5,/40X,'K2 = ',F8.3,
     2 9X,'SKEW K2 = ',F10.5,9X,'KURT K2 = ',F10.5,/40X,' U = ',
     3 F8.3,9X,'SKEW U  = ',F10.5,9X,'KURT U  = ',F10.5,/40X,
     4 'LO = ',F8.3,9X,'SKEW LO = ',F10.5,9X,'KURT LO = ',F10.5,
     5 /40X,'DO = ',F8.3,9X,'SKEW DO = ',F10.5,9X,'KURT DO = ',
     6 F10.5,/3X,'CORRELATION COEFFICIENT BETWEEN K2 AND U IS',
     7 F7.3,///)
C
      RETURN
      END
C
C
C     ********************************************************************
C     SUBROUTINE MODEL3 IS WHERE THE CRITICAL LOCATION IS IDENTIFIED
C     BY FIBONACCI SEARCH USING THE MAXIMUM PROBABILITY OF VIOLATION.
C     THE OUTPUT CONATANS THE EXPECTATION, VARIANCE, SKEW, AND KURTOSIS
C     OF THE DEFICIT FUNCTION, DX, AND THE PROBABILITY OF VIOLATION.
C     ********************************************************************
C
      SUBROUTINE MODEL3(K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,
     1 VARDO,COVK2U,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,KURTK1,KURTK2,
     2 KURTU,KURTLO,KURTDO,A,B,DOSTD,DOSAT,MODEL,ALPHA,CORK2U)
C
      REAL K1,K2,U,LO,DO,KC,KURTXC,KURTDX,KURTK1,KURTK2,KURTU
      REAL KURTLO,KURTDO
      DIMENSION F(500)
      DO 1018 IDIST=1,4
C
      CALL FIBSRCH(K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,VARDO,
     1 COVK2U,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,KURTK1,KURTK2,
     2 KURTU,KURTLO,KURTDO,XC,DOSTD,DOSAT,A,B,ALPHA,YMAX,AA,BB,
     3 IFN,F,MODEL,IFPRT,IDIST)
C
      IF(IDIST.NE.1)GOTO 1009
C
      WRITE(6,1008)A,B,ALPHA,IFPRT,IFPRT,F(IFPRT)
 1008 FORMAT(//////5X,'THE DISTANCE INTERVAL TO BE SEARCHED EXTENDS',
     1' FROM ', F10.5,' TO ',F10.5,' MILES',//5X,'THE DESIRED',
     2' INTERVAL OF UNCERTAINTY IS ',E13.5,' MILES'//5X,I3,
     3 ' FIBONACCI NUMBERS WERE GENERATED IN ORDER TO ACHIEVE',
     4 ' THIS FINAL INTERVAL OF UNCERTAINTY',//5X,' THE ',I3,
     5'TH FIBONACCI NUMBER IS ', E13.5,///)
C
 1009 IF(IDIST.EQ.1)WRITE(6,90)
      IF(IDIST.EQ.2)WRITE(6,100)
      IF(IDIST.EQ.3)WRITE(6,200)
      IF(IDIST.EQ.4)WRITE(6,300)
C
   90 FORMAT(1X,'*********************************',/1X,
     1 'USING EDGEWORTH APPROXIMATION',/1X,
     2 '*********************************')
C
  100 FORMAT(//1X,'************************',/1X,
     1 'USING NORMAL DISTRIBUTION',/1X,'************************')
C
  200 FORMAT(//1X,'****************************',/1X,
     1 'USING LOGNORMAL DISTRIBUTION',/1X,'******************',
     2 '**********')
C
  300 FORMAT(//1X,'**************************',/1X,
     1 'USING GAMMA DISTRIBUTION',/1X,'**************************')
C
      WRITE(6,1010)AA,BB,YMAX
 1010 FORMAT(5X,'THE FINAL INTERVAL OF UNCERTAINTY EXTENDS FROM ',
```

```
     1 E13.5,' TO ',E13.5,' MILES'//5X,'THE MAXIMUM VALUE OF THE',
     2' FUNCTION OBTAINED FOR THIS INTERVAL IS ',E13.5)
C
      X=(AA+BB)/2.
C
      CALL EXPECDX(X,EDX,VARK1,VARK2,VARU,VARLO,VARDO,
     1 K1,K2,U,LO,DO,COVK2U)
C
      CALL VARDX(X,VDX,VARK1,VARK2,VARU,VARLO,VARDO,K1,K2,U,LO,DO,
     X COVK2U)
      STDY=VDX**0.5
C
      CALL M3M4DX(X,VDX,K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,
     1 VARDO,COVK2U,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,KURTK1,KURTK2,
     2 KURTU,KURTLO,KURTDO,SKEWDX,KURTDX)
C
      DSTD=DOSAT-DOSTD
      CALL PRBVIOL(DSTD,EDX,STDY,SKEWDX,KURTDX,
     1 PROBNML,PROBLGN,PROBGAM,PROBEW)
C
      WRITE(6,1015)EDX,VDX,STDY
 1015 FORMAT(//1X,'THE EXPECTATION OF DEFICIT FOR THE FINAL DISTANCE',
     1' IS',F8.4,/1X,'AND THE VARIANCE OF DEFICIT IS',F10.4,
     2 /1X,'WITH A CORRESPONDING STD. DEV. OF ',F10.4)
C
      WRITE(6,1017)PROBNML,PROBLGN,PROBGAM,PROBEW
 1017 FORMAT(//1X,'PROBABILITY OF VIOLATION = ',F8.5,' (NORMAL)',
     1 /26X,'= ',F8.5,' (LOG-NORMAL)',/26X,'= ',F8.5,' (GAMMA)',
     2 /26X,'= ',F8.5,' (EDGEWORTH)')
C
 1018 CONTINUE
C
      WRITE(6,1020)K1,SKEWK1,KURTK1,K2,SKEWK2,KURTK2,U,SKEWU,KURTU,
     1 LO,SKEWLO,KURTLO,DO,SKEWDO,KURTDO,CORK2U
 1020 FORMAT(/1X,'AND AVERAGE STREAM PARAMETER VALUES OF K1 = ',F8.3,
     1 10X,'SKEW K1 =',F10.5,10X,'KURT K1 = ',F10.5,/40X,'K2 = ',F8.3,
     2 9X,'SKEW K2 = ',F10.5,9X,'KURT K2 = ',F10.5,/40X,' U = ',
     3 F8.3,9X,'SKEW U  = ',F10.5,9X,'KURT U  = ',F10.5,/40X,
     4 'LO = ',F8.3,9X,'SKEW LO = ',F10.5,9X,'KURT LO = ',F10.5,
     5 /40X,'DO = ',F8.3,9X,'SKEW DO = ',F10.5,9X,'KURT DO = ',
     6 F10.5,/3X,'CORRELATION COEFFICIENT BETWEEN K2 AND U IS',
     7 F7.3,///)
C
      RETURN
      END
C
C
C     ******************************************************************
C     SUBROUTINE MODEL4 IS WHERE FIBONACCI SEARCH IS USED TO DETERMINE
C     THE MOST LIKELY LOCATION FOR THE CRITICAL DISTANCE UTILIZING
C     EDGEWORTH'S APPROXIMATION, NORMAL, LOG-NORMAL, AND GAMMA DISTRIBUTIONS.
C     THE OUTPUT CONTAINS THE EXPECTATION, VARIANCE, SKEW, AND KURTOSIS
C     OF THE DEFICIT FUNCTION, DX, AND THE PROBABILITY OF VIOLATION.
C     ******************************************************************
C
      SUBROUTINE MODEL4(K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,
     1 VARDO,COVK2U,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,KURTK1,KURTK2,
     2 KURTU,KURTLO,KURTDO,A,B,DOSTD,DOSAT,MODEL,ALPHA,CORK2U)
C
      REAL K1,K2,LO,KC,KURTXC,KURTDX,KURTK1,KURTK2,KURTLO,KURTDO
      REAL MODELGN,NU,LMDA,MODEGM,MODEW
      DIMENSION F(500)
C
      IDIST=1
      CALL FIBSRCH(K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,VARDO,
     1 COVK2U,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,KURTK1,KURTK2,
     2 KURTU,KURTLO,KURTDO,XC,DOSTD,DOSAT,A,B,ALPHA,YMAX,AA,BB,
     3 IFN,F,MODEL,IFPRT,IDIST)
```

```
C
      MODEW=(AA+BB)/2.
C
      WRITE(6,1008)A,B,ALPHA,IFPRT,IFPRT,F(IFPRT)
 1008 FORMAT(//////5X,'THE DISTANCE INTERVAL TO BE SEARCHED EXTENDS',
     1' FROM ', F10.5,' TO ',F10.5,' MILES',//5X,'THE DESIRED',
     2' INTERVAL OF UNCERTAINTY IS ',E13.5,' MILES'//5X,I3,
     3 ' FIBONACCI NUMBERS WERE GENERATED IN ORDER TO ACHIEVE',
     4 ' THIS FINAL INTERVAL OF UNCERTAINTY',//5X,' THE ',I3,
     5'TH FIBONACCI NUMBER IS ', E13.5,///)
C
      CALL M1M2XC(K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,VARDO,
     1 FC,KC,SK1,SK2,SU,SLO,SDO,EXC,VARXC)
C
      STDXC=VARXC**0.5
      CV=STDXC/EXC
      YBAR=0.5*ALOG(EXC**2/(CV**2+1.))
      VARY=ALOG(CV**2+1.)
      MODELGN=EXP(YBAR-VARY)
      NU=EXC**2/VARXC
      LMDA=EXC/VARXC
      MODEGM=(NU-1.)/LMDA
C
      DO 1500 IDIST=1,4
      IF(IDIST.EQ.1)WRITE(6,1009)
 1009 FORMAT(1X,'*********************************',/1X,
     1 'USING EDGEWORTH APPROXIMATION',/1X,
     2 '*********************************')
C
      IF(IDIST.NE.1)GOTO 60
C
      WRITE(6,1010)AA,BB,YMAX
 1010 FORMAT(//5X,'THE FINAL INTERVAL OF UNCERTAINTY EXTENDS FROM ',
     1 E13.5,' TO ',E13.5,' MILES'//5X,'THE MAXIMUM VALUE OF THE',
     2' FUNCTION OBTAINED FOR THIS INTERVAL IS ',E13.5)
C
      X=(AA+BB)/2.
   60 IF(IDIST.EQ.2)WRITE(6,100)EXC
      IF(IDIST.EQ.3)WRITE(6,200)MODELGN
      IF(IDIST.EQ.4)WRITE(6,300)MODEGM
C
  100 FORMAT(//1X,'************************',/1X,
     1 'USING NORMAL DISTRIBUTION',/1X,'************************',
     2 //1X,'THE MOST LIKELY LOCATION FOR THE CRITICAL DISTANCE IS',
     3 F8.4,' MILES.')
C
  200 FORMAT(//1X,'*****************************',/1X,
     1 'USING LOGNORMAL DISTRIBUTION',/1X,'******************',
     2 '**********',//1X,'THE MOST LIKELY LOCATION FOR THE CRITICAL',
     3 ' DISTANCE IS',F8.4,' MILES.')
C
  300 FORMAT(//1X,'*************************',/1X,
     1 'USING GAMMA DISTRIBUTION',/1X,'*************************',
     2 //1X,'THE MOST LIKELY LOCATION FOR THE CRITICAL DISTANCE',
     3 ' IS',F8.4,' MILES.')
C
      IF(IDIST.EQ.1)X=MODEW
      IF(IDIST.EQ.2)X=EXC
      IF(IDIST.EQ.3)X=MODELGN
      IF(IDIST.EQ.4)X=MODEGM
C
      CALL EXPECDX(X,EDX,VARK1,VARK2,VARU,VARLO,VARDO,
     1 K1,K2,U,LO,DO,COVK2U)
C
      CALL VARDX(X,VDX,VARK1,VARK2,VARU,VARLO,VARDO,K1,K2,U,LO,DO,
     X COVK2U)
      STDY=VDX**0.5
C
```

```
      CALL M3M4DX(X,VDX,K1,K2,U,LO,DO,VARK1,VARK2,VARU,VARLO,
     1 VARDO,COVK2U,SKEWK1,SKEWK2,SKEWU,SKEWLO,SKEWDO,KURTK1,KURTK2,
     2 KURTU,KURTLO,KURTDO,SKEWDX,KURTDX)
C
      DSTD=DOSAT-DOSTD
      CALL PRBVIOL(DSTD,EDX,STDY,SKEWDX,KURTDX,
     1 PROBNML,PROBLGN,PROBGAM,PROBEW)
C
      WRITE(6,1015)EDX,VDX,STDY
 1015 FORMAT(//1X,'THE EXPECTATION OF DEFICIT FOR THE FINAL DISTANCE',
     1' IS',F8.4,/1X,'AND THE VARIANCE OF DEFICIT IS',F10.4,
     2 /1X,'WITH A CORRESPONDING STD. DEV. OF ',F10.4)
C
      WRITE(6,1017)PROBNML,PROBLGN,PROBGAM,PROBEW
 1017 FORMAT(//1X,'PROBABILITY OF VIOLATION = ',F8.5,' (NORMAL)',
     1 /26X,'= ',F8.5,' (LOG-NORMAL)',/26X,'= ',F8.5,' (GAMMA)',
     2 /26X,'= ',F8.5,' (EDGEWORTH)')
C
 1500 CONTINUE
C
      WRITE(6,1510)EXC,VARXC,STDXC
 1510 FORMAT(//1X,'THE EXPECTATION OF THE CRITICAL DISTANCE IS',
     1 F8.4,/1X,'AND THE VARIANCE OF XC IS',F10.4,
     2 /1X,'WITH A CORRESOPNDING STD. DEV. OF',F10.4)
C
      WRITE(6,1520)K1,SKEWK1,KURTK1,K2,SKEWK2,KURTK2,U,SKEWU,KURTU,
     1 LO,SKEWLO,KURTLO,DO,SKEWDO,KURTDO,CORK2U
 1520 FORMAT(/1X,'AND AVERAGE STREAM PARAMETER VALUES OF K1 = ',F8.3,
     1 10X,'SKEW K1 =',F10.5,10X,'KURT K1 = ',F10.5,/40X,'K2 = ',F8.3,
     2 9X,'SKEW K2 = ',F10.5,9X,'KURT K2 = ',F10.5,/40X,' U = ',
     3 F8.3,9X,'SKEW U  = ',F10.5,9X,'KURT U  = ',F10.5,/40X,
     4 'LO = ',F8.3,9X,'SKEW LO = ',F10.5,9X,'KURT LO = ',F10.5,
     5 /40X,'DO = ',F8.3,9X,'SKEW DO = ',F10.5,9X,'KURT DO = ',
     6 F10.5,/3X,'CORRELATION COEFFICIENT BETWEEN K2 AND U IS',
     7 F7.3,///)
C
      RETURN
      END
C
C
C     ***************************************************************
C     SUBROUTINE EWPROB IS WHERE EDGEWORTH'S APPROXIMATION IS USED TO
C     CALCULATE THE PROBABILITY.
C     ***************************************************************
C
      SUBROUTINE EWPROB(ZI,KURTDX,SKEWDX,PROB)
C
      REAL KURTDX
      CALL HERMIT(ZI,H0,H1,H2,H3,H4,H5,H6)
      CALL MDNOR(ZI,PROBNML)
      PHI=(1./((2*3.14159)**0.5))*EXP(-0.5*(ZI**2))
      PROB=(1.-PROBNML)+(SKEWDX*H2/6.+((KURTDX-3.)*H3/24.)+((SKEWDX**2)
     1 *H5/72.))*PHI
      RETURN
      END
C
C
C     ***************************************************************
C     SUBROUTINE PRBVIOL IS WHERE THE PROBABILITY OF VIOLATION IS
C     CALCULATED BASED ON EDGEWORTH'S APPROXIMATION, NORMAL, LOG-NORMAL,
C     GAMMA DISTRIBUTIONS.
C     ***************************************************************
C
      SUBROUTINE PRBVIOL(DSTD,EDX,STDY,SKEWDX,KURTDX,
     1 PROBNML,PROBLGN,PROBGAM,PROBEW)
C
      REAL KURTDX
      ZI=(DSTD-EDX)/STDY
```

```
C
C     NORMAL
C
      CALL MDNOR(ZI,PROBNML)
      PROBNML=1.-PROBNML
C
C     LOG-NORMAL
C
      DSTDLOG=ALOG(DSTD)
      CVDX=STDY/EDX
      YBAR=0.5*ALOG(EDX*EDX/(CVDX*CVDX+1.))
      YVAR=ALOG(CVDX*CVDX+1.)
      ZI=(DSTDLOG-YBAR)/SQRT(YVAR)
      CALL MDNOR(ZI,PROBLGN)
      PROBLGN=1.-PROBLGN
C
C     GAMMA
C
      ALPHA=1./(CVDX*CVDX)
      THETA=1./(CVDX*STDY)
      XP=DSTD*THETA
      CALL MDGAM(XP,ALPHA,PROBGAM)
      PROBGAM=1.-PROBGAM
C
C     EDGEWORTH
C
      ZI=(DSTD-EDX)/STDY
      CALL EWPROB(ZI,KURTDX,SKEWDX,PROBEW)
      RETURN
      END
```

APPENDIX J

LISTING OF COMPUTER PROGRAM FOR BUILDING THE MATRIX OF TECHNOLOGICAL COEFFICIENTS TO BE UTILIZED IN THE STOCHASTIC OPTIMAL WASTE LOAD ALLOCATION MODEL 'STOCOEF'

```
      PROGRAM STOCOEF(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7,
     X TAPE8)
C     ******************************************************************
C     THIS PROGRAM IS DESIGNED TO COMPUTE THE TECHNOLOGICAL COEFFICIENTS
C     FOR THE STOCHASTIC WASTE LOAD ALLOCATION LINEAR PROGRAMMING PROBLEM
C
C     SEVERAL DISTRIBUTIONS FOR THE STREAM PARAMETERS K1,K2,U,LO, AND DO
C     MAY BE ASSUMED: (1) NORMAL, (2) LOG-NORMAL, (3) GAMMA, AND
C     (4) WEIBULL
C
C     EACH OF THE TECHNIQUES UTILIZED IN THIS PROGRAM ARE GENERALLY
C     OUTLINED IN THEIR RESPECTIVE SUBROUTINES.  FOR MORE INFORMATION
C     CONCERNING EACH SUBROUTINE, SEE THE COMMENTS PRIOR TO THE LISTING
C     OF EACH ROUTINE IN THE PROGRAM BELOW.
C
C
C     TO EXECUTE THIS PROGRAM, ONE INPUT FILE MUST BE PROVIDED
C     CONTAINING THE FOLLOWING INFORMATION IN THE APPROPRIATE FORMAT:
C
C
C  CARD                        VARIABLES                        FORMAT
C
C   1                      NREACH,IDATTYP,IPOLL                 4I8,D12.1
C                            LSET,DSEED
C
C   2              MEANLO,MEANDO,MEAQO,STDLO,STDDO,             10F8.0
C                        STDQO,CORRK2U
C
C   3                        MEANK1(I)                          10F8.0
C
C   4                        MEANK2(I)                          10F8.0
C
C   5                        MEANU(I)                           10F8.0
C
C   6                        STDK1(I)                           10F8.0
C
C   7                        STDK2(I)                           10F8.0
C
C   8                        STDU(I)                            10F8.0
C
C   9                        IDIST(I)                           10I8
C
C
C  10                       DISLOC(I)                           10F8.0
C
C  11                        Q(I)                               10F8.0
C
C  12                      NCNTRLP(I)                           10I8
C
C  13                      XINBOD(I)                            10F8.0
C
C   OPTIONAL: SEE COMMENTS FOR XCNTRLP
C
C  14                       XCNTRLP(I)                          10F8.0
C
C                      ********************
C                    * DESCRIPTION OF INPUT *
C                      ********************
C
C
C   LSET = NUMBER OF TECHNOLOGICAL COEFFICIENT SETS TO BE GENERATED
C
C   NREACH= NO. OF REACHES
C
C   IDATTYP= INDICIES FOR INPUTTING CONTROL POINT LOCATIONS:
C            (0)=MANUAL
C            (1)=COMPUTER GENER.
C
```

```
C  BY SELECTING IDATTYP=1, THE PROGRAM WILL AUTOMATICALLY SELECT
C  CONTROL POINT LOCATIONS WITHIN EACH REACH WHICH ARE EQUALLY
C  SPACED ACCORDING TO THE NUMBER OF POINTS GIVEN BY THE INPUT
C  VARIABLE "NCNTRLP" (SEE BELOW).
C
C   IPOLL = INDICIES FOR TYPE OF POLLUTION TO BE CONSIDERED:
C           (0)=BOD ONLY
C           (1)=BOD AND DO DEFICITS
C
C   DSEED = SEED VALUE USED IN RANDOM NUMBER GENERATION
C           (ENTER ARBITRARY VALUE UNDER THE GIVEN FORMAT D12.1)
C
C   MEANLO = MEAN VALUE OF THE INITIAL INSTREAM BOD CONCENTRATION, MG/L
C   MEANDO = MEAN VALUE OF THE INITIAL INSTREAM DO DEFICIT CONC., MG/L
C   MEANQO = MEAN VALUE OF THE UPSTREAM FLOW RATE, CFS
C
C   STDLO
C   STDDO = STANDARD DEVIATIONS OF LO,DO,QO
C   STDQO
C
C   CORRK2U = CORRELATION COEFFICIENT BETWEEN K2 AND U
C
C
C   MEANK1(I)
C   MEANK2(I) = ARRAYS CONTIANING THE MEAN VALUES OF K1,K2,AND U
C   MEANU(I)              IN EACH REACH
C
C
C   STDK1(I)
C   STDK2(I) = ARRAYS CONTAINING THE STANDARD DEVIATIONS OF K1,K2,AND U
C   STDU(I)               IN EACH REACH
C
C          NOTE:   K1 = DEOXYGENATION COEFFICIENT  1/DAYS
C                  K2 = REAERATION COEFFCIENT   1/DAYS
C                   U = AVERAGE STREAM VELOCITY  MILES/DAY
C
C
C
C  IDIST(I),I=1,6    SIX ELEMENT ARRAY CONTAINING AN INTEGER
C                    CODE VALUE TO INDICATE THE TYPE OF DISTRIBUTION
C                    TO BE ASSUMED FOR K1,K2,U,LO,DO, AND QO, RESPECTIVELY:
C
C                              IDIST(I)              DISTR. TYPE
C
C                                 1                    NORMAL
C                                 2                    LOG-NORMAL
C                                 3                    GAMMA
C                                 4                    WEIBUL
C
C
C   DISLOC = DISCHARGE LOCATION FOR EACH REACH (MILES)
C
C  DISCHARGE LOCATIONS MUST BE ENTERED IN A SEQUENTIAL ORDER, NOT
C  NECESSARILY BEGGINING WITH 0.0
C
C   Q(I) = DISCHARGE FLOW RATE FOR EACH REACH
C
C   NCNTRLP(I) = NUMBER OF CONTROL POINTS IN EACH REACH
C
C   XINBOD(I) = INFLUENT BOD CONCENTRAION TO EACH DISCHARGE IN MG/L
C
C
C  IF YOU ENTERED IDATTYP=0, YOU MUST INCLUDE ANOTHER CARD TO INPUT
C  THE CONTROL POINT LOCATIONS (CARD 14)
C
C   XCNTRLP(I) = CONTROL POINT LOCATIONS FOR ALL REACHES IN SEQUENCE
C
C
```

```
C*************************************************************************
C
C
      DIMENSION DISLOC(10),QT(10),K1(10),K2(10),U(10),AIOSTD(100)
      DIMENSION XCNTRLP(100),DOCOEF(100,15),Q(10),
     X AIOVAR(100),R(1),WK(3),AWIB(6)
      DIMENSION XINBOD(10),BWIB(6),AIOBAR(100)
      DIMENSION PARAMTR(6),PARMEAN(6),PARSTD(6),
     X AIOFIX(100),AIJBAR(100,15),AIJVAR(100,15),
     X SIGMA(3),RVEC(1,2),WKVEC(1),AGAM(6),BGAM(6),SLN(6),XMLN(6),
     X MEANK1(10),MEANK2(10),MEANU(10),STDK1(10),STDK2(10),STDU(10),
     X AIJSTD(100,15),NCNTRLP(10),NCNT(10)
      INTEGER IDIST(6)
      DOUBLE PRECISION DSEED
      REAL K1,K2,LO,DO
      REAL MEANK1,MEANK2,MEANLO,MEANU,MEANDO,MEANQO
C
C
C
C     READ IN INPUT DATA
C
      READ(5,5)NREACH,IDATTYP,IPOLL,LSET,DSEED
      READ(5,7)MEANLO,MEANDO,MEANQO,STDLO,STDDO,STDQO,CORRK2U
      READ(5,7)(MEANK1(I),I=1,NREACH)
      READ(5,7)(MEANK2(I),I=1,NREACH)
      READ(5,7)(MEANU(I),I=1,NREACH)
      READ(5,7)(STDK1(I),I=1,NREACH)
      READ(5,7)(STDK2(I),I=1,NREACH)
      READ(5,7)(STDU(I),I=1,NREACH)
      READ(5,10)(IDIST(I),I=1,6)
      READ(5,7)(DISLOC(I),I=1,NREACH)
      READ(5,7)(Q(I),I=1,NREACH)
      READ(5,10)(NCNTRLP(I),I=1,NREACH)
      READ(5,7)(XINBOD(I),I=1,NREACH)
    5 FORMAT(4I8,D12.1)
    7 FORMAT(10F8.0)
   10 FORMAT(10I8)
C
C     WRITE OUT INPUT DATA
C
      WRITE(6,15)(MEANK1(I),I=1,NREACH)
   15 FORMAT(1H1//3X,'MEAN VALUES OF K1: ',//10(3X,F6.3))
      WRITE(6,16)(MEANK2(I),I=1,NREACH)
   16 FORMAT(///3X,'MEAN VALUES OF K2: ',//10(3X,F6.3))
      WRITE(6,17)(MEANU(I),I=1,NREACH)
   17 FORMAT(///3X,'MEAN VALUES OF U: ',//10(3X,F6.3))
C
      WRITE(6,18)(STDK1(I),I=1,NREACH)
   18 FORMAT(///3X,'STANDARD DEVIATION VALUES OF K1:',//10(3X,F6.3))
      WRITE(6,19)(STDK2(I),I=1,NREACH)
   19 FORMAT(///3X,'STANDARD DEVIATION VALUES OF K2:',//10(3X,F6.3))
      WRITE(6,20)(STDU(I),I=1,NREACH)
   20 FORMAT(///3X,'STANDARD DEVIATION VALUES OF U:',//10(3X,F6.3))
C
      WRITE(6,21)MEANLO,MEANDO,MEANQO,STDLO,STDDO,STDQO,CORRK2U
   21 FORMAT(///3X,'MEAN AND STANDARD DEVIATION OF FIXED PARAMETERS:',
     X //3X,'MEANLO = ',F7.3,5X,'MEANDO = ',F7.3,5X,'MEANQO = ',F7.3,
     X //3X,'STDLO = ',F7.3,6X,'STDDO = ',F7.3,6X,'STDQO = ',F7.3,
     X /3X,'CORRELATION COEFFICIENT BETWEEN K2 AND U IS ',F7.3)
C
      WRITE(6,22)(IDIST(I),I=1,6)
   22 FORMAT(///3X,'DISTRIBUTION TYPE FOR EACH PARAMETER:',//3X,
     X 'K1 = ',I6,10X,'K2 = ',I6,11X,'U = ',I6,/3X,
     X 'LO = ',I6,10X,'DO = ',I6,10X,'QO = ',I6)
C
      WRITE(6,23)NREACH,IDATTYP,IPOLL,LSET
   23 FORMAT(///3X,'INPUT CONTROLS:',//3X,
     X 'NREACH = ',I6,5X,'IDATTYP = ',I6,/3X,
```

```
     X 'IPOLL = ',I6,6X,'LSET =  ',I6)
C
      WRITE(6,24)(DISLOC(I),I=1,NREACH)
   24 FORMAT(///3X,'DISCHARGE LOCATIONS :'/10(2X,F7.2))
C
      WRITE(6,25)(Q(I),I=1,NREACH)
   25 FORMAT(///3X,'EFFLUENT DISCHARGE RATES (CFS) :',/10(2X,F7.2))
C
      WRITE(6,27)(XINBOD(I),I=1,NREACH)
   27 FORMAT(///3X,'PLANT INFLUENT BOD (MG/L) :',/10(2X,F7.2))
C
      WRITE(6,31)(NCNTRLP(I),I=1,NREACH)
   31 FORMAT(///3X,'NUMBER OF CONTROL POINTS PER REACH :',
     X /10(2X,I7))
C
C     COMPUTE THE CONTROL POINT LOCATTIONS IF NOT ENTERED IN DATA
C     FILE (IDATTYP=1)
C
      IF(IDATTYP .EQ. 1)CALL DATGEN(NCNTRLP,XCNTRLP,DISLOC,NREACH)
C
C     INITIALIZE ARRAYS AND SUM THE NUMBER OF CONTROL POINTS
C
      NT=0
      DO 50 I=1,NREACH
      NT=NT+NCNTRLP(I)
      NCNT(I)=NT
   50 CONTINUE
C
C
      LASTROW=NREACH + NT
      LASTCOL=2.*NREACH
C
      DO 85 I=1,LASTROW
      DO 80 J=1,LASTCOL
      DOCOEF(I,J)=0.0
      AIJBAR(I,J)=0.0
      AIJVAR(I,J)=0.0
      AIJSTD(I,J)=0.0
   80 CONTINUE
      AIOBAR(I)=0.0
      AIOVAR(I)=0.0
      AIOFIX(I)=0.0
      AIOSTD(I)=0.0
   85 CONTINUE
C
      DO 900 ITER=1,LSET
C
C      GENERATE SIMULATED STREAM PARAMETERS AND TECHNOLOGICAL COEFFICIENTS
C
      PARMEAN(4)=MEANLO
      PARMEAN(5)=MEANDO
      PARMEAN(6)=MEANQO
      PARSTD(4)=STDLO
      PARSTD(5)=STDDO
      PARSTD(6)=STDQO
C
      DO 300 IX=1,NREACH
C
      PARMEAN(1)=MEANK1(IX)
      PARMEAN(2)=MEANK2(IX)
      PARMEAN(3)=MEANU(IX)
C
      PARSTD(1)=STDK1(IX)
      PARSTD(2)=STDK2(IX)
      PARSTD(3)=STDU(IX)
      CALL SIMLPAR(CORRK2U,DSEED,PARMEAN,PARSTD,IDIST,
     X K1,K2,U,LO,DO,QO,IX)
C
```

```
  300 CONTINUE
C
C
      CALL TCHCOE1(NREACH,NT,IPOLL,LO,DO,QQ,IDATTYP,
     1 K1,K2,U,DISLOC,Q,NCNTRLP,XCNTRLP,XINBOD,
     2 LSET,ITER,DOCOEF,NCNT,QT,AIOFIX,LASTROW)
C
C     COMPUTE THE SUM OF THE SIMULATED TECHNOLOGICAL COEFFICIENTS
C     AND THEIR SQUARES
C
      JPN=0
      NFROW=0
      DO 700 J=1,NREACH
      JPN=J+NREACH
      IF(J .EQ. 1)NFROW=1
      IF(J .EQ. 1)GOTO 350
      NFROW=NCNT(J-1) + J
  350 DO 500 I=NFROW,LASTROW
      AIJBAR(I,J)=AIJBAR(I,J) + DOCOEF(I,J)
      AIJBAR(I,JPN)=AIJBAR(I,JPN) + DOCOEF(I,JPN)
      AIJVAR(I,J)=AIJVAR(I,J) + DOCOEF(I,J)**2
      AIJVAR(I,JPN)=AIJVAR(I,JPN) + DOCOEF(I,JPN)**2
  500 CONTINUE
  700 CONTINUE
C
C     COMPUTE THE SUM OF THE FIXED PORTION OF BOD AND DO AND THEIR SQUARES
C
      DO 800 I=1,LASTROW
      AIOBAR(I)=AIOBAR(I) + AIOFIX(I)
      AIOVAR(I)=AIOVAR(I) + AIOFIX(I)**2
  800 CONTINUE
C
  900 CONTINUE
C
C     COMPUTE MEAN AND VARIANCE OF THE TECNOLOGICAL COEFFICIENTS
C
      JPN=0
      NFROW=0
      DO 1100 J=1,NREACH
      JPN=J+NREACH
      IF(J .EQ. 1)NFROW=1
      IF(J .EQ. 1)GOTO 950
      NFROW=NCNT(J-1) + J
  950 DO 1000 I=NFROW,LASTROW
      AIJBAR(I,J)=AIJBAR(I,J)/LSET
      AIJBAR(I,JPN)=AIJBAR(I,JPN)/LSET
      AIJVAR(I,J)=AIJVAR(I,J)/LSET - AIJBAR(I,J)**2
      AIJVAR(I,JPN)=AIJVAR(I,JPN)/LSET - AIJBAR(I,JPN)**2
      AIJSTD(I,J)=SQRT(AIJVAR(I,J))
      AIJSTD(I,JPN)=SQRT(AIJVAR(I,JPN))
 1000 CONTINUE
 1100 CONTINUE
C
C     COMPUTE THE MEAN AND VARIANCE OF THE FIXED PORTION OF BOD AND DO
C
      DO 1200 I=1,LASTROW
      AIOBAR(I)=AIOBAR(I)/LSET
      AIOVAR(I)=AIOVAR(I)/LSET - AIOBAR(I)**2
      AIOSTD(I)=SQRT(AIOVAR(I))
 1200 CONTINUE
C
C     WRITE OUT MEAN AND VARIANCE OF THE TECHNOLOGICAL COEFFICIENTS
C     AND FIXED PORTION OF BOD AND DO
C
      DO 1300 I=1,LASTROW
      WRITE(7,1250)(AIJBAR(I,J),J=1,LASTCOL),AIOBAR(I)
 1250 FORMAT(9E10.3)
 1300 CONTINUE
```

```
      DO 1400 I=1,LASTROW
      WRITE(7,1350)(AIJVAR(I,J),J=1,LASTCOL),AIOVAR(I)
 1350 FORMAT(8E10.3)
 1400 CONTINUE
      WRITE(6,1430)
 1430 FORMAT(//3X,'AIJBAR AND AIOBAR:'/)
      DO 1500 I=1,LASTROW
      WRITE(6,1250)(AIJBAR(I,J),J=1,LASTCOL),AIOBAR(I)
 1500 CONTINUE
      WRITE(6,1510)
 1510 FORMAT(//3X,'AIJSTD AND AIOSTD:'/)
      DO 1600 I=1,LASTROW
      WRITE(6,1350)(AIJSTD(I,J),J=1,LASTCOL),AIOSTD(I)
 1600 CONTINUE
      STOP
      END
C
C     ******************************************************************
C
C     SUBROUTINE SIMLPAR IS THE SIMULATION PORTION OF THE PROGRAM WHERE
C     STREAM PARAMETERS K1,K2,U,LO,DO, AND QO ARE GENERATED
C
C     ******************************************************************
C
      SUBROUTINE SIMLPAR(CORRK2U,DSEED,PARMEAN,PARSTD,IDIST,
     1 K1,K2,U,LO,DO,QO,IX)
      DOUBLE PRECISION DSEED
      INTEGER IDIST(6),MODEL
      REAL K1,K2,U,LO,DO
      DIMENSION R(1),WK(3),PARAMTR(6),K1(10),K2(10),PARMEAN(6)
      DIMENSION PARSTD(6),U(10),PROB(30),AWIB(6),BWIB(6)
      DIMENSION SIGMA(3),RVEC(1,2),WKVEC(1),AGAM(6),BGAM(6)
      DIMENSION SLN(6),XMLN(6)
C
      NUMB=6
      NP=0
      IF(IX .GE. 2)NUMB=3
   10 NP=NP+1
      IF(NP .GT. NUMB)GOTO 250
      MODEL=IDIST(NP)
      XMEAN=PARMEAN(NP)
      XSTD=PARSTD(NP)
      GO TO(30,40,50,60),MODEL
C
C      NORMAL DISTRIBUTION
C
   30 IF(NP .EQ. 2 .AND. CORRK2U .NE. 0.0)GOTO 35
      IF(NP .EQ. 3 .AND. CORRK2U .NE. 0.0)GOTO 85
      CALL GGNML(DSEED,1,R)
      PARAMTR(NP)=XMEAN+XSTD*R(1)
C
C     CHECK TO EXCLUDE THE GENERATION OF NEGATIVE PARAMETERS
C
      IF(PARAMTR(NP) .GT. 0.0)GOTO 90
      NP=NP-1
      GO TO 10
C
C       STORE THE MEAN AND STD. DEV. OF K2 WHEN CORRELATION BETWEEN K2 AND
C        VELOCITY IS NONZERO.
C
   35 STDK2=XSTD
      XMEANK2=XMEAN
      GOTO 90
C
C       LOG-NORMAL DISTRIBUTION
C
C
C
C       COMPUTE THE MEAN (XM) AND STD. DEV. (S) OF LOG-TRANSFORMED VARIABLES.
```

```
C
   40 CV=XSTD/XMEAN
      S2=ALOG(CV*CV+1.)
      SLN(NP)=SQRT(S2)
      A=XMEAN*XMEAN/(CV*CV+1.)
      XMLN(NP)=ALOG(A)/2.
      IF(NP.EQ.2 .AND. CORRK2U.NE.0.0)GOTO 45
      IF(NP.EQ.3 .AND. CORRK2U.NE.0.0)GOTO 85
C     CALL GGNLG(DSEED,1,XM,S,R)
C     PARAMTR(NP)=R(1)
      CALL GGNML(DSEED,1,R)
      Y=XMLN(NP)+R(1)*SLN(NP)
      PARAMTR(NP)=EXP(Y)
C
CC    CHECK TO EXCLUDE THE GENERATION OF NEGATIVE PARAMETERS
C
      IF(PARAMTR(NP) .GT. 0.0)GOTO 90
      NP=NP-1
      GO TO 10
C
CC      STORE THE MEAN AND STD. DEV. OF LOG-TRANSFORMED K2 WHEN CORRELATION
CC       BETWEEN K2 AND VELOCITY IS NONZERO.
C
   45 YSTDK2=SLN(NP)
      YMEANK2=XMLN(NP)
      CVK2=CV
      GOTO 90
C
CC     GAMMA DISTRIBUTION
C
   50 XVAR=XSTD*XSTD
      BGAM(NP)=XVAR/XMEAN
      AGAM(NP)=XMEAN/BGAM(NP)
      A=AGAM(NP)
      B=BGAM(NP)
      WK(1)=0.
      CALL GGAMR(DSEED,A,1,WK,R)
      PARAMTR(NP)=R(1)*BGAM(NP)
C
CC    CHECK TO EXCLUDE THE GENERATION OF NEGATIVE PARAMETERS
C
      IF(PARAMTR(NP) .GT. 0.0)GOTO 90
      NP=NP-1
      GO TO 10
C
CC     WEIBULL DISTRIBUTION
C
   60 CV=XSTD/XMEAN
      CALL SEARCH2(XMEAN,CV,AA,BB,IT)
      AWIB(NP)=AA
      BWIB(NP)=BB
      AA=AWIB(NP)
      BB=BWIB(NP)
      CALL GGWIB(DSEED,AA,1,R)
      PARAMTR(NP)=BB*R(1)
C
CC    CHECK TO EXCLUDE THE GENERATION OF NEGATIVE PARAMETERS
C
      IF(PARAMTR(NP) .GT. 0.0)GOTO 90
      NP=NP-1
      GO TO 10
   85 IF(MODEL.EQ.1) GO TO 87
C        COMPUTE THE CORRELATION COEFFICIENT FOR LOG-TRANSFORMED K2 AND
CC        VELOCITY FROM THE CORRELATION COEFFICIENT BETWEEN K2 AND VELOCITY
CC       OF ORIGINAL SCALE.
C
      CVU=CV
      YSTDU=SLN(NP)
```

```
      YMEANU=XMLN(NP)
      F1=ALOG(1.+CORRK2U*CVK2*CVU)
      F2=ALOG(1.+CVK2*CVK2)
      F3=ALOG(1.+CVU*CVU)
      F2=SQRT(F2)
      F3=SQRT(F3)
      YCORR=F1/(F2*F3)
      SIGMA(2)=YCORR*YSTDK2*YSTDU
      SIGMA(1)=YSTDK2*YSTDK2
      SIGMA(3)=YSTDU*YSTDU
      GO TO 88
   87 STDU=XSTD
      XMEANU=XMEAN
      SIGMA(2)=CORRK2U*STDK2*STDU
      SIGMA(1)=STDK2*STDK2
      SIGMA(3)=STDU*STDU
   88 NR=1
      KPAR=2
      IR=1
      WKVEC(1)=0.0
      CALL GGNSM(DSEED,NR,KPAR,SIGMA,IR,RVEC,WKVEC,IER)
      IF(MODEL.EQ.1) GO TO 89
C
C        FIND BIVARIATE NORMAL RANDOM DEVIATES OF LOG-TRANSFORMED K2 AND
C        VELOCITY.  THEN RETRANSFORM THEM BACK TO THE ORIGINAL SCALE.
C
      YK2=RVEC(1,1)+YMEANK2
      YU=RVEC(1,2)+YMEANU
      PARAMTR(NP-1)=EXP(YK2)
      PARAMTR(NP)=EXP(YU)
C
C     CHECK TO EXCLUDE THE GENERATION OF NEGATIVE PARAMETERS
C
      IF(PARAMTR(NP-1) .GT. 0.0 .AND. PARAMTR(NP) .GT. 0.0)GOTO 90
      NP=NP-2
      GOTO 10
C
C        OBTAIN BIAVARIATE NORMAL RANDOM DEVIATES FOR K2 AND VELOCITY.
C
   89 PARAMTR(NP-1)=RVEC(1,1)+XMEANK2
      PARAMTR(NP)=RVEC(1,2)+XMEANU
C
C     CHECK TO EXCLUDE THE GENERATION OF NEGATIVE PARAMETERS
C
      IF(PARAMTR(NP-1) .GT. 0.0 .AND. PARAMTR(NP) .GT. 0.0)GOTO 90
      NP=NP-2
      GOTO 10
   90 CONTINUE
      GOTO 10
  250 K1(IX)=PARAMTR(1)
      K2(IX)=PARAMTR(2)
      U(IX)=PARAMTR(3)
      IF(IX .GT. 1)GOTO 300
      LO=PARAMTR(4)
      DO=PARAMTR(5)
      QO=PARAMTR(6)
  300 RETURN
      END
C
C     ****************************************************************
C     THIS SUBROUTINE CREATES TECHNOLOGICAL COEFFICIENTS FOR WASTE LOAD
C     ALLOCATION MODEL BASED ON FIXED CONTROL POINTS.
C     ****************************************************************
C
      SUBROUTINE TCHCOE1(NREACH,NT,IPOLL,LO,DO,QO,IDATTYP,
     1 K1,K2,U,DISLOC,Q,NCNTRLP,XCNTRLP,XINBOD,
     2 LSET,ITER,DOCOEF,NCNT,QT,AIOFIX,LASTROW)
      DIMENSION DISLOC(10),QT(10),XCNTRLP(100)
```

```
      DIMENSION DOCOEF(100,15),Q(10),NCNTRLP(10),
     X AIOFIX(100),K1(10),K2(10),U(10),NCNT(10)
      REAL LO,K1,K2,XINBOD(10)
C
C
C     CALCULATE TOTAL FLOWS IN EACH REACH
C
      QQT=QO
      DO 50 J=1,NREACH
      QT(J)=0.0
      QQT=QQT+Q(J)
      QT(J)=QQT
   50 CONTINUE
C
C
C     CALCULATE TECHNOLOGICAL COEFFICIENTS FOR DO (BASED ON BOD)
C
      NROW=1
      IM1=0
      DO 700 I=1,NREACH
      IRCNT=0
      IF(I .EQ. 1)GOTO 460
      IM1=I-1
      NROW=NCNT(IM1)+IM1+1
      DO 450 L=1,IM1
      IRCNT=IRCNT+NCNTRLP(L)
  450 CONTINUE
  460 TERM1=1.0
      TERM2=0.0
      DOCOEF(NROW,I)=0.0
      DO 600 J=I,NREACH
      DO 500 K=1,NCNTRLP(J)
      IRCNT=IRCNT+1
      X=XCNTRLP(IRCNT)-DISLOC(J)
      PART1=Q(I)/QT(J)*TERM1*K1(J)/(K2(J)-K1(J))*
     1 (EXP(-K1(J)*X/U(J))-EXP(-K2(J)*X/U(J)))
      PART2=TERM2*Q(I)/QT(J)*EXP(-K2(J)*X/U(J))
      NROW=NROW+1
      DOCOEF(NROW,I)=PART1+PART2
  500 CONTINUE
      IF(J .EQ. NREACH)GOTO 700
      XR=DISLOC(J+1)-DISLOC(J)
      FPART1=TERM1*K1(J)/(K2(J)-K1(J))*
     1 (EXP(-K1(J)*XR/U(J))-EXP(-K2(J)*XR/U(J)))
      FPART2=TERM2*EXP(-K2(J)*XR/U(J))
      NROW=NROW+1
      DOCOEF(NROW,I)=(FPART1+FPART2)*Q(I)/QT(J+1)
      TERM1=TERM1*EXP(-K1(J)*XR/U(J))
      TERM2=FPART1+FPART2
  600 CONTINUE
  700 CONTINUE
C     NR=NT+NREACH
C     DO 705 IR=1,NR
C     WRITE(7,702)(DOCOEF(IR,JC),JC=1,NREACH)
C 702 FORMAT(5(2X,E10.3))
C 705 CONTINUE
C
C     CALCULATE TECHNOLOGICAL COEFFICIENTS FOR DO (BASED ON DO DEFICITS)
C
      MM1=0
      KROW=1
      DO 1000 M=1,NREACH
      KCNTR=0
      IF(M .EQ. 1)GOTO 720
      MM1=M-1
      DO 710 J=1,MM1
      KCNTR=KCNTR+NCNTRLP(J)
  710 CONTINUE
```

```
  720 NCOL=M+NREACH
      TFDO=1.0
      DO 900 N=M,NREACH
      IF(N .EQ. M)GOTO 750
      TFDO=TFDO*EXP(-K2(N-1)*(DISLOC(N)-DISLOC(N-1))/U(N))
  750 DOCOEF(KROW,NCOL)=TFDO*Q(M)/QT(N)
      DO 800 L=1,NCNTRLP(N)
      KCNTR=KCNTR+1
      KROW=KROW+1
      TFDOX=TFDO*EXP(-K2(N)*(XCNTRLP(KCNTR)-DISLOC(N))/U(N))
      DOCOEF(KROW,NCOL)=TFDOX*Q(M)/QT(N)
  800 CONTINUE
      KROW=KROW+1
  900 CONTINUE
      KROW=NCNT(M)+M+1
 1000 CONTINUE
C
C     DETERMINE FIXED PORTION OF BOD AND DO
C
      DO 1200 I=1,LASTROW
      AIOFIX(I)=DOCOEF(I,1)*QO*LO/QT(1) + DOCOEF(I,NREACH+1)*QO*DO/QT(1)
 1200 CONTINUE
      RETURN
      END
C
C
C
C     ********************************************************
C     SUBROUTINE DATGEN IS TO GENERATE CONTROL POINT LOCATIONS
C     ********************************************************
      SUBROUTINE DATGEN(NCNTRLP,XCNTRLP,DISLOC,NREACH)
      DIMENSION NCNTRLP(10),XCNTRLP(100),DISLOC(10)
      MCNTR=0
      DO 100 I=1,NREACH
      IF(I .EQ. NREACH) DISLOC(I+1)=DISLOC(I)+(DISLOC(I)-DISLOC(1))
     1 /(NREACH-1)
      DO 50 J=1,NCNTRLP(I)
      MCNTR=MCNTR+1
      XCNTRLP(MCNTR)=(DISLOC(I+1)-DISLOC(I))/(NCNTRLP(I)+1)*J
     1 +DISLOC(I)
   50 CONTINUE
  100 CONTINUE
      RETURN
      END
C
C     ************************************************
C     SUBROUTINE SEARCH2 IS WHERE THE PARAMETERS OF THE
C     WEIBULL DISTRIBUTION ARE IDENTIFIED USING THE METHOD
C     OF MOMENTS
C     ************************************************
C
      SUBROUTINE SEARCH2(XMEAN,XCV,ALPHA,BETA,IT)
      ALPHA=5.0
      ITMAX=20
      XRIGHT=1.+XCV*XCV
      TOLER=0.01*XRIGHT
      IT=0
   10 IT=IT+1
      IF(IT.GT.ITMAX) GO TO 20
      A1=1./ALPHA
      A2=2.*A1
      A2P1=A2+1.
      A1P1=A1+1.
      T2=GAMMA(A2P1)
      T1=GAMMA(A1P1)
      XLEFT=T2/(T1*T1)
      DIF=XLEFT-XRIGHT
      DIF=ABS(DIF)
```

```
      IF(DIF.LE.TOLER) GO TO 20
      R=XLEFT/XRIGHT
      ALPHA=ALPHA*SQRT(R)
      GO TO 10
   20 CONTINUE
      BETA=XMEAN/T1
C     WRITE(7,25) IT,ALPHA,DIF,BETA
C  25 FORMAT(2X,'SEARCH 2',5X,'IT=',I3,3X,'ALPHA=',E12.6,
C    X 2X,'DIF=',E12.6,2X,'BETA=',E12.6)
      IF(ALPHA.LE.1.) WRITE(7,26)
   26 FORMAT(2X,'*** WARNING : ALPHA IS LESS THAN 1. ***')
      RETURN
      END
```

APPENDIX K

LISTING OF COMPUTER PROGRAM FOR

STOCHASTIC OPTIMAL WASTE LOAD ALLOCATION

USING CHANCE-CONSTRAINED FORMULATION

'WLASTO'

```
      PROGRAM WLASTO(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,
     X TAPE7,TAPE8)
C
C  THIS PROGRAM WAS WRITTEN BY WADE HATHHORN AND DR. Y.K. TUNG WITH
C  FINAL REVISIONS BEING MADE ON SEP. 10, 1986.  THIS PROGRAM IS
C  DESIGNED TO OPTIMALLY ALLOCATE WASTE DISCHARGE FROM SEVERAL
C  INDUSTRIAL USERS IN A STREAM ENVIRONMENT CONTAINING UNCERTAINTY.
C
C  SOLUTIONS FROM THIS PROGRAM ARE DERIVED FROM LINEAR PROGRAMMING
C  TECHNIQUES IN WHICH THE OBJECTIVE FUNCTION IS TO MAXIMIZE
C  WASTE DISCHARGE BASED ON EFFLUENT BOD AND OR DO DEFICIT CONCENTRATIONS.
C  WATER QUALITY CONSTRAINTS ARE PLACED IN THE MODEL BY UTILIZING
C  THE STREETER-PHELPS EQUATION FOR DO DEFICITS AND FIXED CONTROL
C  POINTS.  ADDITONALLY, EQUITY CONSTRAINTS ARE INCLUDED IN AN ATTEMPT
C  TO ADDRESS SOME OF THE ECONOMIC CONSIDERATIONS IN SUCH A
C  DECISION MAKING PROCESS.
C
C  THE ESSENCE OF THIS PROGRAM IS TO SET UP THE OPTIMIZATION PROBLEM
C  IN A LINEAR PROGRAMMING FORMAT.  THIS PROGRAM THEN UTILIZES SEVERAL
C  CALLS TO A 'CANNED PROGRAM - XMP' IN ORDER TO SOLVE THE LINEAR
C  PROGRAMMING PROBLEM.
C
C  IN ORDER TO SOLVE THE PROBLEM OUTLINED BY THIS PROGRAM, INITIAL VALUES
C  FOR THE SOLUTION MAY BE SELECTED OR GENREATED BASED ON THE INFLUENT
C  BOD LOADINGS FROM EACH DISCHARGER INPUT IN THE DATA FILE.  THEN
C  THROUGH SUCCESSIVE ITERATIONS, THE LINEAR PROGRAMMING PROBLEM IS
C  SOLVED UNTIL CONVERGENCE CRITERIA BETWEEN THE ASSUMED AND ACTUAL
C  EFFLUENT (SOLUTIONS TO THE PROBLEM) CONCENTRATIONS ARE MET.
C
C  IN ADDITON TO A COMPILED VERSION OF 'XMP',A SINGLE INPUT FILE
C  AND THE TECHNOLOGICAL COEFFICIENT ARRAYS FROM 'STOCOEF' ON
C  TAPE7 MUST BE PROVIDED IN ORDER TO EXECUTE THIS PROGRAM.  THE INPUT
C  FILE MUST CONTAIN THE FOLLOWING INFORMATION AND FORMAT:
C
C
C
C    CARD        VARIABLES              DESCRIPTION                    FORMAT
C
C      1       NREACH,IPOLL,        NREACH=NO. OF REACHES          6I8,3F8.0
C           ISOLVE,IDIST,IEQUITY    IPOLL=INDICIES FOR TYPE
C             MAXITER,ALPHA               OF POLLUTION TO
C             EPSILON,EQDIF                BE CONSIDERED
C                                          (0)=BOD ONLY
C                                          (1)=BOD AND DO DEFICITS
C
C                                  ISOLVE=IDICIES TO GENERATE
C                                         INITIAL SOLUTIONS
C                                         (0)=INPUT IN DATA FILE
C                                         (1)=COMPUTER GENERATED
C
C                                  IDIST=TYPE OF DISTRIBUTION
C                                        TO ASSUMED FOR ALPHA
C                                        (1) = NORMAL
C                                        (2) = LOG-NORMAL
C
C                                  IEQUITY=TO INDICATE TYPE OF
C                                          EQUITY TO BE CONSIDERED
C                                          (0) = EQUAL % REMOVAL
C                                          (1) = EQUAL EFFLUENT CONCEN.
C
C                                  MAXITER=MAX. NO. OF ITERATIONS
C                                          TO BE PERFORMED TO
C                                          ALLOW CONVERGENCE
C
C                                  ALPHA=RELIABILITY REQUIREMENT
C                                        FOR MEETING WATER QUALITY
C                                        REGULATIONS
C
```

```
C                              EPSILON=CONVERGENCE CRITERIA
C                                      BETWEEN THE ANSWERS
C                                      OBTAINED FOR EACH
C                                      DISCHARGER IN SUCCESSIVE
C                                      ITERATIONS
C
C                             EQDIF=NUMERICAL VALUE CORRESPONDING
C                                   TO THE TYPE OF EQUITY
C                                   CONSIDERED
C
C     NOTE : IF YOU SELECT IEQUITY EQUAL TO :
C            (0) EQDIF REPRESENTS THE ALLOWABLE DIFFERENCE
C                IN PERCENT REMOVAL BETWEEN DISCHARGERS
C
C            (1) EQDIF REPRESENTS THE ALLOWALBLE DIFFERENCE
C                IN EFFLUENT CONCENTRATIONS BETWEEN DISCHARGERS
C                (MG/L)
C
C
C       IF YOU INPUT ISOLVE=0, YOU MUST PROVIDE THE INITIAL
C       SOLUTIONS (ESTIMATION) IN AN ADDTIONAL CARD AT THE
C       BOTTOM OF THE INPUT FILE (SEE CARD 11 FOR DETAILS)
C
C     MAXITER IS AN INTEGER VALUE USUALLY BETWEEN 10 AND 20
C
C     ALPHA VALUE SHOULD BE ENTERED AS A DECIMAL FRACTION
C     EXPRESSING THE ALLOWABLE RELIABILITY FOR MEETING
C     THE WATER QUALITY STANDARDS GIVEN IN THE
C     TECHNOLOGICAL COEFFICIENTS
C
C     EPSILON VALUE SHOULD BE ENTERED AS THE ALLOWABLE PERCENT
C     DIFFERENCE BETWEEN THE ANSWERS OBTAINED IN SUCCESSIVE
C     ITERATIONS  (ENTER IN DECIMAL FRACTION VALUES)
C
C  2          DOSAT(I)          DOSAT=DISSOL. OXYGEN SAT.     10F8.0
C                                     IN EACH REACH MG/L
C
C  3          DOSTD(I)          DOSTD=DISSOL. OXYGEN STD.     10F8.0
C                                     IN EACH REACH MG/L
C
C  4          NCNTRLP(I)        NCNTRLP=NO. OF CONTROL PTS.   10I8
C                                     IN EACH REACH
C
C  5          XINBOD(I)        XINBOD=INFLUENT BOD CONC.      10F8.0
C                                     IN MG/L
C
C  6          UPBTRT(I)         UPBTRT=UPPER BOUNDS FOR       10F8.0
C                                     BOD TREATMENT
C                                     (% REMOVAL AS A
C                                      DECIMAL FRACTION)
C
C  7          LOWBTRT(I)        LOWBTRT=LOWER BOUNDS FOR      10F8.0
C                                     BOD TREATMENT
C                                     (% REMOVAL AS A
C                                      DECIMAL FRACTION)
C
C  8         UPBDOD(I)          UPBDOD=UPPER BOUNDS FOR       10F8.0
C                                     DO DEFICITS IN
C                                     EACH EFFL. MG/L
C
C  9         LOWBDOD(I)         LOWBDOD=LOWER BOUNDS FOR      10F8.0
C                                     DO DEFICITS IN
C                                     EACH EFFL. MG/L
C
C
C     NOTE: IF YOU HAVE A TRIBUTARY AS ONE OF THE DISCHARGE POINTS,
C           YOU MUST ENTER THE APPROPRIATE BOUNDS ON TREATMENT AND
C           DO DEFICITS (.IE. 0.0 TREATMENT AND PREDETERMINED DEFICIT)
```

```
C
C     10         IDISCH(I)              ISDICH=INDICIES FOR        10I8
C                                              TYPE OF DISCHARGE
C                                              POINT
C                                              (0) = TRIBUTARY
C                                              (1) = PLANT EFFL.
C
C  IF YOU ENTERED ISOLVE=0, YOU MUST INCLUDE ANOTHER CARD TO INPUT
C  THE INITIAL SOLUTION (CARD 11)
C
C     11          SOLOLD(I)             SOLOLD=INITIAL SOLUTION      10F8.0
C                                              ESTIMATIONS OF
C                                              EFFLUENT BOD IN
C                                              EACH REACH MG/L
C
C
C     NOTTE:  CARDS 2 THRU 10 CONTAIN VALUES TO BE READ IN AS
C             ARRAYS.   ONE OF EACH OF THE VALUES ON THESE CARDS
C             SHOULD BE ENTERED FOR EACH REACH OR DISCHARGER.
C
C*************************************************************************
C
C
      DIMENSION AIOBAR(100),AIOVAR(100),AIJVAR(100,15),DOD(10)
      DIMENSION RHS(100),DOSAT(10),NCNTRLP(10),NCNT(10),DOSTD(10)
      DIMENSION TCHCOEF(100,15),BOD(20),UPBTRT(10),XINBOD(10),
     X PCTREMV(10),UPBDOD(10),DEF(100),SOLOLD(15),SOLNEW(15)
      REAL LOWERB(20),UPPERB(20),UPPER(20),LJ,UJ
      REAL BASCB(100),BASLB(100),BASUB(100),BOUND,BLOW(100),
     X     CANDA(100,6),CANDCJ(6),CJ,COLA(100),MEMORY(10000),
     X     UZERO(100),XBZERO(100),YQ(100),Z,CJX(20),VALUE(100),
     X    LOWBTRT(10),LOWBDOD(10)
      INTEGER BNDTYP,COLLEN,COLMAX,FACTOR,IOERR,IOLOG,ITER1,ITER2,
     X        LENMA,LENMI,LENMY,LOOK,M,MAPA(50),MAPI(50),MAXA,MAXM,
     X        MAXN,N,NTYPE2,P,PRINT,TERMIN,UNBDIQ,BASIS(100),CAND(6),
     X        CANDI(100,6),CANDL(6),COLI(100),ROWTYP(100),STATUS(100)
      INTEGER IOIN,NCOLSA,IDISCH(10)
      COMMON/XMPCOM/BIG,SMALL,ZL,ZLC,EPS1,EPS2,EPS3,EPS4,EPS5,EPS6
      REAL BIG,SMALL,ZL,ZLC,EPS1,EPS2,EPS3,EPS4,EPS5,EPS6
C
C
C     READ IN DATA FILES:  TAPE5
C
C
      READ(5,5)NREACH,IPOLL,ISOLVE,IDIST,IEQUITY,MAXITER,ALPHA,
     X EPSILON,EQDIF
    5 FORMAT(6I8,3F8.0)
      READ(5,7)(DOSAT(I),I=1,NREACH)
      READ(5,7)(DOSTD(I),I=1,NREACH)
    7 FORMAT(10F8.0)
      READ(5,8)(NCNTRLP(I),I=1,NREACH)
    8 FORMAT(10I8)
      READ(5,7)(XINBOD(I),I=1,NREACH)
      READ(5,7)(UPBTRT(I),I=1,NREACH)
      READ(5,7)(LOWBTRT(I),I=1,NREACH)
      READ(5,7)(UPBDOD(I),I=1,NREACH)
      READ(5,7)(LOWBDOD(I),I=1,NREACH)
      READ(5,8)(IDISCH(I),I=1,NREACH)
      LASTCOL=2*NREACH
      IF(ISOLVE .EQ. 0)READ(5,7)(SOLOLD(I),I=1,LASTCOL)
C
C     WRITE HEADING INFORMATION ON OUTPUT
C
      WRITE(6,10)NREACH,IPOLL,ISOLVE,ALPHA,IEQUITY,EQDIF
   10 FORMAT(1H1/3X,'STOCHASTIC WLA INFORMATION :',
     X //3X,'NREACH = ',I6,10X,'IPOLL = ',I6,
     X /3X,'ISOLVE = ',I6,10X,'ALPHA = ',F6.2,
     X /3X,'IEQUITY = ',I6,9X,'EQDIF = ',F6.2)
```

```
C
      WRITE(6,11)(IDISCH(I),I=1,NREACH)
   11 FORMAT(//3X,'TYPE OF DISCHARGE (TREATMENT PLANT OR TRIBUTARY):',
     X /10(3X,I6))
C
      WRITE(6,12)(NCNTRLP(I),I=1,NREACH)
   12 FORMAT(//3X,'NUMBER OF CONTROL POINTS IN EACH REACH:',
     X /10(3X,I6))
C
      WRITE(6,13)(XINBOD(I),I=1,NREACH)
   13 FORMAT(//3X,'PLANT INFLUENT BOD CONCENTRATIONS IN MG/L:',
     X /10(3X,F9.2))
C
C     COMPUTE TOTAL NUMBER OF CONTROL PONITS AND EQUITY ROWS
C
      NDISCH=0
      NT=0
      DO 14 I=1,NREACH
      NT=NT+NCNTRLP(I)
      NCNT(I)=NT
      NDISCH=NDISCH+IDISCH(I)
   14 CONTINUE
C
C     COMPUTE ALLOWABLE DO DEFICIT IN EACH REACH
C
      IC=1
      DO 16 I=1,NREACH
      IF(I .GT. 1)IC=NCNT(I-1)+I
      IPNCNT=I+NCNT(I)
      DO 15 J=IC,IPNCNT
      DEF(J)=DOSAT(I)-DOSTD(I)
   15 CONTINUE
   16 CONTINUE
C
      LASTROW=NT+NREACH
      NEQROW=(NDISCH**2 - NDISCH)/2
      MBOT=LASTROW+NEQROW
C
C     INITIALIZE ARRAYS
C
      DO 18 I=1,MBOT
      DO 17 J=1,LASTCOL
      TCHCOEF(I,J)=0.0
   17 CONTINUE
   18 CONTINUE
      DO 20 I=1,LASTROW
      DO 19 J=1,LASTCOL
      AIJVAR(I,J)=0.0
   19 CONTINUE
      AIOBAR(I)=0.0
      AIOVAR(I)=0.0
   20 CONTINUE
C
C     READ IN INFORMATION FROM 'STOCOEF':  TAPE7
C
C
C        READ IN MEAN TECHNOLOGICAL COEFFICIENTS
C
      DO 25 I=1,LASTROW
      READ(7,22)(TCHCOEF(I,J),J=1,LASTCOL),AIOBAR(I)
   22 FORMAT(8E10.3)
   25 CONTINUE
C
C        READ IN VARIANCE OF TECHNOLOGICAL COEFFICIENTS
C
      DO 30 I=1,LASTROW
      READ(7,22)(AIJVAR(I,J),J=1,LASTCOL),AIOVAR(I)
   30 CONTINUE
```

```
C
C       COMPUTE EQUITY COEFFICIENTS
C
      CALL EQCOEF(NEQROW,TCHCOEF,RHS,IEQUITY,NREACH,BLOW,EQDIF,
     X LASTROW,IDISCH,XINBOD)
C
C     DO 9999 I=1,MBOT
C     WRITE(8,9998) (TCHCOEF(I,J),J=1,LASTCOL)
C9998 FORMAT(12(1X,E10.3))
C9999 CONTINUE
C     SET UP AN ARRAY FOR UPPER AND LOWER BOUNDS ON DECISION VARIALBLES
C
      KJ=NREACH
      DO 38 IJ=1,NREACH
      KJ=KJ+1
      UPPERB(IJ)=(1-LOWBTRT(IJ))*XINBOD(IJ)
      UPPERB(KJ)=UPBDOD(IJ)
      LOWERB(IJ)=(1-UPBTRT(IJ))*XINBOD(IJ)
      LOWERB(KJ)=LOWBDOD(IJ)
   38 CONTINUE
C
      IF(ISOLVE .EQ. 0)GOTO 50
      DO 40 JC=1,LASTCOL
      RANGE=UPPERB(JC)-LOWERB(JC)
      SOLOLD(JC)=LOWERB(JC)
   40 CONTINUE
   50 CONTINUE
C
      IOIN=5
      IOERR=6
      IOLOG=6
      IOOUT=6
C
C     SET UP VALUES REQUIRED TO EXECUTE 'XMP'
C
      IPRINT=2
      BNDTYP=4
      NTYPE=0
      BOUND=0.
      MAXM=MBOT
      MAXN=2*NREACH + MAXM
      MAXA=MAXM*MAXN
      COLMAX=MAXM
      P=6
      LOOK=2*NREACH
      FACTOR=50
      LENMY=10000
      PRINT=1
C
C     PRINT = (0)  ERROR MESSAGES ONLY
C             (1)  TERMINATION CONDITION MESSAGE
C             (2)  PRINT OBJECTIVE FUNCTION VALUE AFTER
C                  EACH BASIS RE-FACTORIZATION
C             (3)  LOG TRANSFORM AT EVERY ITERATION
C
      NCOLSA=2*NREACH
      M=MAXM
C
      CALL XMAPS(BNDTYP,IOERR,LENMA,LENMI,LENMY,MAPA,MAPI,MAXA,MAXM,
     X           MAXN,MEMORY)
C
C     SET UP ROW TYPES
C
      DO 60 IR=1,MAXM
      ROWTYP(IR)=1
      IF(IR .GT. LASTROW .AND. IR .LE. MBOT)ROWTYP(IR)=2
   60 CONTINUE
C
```

```
C     SET UP OBJECTIVE FUNCTION COEFFICIENTS
C
      DO 80 JC=1,NCOLSA
      CJX(JC)=1.0
      IF(JC .GT. NREACH .AND. IPOLL .EQ. 0)CJX(JC)=0.0
   80 CONTINUE
      ITER=0
      ISTOP=0
   85 ITER=ITER+1
      IF(ITER .GT. MAXITER)WRITE(6,87)MAXITER
   87 FORMAT(//3X,'THE NUMBER OF ITERATIONS EXCEEDS ',I6)
      IF(ITER .GT. MAXITER) GOTO 490
C
C     SET UP RIGHT HAND SIDE
C
      DO 100 IR=1,LASTROW
      COVIR=0.
      VARIR=AIOVAR(IR)
      RHS(IR)=0.0
      BARIR=AIOBAR(IR)
      DO 90 JC=1,LASTCOL
      VARIR=VARIR + AIJVAR(IR,JC)*SOLOLD(JC)**2
      BARIR=BARIR + SOLOLD(JC)*TCHCOEF(IR,JC)
   90 CONTINUE
      STDIR=SQRT(VARIR)
      COVIR=STDIR/BARIR
C
      CALL QUANTIL(IDIST,ALPHA,COVIR,ZALPHA)
C
      RHS(IR)=DEF(IR) - AIOBAR(IR) - STDIR*ZALPHA
  100 CONTINUE
C
C
C     ENTER IN OBJECTIVE FUNCTION COEFFICIENTS AND ADD IN COLUMNS
C     TO CREATE SIMPLEX TABLEAU
C
      N=0
      DO 130 JC=1,NCOLSA
      CJ=CJX(JC)
      UJ=UPPERB(JC)
      LJ=LOWERB(JC)
      COLLEN=MAXM
      DO 120 IR=1,COLLEN
      COLI(IR)=IR
      COLA(IR)=TCHCOEF(IR,JC)
  120 CONTINUE
C
      CALL XADDUB(BNDTYP,IOERR,JC,LENMA,LENMY,LJ,MAPA,MEMORY,UJ)
      CALL XADDAJ(CJ,COLA,COLI,COLLEN,COLMAX,IOERR,JC,
     X            LENMA,LENMY,MAPA,MEMORY,N)
  130 CONTINUE
      DO 140 JC=1,N
  140 STATUS(JC)=0
C
C
C     CREATE SLACK VARIABLES
C
      Z=0.0
      CALL XSLACK(RHS,BASCB,BASIS,BASLB,BASUB,BLOW,BNDTYP,BOUND,
     X COLA,COLI,COLMAX,IOERR,LENMA,LENMI,LENMY,
     X M,MAPA,MAPI,MAXM,MAXN,MEMORY,N,ROWTYP,STATUS,
     X UZERO,XBZERO,Z)
C
C
C
C     PERFORM PRIMAL SIMPLEX ITERATIONS
C
      CALL XPRIML(RHS,BASCB,BASIS,BASLB,BASUB,BNDTYP,BOUND,
```

```
     X CAND,CANDA,CANDCJ,CANDI,CANDL,COLA,COLI,COLMAX,
     X FACTOR,IOERR,IOLOG,ITER1,ITER2,LENMA,LENMI,LENMY,LOOK,
     X M,MAPA,MAPI,MAXM,MAXN,MEMORY,N,NTYPE2,P,PRINT,
     X STATUS,TERMIN,UNBDDQ,UZERO,XBZERO,YQ,Z)
C
C
      WRITE(IOLOG,200) TERMIN
  200 FORMAT(18H TERMINATION CODE=,I4)
C
C     CHECK TERMINATION CODE FOR INFEASIBLE SOLUTIONS AND
C     TERMINATE PROGRAM IF THREE OR MORE INFEASIBLE SOLUTIONS
C     OCCUR
C
      IF(TERMIN .EQ. 3)ISTOP=ISTOP+1
      IF(ISTOP .GE. 3)WRITE(6,203)
  203 FORMAT(/3X,'****WARNING****',//3X,'THE EXECUTION OF XMP HAS ',
     X 'FOUND THREE OR MORE INFEASIBLE SOLUTIONS',/3X,'WITH SINCERE ',
     X 'REGRETS, YOUR PROGRAM HAS BEEN TERMINATED -- OOOPPS')
C
      IF(ISTOP .GE. 3)GOTO 490
C
      IF(IPRINT .EQ. 0 .OR. IPRINT .EQ. 1)GOTO 220
C
C
      CALL XPRINT(BASIS,BNDTYP,BOUND,IOERR,IOLOG,
     X LENMA,LENMY,M,MAPA,MAXM,MAXN,MEMORY,N,NTYPE2,
     X STATUS,XBZERO,Z)
C
C
C     WRITE(6,210) (STATUS(J),J=1,N)
C 210 FORMAT(//5X,"STATUS :"/8(I5,2X))
  220 CONTINUE
C
      DO 260 J=1,NCOLSA
      IF(STATUS(J))230,240,250
  230 SOLNEW(J)=UPPERB(J)
      GOTO 260
  240 SOLNEW(J)=LOWERB(J)
      GOTO 260
  250 IX=STATUS(J)
      SOLNEW(J)=XBZERO(IX)
  260 CONTINUE
C
C     COMPARE SOLUTIONS BETWEEN TWO SUCCESSIVE ITERATIONS
C
      IFLAG=0
      DO 270 J=1,NREACH
      DIFF=ABS(SOLOLD(J) - SOLNEW(J))
      CHANG=DIFF/SOLOLD(J)
      IF(CHANG .GT. EPSILON)IFLAG=IFLAG+1
  270 CONTINUE
      IF(IFLAG .EQ. 0)GOTO 300
C
C     UPDATE SOLUTIONS AND RETURN TO COMPUTE ANOTHER ITERATION
C
      DO 280 J=1,NCOLSA
      SOLOLD(J)=(SOLOLD(J) + SOLNEW(J))/2.
  280 CONTINUE
      GOTO 85
C
C     COMPUTE FINAL ANSWERS
C
  300 CONTINUE
      DO 370 J=1,NREACH
      IF(STATUS(J)) 340,350,360
  340 BOD(J)=UPPERB(J)
      GO TO 370
  350 BOD(J)=LOWERB(J)
```

```
      GO TO 370
  360 IX=STATUS(J)
      BOD(J)=XBZERO(IX)
  370 CONTINUE
      JDOD=0
      NP1=NREACH+1
      DO 410 J=NP1,LASTCOL
      JDOD=JDOD+1
      IF(STATUS(J))380,390,400
  380 DOD(JDOD)=UPPERB(J)
      GOTO 410
  390 DOD(JDOD)=LOWERB(J)
      GOTO 410
  400 IX=STATUS(J)
      DOD(JDOD)=XBZERO(IX)
  410 CONTINUE
      DO 415 LX=1,NREACH
      PCTREMV(LX)=(XINBOD(LX)-BOD(LX))/XINBOD(LX)*100.0
  415 CONTINUE
      WRITE(IOOUT,420)(I,I=1,NREACH)
  420 FORMAT(/9X,6(6X,'REACH',I1))
      WRITE(IOOUT,430)(BOD(J),J=1,NREACH)
  430 FORMAT(/7X,'BOD',10(3X,E9.4))
      WRITE(IOOUT,440)(DOD(J),J=1,NREACH)
  440 FORMAT(/7X,'DOD',10(3X,E9.4))
      WRITE(IOOUT,450)(PCTREMV(J),J=1,NREACH)
  450 FORMAT(/2X,'%REMOVAL',5X,8(F5.2,7X))
      IF(IEQUITY .EQ. 0)WRITE(IOOUT,460)
      IF(IEQUITY .EQ. 1)WRITE(IOOUT,470)
  460 FORMAT(//2X,'THE EQUITY CONSIDERED IS EQUAL % REMOVAL')
  470 FORMAT(//2X,'THE EQUITY CONSIDERED IS EQUAL EFFLUENT CONC.')
      WRITE(6,480)ITER
  480 FORMAT(//2X,'THE NUMBER OF ITERATIONS TO REACH THIS IS ',I6)
  490 CONTINUE
      STOP
      END
C
C
C     ******************************************************************
C     SUBROUTINE QUANTIL IS WHERE THE QUANTILES FOR VARIOUS PROBABILITY
C     DISTRIBUTIONS ARE COMPUTED
C     ******************************************************************
C
      SUBROUTINE QUANTIL(IDIST,ALPHA,COV,ZALPHA)
C
C     NORMAL DISTRIBUTION
C
      P=ALPHA
C
      GOTO (10,10)IDIST
C
   10 CALL MDNRIS(P,Z1,IER)
      ZALPHA=Z1
      IF(IDIST .EQ. 1)GOTO 120
C
C     LOG-NORMAL DISTRIBUTION
C
      C=1. + COV*COV
      CLN=ALOG(C)
      B=Z1*SQRT(CLN)
      ZALPHA=(EXP(B) - SQRT(C))/(COV*SQRT(C))
  120 CONTINUE
      RETURN
      END
C
C     ******************************************************************
C     SUBROUTINE EQCOEF IS WHERE THE EQUITY COEFFICIENTS ARE COMPUTED
C     ******************************************************************
```

```
C
      SUBROUTINE EQCOEF(NEQROW,TCHCOEF,RHS,IEQUITY,NREACH,BLOW,
     X EQDIF,LASTROW,IDISCH,XINBOD)
      DIMENSION TCHCOEF(100,15),BLOW(100),RHS(100),IDISCH(10),
     X XINBOD(10)
C
C
C     DETERMINE EQUITY COEFFICIENTS
C
      ICC=LASTROW
      LLL=1
      MNN=0
      NRR=NREACH-1
      DO 1030 MNN=1,NRR
      LLL=LLL+1
      IF(IDISCH(MNN) .EQ. 0) GO TO 1030
      DO 1020 JNN=LLL,NREACH
      IF(IDISCH(JNN) .EQ. 0) GO TO 1020
      ICC=ICC+1
      IF(IEQUITY .EQ. 0)TCHCOEF(ICC,MNN)=1/XINBOD(MNN)
      IF(IEQUITY .EQ. 0)TCHCOEF(ICC,JNN)=-1/XINBOD(JNN)
      IF(IEQUITY .EQ. 1)TCHCOEF(ICC,MNN)=1.0
      IF(IEQUITY .EQ. 1)TCHCOEF(ICC,JNN)=-1.0
C
C     CREATE RHS FOR EQUITY
C
      RHS(ICC)=EQDIF
C
C     CREATE BLOW FOR LOWER BOUNDS ON EQUITY
C     THESE VALUES ARE USED IN TWO-SIDED CONSTRAINTS
C     DEFINED IN THE MAIN PROGRAM
C
      BLOW(ICC)=-EQDIF
 1020 CONTINUE
 1030 CONTINUE
      RETURN
      END
```