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Abstract: The linear form of the Muskingum model has been widely applied to river flood routing. However, a nonlinear relationship between storage and discharge exists in most actual river systems, making the use of the linear model inappropriate. In this paper, a nonlinear Muskingum model is solved using the state variable modeling technique. Various curve fitting techniques are employed for the calibration of model parameters, and their performances within the model are compared. Both linear and nonlinear models are applied to an example with pronounced nonlinearity between storage and discharge. The results show that the nonlinear Muskingum model is superior to the linear one.

Introduction

It is well-recognized by hydrologists and water resource engineers that river flood routing models have a wide spectrum of sophistication. The Muskingum method (15), which represents a linear reservoir concept, is an example of the simplest form. The full-scale dynamic wave model (the Saint-Venant Equations) (8) is an example of the most sophisticated form. The amount of time and effort required to implement, calibrate, and solve the selected model increases with the degree of model sophistication. Although a sophisticated model usually provides more accurate results, its use is justified only when there are sufficient data of good quality available. Thus, a tradeoff must be made in the selection of a flood routing model based upon the quality of given data, the social or economic importance of the project, the fiscal constraints, and the safety requirements.

Because of its simplicity, among the many models used for flood routing in natural channels and rivers, the Muskingum model has been one of the most frequently used tools. The most common form of the Muskingum model (herein referred to as the linear Muskingum model) is

$$S_t = K[xI_t + (1 - x)O_t] \quad (1)$$

in which S_t = the absolute channel storage at time t; I_t and O_t = the rates of inflow and outflow at time t, respectively; K = the storage time constant for the river reach, which has a value reasonably close to the flow travel time within the river reach; and x = a weighting factor varying between 0 and 0.5. Strupczewski and Kundzewicz (31) have recently shown that the theoretical values of x range from $-\infty$ to 0.5. To perform channel flood routing, Eq. 1 is solved in conjunction with the continuity equation

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$$\dot{S}_t = \frac{dS_t}{dt} = I_t - O_t \qquad (2)$$

in which $\dot{S}_t$ = the time rate of change of channel storage at time t. The numerical solution of Eqs. 1 and 2 results in the well-known Muskingum routing equation as

$$O_t = C_0 I_t + C_1 I_{t-1} + C_2 O_{t-1} \qquad (3)$$

in which C_0, C_1, and C_2 = coefficients that are functions of K, x, and discretized time interval Δt; $C_0 + C_1 + C_2 = 1$.

After the introduction of the linear Muskingum model, there have been a significant number of studies done both on the model and the implications involved in its use (2,9–10,11,13,14,22–25,31,32,34). The application of the Muskingum model basically involves two steps: calibration and prediction. The calibration procedure, in essence, is centered on model parameter identification using historical inflow-outflow data. Conventionally, parameters K and x in the linear model are graphically estimated by a trial and error procedure, which is subjective and inefficient. Several methods utilizing curve-fitting techniques such as least squares, linear programming, and other statistical methods were recently proposed to enhance the efficiency of the calibration procedure (9,19,29,30). Finally, prediction with the model is simply a straightforward application of the routing equation given by Eq. 3.

In natural channel reaches, it is not uncommon to observe a nonlinear storage-discharge relationship as opposed to a linear one assumed by Eq. 1. Under such circumstances, the use of the linear Muskingum model could result in significant error in the prediction of flood levels. There have been several methods proposed to address this nonlinear behavior by considering the values of K and x, in the linear model, to vary both with respect to time and space (13,26,27). Napiorkowski, et al. (22) recently derived a lumped nonlinear state model from hydrodynamics. The linearization of the resulting model was found to be equivalent to the linear Muskingum model. Their physical-based approach yields functional relationships between model parameters and hydrodynamic characteristics of the system.

Alternatively, the formulation of the linear model can be modified to account for nonlinearity by writing

$$S_t = \alpha[xI_t + (1 - x)O_t]^m \qquad (4)$$

in which α and m = constants. Eq. 4, as compared with Eq. 1, has more degrees of freedom, which presumably would yield a closer fit to the nonlinear relation between storage and discharge. However, because of the presence of nonlinearity in the equation, the calibration procedure becomes more complicated. Furthermore, the routing procedures for flood prediction will no longer be as straightforward as those for using the linear model. Nonlinear forms of the Muskingum models such as Eq. 4 and others can be found in hydrology texts. However, with the exception of Gill's recent works (9), the solution procedures for such nonlinear models have never been mentioned or developed. The routing technique, proposed by Gill, for solving Eq. 4 requires a trial and error solution of a system of nonlinear equations at each time step. The technique, could be very time consuming if several time steps are involved.

In this paper, a routing technique is proposed for the nonlinear model expressed by Eq. 4 using the concept of state variable modeling. By taking advantage of this concept, the trial and error procedure to obtain a solution is eliminated. Three techniques for parameter estimations are employed and their performances are compared.

State Variable Modeling Concept

The concept of state variable modeling was developed primarily to analyze automatic control systems in the field of electrical engineering (3). It is capable of describing systems which are linear or nonlinear, time-variant or time-invariant, deterministic or stochastic, while having multiple inputs and outputs at the same time (18). For a system to be solvable by the state variable modeling analysis, it must be lumped. In other words, a system must be represented in only one dimension such as time or space and must be describable by ordinary differential or difference equations. Water resource systems are usually distributed, but they can be approximated by dividing the entire system into subsystems, which may be individually treated as a lumped system. Also, water resource systems are dynamic in nature with the inputs, outputs, and throughputs varying with respect to time.

State variable modeling follows the "modern system" theory in which the input space is first related to the state space through the state equation (Fig. 1). Then the state space, and in some cases in input space, is related to the output space through the output equation. The state equation is used to describe the change in the state of system with respect to time in response to various inputs. The output equation is used to relate the output to the state of the system and, in some cases, to the inputs. In state variable modeling, the system structure is given explicit representation as a state vector $\underline{X}$, where $\underline{X} = (X_1, X_2, \ldots X_n)$ and the state variables $X_1, X_2, \ldots, X_n$ are functions of time or space, or both.

In water resources systems, the state variables are usually expressed in volumetric or mass units and can represent, e.g., the volume of water or the amounts of pollutant contained in various parts of the system. The input and output variables commonly correspond to volume or mass flow rates, which may be rainfall intensity or rate of discharge of pollutant. The state of a system is a measure of the level of activity in each of its components and can be thought of as the interface between the past and the future of the state of the system.

The state variable model for continuous time can be formulated as follows

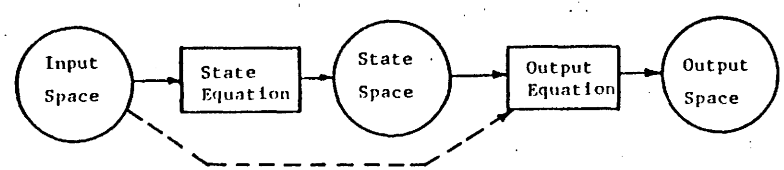

FIG. 1.—Modern Approach to Dynamic System Modeling

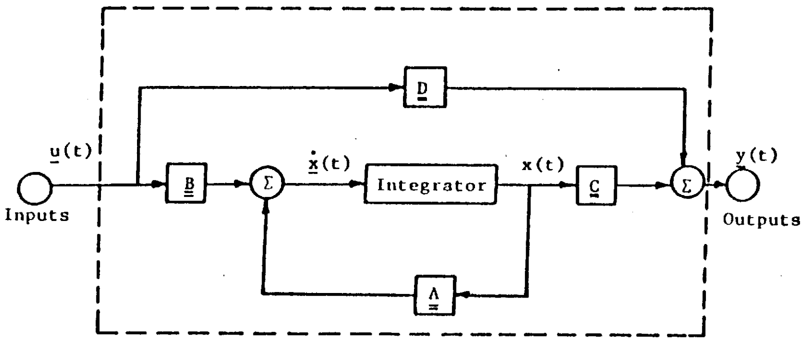

FIG. 2.—Schematic Diagram of State Variable Model

$$\dot{\underline{X}}_t = \underline{\underline{A}}\underline{X}_t + \underline{\underline{B}}\underline{U}_t \qquad (5)$$

and $$\underline{Y}_t = \underline{\underline{C}}\underline{X}_t + \underline{\underline{D}}\underline{U}_t \qquad (6)$$

in which Eqs. 5 and 6 are the state equation and output equation, respectively; $\dot{\underline{X}}_t = d\underline{X}_t/dt$ = the time rate of change of the state vector; $\underline{U}_t$ = the input vector; $\underline{Y}_t$ = the output vector; and $\underline{\underline{A}}$, $\underline{\underline{B}}$, $\underline{\underline{C}}$, and $\underline{\underline{D}}$ = matrices that can be constant or functions of time or space, or both. The system representation given by Eqs. 5 and 6 is shown schematically in Fig. 2. The time rate of change of the system state, $\dot{\underline{X}}_t$, is formed as the sum of modified inputs, $\underline{\underline{B}}\underline{U}_t$, and the modified current state, $\underline{\underline{A}}\underline{X}_t$. Also, the state feedback has a major role in determining the future behavior of the system. The rate of change of the state vector, $\dot{\underline{X}}_t$, is continuously integrated with the current state to produce the new state. The output, $\underline{Y}_t$, is formed by summing the new state which has been scaled by matrix $\underline{\underline{C}}$ with a direct contribution from modified input, $\underline{\underline{D}}\underline{U}_t$. These features of state variable modeling make it particularly attractive because, once the system parameters are identified, the only requirements for a solution are the initial conditions of the system and the input to the system.

There have been a number of applications of state variable modeling concepts to wastewater treatment water quality control (6,36), operation of hydroelectric power stations (5), rainfall-runoff process modeling (4,21,33), reservoir operation (17), and flow routing in storm sewers and channels (1,20,22).

State Variable Formulation for Nonlinear Muskingum Model

The derivation of the state variable formulation for the nonlinear Muskingum model, Eq. 4, is straightforward. By rearranging and manipulating Eq. 4, the rate of outflow at time t, O_t, can be expressed in terms of channel storage, S_t, and inflow rate, I_t, as

$$O_t = \left(\frac{1}{1-x}\right)\left(\frac{S_t}{\alpha}\right)^{1/m} - \left(\frac{x}{1-x}\right) I_t \qquad (7)$$

Eq. 7 forms an output equation in the state variable model. Combining

Eq. 7 and the continuity equation, Eq. 2, the state equation can be expressed as

$$\dot{S}_t = -\left(\frac{1}{1-x}\right)\left(\frac{S_t}{\alpha}\right)^{1/m} + \left(\frac{1}{1-x}\right) I_t \qquad (8)$$

where the state variable for the system is the channel storage and the input is the inflow at the upstream end of the channel reach.

Once the state variable model is formulated, the output from the system can be obtained by solving the state equation and the output equation recursively. Although water resource systems actually operate continuously in time, the data are usually analyzed using discrete-time intervals. The solution procedure for the discrete-time state variable nonlinear Muskingum model, thus, involves the following five steps:

Step 1.—The inflow hydrograph to the channel reach is discretized into several time stages where time intervals need not be equal.

Step 2.—From the initial state of system storage, S_1, and initial inflow rate to the channel reach, I_1, the time rate of change of storage

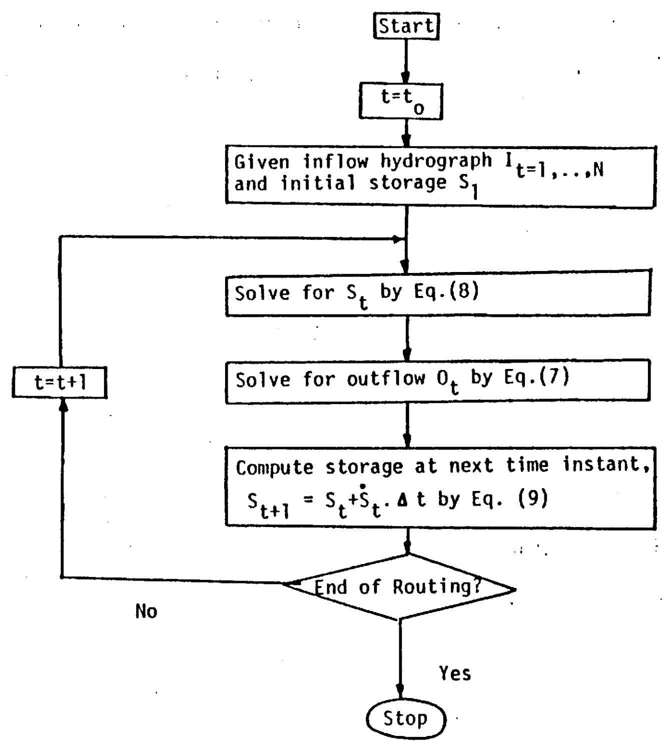

FIG. 3.—Flow Chart for Discrete Time-State Variable Model for Nonlinear Muskingum Routing

volume in the channel reach at the initial state, $\dot{S}_1$, can be evaluated by the state equation, Eq. 8.

Step 3.—The state of the system, i.e., channel storage, at the next time stage, S_2, is estimated or approximated as

$$S_2 \approx S_1 + \dot{S}_1 \Delta t \quad (9)$$

Step 4.—The magnitude of the outflow rate at the current stage can then be calculated by solving the output equation, Eq. 7, using current values of inflow rate and channel storage at the same stage.

Step 5.—Using current information on inflow and channel storage, Steps 2–4 are repeated recursively until the last stage is reached.

A flow chart for the above algorithm is shown in Fig. 3.

Parameter Estimation

The nonlinear Muskingum model considered in this paper consists of three parameters, α, x, and m, which are to be estimated from observed stream flow data. Gill (9) proposed a three-point estimation technique involving the solution of a system of simultaneous nonlinear equations at each time point. The selection of these three points for parameter estimation is arbitrary and is left to the judgment of the individual analyst. In this paper, three parameter estimation techniques are employed to minimize the sum of the squares of deviations between observed channel storage and computed channel storage over the total data points, i.e.

$$\underset{\alpha,x,m}{\text{Minimize}} \quad F = \sum_{t=1}^{n} \{\bar{S}_t - \hat{S}_t\}^2 \quad (10)$$

or, equivalently

$$\underset{\alpha,x,m}{\text{Minimize}} \quad F = \sum_{t=1}^{n} \{\bar{S}_t - \alpha[xI_t + (1-x)O_t]^m\}^2 \quad (11)$$

in which $\bar{S}_t$ = the observed channel storage at time t, which can be calculated from given historical inflow and outflow hydrographs selected for calibration; and $\hat{S}_t$ = the computed channel storage determined by the nonlinear Muskingum model.

Hooke-Jeeves Pattern Search in Conjunction with Linear Regression (HJ + LR).—The nonlinear Muskingum model considered herein can be reduced to a linear form if the value of the weighting factor, x, is known or assumed. In other words, the nonlinear Muskingum model can be expressed (via logarithms) as

$$\ln(S_t) = \ln(\alpha) + m \cdot \ln[x^* I_t + (1 - x^*)O_t] \quad (12)$$

in which a weighting factor x takes as assumed value x^*. If the value of x is given or assumed, the logarithms of channel storage and weighted flow have a linear relationships. Under such circumstances, the parameter values for α dnd m can be estimated by using the simple linear regression technique. By using this technique, the values of α and m

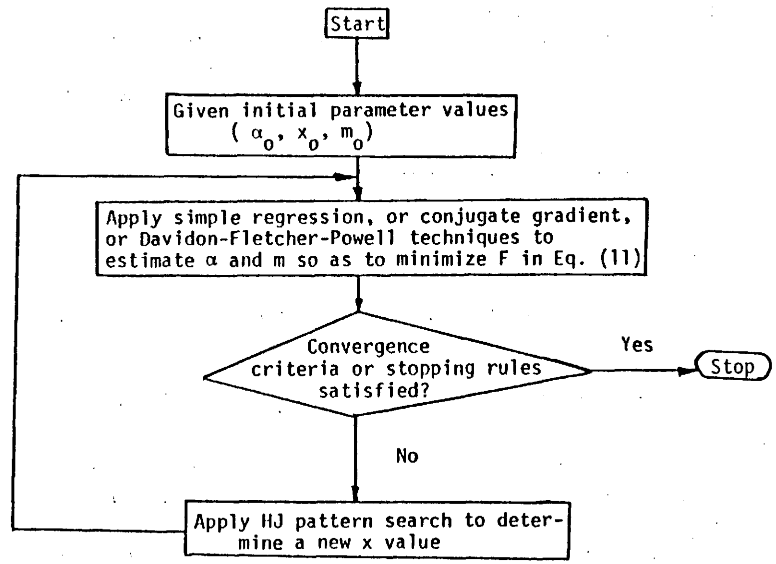

FIG. 4.—Flow Chart of Parameter Estimation for Nonlinear Muskingum Model

would minimize the sum of the square of deviations between observed and computed channel storages, based on a given value of weighting factor x^*. Since the weighting factor x itself is an unknown parameter, the values of all three parameters can be estimated by using a combination of direct search techniques and simple linear regression (LR). The scheme employed herein for identifying the value of x is called the pattern search technique developed by Hooke and Jeeves (12). The technique, coded herein as (HJ), is based on the philosophy that any set of moves that have been successful in improving objective function values in early trials will be worth repeating. The entire methodology for parameter estimation involves sequential applications of this (HJ + LR) method in an iterative manner. The flow chart of the (HJ + LR) algorithm is shown in Fig. 4.

The HJ method starts cautiously with short excursions from a starting point. Then, the step sizes grow with each repeated success. Subsequent failure indicates that shorter step sizes are in order. If a change in direction is required, the technique will start over again with a new pattern. In the vicinity of the peak or valley of the response surface, the step sizes become very small to avoid overlooking any promising directions. The technique has been applied by Tung and Mays (33) to identify parameters in a nonlinear hydrologic system model proposed by Prasad (28).

Hooke-Jeeve Pattern Search in Conjunction with Conjugate Gradient Method (HJ + CG).—The optimal estimation of unknown parameters in the nonlinear Muskingum model can also be derived by solving the objective function, Eq. 11, with an unconstrained optimization technique or a combination of direct search and unconstrained optimization. Similar to the previous method described, the JH direct search technique is employed to estimate the value of weighting factor x. Two unconstrained optimization schemes, i.e., the conjugate gradient (CG) and Davidon-Fletcher-Powell (DFP) methods, are applied to estimate the value of α and m. The descriptions of DFP are presented in the next section.

Applications of the two unconstrained optimization techniques require computation of the gradients of the objective function with respect to unknown parameters under estimation. The gradient of the objective function with respect to α and m, for a given value of x^* determined by the HJ method, can be expressed as

$$\frac{\partial F}{\partial \alpha} = -2 \sum_{t=1}^{n} \{\tilde{S}_t - \alpha[x^* I_t + (1 - x^*)O_t]^m\} \cdot [x^* I_t + (1 - x^*)O_t]^m \quad (13)$$

$$\frac{\partial F}{\partial m} = -2 \sum_{t=1}^{n} \{\tilde{S}_t - \alpha[x^* I_t + (1 - x^*)O_t]^m\}\{[x^* I_t + (1 - x^*)O_t]^m\}$$

$$\cdot \ln [x^* I_t + (1 - x^*) O_t] \quad (14)$$

The terms $\partial F/\partial\alpha$ and $\partial F/\partial m$ form the two elements of the gradient vector $\underline{G} = (\partial F/\partial\alpha, \partial F/\partial m)$. The gradient will be evaluated and serve as the basis for determining the new direction vector, along which the search for optimalty is pursued. The Fletcher-Reeves algorithm (7) of the CG method starts with any initial values for unknown parameters, $\underline{Z}_1 = (\alpha_1, m_1)$ and continues with iteration index $k = 1$ as follows:

Step 1.—Evaluate the gratient, $\underline{G}_k$, and set the vector of search direction, $\underline{D}_k = -\underline{G}_k$.

Step 2.—Minimize $F(\underline{Z}_k + \beta_k \underline{D}_k)$ with respect to $\beta_k \geq 0$ to obtain the next starting point, $\underline{Z}_{k+1} = \underline{Z}_k + \beta_k^* \underline{D}_k$, in which β_k^* = the value of the step size that minimizes the objective function value along the direction defined by $\underline{D}_k$.

Step 3.—Evaluate the gradient, $\underline{G}_{k+1}$, at the new starting point, $\underline{Z}_{k+1}$, and redefine the direction vector as

$$\underline{D}_{k+1} = -\underline{G}_{k+1} + \gamma_k \underline{D}_k \quad (15)$$

in which $$\gamma_k = \frac{\underline{G}'_{k+1}\underline{G}_{k+1}}{\underline{G}'_k \underline{G}_k} \quad (16)$$

where the prime indicates the transpose of the vector.

Step 4.—Go to Step 2 until convergence criteria or stopping rules are satisfied. Useful stopping rules that are commonly used in search techniques for preventing excessive computations are the specification of a maximum number of iterations and step size reductions.

Descriptions of the CG method are given by Luenberger (16).

The basic structure of the (HJ + CG) method for parameter estimation is very similar to the (HJ + LR) method. As previously described, the only difference is between the two methods employed to estimate parameters and m.

Hooke-Jeeves Pattern Search in Conjunction with the Davidon-Fletcher-Powell Method (HJ + DFP).—Similar to the two previous methods, (HJ + LR) and (HJ + CG), this technique for estimating α and m is called the Davidon-Fletcher-Powell (DFP) method. The DFP method is a quasi-Newton method that simultaneously generates the search direction while constructing and updating the inverse of the Hessian matrix. The procedure of the DFP method is as follows:

Step 1.—Select any symmetric positive matrix $\underline{\underline{S}}$ and initial point $\underline{Z}_1$, then begin the iterations with index $k = 1$.

Step 2.—Set search direction $\underline{D}_k = -\underline{\underline{S}}_k G_k$.

Step 3.—Minimize $F(\underline{Z}_k + \beta_k \underline{D}_k)$ with respect to $\beta_k \geq 0$ to obtain $\underline{Z}_{k+1}$, $\underline{P}_k = \beta_k^* \underline{D}_k$, and $\underline{G}_{k+1}$.

Step 4.—Set $\underline{Q}_k = \underline{G}_{k+1} - \underline{G}_k$ and updating matrix $\underline{\underline{S}}$ as

$$\underline{\underline{S}}_{k+1} = \underline{\underline{S}}_k + \frac{\underline{P}_k \underline{P}'_k}{\underline{P}'_k \underline{Q}_k} - \frac{\underline{S}_k \underline{Q}_k \underline{Q}'_k \underline{S}_k}{\underline{Q}'_k \underline{S}_k \underline{Q}_k} \quad (17)$$

Step 5.—Check with convergence criteria and stopping rules before returning to Step 2.

Again, for descriptions of the DFP method readers are referred to Luenberger (16).

Applications

The nonlinear Muskingum model, Eq. 4, is applied to channel flood routing using an example from Wilson (35). The state variable modeling technique is used as a tool for performing flood routing. Parameter estimation techniques previously described are used and their performance in calibration are compared. The reasons for selecting this example for demonstration are twofold: (1) The example presents a pronounced nonlinearity between weighted flow and storage volume; and (2) the example has been studied previously for testing the different routing methodologies developed by Gill (9). Therefore, the performance of parameter estimation procedures proposed herein can be compared with Gill's previous study.

The observed inflow and outflow hydrographs for the example are tabulated in Cols. 2 and 3 of Table 1 and are also shown in Figs. 5(a–b). The parameter values in Eq. 4 estimated by different techniques are given in Table 2. For purpose of comparison, the parameter values in the linear Muskingum model, as derived by Gill (9) using the least squares method, are also presented in the last row of Table 2. The estimated value of weighting factor, x, differs very little among the various methods, but the values of α and m vary quite significantly. Finally, the two methods, (JH + CG) and (HJ + DFP), produce almost identical results.

To evaluate the performance of different parameter estimation techniques in the calibration process, the inflow hydrograph is routed to produce a computed outflow hydrograph for a given parameter set. Then, the computed and observed outflow hydrographs are compared and their deviations are calculated. The computed outflow hydrographs obtained by the state variable modeling of the nonlinear Muskingum model using different parameter values are tabulated in Cols. 4–7 of Table 1 and are

TABLE 1.—Comparison of Performance of Muskingum Model Using Different Parameter Estimation Procedures

Time (hrs) (1)	Inflow (cms) (2)	Observed outflow (cms) (3)	Computed Outflow (cms) (HG + LR) Eq. 4 (4)	(HJ + CG) Eq. 4 (5)	(HJ + DFP) Eq. 4 (6)	Gill Eq. 4 (7)	(LS) Eq. 1 (8)
0	22	22	22.0	22.0	22.0	22.0	22.0
6	23	21	22.0	22.0	22.0	22.0	22.3
12	35	21	22.4	22.4	22.4	22.8	26.4
18	71	26	26.3	26.8	26.7	29.6	44.6
24	103	34	35.5	34.9	34.8	39.1	78.3
30	111	44	47.4	44.5	44.7	47.6	103.6
36	109	55	60.7	56.7	56.9	58.0	109.9
42	100	66	71.7	67.3	67.7	67.1	106.5
48	86	75	79.8	75.9	76.3	74.8	96.5
54	71	82	84.6	81.9	82.2	80.4	82.5
60	59	85	85.6	84.5	84.7	83.2	68.4
66	47	84	82.9	83.4	83.5	82.8	56.2
72	39	80	77.9	79.9	79.8	80.1	45.2
78	32	73	70.5	73.6	73.3	74.5	37.5
84	28	64	62.1	65.8	65.5	67.2	31.2
90	24	54	53.1	56.9	56.5	58.1	27.1
96	22	44	44.6	47.8	47.5	48.1	23.6
102	21	36	36.7	38.9	38.7	37.6	21.8
108	20	30	30.4	31.5	31.4	28.2	20.8
114	19	25	25.8	25.8	25.9	21.9	19.8
120	19	22	22.5	22.0	22.1	19.1	18.9
126	18	19	20.6	20.1	20.2	19.0	18.1
Error[a]			40.10	25.20	24.8	46.40	473.90
Error[b]			132.75	49.64	45.54	143.60	17,054.01

[a]Sum of absolute deviations.
[b]Sum of square of deviations.

TABLE 2.—Values of Parameters α, x, and m Estimated by Different Methods for Various Models

Model (1)	Method (2)	Parameters α (3)	x (4)	m (5)
Eq. 4	HJ + LR	0.1700	0.2400	1.7012
Eq. 4	HJ + CG	0.0669	0.2685	1.9291
Eq. 4	HJ + DFP	0.0764	0.2677	1.8978
Eq. 4	Gill	0.0100	0.2500	2.3470
Eq. 1	Gill[a]	4.6110	0.2540	1.000

[a]Least square method is used.

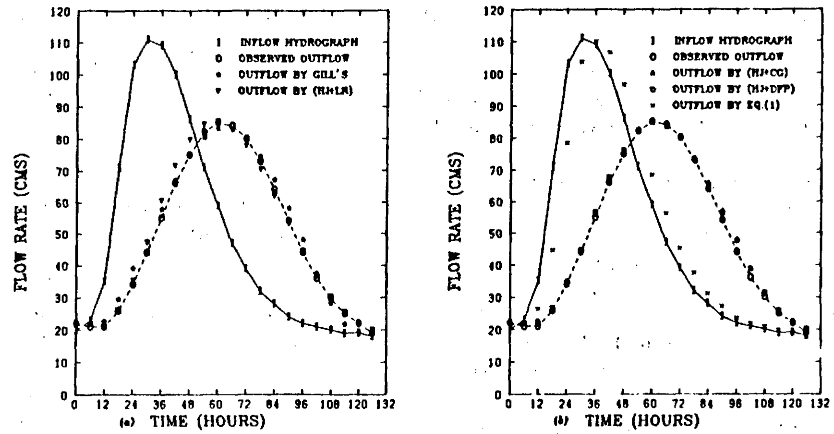

FIG. 5.—(*a*) Inflow Hydrograph, Observed Outflow Hydrograph, and Computed Outflow Hydrographs; (*b*) Inflow Hydrograph, Observed Outflow Hydrograph, and Computed Outflow Hydrographs

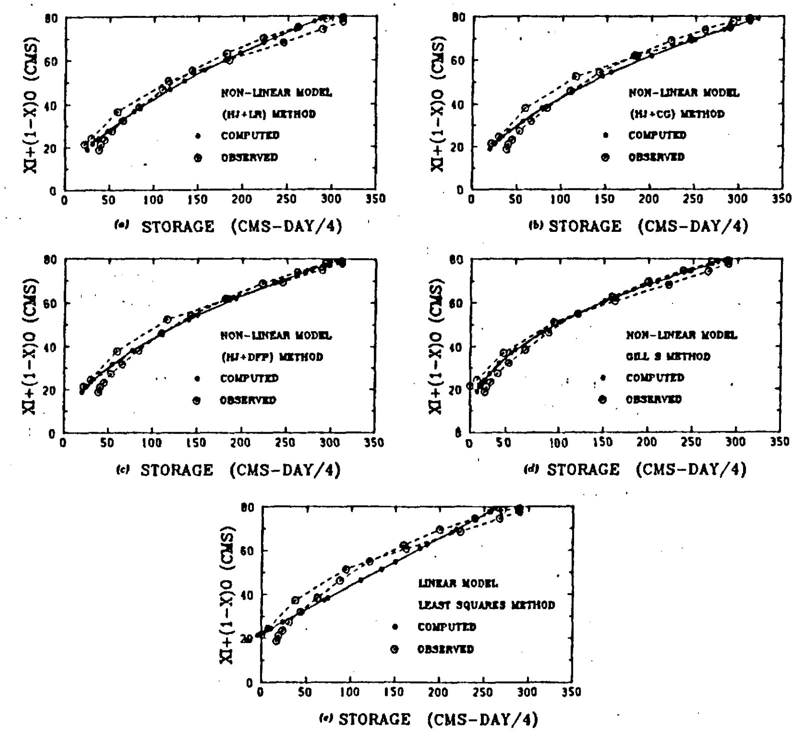

FIG. 6.—(*a*) Plots of Weighted Flows versus Storage; (*b*) Plots of Weighted Flow versus Storage; (*c*) Plots of Weighted Flow versus Storage; (*d*) Plots of Weighted Flow versus Storage; (*e*) Plots of Weighted Flow versus Storage

plotted in Fig. 5. Col. 8 is obtained by solving the linear Muskingum model with the conventional method. In Table 2, the computed outflow hydrograph in Col. 7 is derived by the state variable modeling technique with the parameter values estimated by Gill (9) because the computed outflow hydrograph was not directly available in Gill's paper for comparison.

Two criteria are used herein for evaluating the performance of different parameter estimation techniques and models: (1) The sum of the absolute value of deviations between the computed and observed outflows; this deviation is termed "error"; and (2) the sum of the square of errors. The magnitudes of the two error criteria described above for different parameter estimation techniques and models are given in the last two rows of Table 1. As can be observed, the two methods, (HJ + CG) and (HJ + DFP), outperform all other parameter estimation techniques considered in this presentation. The method of (HJ + LR) performed slightly better than Gill's method. The linear form of the Muskingum model (see Col. 8) yields the least desirable results of all methods considered because, as indicated in Figs. 6(*a–e*), the system has an appreciable nonlinearity between weighted flow and channel storage, which makes the linearity assumption inappropriate. This example highlights the limitation of using the linear Muskingum model in channel flood routing when the system's behavior is actually nonlinear.

Summary and Conclusions

The Muskingum model commonly applied to river and channel flood routing may experience severe limitations because of its inherent assumption of a linear relationship between channel storage and weighted flow. Although nonlinear forms of the Muskingum model have been proposed, the routing procedure is still lacking. This study presents a routing technique for one type of the nonlinear Muskingum model, Eq. 4, using the concept of state variable modeling. The state variable routing technique is direct and eliminates monotonous trial and error procedures.

When a nonlinear flood routing model is considered, the task of parameter estimation, in the calibration process, becomes more involved. Three parameter estimations procedures are devised using the Hooke-Jeeve (HJ) pattern search technique in conjunction with simple linear regression (LR), the conjugate gradient (CG), and the Davidon-Fletcher-Powell (DFP) techniques. Comparisons were made of the model parameter estimation techniques developed and Gill's procedure (9), including the use of the linear model. It was found that methods (HJ + CG) and (HJ + DFP) yield better results than the other methods considered in this study. The results of applying the linear model to the given example were far from desirable. This demonstrates the severe limitation of the linear model and that care should be exercised if a system's behavior appears to be nonlinear.

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