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Models for Evaluating Flow Conveyance Reliability of Hydraulic Structures

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Two generalized dynamic reliability models (integrating both hydrologic and hydraulic uncertainties) based on binomial and Poisson distributions are developed for evaluating flow conveyance reliability of hydraulic structures. The two generalized models correct the deficiencies of dynamic reliability models developed previously. Relative performance of the generalized models and two conventional reliability models were examined. It is found that the total risk is significantly underestimated by the conventional approaches when hydraulic uncertainty is moderate or large.

INTRODUCTION

In the development of water resource engineering projects it usually includes the design of various types of hydraulic structures such as pipe systems for water supply, sewer network systems for sewage and runoff collection, and levee and dike systems for flood control and protection, etc. In addition to the determination of capacity and layout of hydraulic structures under normal design processes, one related but equally important task that faces engineers is the evaluation of the operational aspect of performance reliability of hydraulic structures or systems to be designed. The general practice is that hydraulic structures are designed with reference to natural events that could be imposed on the structure during its expected service life. This, then, involves a hydrologic determination of the flow magnitude of a design event.

Risk is an element that exists in all aspects of engineering design and planning, including water resources engineering. Total risk in water resource engineering design can be attributed mainly to the inherently random behavior of hydrologic processes, the lack of perfect knowledge about hydrologic processes involved, and the lack of complete control of design and operation of hydraulic structures. This lack of knowledge is generally referred to as uncertainty. *Yevjevich* [1977] has made a clear distinction between the inherent risk and uncertainty in constituting a total risk.

In general, uncertainties in water resources engineering projects can be divided into four basic categories: hydrologic, hydraulic, structural, and social and economical. Detailed discussions of these uncertainties were given by *Tung and Mays* [1980*a*]. The existence of various aspects of uncertainty highlights the interdisciplinary nature of water resource planning and design projects. Collaborations with experts in other disciplines are essential. There have been many reliability models developed in the literature. The majority of them emphasized on hydrologic risk, including various hydrologic uncertainties [*Bernier*, 1967; *Davis et al.*, 1972; *Vicens et al.*, 1975; *Wood and Rodriquez-Iturbe*, 1975*a*, *b*; *Bodo and Unny*, 1976; *Castano et al.*, 1978]. Only a few models were extended to incorporate other aspects of uncertainty in risk evaluation [*Tang et al.*, 1975; *Sidarovszky et al.*, 1976; *Duckstein and Borgardi*, 1981; *Tung and Mays*, 1980*b*, 1981]. The scope of this paper is limited to the integration of both hydraulic and hydrologic uncertainties in risk analysis. The issue, such as how to analyze uncertainties, will not be addressed.

FUNDAMENTALS

Analysis of system performance reliability of any kind generally requires the consideration of interaction between loadings and resistances. Loadings are commonly referred to as those external stresses to be imposed on the system while resistance usually represents capacities or strengths of the system to withstand loadings. Under the context of water resources engineering design, the loadings can be the magnitude of hydrologic events and resistances can be the flow capacity of hydraulic structures. In view of uncertainties that exist in both the hydrologic and hydraulic aspects, the loadings as well as resistances should be treated as random variables with associated probability distributions. Therefore from the operational viewpoint, the performance reliability in terms of flow conveyance capability of a hydraulic structure is the probability that structural flow capacity can accommodate the flow magnitude generated by external hydrologic processes such as rainfall or flood, i.e.,

$$R = Pr\left[\ l \leq r\right] \tag{1}$$

where R is the flow conveyance reliability; $Pr[\ \ ]$ refers to the probability; and l and r represent the loading and resistance, respectively. The conventional approach in evaluating flow conveyance reliability of a hydraulic structure ignores the existence of hydraulic uncertainty by treating design flow capacity of hydraulic structures as being deterministic. There have been several reliability models developed which integrate uncertainties attributed to both the hydraulic and hydrologic aspects using (1). However, the majority of them are static reliability models which do not consider the repeated nature of hydrologic loading [*Yen and Ang*, 1971; *Tang and Yen*, 1972; *Tang et al.*, 1976; *Tung and Mays*, 1980*b*].

In reality, a hydraulic structure is subjected to a repeated application of loading over its expected service life. As a result, the use of a dynamic reliability model is a more appropriate approach to the problem. Development and application of dynamic reliability models integrating both hydrologic and hydraulic uncertainties have been very recent [*Tung and Mays*, 1980*b*, 1981]. The dynamic reliability model consists of such elements as the design return period (or the magnitude of design event), the service life of the hydraulic structure, a safety factor, and the probability density functions of the resistance as well as the loading. The original dynamic reliability model developed by *Tung and Mays* [1980*a*] considered the

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annual maximum flood series as loading. For this reason the model has a drawback in that it is unable to clearly define the rate of occurrence of the design event in the model. *Lee and Mays* [1983] recently addressed the problem and attempted to improve the model formulation using the conditional probability. Unfortunately, their model is still not complete because it does not lead to a formulation for the total probability of survival or failure. Two generalized formulations for the dynamic reliability model based on binomial and Poisson probabilities are described in the next section.

GENERALIZED DYNAMIC RELIABILITY MODELS

Ignoring the hydraulic uncertainties in the risk evaluation, there are two commonly used dynamic reliability models. The first model is developed using the binomial law as

$$R_1(n, T, SF) = \left[1 - \frac{1}{T(SF)}\right]^n \tag{2}$$

where $R_1(n, T, SF)$ is the flow conveyance reliability of a hydraulic structure over an expected service life of n years using a design return period of T years associated with a safety factor, SF. Equation (2) was used by *Yen* [1970] to develop a series of curves relating risk to the structure return period and the expected service life with a safety factor of one for hydraulic structures.

The second model utilizes the Poisson distribution for the reliability evaluation and can be expressed as

$$R_2(t, T, SF) = \exp\left[-t/T(SF)\right] \tag{3}$$

in which t is the time period of interest which could be the expected service life of the hydraulic structure. On the basis of (3), *Hall and Howell* [1963] presented a risk evaluation procedure. It is known that for a fixed service life (t or n) and safety factor (SF), the flow conveyance reliability calculated by (3) will asymptotically converge to that calculated by (2) as T gets larger. *Chow and Takase* [1977] developed a model using the nonparametric approach which can also be applied to estimate the risk of hydraulic structures without considering hydraulic uncertainties.

When hydraulic uncertainties are considered in reliability computation, two generalized dynamic reliability models can be derived from (1). Assuming the probability density functions of the hydrologic loading as well as hydraulic resistance are available from which the magnitude of the design hydrologic event l_T^* with a return period of T year can be calculated without error. Then, the magnitude of the future hydrologic event can be partitioned into two complementary subsets, i.e., $l \le l_T^*$ and $l > l_T^*$, with each representing different recurrence interval hydrologic processes. As the result, the structural flow conveyance reliability subjected to the ith hydrologic loading occurring in the future can be expressed using the law of total probability as

$$\begin{aligned} R_i = Pr(l_i \le r) &= Pr(l_i \le r \mid l_i > l_T^*)Pr(l_i > l_T^*) \\ &\quad + Pr(l_i \le r \mid l_i \le l_T^*)Pr(l_i \le l_T^*) \\ &= Pr(l_T^* \le l_i \le r) + Pr(l_i \le r, l_i \le l_T^*) = P_1 + P_2 \end{aligned} \tag{4}$$

More explicitly, the terms P_1 and P_2 can be expressed as

$$P_1 = \int_{l_T^*}^{\infty} \int_{l_T^*}^{r} f(r, l)\, dl\, dr \tag{5}$$

$$P_2 = \int_0^{l_T^*} \int_0^{r} f(r, l)\, dl\, dr + \int_{l_T^*}^{\infty} \int_0^{l_T^*} f(r, l)\, dl\, dr \tag{6}$$

where $f(r, l)$ is the joint probability density function of the resistance and loading. If r and l are considered independent, then $f(r, l) = f(r)f(l)$, in which case $f(r)$ and $f(l)$ are the marginal probability density functions for the flow capacity of hydraulic structure and the magnitude of hydrologic event based on the annual maximum series, respectively.

With the above partitioning of the future flood events into two complementary subsets, the justification of the rate of occurrence of the design hydraulic event becomes very clear, which was not the case in the original dynamic reliability formulation proposed by *Tung and Mays* [1980*a*]. Furthermore, an examination of (4) reveals that the dynamic reliability model recently proposed by *Lee and Mays* [1983] does not satisfy the law of total probability.

By using the binomial law, the flow conveyance reliability of hydraulic structure, with its flow capacity determined on the basis of T-year design hydrologic event and a specified safety factor SF, under n loadings or over n year service period can be expressed as

$$R_3(n, T, SF) = R_n = \sum_{x=0}^{n} \binom{n}{x} P_1^{\,x} P_2^{\,n-x} \tag{7}$$

It can easily be shown that when only inherent hydrologic uncertainty is considered as traditionally done, (7) is reduced to (2).

Alternatively, the second generalized dynamic formulation can be developed using Poisson distribution. By using Poisson distribution the probability of n occurrences of a hydrologic event over a period of $[0,t)$, $\Phi_n(t)$, regardless of their types (i.e., either $l \le l_T^*$ or $l > l_T^*$), can be expressed as

$$\Phi_n(t) = \frac{e^{-t}t^n}{n!} \tag{8}$$

Combining (8) and (7) leads to a different formulation of a dynamic reliability model as

$$\begin{aligned} R_4(t, T, SF) &= \sum_{n=0}^{\infty} \Phi_n(t) R_n \\ &= \sum_{n=0}^{\infty} \frac{e^{-t}t^n}{n!} \sum_{x=0}^{n} \binom{n}{x} P_1^{\,x} P_2^{\,n-x} \end{aligned} \tag{9}$$

Again, (9) can be reduced to (3) when hydraulic uncertainties are ignored.

It should be made aware that both (7) and (9) are the two general formulations of dynamic reliability models for the random-independent resistance case. Random-independent resistance means that the random behavior of structural resistance is independent of the loading condition. Flow capacity of some hydraulic structures may show a degrading trend with time because of aging. In such circumstances, a random-fixed resistance dynamic reliability model is appropriate; however, the model formulation of this type (which is currently under development) is much more complicated than the ones for random-independent resistance cases. Although the two dynamic reliability models proposed by *Tung and Mays* [1980*a*] and by *Lee and Mays* [1983] were for the random-fixed resistance situation, the inherent drawbacks of the two models previously mentioned are still in existence.

RELIABILITY COMPUTATION USING GENERALIZED MODELS

Using the two generalized reliability models for reliability evaluation, the key is to compute P_1 and P_2 given by (5) and (6). One way to evaluate P_1 and P_2 is to perform integration

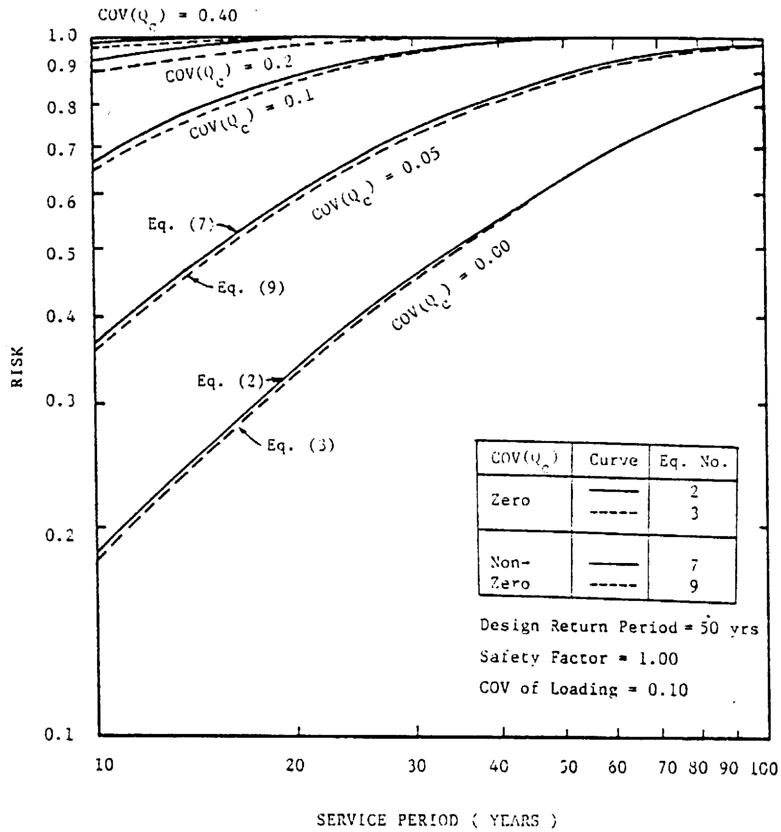


Fig. 1. Comparison of reliability models.

analytically if possible or to use numerical integration procedures, which could be the most likely case in a practical situation. In this paper an alternative approach is taken which becomes more convenient in calculating P_1 and P_2 when both the resistance and loading have lognormal or normal distribution.

The following derivation assumes that the resistance and loading are the two independent random variables. This independency assumption is not necessarily required. To evaluate P_1, let us define two new random variables $u_1 = r - l$ and $v_1 = l - l_T^*$. Then the two new random variables would have the following statistical properties:

$$u_1 \sim (\mu_r - \mu_l, \sigma_r^2 + \sigma_l^2) \tag{10a}$$

$$v_1 \sim (\mu_l - l_T^*, \sigma_l^2) \tag{10b}$$

in which μ_x and σ_x^2 represents the mean and variance of random variable x. Furthermore, the random variables u_1, and v_1 possess a correlation coefficient $\rho(u_1, v_1)$ as

$$\rho(u_1, v_1) = \frac{-1}{\left(1 + \left(\frac{\sigma_r}{\sigma_l}\right)^2\right)^{1/2}} \tag{11}$$

As a result, the expression for P_1 given by (5) can be written as

$$P_1 = Pr[u_1 \geq 0, v_1 \geq 0 \mid \rho(u_1, v_1)] \tag{12}$$

Similarly, under the assumption of independency between the loading and resistance, the expression for P_2 given by (6) can be written as

$$P_2 = Pr[u_2' \geq 0, v_2' \geq 0 \mid \rho(u_2', v_2')] + Pr[u_2'' \geq 0, v_2'' \geq 0 \mid \rho(u_2'', v_2'')] \tag{13}$$

in which

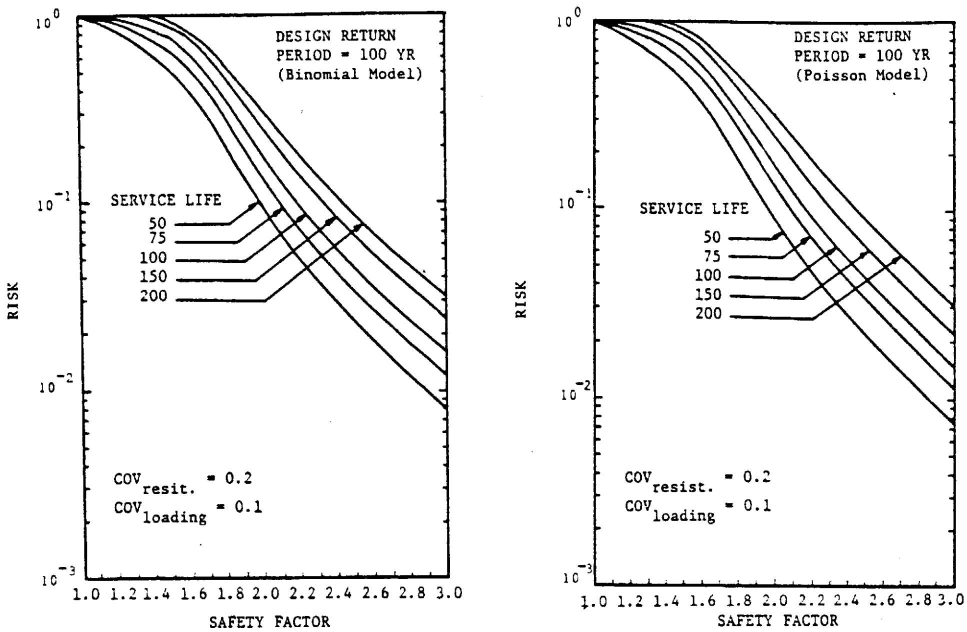
$$u_2' = r - l \qquad v_2' = l_T^* - r \qquad u_2'' = r - l_T^*$$
$$v_2'' = l_T^* - l \qquad \rho(u_2', v_2') = \frac{-1}{\left(1 + \left(\frac{\sigma_l}{\sigma_r}\right)^2\right)^{1/2}} \tag{14}$$

and $\rho(u_2'', v_2'') = 0$. If the resistance and loading are independent normal or lognormal random variables, the probability evaluation for P_1 and P_2 using (12) and (13) can easily be made because the random variables u and v have a bivariate-normal distribution [*Abramowitz and Stegun*, 1970].

PERFORMANCE EVALUATION OF MODELS

A question that naturally arises at this point is, Given these various dynamic reliability models for risk evaluation, how do they perform relatively? To compare the relative performance of the two conventional dynamic reliability models, (2) and (3), and the two generalized reliability models, (7) and (9), the statistical properties of the resistance and loading in risk evaluation are assumed from the previous levee reliability study [*Tung and Mays*, 1981] for the Guadalupe River near Victoria, Texas. The loading is the annual maximum flood series and the resistance is the flow capacity of the levee.

Due to a difference in the way in which the resistance is considered between the two previous models which considered the random-fixed resistance and the two models in the paper which consider the random-independent resistance, a comparison of the resulting risks would be inappropriate and therefore will not be made. Both the loading (annual maximum flood) as well as resistance (levee flow capacity) are assumed to be independent with lognormal distribution for simplifying ana-



(a) Generalized Dynamic Reliability Model Using Binomial Distribution (Eq. 7)

(b) Generalized Dynamic Reliability Model Using Poisson Distribution (Eq. 9)

Fig. 2. Risk-safety factor curves derived from the two generalized dynamic reliability models using lognormal loading and lognormal resistance.

lytical evaluation. Any combination of probability distributions for the loading and resistance are possible. The mean and standard deviation of the log-transformed annual maximum flood flow are 9.70 and 0.93, respectively, while the coefficient of variation of log-transformed levee flow capacity is 0.2. The mean value of levee flow capacity is determined using the characteristic safety factor [*Yen*, 1979] defined as

$$\bar{r} = SF \cdot l_T^* \quad (15)$$

where $\bar{r}$ is the mean value of the resistance.

Figure 1 shows the relationship between risk and service life of the levee using a design return period of 50 years and a safety factor of one. As can be seen, the two conventional reliability models, (2) and (3), which ignore the hydraulic uncertainties significantly underestimate the total flow conveyance risk associated with the levee structure, especially when the level of hydraulic uncertainty, in terms of coefficient of variation of levee flow capacity, is moderate or large. Furthermore, as the service life of the structure becomes longer, the risk calculated by the conventional models will asymptotically approach to that of the generalized dynamic reliability models.

Figure 2 shows the resulting risk-safety factor curves for the levee with a design return period of 100 years derived from the two generalized reliability models, (7) and (9). Table 1 provides numerical values of overtopping risk for the selected service lives, safety factors, and uncertainty levels of levee flow capacity computed by the two generalized models. They are practically identical with slightly less risk given by (9), which uses the Poisson distribution. From the computational viewpoint, (7) is recommended for risk calculation for its simplicity.

Application of a dynamic reliability model for risk evaluation of a hydraulic structure enables the examination of interrelationships between risk, safety factor, expected service life of the structure, and design return period. The interaction is dictated by the statistical properties that describe the random characteristics of the loading and resistance. A series of risk-safety factor curves derived from (7) with various levels of hydraulic uncertainty, expressed in terms of coefficient of variation of levee flow capacity, are shown in Figure 3. The curve corresponding to zero coefficient of variation represents that hydraulic uncertainty does not exist or is ignored as assumed in the conventional reliability models. The conventional reliability models underestimate the total associated risk. The discrepancy in underestimation increases rapidly as the value of the safety factor and uncertainty level of hydraulic flow capacity get larger.

Risk-safety factor curves shown in Figure 3 also show an implication in engineering design processes using different risk evaluation procedures. By referring to Figure 3 it can be observed that the required specification for the safety factor increases very rapidly as the hydraulic uncertainty increases in order to achieve the same risk level as if hydraulic uncertainty did not exist. The existence of a moderate or large uncertainty in hydraulic flow capacity determination would require specification of a larger safety factor than the case when the hydraulic uncertainty is small for improving the total risk to the same degree.

Summary and Conclusions

Two generalized dynamic reliability models considering both inherent hydrologic and hydraulic uncertainties were developed. The two models can be reduced to the conventional risk models which only consider hydrologic uncertainty. This development is one step forward in reaching a more complete and general model for evaluating risk and reliability of hydraulic structure design. Furthermore, the models provide insight into the interaction among the safety factor, design return period, expected service life of hydraulic structures, and statistical characteristics of resistance and loading and their effects on the total risk.

Numerical examples are presented in the paper to compare

TABLE 1. Numerical Comparisons of Calculated Risk Using (7) and (9)

Design Return Period, years	Expected Service Life, years	Safety Factor	Covariance of Levee Flow Capacity			
			0.4		0.2	
			(7)	(9)	(7)	(9)
50	10	1.0	9.851×10^{-1}	9.677×10^{-1}	9.193×10^{-1}	8.920×10^{-1}
		1.5	7.686×10^{-1}	7.437×10^{-1}	1.505×10^{-1}	1.493×10^{-1}
		2.0	5.352×10^{-1}	5.217×10^{-1}	2.134×10^{-2}	2.132×10^{-2}
		2.5	3.920×10^{-1}	3.845×10^{-1}	5.050×10^{-3}	5.047×10^{-3}
	50	1.0	1.000	1.000	1.000	1.000
		1.5	9.993×10^{-1}	9.989×10^{-1}	5.575×10^{-1}	5.546×10^{-1}
		2.0	9.783×10^{-1}	9.750×10^{-1}	1.022×10^{-1}	1.021×10^{-1}
		2.5	9.169×10^{-1}	9.117×10^{-1}	2.499×10^{-2}	2.498×10^{-2}
100	75	1.0	1.000	1.000	1.000	1.000
		1.5	1.000	1.000	6.495×10^{-1}	6.469×10^{-1}
		2.0	9.959×10^{-1}	9.950×10^{-1}	1.304×10^{-1}	1.303×10^{-1}
		2.5	9.724×10^{-1}	9.700×10^{-1}	3.272×10^{-2}	3.268×10^{-2}
	100	1.0	1.000	1.000	1.000	1.000
		1.5	1.000	1.000	7.528×10^{-1}	7.504×10^{-1}
		2.0	9.993×10^{-1}	9.992×10^{-1}	1.700×10^{-1}	1.699×10^{-1}
		2.5	9.917×10^{-1}	9.907×10^{-1}	4.339×10^{-2}	4.334×10^{-2}
200	150	1.0	1.000	1.000	1.000	1.000
		1.5	1.000	1.000	8.389×10^{-1}	8.371×10^{-1}
		2.0	1.000	1.000	2.176×10^{-1}	2.205×10^{-1}
		2.5	9.990×10^{-1}	9.989×10^{-1}	5.747×10^{-2}	5.753×10^{-2}
	200	1.0	1.000	1.000	1.000	1.000
		1.5	1.000	1.000	9.124×10^{-1}	9.111×10^{-1}
		2.0	1.000	1.000	2.790×10^{-1}	2.827×10^{-1}
		2.5	9.999×10^{-1}	9.999×10^{-1}	7.588×10^{-2}	7.597×10^{-2}

relative performance of the conventional risk models and the generalized models developed herein. It is generally observed that hydraulic uancertainty can not be ignored in risk evaluation when its level is moderate or large. Furthermore, a higher price, in terms of a larger value of safety factor, is needed to improve risk level when the hydraulic uncertainty is large relative to hydrologic uncertainty.

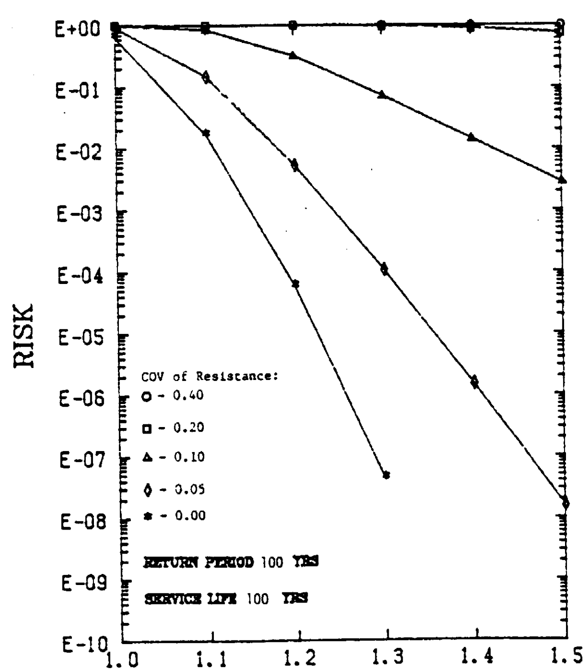


Fig. 3. Risk-safety factor curves for various resistance uncertainty levels using (7).

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REFERENCES

Abramowitz, M., and I. A. Stegun, Handbook of mathematical functions, *Appl. Math. Ser. 55*, Nat. Bur. of Stand., Washington, D. C., 1964.

Bernier, J., Les methodes bayesieneu en hydrologie statistique, in *Proceedings, 1st International Hydrology Symposium*, Water Resources Publications, Fort Collins, Colo., 1967.

Bodo, B., and T. E. Unny, Model uncertainty in flood frequency analysis and frequency-based design, *Water Resour. Res., 12*(6), 1109–1117, 1976.

Castano, E., L. Duckstein, and I. Bogardi, Choice of distribution functions for hydrologic design, *Water Resour. Res., 14*(4), 643–652, 1978.

Chow, V. T., and N. Takase, Design criteria for hydrologic extremes, *J. Hydraul. Div. Am. Soc. Civ. Eng., 103*(HY4), 425–436, 1977.

Davis, D. R., C. C. Kisiel, and L. Duckstein, Bayesian decision theory applied to design in hydrology, *Water Resour. Res., 8*(1), 33–41, 1972.

Duckstein, L., and I. Bogardi, Application of reliability theory to hydraulic engineering design, *J. Hydraul. Div. Am. Soc. Civ. Eng., 107*(HY7), 799–815, 1981.

Hall, W. A., and D. T. Howell, Estimating flood probabilities within specific time intervals, *J. Hydrol., 1*(1), 265–271, 1963.

Lee, H. L., and L. W. Mays, Improved risk and reliability model for hydraulic structures, *Water Resour. Res., 19*(6), 1415–1422, 1983.

Szidarovszky, F., I. Borardi, L. Duckstein and D. Davis, Economic uncertainties in water resources project design, *Water Resour. Res., 12*(4), 573–580, 1976.

Tang, W. H., and B. C. Yen, Hydrologic and hydraulic design under uncertainties, paper presented at Proceedings of the International Symposium on Uncertainties in Hydrologic and Water Resources Systems, Int. Symp. on Uncertainties in Hydrol. and Water Res. Syst., Tucson, Ariz., December 1972.

Tang, W. H., L. W. Mays, and B. C. Yen, Optimal risk-based design of storm sewer networks, *J. Environ. Eng. Div. Am. Soc. Civ. Eng., 101*(EE3), 381–398, 1975.

Tang, W. H., L. W. Mays, and H. G. Wenzel, Discounted flood risks in least-cost design of storm sewer networks, paper presented at Proceedings of the International IAHR Symposium on Stochastic Hydraulics, Int. IAHR Symp. on Stochastic Hydraul., Lund, Sweden, August 1976.

Tung, Y. K., and L. W. Mays, Optimal risk-based design of water resource engineering projects, *Tech. Rep. CRWR-171*, Cent. for Res. in Water Resour., Univ. of Tex. at Austin, 1980*a*.

Tung, Y. K., and L. W. Mays, Risk analysis for hydraulic design, *J. Hydraul. Div. Am. Soc. Civ. Eng.*, *106*(HY5), 893–913, 1980*b*.

Tung, Y. K., and L. W. Mays, Risk models for flood levee design, *Water Resour. Res.*, *17*(4), 833–841, 1981.

Vicens, G. J., I. Rodriques-Iturbe, and J. C. Schaake, Jr., A Bayesian framework for the use of regional information in hydrology, *Water Resour. Res.*, *8*(1), 33–41, 1975.

Wood, E. F., and I. Rodriques-Iturbe, Bayesian inference and decision making for extreme hydrologic events, *Water Resour. Res.*, *11*(4) 533–542, 1975*a*.

Wood, E. F., and I. Rodriques-Iturbe, A Bayesian approach to analyzing uncertainty among flood frequency models, *Water Resour. Res.*, *11*(6), 839–843, 1975*b*.

Yen, B. C., Risks in hydrologic design of engineering projects, *J. Hydraul. Div. Am. Soc. Civ. Eng.*, *96*(HY4), 959–966, 1970.

Yen, B. C., Safety factor in hydrologic and hydraulic engineering design, in *Reliability in Water Resources Management*, edited by McBean et al., Water Resources Publications, Fort Collins, Colo., 1979.

Yen, B. C., and A. H. S. Ang, Risk analysis in design of hydraulic projects, paper presented at 1st International Symposium on Stochastic Hydraulics, Univ. of Pittsburgh, Pittsburgh, Pa., June 1971.

Yevjevich, V., Risk and uncertainty in design of hydraulic structures, in *Stochastic Processes in Water Resources Engineering*, Water Resources Publications, Fort Collins, Colo., 1977.

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