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1985
Symposium Proceedings WWRC-85-38

In

Volume 1

Hydraulics and Hydrology
in the Small Computer Age
Proceedings of the Specialty Conference
American Society of Civil Engineers
Hydraulics Division

Lake Buena Vista, Florida
August 12-17, 1985

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INTRODUCTION

Water distribution systems play a vitally important role in preserving and providing a desirable life quality to the public. It has been in the past that much of the effort in the design of water distribution systems emphasizes on the aspect of least cost. The amount of effort and attention given to develop a procedure for system performance reliability evaluation has not attained a comparable scale. Realizing the fact that many of the aged infrastructures in our communities, including a water distribution system, have been deteriorated to the point that their serviceabilities have drawn much attention. There has been a growing awareness that it is equally important to have a public water distribution system possessing a high service reliability. There is very little work done in attempting to quantify the system reliability of a water distribution network. This paper reviews and applies some techniques which were found potentially applicable for evaluating water distribution network reliability (Billinton and Allen, 1983; Ang and Tang, 1975).

DEFINITION OF SYSTEM RELIABILITY

At the present time there is no universally accepted definition or measure of the reliability of water distribution systems. A water distribution system has a unique feature differing from many network systems, i.e. the supply sources and demand destinations are multiple. To facilitate the comparison of various techniques, a numerical measurable system service reliability is defined herein as the probability that flow can reach all the demand points in the network. On the other hand, service unreliability of the system is defined as the probability that any demand point in the network cannot be reached or serviced by the flow.

METHODOLOGIES FOR NETWORK RELIABILITY EVALUATION

There have been a number of techniques developed for evaluating system reliability with complex configuration. The basic principle behind the methods is to transform the logical operation of the system, or the topology of the system into a structure that consists of only a simple series and/or parallel components, paths or branches. Most of the techniques are one form of the others deriving from different lines of considerations. Consequently, the amount of effort

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involved in analyses and computations vary. This section briefly describes six techniques developed for reliability evaluation of complex network systems and, at the same time, comments about their practical applicabilities to reliability analysis of water distribution systems with multiple-user (or multiple-outlets).

Conditional Probability Approach. The approach starts with a selection of key elements (or components) whose states (survival or failure) would decompose the entire system into simple series and/or parallel subsystems to which the reliability or risk of subsystems can be easily evaluated. Then, the reliability or risk of the entire system is obtained by combining the subsystems using the conditional probability rule as:

$$\begin{aligned}&P(\text{system success or failure}) = P(\text{system success or}\\&\text{failure if component X is good}) \cdot P(\text{X is good}) +\\&P(\text{system success or failure if component X is bad})\\&\cdot P(\text{X is bad})\end{aligned} \tag{1}$$

in which P() is the notation for probability. Except for a very simple and small network, a nested conditional probability operation is inevitable. Efficient evaluation of system reliability of a complex network hinges entirely on a proper selection of key elements which generally would be a difficult task when one deals with a moderate or large water distribution network. The technique also cannot be easily adopted to computerization for problem solving.

Cut-Set Method. In general, the cut-set method is powerful for evaluating system reliability for two reasons: (a) it can be easily programmed on a digital computer for fast and efficient solutions of any general network; (b) the cut sets are directly related to the modes of system failure and therefore identify the distinct and discrete ways in which a system may fail. The cut set in this method is defined as a set of system components which, when failed, causes failure of the system. In a water distribution system, a cut set will be the set of system components including pipe sections, pumps, storage facilities, etc., which, when failed jointly, would disrupt the service to certain users. The cut set method utilizes the minimum cut sets for calculating the system reliability or service reliability to the users in the network. The minimum cut set is a set of system components which, when failed, causes failure of the system but when any one component of the set has not failed, does not cause system failure. A minimum cut set implies that all components of the cut set must be in the failure state to cause system failure. Therefore, the components of the minimum cut set are effectively connected in parallel and each cut set is connected in series. As a result, the probability of service failure to a certain user in the network can be expressed as:

$$Q_i = P(O_i') = P\left\{\text{service to user i fails}\right\} = P\left\{\bigcup_k C_{ik}\right\} \tag{2}$$

in which C_{ik} is the k-th minimum cut set associated with the i-th user in the water distribution network. In case the number of the minimum cut set is large, Eq. (2) can be approximated by:

$$Q_i^U = \sum_k P(C_{ik}) \tag{3}$$

which is an upper bound for the probability of service failure. A second approximation can be made by adding a second term to Eq. (3) as:

$$Q_i^L = \sum_k P(C_{ik}) - \sum_{k<l}\sum P(C_{ik}C_{il}) \tag{4}$$

which yields a lower bound to the probibility of service failure. The true probability of failure, if calculated by Eq. (2), should lie between (Q_i^L, Q_i^U). As a result, the second term on the right-hand side of Eq. (4) can be used to examine whether the approximation is adequate or not. For a system consisting of high reliability components, using either Eqs. (3) or (4), will be sufficient to approximate the true risk. Hence, risk computation can be greatly simplified. Once the probabilities of service failure to all demand modes are computed, the system unreliability then can be calculated as:

$$Q_s = P\left\{ O_1' \cup O_2' \cup \ldots \cup O_N' \right\} \tag{5}$$

when O_i' is the event that demand for i-th user in the network is not satisfied. Similar to the approximation mode previously the systems service unreliability can be calculated as:

$$Q_s^U = \sum_{i=1}^{N} P(O_i') = \sum_{i=1}^{N} Q_i \tag{6}$$

in which Q_i can be calculated by Eqs. (2), or (3), or (4). The difference in system service unreliability between its upper and lower bounds can be calculated by:

$$\Delta_s = \sum_{i<j}\sum P\left\{ O_i' O_j' \right\} = \sum_{i<j}\sum P\left\{ \bigcup_k C_{ik}, \bigcup_l C_{il} \right\} = \sum_{i<j}\sum P\left\{ \bigcup_p C_p(i,j) \right\} \tag{7}$$

where $C_p(i,j)$ is the p-th cut set common to both users i and j. The service reliability of the system can be obtained by subtracting Q_s from one.

Tie Set Analysis. The tie set analysis is a complement of the cut set method. A tie set is a minimal path of the system in which system components are connected in series. Consequently, a tie set fails or service is interrupted if any one component in the tie set fails. All tie sets are effectively connected in parallel. The service reliability to the i-th user in the network is:

$$\begin{aligned} R_i = P(O_i) &= P\left\{ \text{Demand of the i-th user in the network is met} \right\} \\ &= P\left\{ \bigcup_k T_{ik} \right\} \end{aligned} \tag{8}$$

in which T_{ik} is the k-th tie set associated with the i-th user. Although Eqs. (2) and (8) have the same mathematical expression, however, the approximations employed in the cut set method cannot be applied because, in most cases, the service reliability of system components in each tie set is high. Using the approximation schemes would lead to significant error. More terms (or perhaps every single term) in the expansion of probability of a union have to be included. As a result, computation involved is much more cumbersome than that of the cut set method. To evaluate service reliability of the entire system, it can be calculated as:

$$R_s = P\left\{ \bigcap_{i=1}^{N} O_i \right\} = P(O_1)P(O_2|O_1)\ldots P(O_N|O_{N-1})$$

$$= \frac{\prod_{i=1}^{N-1} P(O_i O_{i+1})}{\prod_{i=2}^{N-1} P(O_i)} = \frac{\prod_{i=1}^{N-1} P\left\{ \bigcup_{p} T_p(i,i+1) \right\}}{\prod_{i=2}^{N-1} P(O_i)} \qquad (9)$$

in which $T_p(i,i+1)$ is the p-th minimal tie set consisting components which belong to either tie sets for users i and i+1. As can be imagined, the evaluation for the joint probability on the numerator could be very time consuming if the number of the tie set is large. One other disadvantage of the tie set method, other than the computational aspect, is that failure modes are not directly identified. Direct identification of failure modes is sometimes essential if the limit amount of a resource is available to place emphasis on a few dominant failure modes.

Connection Matrix Method. The connection matrix is a matrix showing the components that connect between the nodes of the network. The essence of the method is to transform this basic connecting matrix into one which defines the transmission of flow between the two nodes of interest. In other words, the connection matrix technique leads to an identification of minimal tie set connecting the two nodes of interest. There are two methods developed for matrix operation which employ Boolean algebra. Although the method is a formal method in its own right, it can also be regarded as a means of deducing the tie sets. Therefore, the processes involved in system reliability analysis are basically the same as the tie set method.

Event Tree Technique. An event tree is a pictorial representation of all events that can possibly occur in a system. It has been applied mainly to safety oriented systems involving standby mode with sequential logic and switching. However, it is also applicable to systems in which the components are continuously operated like water distribution systems. As it is applied to a continuously operated system, the events that can occur, i.e., the components that can fail, can be considered in any arbitrary order. An event tree for a system can be deduced through the consideration of all possible states (e.g., failure or functioning) of each component in the network. The most time consuming part of the techniques is the deduction of the event tree which could become a formidable task when the network to be dealt with is large. The number of possible paths in a complete event tree is 2^n in which n is the number of components in the system. In some situations, a reduced event tree can be constructed with less paths if the analyst can predetermine the final outcome of the system without knowing the status of all components involved. The process can be computerized to save the manpower.

Having deduced the outcome of each path, it is rather easy to evaluate the probability of the occurrence of each outcome. Since paths in the event tree are mutually exclusive, the probability of a particular system outcome can be evaluated by summing the probabilities leading to that outcome. The construction of an event tree would also be used as a means for deducing the tie and cut sets. However, it would not be as efficient as other techniques developed for that sole purpose.

Fault Tree Analysis. This method has been widely used to evaluate system reliability of standby and mission oriented systems. It is rarely used for the topological type of systems. For a complete water distribution system involving a pipe network, pumps, valves, storages, and others, the evaluation of reliability of an entire system is a problem concerning both mission oriented and topological.

Fault trees use a logic that is essentially the reverse of that used in event trees. In this method a particular failure condition is considered and a tree is constructed that identified various combinations of causes leading to the considered failure. It can be used for both quantitative and qualitative evaluation of system reliability.

EXAMPLE

To demonstrate the applicability of techniques described previously, a simple water distribution network as shown in Fig. 1 is used. The distribution system involves eight pipe sections of equal length and four demand points (nodes 3,4,6 and 7). The system service reliability is defined as the probability that demands for all users are met. All pipe sections are assumed to behave independently and all have the same probability of failure. Assuming that all pipe sections have five percent of failure probability, the service reliability of the entire system is tabulated in Table 1 resulting from the five different methods. As can be seen, using a first-order approximation in the cut set method, i.e., Eqs. (3) and (6), yields a result that is practically the same as other techniques which generally are more time consuming.

$q=0.05$ $R=0.95$

SUMMARY AND CONCLUSIONS

This paper briefly describes six techniques developed for evaluating reliability of systems with complex configuration. Also, a brief comment about the computational aspects and applicability of each technique as applied to water distribution system reliability evaluation are made. Five techniques, except the connection matrix method, are applied to a simple looped water distribution system involving four demand points and eight pipe sections. It is found that all methods yield practically the same system reliability. However, from the computational viewpoint, the cut set method with a first-order approximation is the most efficient.

Examination of various reliability evaluation techniques for water distribution systems made herein consider that all system components behave independently which may not be the case in reality. Failure of one component in a water distribution system would increase the load to the others, henceforth probably increase the chance of failure of the others. Quantification of such dependency remains a challenging task.

REFERENCES

Ang, A. S-H and W.H. Tang. Probability Concepts in Engineering Planning and Design, Volume II, 1975, Wiley and Sons.

Billinton, R. and R.N. Allan, Reliability Evaluation of Engineering System: Concepts and Techniques, 1983, Plenum Press.

Table 1. Total System Reliability By Different Methods.

Method	Cut Set	Tie Set	Cond. Prob.	Event Trees	Fault Trees
Reliability	0.9341	0.9352	0.9319	0.9355	0.9354

Figure 1. Sample Water Distribution Network